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Coordination and discrimination in contracting with externalities: divide and conquer?

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Abstract

The paper studies bilateral contracting between N agents and one principal, whose trade with each agent generates externalities on other agents. It examines the effects of prohibiting the principal from (i) coordinating agents on her preferred equilibrium, and (ii) making different contracts available to different agents. These effects depend on whether an agent is more or less eager to trade when others trade more. The prohibitions reduce the aggregate trade in the former case, and have little or no effect in the latter case. The inefficiencies under different contracting regimes are linked to the sign of the relevant externalities, and are shown to be typically reduced by both prohibitions.

JEL classification: C72; G34; L42; 033

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1. Introduction

Many situations of bilateral contracting involve externalities. To give a few examples, a shareholder tendering his shares to a corporate raider generates an externality on the firm's other shareholders, a creditor exchanging debt for equity in a distressed firm generates an externality on the firm's other creditors, a participant in an Internet marketplace generates a "network" externality on other participants in the marketplace, a private contributor to a public good creates an externality on other consumers of the good, a buyer signing an exclusive dealing contract that

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hinders competition imposes an externality on other buyers, a firm purchasing an intermediate input from a manufacturer imposes an externality on competing downstream firms. In all these situations, it has been shown that even when all agents participate in contracting, externalities may not be internalized and thus may yield inefficient outcomes. However, the general nature of arising inefficiencies, and their dependence on the contracting procedure, have not been well understood.

Segal [29] introduces a general model of contracting with externalities, which unifies the above examples and demonstrates the general nature of arising inefficiencies. In the model, one principal contracts with N agents, and each agent's trade with the principal has externalities on other agents. However, the analysis of [29] assumed that the principal can (i) discriminate by making different offers available to different agents, and (ii) coordinate agents on her preferred equilibrium. The aim of the present paper is to examine bilateral contracting with externalities when one or both assumptions are reversed.

Discrimination is defined in this paper as a situation where some contracts are available to some, but not all, agents. For example, restricted takeover bids, which do not invite all shareholders to tender (see, e.g., [9]), are discriminatory. On the other hand, "any-and-all" bids, which invite all shareholders to tender at a given price, are non-discriminatory, even though they may induce an equilibrium in which some shareholders tender and others do not. The paper examines how a ban on discrimination affects contracting.

The paper also examines how contracting is affected by the principal's inability to coordinate agents on her preferred equilibrium. In particular, it is argued that if the principal expects agents to play the equilibrium that is worst for her, she can restrict attention to implementation in a *unique* Nash equilibrium. The paper examines how such restriction affects contracting outcomes. (It also examines contracting under the more optimistic assumption that agents coordinate on an equilibrium that is best *for them*.) Thus we consider four contracting regimes in total, differing in whether the principal can discriminate and whether she can coordinate agents to her advantage. The contracting outcomes under these four regimes are compared to each other and to the socially efficient allocations.

The paper identifies a key property of the agents' payoffs that accounts for the qualitative features of contracting under the four different regimes. This property is whether an agent is more or less eager to trade with the principal when he expects other agents to trade more. The same property determines whether the externality imposed on an agent by increasing other agents' trades is increasing or decreasing in his own trade. These two cases are labeled as those of "increasing" and "decreasing" externalities, respectively.

The key ideas behind the paper's results can be illustrated using a simple example with two identical agents, where each can trade an indivisible unit with the principal. The principal offers each agent a price, and upon observing both prices, the agents simultaneously and independently decide whether to trade. (This contracting is bilateral in the sense that the price offered to one agent cannot depend on the other agent's decision.) First, to examine the role of discrimination, suppose that the principal wants to trade exactly one unit. The optimal way to do this is by offering only one agent, say agent 1, a price that makes him indifferent between trading and not. If the principal is not allowed to discriminate, however, the option to trade at the same price must also be made available to agent 2. Will agent 2 exercise this option? With increasing externalities, agent 2, who expects agent 1 to trade, will be even more eager to trade himself. More generally, at any price that induces one agent to trade, the other agent will also trade. This implies that with increasing externalities, the principal cannot implement trade with a single agent. With decreasing externalities, on the other hand, if agent 1 is just willing to trade at a given price, agent 2, expecting agent 1 to trade, will prefer not to. Thus, in this case the principal can implement an asymmetric outcome without explicitly discriminating between agents.

The paper extends these findings to a general setting with many agents and many possible trade levels. It examines how offering the same menu to all agents could induce a Nash equilibrium in which identical agents make different choices. Even though the agents are identical ex ante, their preferences may differ ex post due to different expectations of other agents' equilibrium trades. Thinking of these expectations as the agents' "types", the principal's menu design problem can be analyzed using standard techniques from the theory of second-degree price discrimination [5,10,20]. We find that with strictly increasing externalities, a nondiscriminating principal can only implement symmetric outcomes. In contrast, with decreasing externalities, the principal can implement asymmetric equilibria. In this case, the principal's inability to discriminate proves not to affect the contracting outcome either when each agent has an indivisible unit trade or asymptotically with a large number of agents.

The paper's analysis of coordination can also be illustrated using the above two-agent example. For this purpose, suppose that the principal wishes to trade with both agents. Her problem is depicted in Fig. 1, where the coordinates for each square are the two agents' trades: the bottom left corner corresponds to no trade with either agent, the top right corner to trade with both, bottom right to trade with agent 1 only, and top left to trade with agent 2 only.

When the principal can coordinate the agents on her desired Nash equilibrium, she offers them a price that makes each agent just willing to trade when he expects the other to trade. Graphically, the price compensates each agent for moving along his

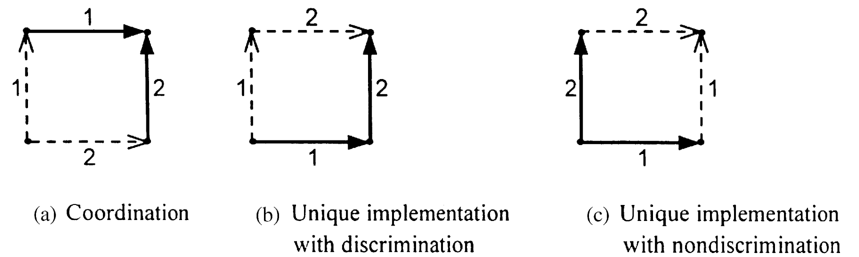

Fig. 1. Simple two-agent example with increasing externalities.

respective solid arrow in Fig. 1(a). But will the induced game between the agents have another Nash equilibrium? This depends on whether externalities are increasing or decreasing. With decreasing externalities, an agent would be even more eager to trade if he expected the other agent not to, hence trading is a dominant strategy given the price defined above. In fact, it turns out that in the general setting with decreasing externalities, the principal can ensure uniqueness of Nash equilibrium at no extra cost. On the other hand, with increasing externalities, an agent would be less eager to trade if he expected the other agent not to. Hence, given the offered price, *not* trading is a dominant strategy, and so there exists a Nash equilibrium in which neither agent trades. The principal can only rule out this equilibrium by offering the agents a better price.

Focusing on the case of increasing externalities, let us identify the principal's optimal strategy for implementing trade with both agents in a unique Nash equilibrium. First, to rule out "no trade" as a possible equilibrium, the principal must offer at least one agent, say agent 1, a price that induces him to trade even if he expects the other agent not to. Then she can offer agent 2 a price that induces him to trade if he expects agent 1 to trade. Graphically, the agents are compensated for moving along the respective solid arrows in Fig. 1(b). Note that agent 1, who is induced to move first, receives a better price than agent 2, since due to increasing externalities he needs a larger inducement to trade.[1] This is a simple example of a "divide-and-conquer" strategy. Observe that trading with both agents now costs the principal more than in case (a), therefore she is less likely to implement it.

The paper characterizes the general form of the principal's optimal "divide-and-conquer" strategy for unique implementation with increasing externalities. The characterization makes use of the fact that in this case, any menu profile offered by the principal induces a supermodular game among agents [22,32]. The optimal strategy in general involves constructing an increasing path from "no trade" to the trade profile to be implemented. Each point on this path (other than the final point) must be ruled out as a possible Nash equilibrium by bribing exactly one agent into increasing his trade along the path. In the simple case where all agents have indivisible unit trades, a "divide-and-conquer" path is constructed by ordering agents in an arbitrary fashion, and offering each agent a price that is just accepted if he believes that all preceding agents trade and all subsequent ones do not. Due to increasing externalities, "later" agents are more eager to trade than "earlier" agents, and thus are offered smaller bribes by the principal.

Specific "divide-and-conquer" strategies have been suggested for the use by an incumbent firm to coordinate buyers on accepting exclusive contracts [13,30], by an inferior raider to coordinate shareholders on a takeover equilibrium [9], by a leading bank to coordinate investments in a developing economy [6], by an Internet marketplace to attract participants [4,27]. All these papers consider settings with externalities in which a principal optimally coordinates agents by offering some of them better deals than others. The present paper contributes to the literature by

[1] The temporal wording is just a metaphor describing best-response dynamics from in the agents' simultaneous-move game.

identifying the general nature of the contracting problem and by solving for the general form of the "divide-and-conquer" strategy.

The "divide-and-conquer" strategy discriminates among identical agents by offering them different transfers for identical trades. If the principal is not allowed to discriminate, this strategy is no longer feasible. Back in the simple two-agent example, in order to uniquely implement trading with both agents, the principal must offer a price that induces one agent to trade even if he expects the other agent not to, and both will want to trade at this price. Thus, the two agents must be compensated for moving along the respective solid arrows in Fig. 1(c). Since trade costs the principal even more than in case (b), she is even less likely to implement it. In fact, the paper establishes in considerable generality that the principal's inability both to coordinate and to discriminate reduces her optimal choice of aggregate trade.

The paper also compares the contracting outcome under the four regimes to the socially efficient outcomes. It is intuitively clear that the divergence between the principal's incentive to trade and the social incentive is due to externalities. Recall, however, that the externality imposed on an agent by others' trades in general depends on the agent's own trade. Which externalities cause distortions depends on the contracting regime.

The analysis of relevant externalities in the two-agent example can again be illustrated using Fig. 1. Thinking of the two agents as moving from the "no trade" corner to the "both trade" corner, changes in their payoffs that are not compensated by the principal are precisely the externalities that give rise to inefficiency. Consider first case (a), where the principal coordinates agents. Suppose that agent 1 moves from "no trade" to "both trade" via the top left corner, while agent 2 moves there via the bottom right corner. As argued above, the principal compensates the agents only for the moves represented by solid arrows. Therefore, the dotted arrows represent the relevant externalities, which are those imposed on a non-trader by the other agent's trade. When these *externalities on non-traders* are absent, the contracting outcome is efficient in case (a) regardless of any externalities on traders. When they are positive/negative, the principal's incentive to trade is insufficient/excessive from the social viewpoint.

Consider now case (c), where the principal must implement uniquely and cannot discriminate. Suppose now that agent 1 moves from "no trade" to "both trade" via the bottom right corner, while agent 2 moves there via the top left corner. The picture shows that the relevant externalities are now those imposed on a trader by the other agent's trade. Thus, distortion is now due to *externalities on traders*.

Finally, consider the case of unique implementation with discrimination (case (b)). Suppose that both agents move from "no trade" to "both trade" via the bottom right corner. Note that agent 2 is not compensated for the horizontal move, while agent 1 is not compensated for the vertical move. (By symmetry, the uncompensated moves are equivalent to the moves represented by the dotted arrows.) It follows that both externalities on traders and on non-traders could now matter. More precisely, we show that in this case it is *externalities at efficient outcomes* that determine the direction of distortion. When such externalities are absent, the principal has no incentive to deviate from an efficient outcome. When they are positive/negative, she will deviate by increasing/reducing the aggregate trade. The results linking the direction of distortion to the sign of the relevant externalities are established in considerable generality.

The remainder of the paper is organized as follows. Section 2 introduces the general model of contracting with externalities, describes some of its applications, and spells out the paper's contracting assumptions. Section 3 studies the principal's optimal implementation in Nash equilibrium, both with and without discrimination. Section 4 studies the principal's optimal implementation in a unique Nash equilibrium, both with and without discrimination. Section 5 concludes by discussing the welfare implications of our analysis.

2. Setup

2.1. The model

The paper studies the model of contracting with externalities introduced in [29]: one party, called "the principal", can offer contracts to N other parties, called "agents". (With a slight abuse of notation, N will represent the *set* as well as the *number* of agents.) The principal's "trade" with each agent i is denoted by $x_i \in \mathscr{X}_i$, where $\mathscr{X}_i$ is a compact subset of the set $\mathbb{R}_+$ of nonnegative real numbers, with $0 \in \mathscr{X}_i$. The agents' trade profile is then described by a vector $x = (x_1, \ldots, x_N) \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N$. Externalities among agents arise because each agent's utility depends not only on his own trade with the principal, but also on other agents' trades. Namely, each agent i's payoff is $u_i(x) - t_i$, and the principal's payoff is $f(x) + \sum_i t_i$, where $t_i \in \mathbb{R}$ is the monetary transfer from agent i to the principal. The default ("no trade") point for each agent i is $(x_i, t_i) = (0, 0)$.

The following properties of agents' payoffs have been defined in [29] (for the sake of consistent terminology, *increasing* and *decreasing* will be used in the weak sense, to denote nondecreasing and nonincreasing respectively):

Definition 1. Externalities are positive [negative] at $\widehat{x} \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N$ if for all $i \in N, u_i(\widehat{x}_i, x_{-i})$ is increasing [decreasing] in $x_{-i} \in \mathscr{X}_{-i}$. Externalities are positive [negative] if they are positive [negative] at all $\widehat{x} \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N$.

Definition 2. Externalities are (strictly) increasing [decreasing] if for all $i \in N, u_i(x_i, x_{-i})$ has (strictly) increasing differences in (x_i, x_{-i}) [in $(x_i, -x_{-i})$] in the sense of Topkis [32]; specifically, for all $x_i, x_i' \in \mathscr{X}_i$ with $x_i' > x_i$, $u_i(x_i', x_{-i}) - u_i(x_i, x_{-i})$ is (strictly) increasing [decreasing] in $x_{-i} \in \mathscr{X}_{-i}$.

The property of increasing (decreasing) externalities involves the sign of a cross-partial difference, and can be interpreted in two ways. One interpretation is that an agent is more (less) eager to trade more when other agents trade more. The other is that the externalities at higher trades are larger (smaller) than at lower trades—formally, $u_i(x_i, x'_{-i}) - u_i(x_i, x_{-i})$ is increasing (decreasing) in $x_i \in \mathscr{X}_i$ when $x'_{-i} > x_{-i}$.

Many of the paper's results will use additional structure of the parties' payoffs and trade domains. In particular, just as in [29], the following two properties will prove useful for identifying the direction of distortion under various contracting regimes:

Condition D. Either $\mathscr{X}_i = [0, \bar{x}_i]$ or $\mathscr{X}_i = \{kz : k = 0, 1, \ldots, \bar{k}_i\}$ for all i.

Condition W. $f(x) + \sum_i u_i(x) = W(\sum_i x_i)$.

Condition D (for "domains"), requires that all agents' trades be measured in the same increments, which could be either infinitesimal or finite. Condition W (for "welfare") states that total surplus only depends on the aggregate trade $X = \sum_i x_i$, and not on its allocation across agents. The set of surplus-maximizing aggregate trades can then be defined as $M^* = \arg\max_{\sum_i \mathscr{X}_i} W(X)$ (where $\sum_i \mathscr{X}_i = \{\sum_i x_i : x \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N\}$). The set M^* will serve as a benchmark against which contracting outcomes are compared, and the contracting parties' failure to maximize their joint surplus will be referred to as "inefficiency".

The analysis of nondiscrimination will focus on the case where all agents are "identical", which is captured by the following condition:

Condition I. $\mathscr{X}_i = \mathscr{X}$ for all i. Furthermore, for any trade profile x and any permutation π of N, letting x_π denote the permuted trade profile $(x_{\pi(1)}, \ldots, x_{\pi(N)})$, we have $u_i(x_\pi) = u_{\pi(i)}(x)$ for all i, and $f(x_\pi) = f(x)$.

In particular, Condition I implies that there exists a single function $u : \mathscr{X}^N \to \mathbb{R}$ such that $u_i(x_i, x_{-i}) = u(x_i; x_{-i})$ for all $i \in N$ and $x \in \mathscr{X}^N$, and this function is invariant to permutations of x_{-i}.

As an example of payoffs satisfying both Conditions I and W, consider $u_i(x) = x_i \alpha(\sum_j x_j) + \beta(\sum_j x_j)$ and $f(x) = F(\sum_j x_j)$ (this is a symmetric version of Condition L in [29]). In this case $W(X) = F(X) + X\alpha(X) + N\beta(X)$.

Finally, some of our results hold only in the simple settings satisfying

Condition S. Condition I holds and $\mathscr{X} = \{0, 1\}$.

Under Condition S (for "symmetric single-unit trades"), the parties' payoffs can be written as $u_i(x) = U(x_i, \sum_{j \neq i} x_j)$, and $f(x) = F(\sum_j x_j)$. Note that Condition S implies all the other conditions.

2.2. Some applications

Consider a few applications of the general model (see [29] for many others):

Takeovers (Grossman and Hart [8], Bagnoli and Lipman [1], Holmstrom and Nalebuff [12], Burkart et al. [3]): The principal is a corporate raider, who makes a tender offer to N shareholders (agents). $x_i \geqslant 0$ is the number of shares tendered by shareholder i, and $(-t_i)$ is the raider's payment to this shareholder. Let $v(X)$ denote the firm's expected public value per share as a function of the total number X of shares tendered. Finally, let $c(X)$ denote the raider's "transaction cost" of acquiring X shares (it could also be negative, reflecting her private benefit of control). Then the parties' utilities net of monetary transfers are given by $f(x) = Xv(X) - c(X)$ and $u_i(x) = (\overline{x}_i - x_i)v(X)$, where $\overline{x}_i$ is shareholder i's endowment of shares. These payoffs satisfy Condition W.

Say that the raider is *superior* if $v(X)$ is increasing in X, which may be due to the raider's greater ability and/or incentive to enhance the firm's value when she holds a larger stake in the firm. In this case, externalities are positive, and they are decreasing when in addition shareholdings are indivisible, i.e., $\mathscr{X}_i = \{0, \overline{x}_i\}$ for all i. On the other hand, say that the raider is *inferior* if $v(X)$ is decreasing in X, which may be due to the raider's greater ability to "loot" the firm's assets or "freeze out" other shareholders when she owns a larger stake in the firm. In this case, externalities are negative, and they are also increasing when in addition shareholdings are indivisible.

Vertical contracting (Hart and Tirole [11], O'Brien and Shaffer [26], McAfee and Schwartz [21], Katz and Shapiro [17], Kamien et al. [16]): The principal supplies an intermediate good to N agents (downstream firms), who use it to produce substitutable consumer goods. x_i is firm i's purchase of the intermediate good, and t_i is its payment to the supplier. Due to downstream competition, each firm i's utility $u_i(x_i, x_{-i})$ is decreasing in other firms' purchases x_{-i}. Unlike the first three papers, Katz and Shapiro [17] and Kamien et al. [16] study models in which each downstream firm has access to an inferior technology that does not use the principal's input, therefore externalities on nontraders are strictly negative. The two papers also assume that the intermediate good is a fixed input (a licence to use the principal's patent), and that downstream firms are identical. Then Condition S holds, and with it all the other conditions. These models may exhibit either increasing or decreasing externalities (for example, [17] considers both cases).

Exclusive dealing (Rasmusen et al. [28], Innes and Sexton [13], Segal and Whinston [30]): The principal is an incumbent monopolist who offers exclusive dealing contracts to N identical buyers (agents). The contract obliges a buyer not to buy from a rival seller. Let $x_i = \{0, 1\}$ indicate whether buyer i signs such a contract, and $(-t_i)$ be the compensation paid to him by the incumbent. After observing the number X of signers, a potential entrant decides whether to enter. Due to the entrant's economies of scale, the probability of entry, $\rho(X)$, is a decreasing function of X. In the case of no entry, in the second stage the incumbent makes the monopoly profit π^{m} on each buyer by charging him the monopoly price p^{m}. In the case of entry, the entrant and incumbent compete for the buyers who have not signed in the first stage, and the incumbent, whose marginal cost is higher than the entrant's, makes no profit on these buyers. The incumbent still charges p^{m} to the buyers who have signed exclusives, and earns π^{m} on each of them.

Since all buyers are identical, Condition S holds, and with it all the other conditions. The incumbent's net profit can be written as $f(x) = [\rho(X)X + (1 - \rho(X))N]\pi^{\mathrm{m}}$. Normalizing each buyer's surplus under price p^{m} to zero, and letting b denote his surplus under the competitive price, his utility can be written as $u_i(x) = (1 - x_i)\rho(X)b$. This model exhibits negative increasing externalities.

Network externalities (Katz and Shapiro [18], Ellison and Fudenberg [7]): The principal is the seller of a good for which each agent (buyer) has a unit demand. $x_i \in \{0, 1\}$ denotes buyer i's purchase of the good, and t_i denotes his payment to the seller. Since all buyers are assumed to be identical, Condition S is satisfied, and with it all the other conditions. When the seller's good exhibits positive "network externalities", the utility $u(1, x_{-i})$ of each consumer i of the good is increasing in other buyers' purchases x_{-i}. A consumer who does not buy the seller's good uses an old substitute technology, which we assume here to be "unsponsored", i.e., supplied by a competitive market. If the old technology also exhibits network effects, its users' utility, $u(0, x_{-i})$, is decreasing in x_{-i}. Thus, a consumer who buys from the seller has a positive externality on other consumers who buy, but a negative externality on those who do not, which implies that the model exhibits increasing externalities.

2.3. Contracting

A *contract* between the principal and agent i is a bundle $(x_i, t_i) \in \mathscr{X}_i \times \mathbb{R}$. We consider a two-stage contracting game of the following kind: In the first stage, the principal offers each agent i a *menu* of contracts $S_i \subset \mathscr{X}_i \times \mathbb{R}$, which must include the *null contract*: $(0, 0) \in S_i$. In the second stage, each agent chooses a contract $(x_i, t_i) \in S_i$.

We assume that before making his choice, each agent observes the menus offered to all agents. This assumes a commitment on the principal's part not to negotiate contracts secretly.[2] At the same time, we rule out contracts that make one agent's trade contingent on other agents' choices. Such contingent contracts (analyzed in [29, Section 5]) are seldom observed in the applied settings described in the previous subsection. This may be explained by the difficulty of enforcing contingent contracts, especially when the number of agents is large (see also [13, footnote 20]).

The menus offered by the principal constitute the agents' strategy spaces in the induced second-period symmetric-information game. An outcome $(x, t) \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N \times \mathbb{R}^N$ is said to be (uniquely) implementable if there exists a menu profile $(S_i)_{i \in N}$ such that (x, t) constitutes a (unique) Nash equilibrium of the induced game. It is said to be (uniquely) implementable *with nondiscrimination* if, in addition, $S_i = S$ for all $i \in N$, for a *single* menu $S \subset \mathbb{R}_+ \times \mathbb{R}$. Section 3 will study the principal's preferred implementable outcomes with and without discrimination. Section 4 will study the principal's preferred uniquely implementable outcomes with and without

[2] For analyses of contracting when the principal lacks such commitment, see [29, Section 4] and [31].

discrimination. The sets of the principal's preferred implementable aggregate trades in the four cases will be denoted by $M^{\mathrm{u/s}}_{\mathrm{d/n}}$, where the subscript denotes whether discrimination is allowed or not (d or n, respectively), and the superscript denotes whether unique or simple (i.e., nonunique) implementation is required (u or s, respectively).

3. Simple implementation

3.1. Discrimination

The principal's preferred implementable outcomes have been studied in [29, Section 2], whose main result is outlined here. It is easy to see that for simple implementation of an outcome $(x,t)\in\mathscr{X}_1\times\cdots\times\mathscr{X}_N\times\mathbb{R}^N$, the principal only needs to offer each agent i the menu containing only the agent's equilibrium contract along with the null contract: $S_i=\{(0,0),(x_i,t_i)\}$. Thus, the outcome is implementable if and only if it is a Nash equilibrium of the game induced by such menus, i.e., it satisfies the following participation constraints:

$$u_i(x)-t_i\geqslant u_i(0,x_{-i}) \qquad (IR_i) \text{ for all } i\in N.$$

At the principal's preferred implementable outcome, all these participation constraints bind, since otherwise she could raise some transfer t_i without violating the constraints. Expressing transfers from the binding constraints and substituting them into the principal's objective function, we see that the principal chooses the trade profile $x\in\mathscr{X}_1\times\cdots\times\mathscr{X}_N$ to maximize the difference between total surplus and the sum of agents' reservation utilities:

$$f(x)+\sum_i u_i(x)-\sum_i u_i(0,x_{-i}). \tag{1}$$

This implies that inefficiency arises whenever externalities on nontraders (i.e., at $\hat{x}=0$) are present.

Formally, we compare the set $M^{\mathrm{s}}_{\mathrm{d}}$ of the principal's preferred implementable aggregate trades to the set M^* of socially efficient aggregate trades. The assumptions required to ensure that these two sets are single valued (such as strict concavity of the objective function and convexity of the feasible set) would not be natural in many applications. Instead, the two sets can be compared using the *strong* (*induced*) *set order* [24,32]. Namely, for two subsets A,B of a lattice, it is said that $A\leqslant B$ if whenever $a\in A, b\in B$, and $a\geqslant b$, we also have $a\in B$ and $b\in A$. This paper only deals with the case where $A,B\subset\mathbb{R}$, in which case $A\leqslant B$ if and only if $A\backslash B$ lies below $A\cap B$, which in turn lies below $B\backslash A$. (Note that when A and B are singletons, the ordering coincides with the standard ordering of real numbers.) [29, Proposition 2] establishes the following result:

Proposition 1. *If Conditions W, D hold, then with positive* [*negative*] *externalities at* $(0,\ldots,0)$, $M^{\mathrm{s}}_{\mathrm{d}}\leqslant[\geqslant]M^*$.

3.2. Nondiscrimination

Recall that nondiscrimination is defined as the requirement that the principal offer the same menu S to all agents. It is easy to see that for simple implementation of an outcome $(x,t) \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N \times \mathbb{R}^N$, the menu only needs to contain the contracts chosen by *some* agent in equilibrium, along with the null contract: $S = \{(0,0)\} \cup \{(x_i,t_i) : i \in N\}$. The outcome is implementable with nondiscrimination if and only if it is a Nash equilibrium of the game induced by offering menu S to all agents. This equilibrium condition in turn consists of the requirements that (i) no agent i prefers to choose the null contract (0,0), which is ensured by the participation constraints (IR_i) above, and (ii) no agent i prefers a contract intended for another agent j, which is ensured by the following incentive constraints:

$$u_i(x_i, x_{-i}) - t_i \geqslant u_i(x_j, x_{-i}) - t_j \qquad (IC_{ij}) \text{ for all } i,j \in N.$$

We focus on the effects of nondiscrimination when all agents are ex ante identical, i.e., Condition I holds. In this situation, differences in agents' preferences over contracts can be attributed entirely to their different expectations of others' equilibrium trades. Thus, we can think of x_{-i} as the "type" of agent i. With increasing (decreasing) externalities, an agent with a higher type has a higher (lower) incentive to trade. In both these cases, the agents' incentive and participation constraints can be analyzed using standard ideas from the theory of second-degree price discrimination [5,10,20].

The incentive constraints of identical agents are trivially satisfied at a symmetric outcome. However, the principal may prefer to implement an asymmetric outcome. Indeed, if she could discriminate as in the previous subsection, she would maximize the difference between total surplus and the sum of agents' reservation utilities. Under Condition W, the allocation of a given aggregate trade X among agents does not affect total surplus, but the principal might still want to allocate X unequally among agents when their reservation utilities $u_i(0, x_{-i})$ are not convex in x_{-i}, or when their domain $\mathscr{X}$ is not convex, in particular when it is finite. For example, in Takeovers, a raider may optimally want to gain control by buying as few shares as possible, and given indivisibility of shares, this may require buying from some but not all shareholders.[3] This raises the question of whether the principal can implement the same outcome without discrimination, by inducing ex ante identical agents to voluntarily select different contracts from the same menu. The answer depends on whether externalities are increasing or decreasing.

3.2.1. Nondiscrimination with strictly increasing externalities

Here we show that with strictly increasing externalities and under Condition I, if all agents are offered the same menu, in any equilibrium they must choose the same contract from this menu. To see this, let $(x,t) \in S^N$ be a Nash equilibrium of the game induced by offering all agents the same menu $S \subset \mathscr{X} \times \mathbb{R}$, and suppose in

[3] When ensuring unique implementation, the principal will also want to discriminate by charging different payments for identical trades, as will be shown in Section 4.

negation that $x_i > x_j$ for some agents $i, j \in N$. In such case, agent j's "type" (x_i, x_{-i-j}) is higher than agent i's "type" (x_j, x_{-i-j}), and by standard incentive theory arguments agent j chooses a (weakly) higher trade from any menu,[4] which contradicts our assumption. Therefore, all agents must have the same equilibrium trades. Incentive constraints then imply that all agents must also have the same transfers. Formally,[5]

Lemma 1. *With strictly increasing externalities and under Condition I,* $(x,t) \in \mathscr{X}^N \times \mathbb{R}^N$ *is implementable without discrimination if and only if* $(x_i, t_i) = (\overline{x}, \overline{t})$ *for all i, and* $u(\overline{x}; \overline{x}, \ldots, \overline{x}) - \overline{t} \geqslant u(0, \overline{x}, \ldots, \overline{x})$.

The inequality in Lemma 1 describes the agents' participation constraint. The principal optimally sets transfer $\overline{t}$ to make this constraint bind. Substituting this transfer into the principal's objective function, the latter can be written as a function of the aggregate trade X:

$$\begin{aligned}\pi_{\mathrm{n}}^{\mathrm{s}}(X) = & f(X/N, \ldots, X/N) + Nu(X/N; X/N, \ldots, X/N) \\ & - Nu(0; X/N, \ldots, X/N).\end{aligned} \tag{2}$$

The set of the principal's preferred implementable aggregate trades with nondiscrimination is then defined as $M_{\mathrm{n}}^{\mathrm{s}} = \arg\max_{X \in N\mathscr{X}} \pi_{\mathrm{n}}^{\mathrm{s}}(X)$ (where $N\mathscr{X} = \{Nx : x \in \mathscr{X}\}$).

To illustrate Lemma 1, consider the setting of Takeovers with an inferior raider. If the raider cannot discriminate, all shareholders will tender the same number of shares. Similarly, without discrimination, the incumbent firm cannot sign Exclusive dealing contracts with some but not all of the identical buyers, and the sponsor of a technology exhibiting Network externalities cannot sell it to some but not all of the identical consumers.

The effect of restricting the principal to trade equally with all agents on the aggregate trade is in general hard to predict. For example, under Condition S, the principal is forced to choose between trading with all agents and trading with none. This choice will depend on the particulars of her objective function. Thus, we do not have a general result about the welfare effect of nondiscrimination under increasing externalities.

What we *can* do, however, is compare the principal's choice to the socially optimal choice *conditional on trading equal amounts with all agents.* For this purpose, denote the set of efficient aggregate trades subject to the constraint that they are equally allocated among agents by $\overline{M}^* = \arg\max_{X \in N\mathscr{X}} \overline{W}(X)$, where

$$\overline{W}(X) = f(X/N, \ldots, X/N) + Nu(X/N; X/N, \ldots, X/N). \tag{3}$$

[4] This could be shown formally using the Monotone Selection Theorem [32, Theorem 2.8.4].

[5] Two expositional comments are in order. First, straightforward formal proofs are omitted. Second, characterizations of implementable outcomes are stated as "lemmas", while comparisons of outcomes under different contracting regimes are stated as "propositions".

Note that the first two terms in the principal's objective function (2) constitute the total surplus $\overline{W}(X)$. Therefore, the principal who is restricted to trade equal amounts with all agents will distort this amount to reduce agents' reservation utilities. Formalizing this argument using Topkis's Monotonicity Theorem [32, Theorem 2.8.1] yields

Proposition 2. *With strictly increasing positive* [*negative*] *externalities at* $(0, \ldots, 0)$, *under Condition I*, $M_{\mathrm{n}}^{\mathrm{s}} \leqslant [\geqslant] \overline{M}^*$.

3.2.2. Nondiscrimination with decreasing externalities

With decreasing externalities, the principal may be able to implement an equilibrium in which ex ante identical agents select different contracts from the same menu.[6] To see exactly what is implementable, we once again characterize the incentive and participation constraints using standard arguments from the theory of second-degree price discrimination. Without loss of generality, we order the agents so that their equilibrium trades are increasing: $x_1 \leqslant \cdots \leqslant x_N$.

With decreasing externalities, each agent i's payoff has the negative single-crossing property in his trade x_i and other agents' trades x_{-i}. This implies that if agent $i \geqslant 2$ prefers x_i to $x_{i-1} \leqslant x_i$, i.e., $(IC_{i,i-1})$ holds, then agent $i+1$ must have the same preference. If we know in addition that agent $i+1$ prefers x_{i+1} to x_i, i.e., $(IC_{i+1,i})$ holds, then by transitivity he also prefers x_{i+1} to x_{i-1}. Formally, this means that $(IC_{i,i-1})$ and $(IC_{i+1,i})$ together imply $(IC_{i+1,i-1})$. This argument establishes that all nonadjacent downward incentive constraints are implied by the adjacent downward incentive constraints, and thus can be discarded. A similar argument implies that all (IR_i) for $i \geqslant 2$ are implied by (IR_1) and (IC_{i1}), and therefore can also be discarded. Furthermore, note that if the principal only faced (IR_1) and downward adjacent (IC)'s, then she would optimally set transfers to make all these constraints bind. But then all upward incentive constraints would also be satisfied (recall that the trade profile is assumed to be increasing). This argument implies

Lemma 2. *Suppose we have decreasing extenalities and Condition I holds. If* $(x, t) \in \mathcal{X}^N \times \mathbb{R}^N$ *with* $x_1 \leqslant \cdots \leqslant x_N$ *is the principal's preferred implementable outcome with nondiscrimination, then* (IR_1) *and* $(IC_{i,i-1})$ *for* $i \in \{2, \ldots, N\}$ *bind, and all the other incentive and participation constraints can be discarded.*

In the simple case where Condition S holds, nondiscrimination requires the principal to offer a single price to all agents. In this case, all nontrivial downward incentive constraints, i.e., (IC_{ij}) with $x_i = 1$ and $x_j = 0$, coincide with the participation constraints (IR_i). With decreasing externalities, moreover, the above lemma establishes that upward incentive constraints can be ignored as well, hence nondiscrimination does not matter:

[6] Note that in such equilibrium different agents in general receive different utilities: with positive (negative) externalities, agents who choose larger trades x_i face lower x_{-i} and therefore receive lower (higher) utilities than agents who choose smaller trades.

Proposition 3. *Under Condition S with decreasing externalities*, $M_{\mathrm{n}}^{\mathrm{s}} = M_{\mathrm{d}}^{\mathrm{s}}$.

To illustrate this result, consider the setting of Takeovers with a superior raider, which exhibits decreasing externalities. The proposition implies that the raider can implement an equilibrium in which any given number X of shares are tendered and all shareholders' participation constraints bind by offering a single nondiscriminatory price to all agents. This observation has been made by Burkart et al. [3, Section 2].[7]

In the general case, the nondiscrimination requirement will hurt the principal, since the binding downward adjacent incentive constraints $(IC_{i,i-1})$ are stronger than participation constraints (IR_i). To characterize the principal's program in this case, express transfers from the binding constraints (IR_1) and $(IC_{i,i-1})$ for $i \geqslant 2$, and substitute them into the principal's objective function. Then her program, up to a permutation of agents, can be written as follows (with the convention $x_0 = 0$):

$$\max_{x \in \mathscr{X}^N,\ x_1 \leqslant \cdots \leqslant x_N} f(x) + \sum_{i=1}^{N} \sum_{k=1}^{i} [u(x_k; x_{-k}) - u(x_{k-1}; x_{-k})].$$

While a general characterization of the solution to this program is difficult, we can describe intuitively what happens asymptotically as the number of agents grows large. Given appropriate continuity in agents' payoffs, a single agent's trade then has a vanishing effect on other agents' payoffs. In this case, the nondiscrimination requirement will have little effect on the principal's profit and the contracting outcome. To see this, note that the principal's objective function can be asymptotically approximated by replacing the terms $u(\cdot; x_{-k})$ with $u(\cdot; x_{-i})$. Upon summing over k and cancelling terms, the objective function becomes

$$f(x) + \sum_{i=1}^{N} [u(x_i; x_{-i}) - u(0, x_{-i})],$$

which coincides with the principal's objective function (1) with discrimination. A formalization of this argument in the asymptotic setting defined in [29, Section 7] establishes that as $N \to \infty$, the principal's maximum profit without discrimination converges to the same value as that with discrimination. This also implies that the set of profit-maximizing aggregate trades without discrimination converges to the set of "nonpivotal" aggregate trades that obtain asymptotically with discrimination (a formal proof is available upon request).

[7] This conclusion is also related to the findings of McAfee and Schwartz [21], who study the role of nondiscrimination in vertical contracting. They show that under their Assumption 3 (which has the flavor of decreasing externalities), a ban on discrimination does not have any effect, since downstream firms can voluntarily select different contracts. Their contracting game, however, is substantially different from that considered in this paper. The main difference is that in their game, the principal lacks commitment power, and hopes to use a ban on discrimination as a commitment device.

4. Unique implementation

This section studies the outcome of contracting when the principal must implement an outcome in a unique Nash equilibrium. To motivate the analysis, consider an example in the setting of Takeovers described in Section 2.2. Suppose that each shareholder holds exactly one share, and suppose for simplicity that the raider's private benefits of control are such that she always desires to acquire all shares. The raider's optimal implementable outcome can then be implemented by offering each agent to tender his share at price $v(N-1)$. This price ensures that "all tender" is a Nash equilibrium, while making all shareholders' participation constraints bind. However, the game induced by this offer could have other, undesirable, equilibria.

Consider first the case where the raider is superior, i.e., $v(\cdot)$ is decreasing. This case exhibits decreasing externalities: a shareholder is more eager to tender when he expects fewer shares to be tendered. Therefore, a shareholder would strictly prefer to tender at price $v(N-1)$ if he expected another shareholder not to tender—formally, tendering at this price is a weakly dominant strategy. This implies that "all tender" is a unique Nash equilibrium. More generally, in this section we show that with decreasing externalities, unique implementation does not impose an extra cost on the principal.

Now consider the case where the raider is inferior, i.e., $v(\cdot)$ is increasing. This case exhibits increasing externalities: a shareholder is less eager to tender if he expects fewer shares to be tendered. Therefore, *not* tendering at price $v(N-1)$ is a weakly dominant strategy for each shareholder, and thus there exists a Nash equilibrium in which nobody tenders. (Furthermore, due to negative externalities in this example, the equilibrium in which nobody tenders is preferred by all agents to the equilibrium in which everybody tenders.) This multiplicity of equilibria has been noted by Grossman and Hart [8] and Burkart et al. [3], and similar observations have been made by Katz and Shapiro [17,18] and Segal and Whinston [30] in the context of Vertical contracting, Network externalities, and Exclusive dealing, respectively.

In this section we show that with increasing externalities, multiplicity of equilibria constitutes a serious problem for the principal, and characterize the principal's optimal mechanisms ensuring unique implementation. We also show that in some settings, the principal could not do any better by offering mechanisms that have multiple equilibria, assuming that agents coordinate either on the worst equilibrium for her, or on the best one for them.

Before proceeding, we need to take care of two technical problems. The first problem is that the principal could rule out unwanted equilibria at an arbitrarily small extra cost by offering noncompact menus. For example, suppose that, in the takeover example, the inferior raider offers all shareholders the menu $S=\{(0,t): t\in(-\delta,0]\}\cup\{(1,-v(N-1)-\delta)\}$ to all agents, with an arbitrarily small $\delta>0$. That is, the raider offers to buy shares at price $v(N-1)+\delta$, and any shareholder who does not tender can ask for a payment up to, but not including, δ. In the game induced by this menu, any nontendering strategy is strictly dominated by another nontendering strategy in which the shareholder asks the raider for a slightly higher payment. This rules out any equilibrium in which a shareholder does not tender, and ensures that all tender in the unique Nash equilibrium. This trick, which is similar to the use of noncompactness in implementation theory pioneered by Maskin's integer games [19], does not seem appealing in our context. In particular, for an arbitrarily small $\varepsilon \in (0, \delta)$ there exists an ε-Nash equilibrium in which all shareholders do not tender and ask the raider for payment $\delta - \varepsilon$.

To rule out such tricks, we restrict the principal to offer compact menus S_i.[8] Furthermore, to avoid technical complications, we restrict attention to the case where the trade domains $\mathscr{X}_i$ are finite. In this case, compactness of S_i is equivalent to the requirement that the set of transfers offered for each trade $x_i \in \mathscr{X}_i$ is compact. We can further simplify exposition by eliminating strictly dominated contracts and thus restricting attention to menus that offer at most one transfer for each trade, henceforth called *tariffs*.

A second "open set" problem encountered in the study of unique implementation is that the set $\mathfrak{S} \subset \mathscr{X}_1 \times \cdots \times \mathscr{X}_N \times \mathbb{R}^N$ of uniquely implementable outcomes is not closed, and the principal does not have a profit-maximizing outcome in $\mathfrak{S}$. We resolve this problem, just as the previous one, by brute force. Specifically, we consider outcomes that are *nearly* in $\mathfrak{S}$, i.e., belong to the closure of $\mathfrak{S}$. A profit-maximizing outcome within this set will exist.[9]

4.1. Increasing externalities

The analysis of the case of increasing externalities is greatly simplified by observing that in this case any profile of tariffs induces a *supermodular game* among the agents [22,32], when their strategies are ordered according to their trades: $(x_i', t_i') \geqslant (x_i, t_i)$ if and only if $x_i' \geqslant x_i$. Indeed, under this ordering, the agents' strategy sets S_i are complete lattices (in fact, chains), and each agent i's payoff $u_i(x_i, x_{-i}) - t_i$ has increasing differences in his own strategy (x_i, t_i) and other agents' strategies (x_{-i}, t_{-i}).

4.1.1. Discrimination with increasing externalities

The logic of our analysis can be illustrated using the case of two agents. The tariffs offered to the two agents induce a supermodular game in which the agents' strategies can be described by their trade choices (x_1, x_2) (see Fig. 2). Suppose that the principal wants to ensure that $\widehat{x} = (\widehat{x}_1, \widehat{x}_2)$ is a unique Nash equilibrium of this game. If $\widehat{x} \neq (0,0)$, then $x^0 = (0,0)$ must not be a Nash equilibrium, hence there must be an agent, say agent 1, whose minimum best response to $x_2^0 = 0$ is $x_1^1 > 0$. Let $x^1 \equiv (x_1^1, 0)$. If $x^1 \neq \widehat{x}$, then it also must not be a Nash equilibrium, hence agent 2's

[8] In the same spirit, Jackson [14] argues for restricting attention to so-called "bounded" mechanisms, in which an agent always has a best response.

[9] One way to justify this outcome is by restricting the principal to offer payments in multiples of a small $\delta > 0$. The corresponding implementable set $\mathfrak{S}$ is closed, and a profit-maximizing point within this set exists. Furthermore, by Berge's "maximum theorem" (see, e.g., [34]), as $\delta \to 0$, any limit of a sequence of optimal points in $\mathfrak{S}$ is an optimal point in the closure of $\mathfrak{S}$.

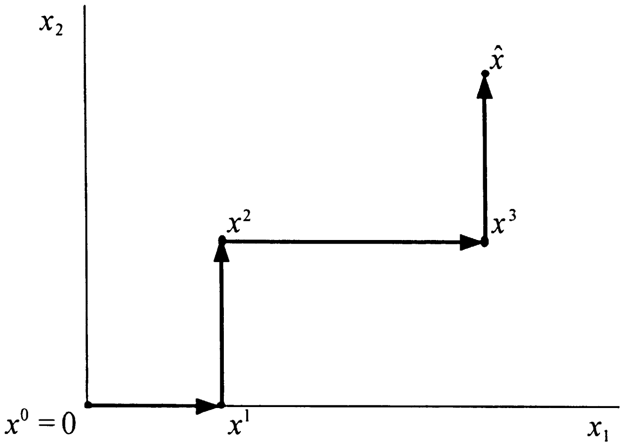

Fig. 2. Round-robin optimization.

minimum best response to x_1^1 must be some $x_2^2 > 0$. Next, if $x^2 \equiv (x_1^1, x_2^2) \neq \widehat{x}$, it also must not be a Nash equilibrium, hence x_1^1 cannot be a best response to x_2^2. Let x_1^3 denote agent 1's minimum best response to x_2^2. The property of increasing externalities implies that x_1^3 cannot be below x_1^1, which is agent 1's minimum best response to $0 < x_2^2$. Therefore, $x_1^3 > x_1^1$. Let $x^3 \equiv (x_1^3, x_2^2)$. Continuing the process, we obtain an increasing sequence of strategy profiles obtained by alternating application of the two agents' minimum best responses. Topkis [32, Subsection 4.3.1] calls this procedure "round-robin optimization", and shows that in a supermodular game (with arbitrary many players) it is increasing and converges to the game's lowest Nash equilibrium, in our case $\widehat{x}$.[10,11] (Since the strategy sets in our game are finite, convergence occurs in a finite number of steps.) Topkis's result also implies the converse: if round-robin optimization converges to $\widehat{x}$, then $\widehat{x}$ can be implemented as a unique Nash equilibrium by eliminating any trades above $\widehat{x}_1$ and $\widehat{x}_2$, respectively, from the two agents' respective menus. This discussion suggests that an outcome is uniquely implementable if and only if a menu profile can induce a round-robin optimization procedure converging to it. A formalization of this argument yields

Lemma 3. *Suppose that we have increasing externalities, and that agents' trade domains are finite, i.e.,* $|\mathscr{X}_i| < \infty$ *for all* $i \in N$. *Then* $(x, t) \in \mathscr{X}_i \times \cdots \times \mathscr{X}_N \times \mathbb{R}^N$ *is uniquely implementable if and only if there exists a path* $(x^r, t^r)_{r=0}^R \subset \mathscr{X}_i \times \cdots \times \mathscr{X}_N \times \mathbb{R}^N$ *such that*

[10] The Nash equilibrium set of a supermodular game is a lattice, hence the highest and lowest equilibria (in all agents' strategies at once) exist (see [22,32]). This fact will be used extensively in our analysis.

[11] Similar results are obtained in [33], whose "sequential Cournot tatônnement" and "simultaneous Cournot tatônnement" correspond to Topkis's "round-robin optimization" and "simultaneous optimization", respectively.

(a) $(x^0, t^0) = (0, 0)$ *and* $(x^R, t^R) = (x, t)$;
(b) *for each* $r \in \{1, \ldots, R\}$, *there exists* $i(r) \in N$ *such that* $(x^r_{-i(r)}, t^r_{-i(r)}) = (x^{r-1}_{-i(r)}, t^{r-1}_{-i(r)})$ *and* $x^r_{i(r)} \geqslant x^{r-1}_{i(r)}$;
(c) *for each* $r \in \{1, \ldots, R\}$,

$$u_{i(r)}(x^r) - t^r_{i(r)} > u_{i(r)}(x^{r-1}) - t^{r-1}_{i(r)}.$$

Proof. *Necessity*: Suppose that there exists a tariff profile $(S_i)_{i \in N}$ such that (x, t) is a unique Nash equilibrium of the induced supermodular game. Let $(s^r)_{r=0}^{\infty}$ be the round-robin optimization sequence for this game as constructed in [32, Algorithm 4.3.1]. By construction and [32, Theorem 4.3.2], the finite subsequence of $(s^r)_{r=0}^{\infty}$ obtained by omitting repetitions satisfies (a)–(c).

Sufficiency: Using (a)–(c) and the supermodularity of payoffs, it can be seen by induction on $r = 0, \ldots, R$ that (x^r, t^r) is a unique Nash equilibrium of the game induced by the menus $S_i = \bigcup_{q=0}^{r} \{(x_i^q, t_i^q)\}$ for all $i \in N$. Therefore, (x^R, t^R) is uniquely implementable. $\square$

Note that for any sequence $(x^r, t^r)_{r=0}^{R}$ satisfying conditions (a)–(c), the principal could raise her profits, for example, by raising $t^R_{i(R)}$ by a slightly, while preserving the strict inequalities (c). Thus, the principal does not have a profit-maximizing uniquely implementable outcome. To sidestep this technical problem, we allow the principal to choose *nearly* uniquely implementable outcomes, which are obtained by relaxing inequalities (c) to weak inequalities:

$$u_{i(r)}(x^r) - t^r_{i(r)} \geqslant u_{i(r)}(x^{r-1}) - t^{r-1}_{i(r)} \quad \text{for each } r \in \{1, \ldots, R\}. \tag{4}$$

Indeed, if (4) holds, then (c) can be satisfied by giving agent $i(r)$ an arbitrarily small extra bribe at each move r.

Once the principal is allowed to choose a nearly uniquely implementable outcome, then for any given round-robin trade path $(x^r)_{r=0}^{R}$ satisfying conditions (a), (b) in Lemma 3, she optimally chooses transfers to make inequalities (4) bind. (Indeed, if (4) did not bind for some $\hat{r}$, the principal could raise $t^r_{i(\hat{r})}$ for all $r \geqslant \hat{r}$ by the same small $\varepsilon > 0$ while preserving all inequalities.) Graphically, at each step r on the path, represented by an arrow in Fig. 2, the principal exactly compensates agent $i(r)$ for moving along the path. Expressing transfers from the binding inequalities (4), the principal's profits can be written as

$$f(x^R) + \sum_{r=1}^{R} [u_{i(r)}(x^r) - u_{i(r)}(x^{r-1})]. \tag{5}$$

One might wonder in what circumstances the principal's profits do not depend on her choice of round-robin trade path $(x^r)_{r=0}^{R}$ leading to a given trade profile x. It turns out that a necessary and sufficient condition for such independence is that the agents' payoffs admit a "potential" as defined by Monderer and Shapley [25]:

Definition 3. $P : \mathscr{X}_1 \times \cdots \times \mathscr{X}_N \to \mathbb{R}$ is a potential for the agents' payoffs if

$$u_i(x_i', x_{-i}) - u_i(x_i, x_{-i}) = P(x_i', x_{-i}) - P(x_i, x_{-i}) \text{ for all } i \in N, x_i, x_i' \in \mathscr{X}_i, x_{-i} \in \mathscr{X}_{-i}.$$

Observe that P is a potential if and only if the payoff of each agent i can be written in the form $u_i(x) = P(x) + g_i(x_{-i})$. Note also that when a potential function exists, we have increasing externalities if and only if it is supermodular.

Lemma 4. *The principal's profit* (5) *is a function* $\Pi(x^R)$ *that does not depend on the round-robin trade path* $(x^r)_{r=0}^R$ *leading to* x^R *if and only if the agents' payoff admit a potential.*

Proof. *Necessity*: Under the lemma's assumption, the function $P(x) \equiv \Pi(x) - f(x)$ constitutes a potential for the agents' payoffs. Indeed, take any $x_i, x_i' \in \mathscr{X}_i$ with $x_i' > x_i$, and any $x_{-i} \in \mathscr{X}_{-i}$. We can construct a round-robin trade path $(x^r)_{r=0}^R$ such that $x^R = (x_i, x_{-i})$. Let $x^{R+1} = (x_i', x_{-i})$. Since $(x^r)_{r=0}^{R+1}$ is also a round-robin trade path with $i(R+1) = i$, using (5), the change in the principal's profits can be written as

$$\Pi(x^{R+1}) - \Pi(x^R) = f(x^{R+1}) - f(x^R) + u_i(x^{R+1}) - u_i(x^R),$$

which implies

$$u_i(x_i', x_{-i}) - u_i(x_i, x_{-i}) = P(x_i', x_{-i}) - P(x_i, x_{-i}).$$

Sufficiency: If P is a potential function for the agents' payoffs, then $u_{i(r)}(x^r) - u_{i(r)}(x^{r-1}) = P(x^r) - P(x^{r-1})$, and therefore the principal's profits (5) equal

$$f(x^R) + \sum_{r=1}^{R} [P(x^r) - P(x^{r-1})] = f(x^R) + P(x^R) - P(0) \equiv \Pi(x^R)$$

regardless of the round-robin trade path leading to x^R. $\square$

The simplest setting satisfying the condition of Lemma 4 is that of Condition S. In this case, any round-robin trade path implementing trade with X agents has these agents switch sequentially from $x_i = 0$ to 1. The principal's profits (5), regardless of the switching order, can be written as

$$F(X) + \sum_{r=1}^{X} [U(1, r-1) - U(0, r-1)].$$

The principal will choose the aggregate trade $X \in \{0, \ldots, N\}$ to maximize this expression. To compare the principal's preferred aggregate trade here to what she would have implemented if she could coordinate agents, consider the following intuition. Start with the cheapest nearly uniquely implementable outcome in which the principal trades with the first $X-1$ agents, and suppose that she contemplates trading with one more agent, say agent X. She would optimally do this by signing up agent X at the price $t_X = U(1, X-1) - U(0, X-1)$, while holding her contracts with other agents fixed, in order to preserve the round-robin procedure involving the first X agents. When the principal could coordinate agents, on the other hand, signing up agent X at the same price t_X as before would relax other agents' participation constraints by virtue of increasing externalities, thus allowing the principal to raise the payments of the first X agents. Therefore, the principal's incentive to increase trade is lower with unique implementation than with simple implementation. This observation can be generalized to all settings in which the agents' payoffs admit a potential that, along with the principal's payoff, depends only on the aggregate trade:[12]

Proposition 4. *If externalities are increasing, there exists a potential function that depends only on* X, $f(x) = F(X)$, *and Condition D holds, then* $M_d^u \leqslant M_d^s$.

Proof. Let $\pi_d^{s/u}(X)$ denote the principal's maximum profit from implementing aggregate trade X, simply or uniquely, respectively, with discrimination. Take $X, X' \in \sum_i \mathscr{X}_i$ such that $X \leqslant X'$, and suppose that $\pi_d^s(X)$ is achieved by $x \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N$. Condition D implies the existence of $x' \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N$ such that $x' \geqslant x$ and $\sum_i x'_i = X'$. Since x' is one way to implement aggregate trade X', we can write, using expression (1) for the principal's profit:

$$\begin{aligned}
\pi_d^s(X') - \pi_d^s(X) \geqslant\; & F(X') - F(X) + \sum_i [u_i(x') - u_i(0, x'_{-i})] \\
& - \sum_i [u_i(x) - u_i(0, x_{-i})] \\
\geqslant\; & F(X') - F(X) + \sum_i [u_i(x') - u_i(x_i, x'_{-i})] \\
\geqslant\; & F(X') - F(X) + \sum_i [u_i(x'_1, \ldots, x'_i, x_{i+1}, \ldots, x_N) \\
& - u_i(x'_1, \ldots, x'_{i-1}, x_i, \ldots, x_N)] \\
=\; & F(X') - F(X) + P(X') - P(X) = \pi_d^u(X') - \pi_d^u(X),
\end{aligned}$$

where the second and third inequalities are by increasing externalities, and the equality is by Lemma 4. The result follows by Topkis's Monotonicity Theorem. □

When the potential property fails, the principal's optimal round-robin trade path could be obtained by solving an optimal control problem. However, the inefficiencies at the principal's preferred nearly uniquely implementable outcome can be characterized without solving this problem:

[12] This comparison cannot be generalized further due to interactions of variables in the principal's objective function (see [29, Example 1] for a discussion of such interactions). However, the comparison holds in an asymptotic setting as the number of agents goes to infinity (proof available from the author).

Proposition 5. *Suppose that we have increasing externalities, Conditions W, D hold, and $|\mathcal{X}_i| < \infty$ for all $i \in N$. Then with positive [negative] externalities at every $x^* \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_N$ such that $\sum_i x_i^* \in M^*$, we have $M_{\mathrm{d}}^{\mathrm{u}} \leqslant [\geqslant] M^*$.*

Proof. *Negative externalities*: Let $\pi_{\mathrm{d}}^{\mathrm{u}}(X)$ denote the principal's maximum profit from nearly uniquely implementing aggregate trade X with discrimination. Take any $X \in M_{\mathrm{d}}^{\mathrm{u}}$ and $X^* \in M^*$ such that $X \leqslant X^*$. Suppose that $\pi_{\mathrm{d}}^{\mathrm{u}}(X)$ is achieved by a round-robin path $(x^r)_{r=0}^R$, with $\sum_i x_i^R = X$. Under Condition D, there exists $x^* \geqslant x^R$ such that $\sum_i x_i^* = X^*$. Consider the path $(\tilde{x}^r)_{r=0}^{R+N}$ such that $\tilde{x}^r = x^r$ for $r \leqslant R$, and $\tilde{x}^r = (x_{i(r)}^*, \tilde{x}_{-i(r)}^{r-1})$ and $i(r) = r - R$ for $R < r \leqslant R + N$. Since this path satisfies conditions (a) and (b) in Lemma 3 and constitutes one way to implement aggregate trade X^*, we must have

$$
\begin{aligned}
\pi_{\mathrm{d}}^{\mathrm{u}}(X^*) - \pi_{\mathrm{d}}^{\mathrm{u}}(X) &\geqslant f(x^*) - f(x^R) + \sum_{r=R+1}^{R+N} [u_{i(r)}(\tilde{x}^r) - u_{i(r)}(\tilde{x}^{r-1})] \\
&= f(x^*) - f(x^R) + \sum_{r=R+1}^{R+N} \left\{ \sum_j [u_j(\tilde{x}^r) - u_j(\tilde{x}^{r-1})] \right. \\
&\quad \left. - \sum_{j \neq i(r)} [u_j(\tilde{x}^r) - u_j(\tilde{x}^{r-1})] \right\} \\
&= W(X^*) - W(X) - \sum_{r=R+1}^{R+N} \sum_{j \neq i(r)} [u_j(\tilde{x}_j^r, \tilde{x}_{-j}^r) - u_j(\tilde{x}_j^{r-1}, \tilde{x}_{-j}^{r-1})] \\
&\geqslant W(X^*) - W(X) - \sum_{r=R+1}^{R+N} \sum_{j \neq i(r)} [u_j(x_j^*, \tilde{x}_{-j}^r) - u_j(x_j^*, \tilde{x}_{-j}^{r-1})] \\
&\geqslant W(X^*) - W(X),
\end{aligned}
$$

where the second equality obtains by Condition W, the second inequality by increasing externalities since $\tilde{x}_j^r = \tilde{x}_j^{r-1} \leqslant x_j^*$ for $j \neq i(r)$, and the last inequality by negative externalities at x^*. Since by assumption $\pi_{\mathrm{d}}^{\mathrm{u}}(X^*) - \pi_{\mathrm{d}}^{\mathrm{u}}(X) \leqslant 0$ and $W(X^*) - W(X) \geqslant 0$, we thus have

$$\pi_{\mathrm{d}}^{\mathrm{u}}(X^*) - \pi_{\mathrm{d}}^{\mathrm{u}}(X) = W(X^*) - W(X) = 0,$$

and therefore $X^* \in M_{\mathrm{d}}^{\mathrm{u}}$ and $X \in M^*$.

Positive externalities: Take any $X \in M_{\mathrm{d}}^{\mathrm{u}}$ and $X^* \in M^*$ such that $X^* \leqslant X$. Under Condition D, $\pi_{\mathrm{d}}^{\mathrm{u}}(X)$ can be achieved by a path $(x^r)_{r=0}^R$ where $R = X/z$ and $\sum_i x_i^r = rz$ for all r. Let $R^* = X^*/z$. Note that the path $(x^r)_{r=0}^{R^*}$ constitutes one way to implement aggregate trade X^*, and therefore

$$\pi_{\mathrm{d}}^{\mathrm{u}}(X) - \pi_{\mathrm{d}}^{\mathrm{u}}(X^*) \leqslant f(x^R) - f(x^{R^*}) + \sum_{r=R^*+1}^{R} [u_{i(r)}(x^r) - u_{i(r)}(x^{r-1})].$$

Using similar arguments to those in the previous part, based on increasing externalities and positive externalities at x^{R^*}, this implies

$$\pi_{\mathrm{d}}^{\mathrm{u}}(X) - \pi_{\mathrm{d}}^{\mathrm{u}}(X^*) \leqslant W(X) - W(X^*),$$

which in turn implies that $X^* \in M_{\mathrm{d}}^{\mathrm{u}}$ and $X \in M^*$. $\square$

The proposition establishes that when the principal must ensure unique implementation, distortion is determined by externalities on efficient traders. In particular, a situation with no such externalities can be thought of as one with both positive and negative externalities at once, for which case Proposition 5 implies that $M_{\mathrm{d}}^{\mathrm{u}} = M^*$. These conclusions are qualitatively similar to those established in [29, Section 4] for the case where the principal makes privately observed offers and lacks public commitment, and the agents have passive beliefs. Intuitively, in both cases the principal who deviates by offering a different contract to one agent need not change her offers to other agents. With privately observed offers, this is because the deviation is not observed by other agents. With publicly observed offers and unique implementation, this is because the round-robin procedure must be preserved for the remaining agents. (Contrast this to simple implementation, where a publicly observed deviation to one agent affects other agents' participation constraints, and thus makes the principal adjust her offers to other agents as well.) Therefore, the principal's incentive to deviate from an efficient outcome should be attributed to the externalities at this outcome.

Propositions 4 and 5 can be illustrated with the following examples. In Takeovers with an inferior raider, the externalities are increasing and negative, but they are absent at the point at which the raider acquired all the shares. Indeed, a shareholder who has sold his shares does not care anymore what will happen to the firm. Thus, if the raider's private benefits of control exceed the harm to the firm's value so that it is efficient for her to acquire all the shares, there will be no externalities at the efficient outcome, and by Proposition 5 it will be uniquely implemented by the raider. On the other hand, if efficiency requires some or all shareholders to keep their shares, by Proposition 5 the raider will acquire a socially excessive number of shares due to negative externalities at such outcomes. Still, under the assumptions of Proposition 4, she will acquire fewer shares than if she could coordinate shareholders on her preferred equilibrium.

Similarly, in the setting of Exclusive dealing, the externalities are increasing and negative, but they are absent at the point at which all buyers sign exclusive dealing contracts with the incumbent firm. Indeed, a buyer who has signed an exclusive does not care about future competition anymore. Thus, if it is efficient for all buyers to sign exclusives, then there are no externalities at this efficient outcome, and by Proposition 5 it will be uniquely implemented by the firm. Otherwise, there will be negative externalities on efficient traders, and therefore too much exclusive dealing from the social viewpoint. At the same time, under Condition S, by Proposition 4 there will be less exclusion than if the incumbent firm could coordinate buyers on its preferred equilibrium.

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Finally, in the setting of Network externalities, the externalities from the adoption of a new (sponsored) technology are increasing, and can be of the opposite signs for the users of the unsponsored and sponsored technology. Suppose for simplicity that the efficient outcome involves standardization on one of the technologies. If the efficient technology exhibits no network externalities, then by Proposition 5 this efficient outcome will be uniquely implemented (regardless of any network externalities associated with the other technology). On the other hand, if the efficient technology exhibits positive network externalities, by Proposition 5 it will be adopted insufficiently—i.e., there will be insufficient standardization. Also, Proposition 4 implies that under Condition S, the sponsored technology will be adopted less than if the sponsor could coordinate consumers on her preferred equilibrium. (In the latter case, by Proposition 1 the sponsored technology will always be adopted excessively due to negative externalities on non-traders.)

4.1.2. *Nondiscrimination with increasing externalities*

Here we examine how nondiscrimination affects unique implementation when agents are ex ante identical, i.e., Condition I holds. Observe that the principal's optimal uniquely implementable outcome analyzed above involved discrimination whenever a positive trade was implemented and externalities were strictly increasing. Indeed, the principal optimally ensured that a certain trade profile could not arise in a Nash equilibrium by bribing just one agent into trading more, without offering the same bribe to other agents. When the principal is not allowed to discriminate and must offer the same menu to all agents, she can no longer use this "divide-and-conquer" strategy.

To understand the logic of the analysis in the case of nondiscrimination, consider again the case of two identical agents. The principal offers both agents the same menu (tariff), which induces a symmetric supermodular game where the agents' strategies can be described by their trade choices (x_1, x_2) (see Fig. 3). Suppose that the game has a unique Nash equilibrium. By symmetry of the game, this equilibrium must be symmetric; denote it by $(\widehat{x}, \widehat{x})$. In particular, if $\widehat{x} > 0, (0,0)$ must not constitute a Nash equilibrium, hence there must be an agent whose minimum best

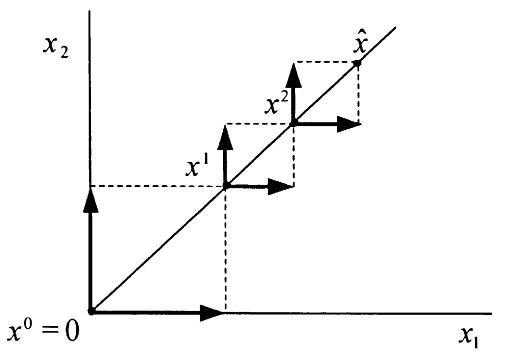

Fig. 3. Simultaneous optimization.

response to $x^0 = 0$ is some $x^1 > 0$. By symmetry of the game, x^1 must then be *each* agent's minimum best response to x^0. Furthermore, if $x^1 \neq \hat{x}$, (x^1, x^1) also should not be a Nash equilibrium, hence the minimum best response to x^1, denoted by x^2, must differ from x^1. By the property of increasing externalities, x^2 cannot be below x^1, which is the minimum best response to 0. Hence $x^2 > x^1$. Continuing the process, we obtain an increasing sequence of symmetric strategy profiles, obtained by simultaneous iterative application of the agents' minimum best responses. Topkis [32, Subsection 4.3.2] calls this procedure "simultaneous optimization", and shows that in a general supermodular game, with arbitrary many players, it is increasing and converges to the game's lowest Nash equilibrium, which in our case is $(\hat{x}, \hat{x})$. (Note again that since strategy sets are finite, convergence occurs in a finite number of steps.) Topkis's result also implies the converse: if simultaneous optimization converges to $(\hat{x}, \hat{x})$, then $(\hat{x}, \hat{x})$ can be implemented as a unique Nash equilibrium by eliminating any trades above $\hat{x}$ from the menu offered to the agents. This discussion suggests that an outcome is uniquely implementable with nondiscrimination if and only if a menu can induce a simultaneous optimization procedure converging to it. A formalization of this argument yields

Lemma 5. *Suppose that we have increasing externalities, Condition I holds, and* $|\mathscr{X}| < \infty$. *Then* $(x, t) \in \mathscr{X}^N \times \mathbb{R}^N$ *is uniquely implementable with nondiscrimination if and only if* $(x_i, t_i) = (\bar{x}, \bar{t})$ *for all* $i \in N$, *and there exists a path* $(x^r, t^r)_{r=0}^R \subset \mathscr{X} \times \mathbb{R}$ *such that*

(a) $(x^0, t^0) = (0, 0)$ *and* $(x^R, t^R) = (\bar{x}, \bar{t})$;
(b) *for each* $r \in \{1, \ldots, R\}$, $x^r \geqslant x^{r-1}$;
(c) *for each* $r \in \{1, \ldots, R\}$,
$$u(x^r; x^{r-1}, \ldots, x^{r-1}) - t^r > u(x^{r-1}; x^{r-1}, \ldots, x^{r-1}) - t^{r-1}.$$

Proof. *Necessity*: Suppose that there exists a tariff S such that (x, t) is a unique Nash equilibrium of the supermodular game induced by offering each player tariff S. Since the game is symmetric, (x, t) must be symmetric, i.e., $(x_i, t_i) = (\bar{x}, \bar{t})$ for all i. Let $(s^r)_{r=0}^{\infty}$ be the (symmetric) simultaneous optimization sequence for this game, as constructed in [32, Algorithm 4.3.2]. By construction and [32, Theorem 4.3.4], the finite subsequence of $(s^r)_{r=0}^{\infty}$ obtained by omitting repetitions satisfies (a)–(c).

Sufficiency: Using (a)–(c) and the supermodularity of payoffs, it can be seen by induction on $r = 0, \ldots, R$ that (x^r, t^r) is a unique Nash equilibrium of the game induced by offering each agent the menu $S = \bigcup_{q=0}^{r} \{(x^q, t^q)\}$. Therefore, (x^R, t^R) is uniquely implementable with nondiscrimination. $\square$

Just as in the case with discrimination, the principal does not have a profit-maximizing uniquely implementable outcome because inequalities (c) are strict. We sidestep this problem by allowing the principal to choose *nearly* uniquely implementable outcomes, which are obtained by relaxing (c) to weak inequalities:

$$u(x^r; x^{r-1}, \dots, x^{r-1}) - t^r \geqslant u(x^{r-1}; x^{r-1}, \dots, x^{r-1}) - t^{r-1} \quad \text{for each } r \in \{1, \dots, R\}.$$

Then for any given trade sequence $(x^r)_{r=0}^R$ satisfying conditions (a) and (b) in Lemma 5, the principal will optimally choose transfers $(t^r)_{r=0}^R$ that make the above inequalities bind. Graphically, at each step of the sequence, the principal exactly compensates each agent for moving along the respective arrow depicted in Fig. 3. Expressing transfers from the binding inequalities, the principal's profits can be written as

$$f(x^R, \dots, x^R) + N \sum_{r=1}^{R} [u(x^r; x^{r-1}, \dots, x^{r-1}) - u(x^{r-1}; x^{r-1}, \dots, x^{r-1})].$$

The principal will choose a simultaneous-optimization trade sequence $(x^r)_{r=0}^R$ satisfying (a) and (b) to maximize this expression. It turns out that she can restrict attention to sequences that skip no feasible trade levels. Indeed, if there existed $x' \in \mathcal{X}$ such that $x^k < x' < x^{k+1}$ for some $k \in \{0, \dots, R-1\}$, by increasing externalities we would have

$$\begin{aligned}&[u(x'; x^k, \dots, x^k) - u(x^k; x^k, \dots, x^k)] + [u(x^{k+1}; x', \dots, x') - u(x'; x', \dots, x')]\\ &\quad \geqslant u(x^{k+1}; x^k, \dots, x^k) - u(x^k; x^k, \dots, x^k),\end{aligned}$$

and therefore the principal's profits from the sequence $(x^0, \dots, x^k, x', x^{k+1}, \dots, x^R)$ would be at least as high as those from the original sequence. Hence, letting $\mathcal{X} = \{y^j\}_{j=0}^J$ with $0 = y^0 < y^1 < \dots < y^J$ without loss of generality, the principal's maximum profit from uniquely implementing aggregate trade $X = Ny^R$ is given by

$$\begin{aligned}\pi_{\text{n}}^{\text{u}}(Ny^R) = &f(y^R, \dots, y^R) + N \sum_{r=1}^{R} [u(y^r; y^{r-1}, \dots, y^{r-1})\\ &- u(y^{r-1}; y^{r-1}, \dots, y^{r-1})]. \end{aligned} \tag{6}$$

The set of the principal's preferred nearly uniquely implementable aggregate trades can then be defined as $M_{\text{n}}^{\text{u}} = \arg\max_{X \in N\mathcal{X}} \pi_{\text{n}}^{\text{u}}(X)$.

We can now show that, just as when the principal could discriminate, the requirement of unique implementation reduces aggregate trade:

Proposition 6. *Suppose that we have strictly increasing externalities, Condition I holds, and $|\mathcal{X}| < \infty$. Then $M_{\text{n}}^{\text{u}} \leqslant M_{\text{n}}^{\text{s}}$.*

Proof. Using (2) and increasing externalities,

$$\begin{aligned}\pi_{\text{n}}^{\text{s}}(Ny^{j+1}) = &f(y^{j+1}, \dots, y^{j+1}) + N[u(y^{j+1}; y^{j+1}, \dots, y^{j+1}) - u(0; y^{j+1}, \dots, y^{j+1})]\\ &\geqslant f(y^{j+1}, \dots, y^{j+1}) + N[u(y^{j+1}; y^j, \dots, y^j) - u(0; y^j, \dots, y^j)].\end{aligned}$$

Therefore, using (2) and (6),

$$\pi_{\mathrm{n}}^{\mathrm{s}}(Ny^{j+1}) - \pi_{\mathrm{n}}^{\mathrm{s}}(Ny^{j}) \geqslant f(y^{j+1}, \ldots, y^{j+1}) - f(y^{j}, \ldots, y^{j}) + N[u(y^{j+1}; y^{j}, \ldots, y^{j}) - u(y^{j}; y^{j}, \ldots, y^{j})] = \pi_{\mathrm{n}}^{\mathrm{u}}(Ny^{j+1}) - \pi_{\mathrm{n}}^{\mathrm{u}}(Ny^{j}).$$

Topkis's Monotonicity Theorem implies the result. □

We can also examine the effect of nondiscrimination given that unique implementation is required. Since nondiscrimination under increasing externalities requires the principal to trade equal amounts with all agents, the general welfare effect of this requirement is impossible to predict (just as it was in the case of simple implementation). However, unlike in the case of simple implementation, here nondiscrimination has an effect even if the principal would otherwise also implement equal *trades* with equal agents. This is because nondiscrimination prevents the principal from charging different *payments* for equal trades using a "divide-and-conquer" strategy. Intuitively, *conditional on having equal trades with all agents*, nondiscrimination makes increasing trade more expensive for the principal, and therefore reduces equilibrium trades. To establish this formally, let $\overline{M}_{\mathrm{d}}^{\mathrm{u}}$ denote the set of the principal's preferred nearly uniquely implementable aggregate trades with discrimination conditional on trading equally with all agents. Then we obtain

Proposition 7. *Suppose that we have increasing externalities, Condition I holds, and* $|\mathscr{X}| < \infty$. *Then* $M_{\mathrm{n}}^{\mathrm{u}} \leqslant \overline{M}_{\mathrm{d}}^{\mathrm{u}}$.

Proof. Let $\overline{\pi}_{\mathrm{d}}^{\mathrm{u}}(X)$ denote the principal's maximum profit from nearly uniquely implementing trade profile $(X/N, \ldots, X/N)$ with discrimination. Suppose that the profit $\overline{\pi}_{\mathrm{d}}^{\mathrm{u}}(Ny^{j})$ is achieved by a path $(x^{r})_{r=0}^{R} \subset \mathscr{X}^{N}$, with $x^{R} = (y^{j}, \ldots, y^{j})$. Consider the path $(\tilde{x}^{r})_{r=0}^{R+N}$ such that $\tilde{x}^{r} = x^{r}$ for $r \leqslant R$, and $\tilde{x}^{r} = (y^{j+1}, \tilde{x}_{-i(r)}^{r-1})$ and $i(r) = r - R$ for $R < r \leqslant R + N$. Since this path satisfies conditions (a) and (b) in Lemma 3 and constitutes one way to nearly uniquely implement aggregate trade Ny^{j+1} with discrimination,

$$\begin{aligned}
&\overline{\pi}_{\mathrm{d}}^{\mathrm{u}}(Ny^{j+1}) - \overline{\pi}_{\mathrm{d}}^{\mathrm{u}}(Ny^{j}) \\
&\quad \geqslant f(y^{j+1}, \ldots, y^{j+1}) - f(y^{j}, \ldots, y^{j}) + \sum_{i=1}^{N} [u_i(\tilde{x}^{R+i}) - u_i(\tilde{x}^{R+i-1})] \\
&\quad \geqslant f(y^{j+1}, \ldots, y^{j+1}) - f(y^{j}, \ldots, y^{j}) + N[u(y^{j+1}; y^{j}, \ldots, y^{j}) - u(y^{j}; y^{j}, \ldots, y^{j})] \\
&\quad = \pi_{\mathrm{n}}^{\mathrm{u}}(Ny^{j+1}) - \pi_{\mathrm{n}}^{\mathrm{u}}(Ny^{j}),
\end{aligned}$$

where the second inequality follows from increasing externalities. Topkis's Monotonicity Theorem implies the result. □

Next, we characterize the contracting inefficiencies conditional on having equal trades with all agents. These inefficiencies turn out to be driven by the externalities either at or just above the symmetric efficient point:

Proposition 8. *Suppose that we have increasing externalities, Condition I holds, and* $|\mathscr{X}| < \infty$. *If for all* $j = 1, \dots, J$ *such that* $Ny^j \in \overline{M}^*$ *we have negative externalities at* $(y^j, \dots, y^j)$ *[positive externalities at* $(y^{j+1}, \dots, y^{j+1})$ *whenever* $j < J$*], then* $M_{\mathrm{n}}^{\mathrm{u}} \geqslant [\leqslant] \overline{M}^*$.

Proof. Consider the case of positive externalities. Take $X = Ny^j \in M_{\mathrm{n}}^{\mathrm{u}}$ and $X^* = Ny^{j^*} \in \overline{M}^*$ such that $X^* < X$. We can write

$$\pi_{\mathrm{n}}^{\mathrm{u}}(X) - \pi_{\mathrm{n}}^{\mathrm{u}}(X^*) = f(y^j, \dots, y^j) - f(y^{j^*}, \dots, y^{j^*}) + N \sum_{k=j^*}^{j-1} [u(y^{k+1}; y^k, \dots, y^k) - u(y^k; y^k, \dots, y^k)].$$

For all $k \in \{j^*, \dots, j-1\}$, using the fact that $y^{k+1} \geqslant y^{j^*+1}$, increasing externalities, and positive externalities at $(y^{j^*+1}, \dots, y^{j^*+1})$, we have

$$\begin{aligned} & u(y^{k+1}; y^k, \dots, y^k) \\ & \leqslant u(y^{k+1}; y^{k+1}, \dots, y^{k+1}) + \left[u(y^{j^*+1}; y^k, \dots, y^k) - u(y^{j^*+1}; y^{k+1}, \dots, y^{k+1})\right] \\ & \leqslant u(y^{k+1}; y^{k+1}, \dots, y^{k+1}). \end{aligned}$$

Substituting in the previous expression, we see that

$$\begin{aligned} \pi_{\mathrm{n}}^{\mathrm{u}}(X) - \pi_{\mathrm{n}}^{\mathrm{u}}(X^*) \leqslant & f(y^j, \dots, y^j) - f(y^{j^*}, \dots, y^{j^*}) \\ & + N \sum_{k=j^*}^{j-1} [u(y^{k+1}; y^{k+1}, \dots, y^{k+1}) - u(y^k; y^k, \dots, y^k)] \\ = & \overline{W}(Ny^j) - \overline{W}(Ny^{j^*}), \end{aligned}$$

where $\overline{W}(\cdot)$ is given by (3).

Since by assumption $\pi_{\mathrm{n}}^{\mathrm{u}}(X) - \pi_{\mathrm{n}}^{\mathrm{u}}(X^*) \geqslant 0$ and $\overline{W}(X) - \overline{W}(X^*) \leqslant 0$, we thus have

$$\pi_{\mathrm{n}}^{\mathrm{u}}(X) - \pi_{\mathrm{n}}^{\mathrm{u}}(X^*) = \overline{W}(X) - \overline{W}(X^*) = 0,$$

and therefore $X^* \in M_{\mathrm{n}}^{\mathrm{u}}$ and $X \in \overline{M}^*$. The proof for negative externalities is similar. □

This result can be illustrated and somewhat strengthened under Condition S, when the choice is between trading with everybody $(X = N)$ and trading with nobody $(X = 0)$. When the principal trades with everybody in the simultaneous-optimization procedure, she receives $U(1, 0) - U(0, 0)$ from each agent, thus her profit is

$$\pi_{\mathrm{n}}^{\mathrm{u}}(N) = F(N) + N[U(1, 0) - U(0, 0)].$$

The principal prefers trading with everybody rather than nobody when

$$\pi_{\mathrm{n}}^{\mathrm{u}}(N) - \pi_{\mathrm{n}}^{\mathrm{u}}(0) = F(N) - F(0) + N[U(1,0) - U(0,0)] \geqslant 0.$$

First, observe that due to increasing externalities, when $X = N$ is implemented, all agents but one must be now paid more than if the principal could discriminate, thus the principal is less likely to trade with everybody, which illustrates Proposition 7. Second, it is socially efficient to trade with everybody when

$$W(N) - W(0) = F(N) - F(0) + N[U(1, N-1) - U(0,0)] \geqslant 0.$$

We see that the difference between the principal's private incentive to trade and the social incentive to trade is exactly $N[U(1,0) - U(1, N-1)]$. Formalizing this argument using Topkis's Monotonicity Theorem, we have

Proposition 9. *With increasing externalities, under Condition S, we have* $M_{\mathrm{n}}^{\mathrm{u}} \leqslant [\geqslant] \overline{M}^*$ *if* $U(1, N-1) \geqslant [\leqslant] U(1,0)$.

The proposition establishes that the contracting distortion with unique implementation and nondiscrimination under Condition S is determined by the externalities *on traders*. If these externalities are absent, i.e., $U(1, N-1) = U(1,0)$, the Proposition implies that the principal makes the socially efficient choice between trading with all agents and trading with none. This unifies the findings that unique implementation with nondiscrimination yields efficient outcomes in the context of Takeovers with an inferior raider [3], Exclusive dealing [13,30], and Network externalities when the sponsored technology exhibits no externalities (i.e., is backward compatible) [7]. On the other hand, if the sponsored technology exhibits positive network externalities, it is less likely to be adopted than socially optimal. Contrast this to simple implementation, where by Propositions 1 and 2 there is too much adoption due to negative externalities on nontraders (users of non-sponsored technology).

4.1.3. Assumptions about equilibrium selection

The requirement of unique implementation may be overly restrictive. For example, the principal might prefer to design a game that has multiple equilibria if her profits are high in all of these equilibria. It turns out, however, that with increasing externalities, if the principal is so pessimistic that she always expects agents to coordinate on the worst Nash equilibrium for her, she might as well use unique implementation. The argument is based on the following observation about supermodular games. Take a supermodular game with strategy sets $(S_i)_{i \in N}$, and a strategy profile $\hat{s} \in S_1 \times \cdots \times S_N$. The game *truncated at* $\hat{s}$ is defined as the game with strategy spaces $\tilde{S}_i = \{s_i \in S_i : s_i \leqslant \hat{s}_i\}$ for all $i \in N$, and the same payoffs as the original game. The following result holds:

Lemma 6. *If* $\hat{s}$ *is the lowest Nash equilibrium of a supermodular game, then it is a unique Nash equilibrium of the game truncated at* $\hat{s}$.

This lemma follows from the observation that truncation does not affect Topkis's round-robin optimization procedure [32] leading to $\hat{s}$, hence $\hat{s}$ remains the lowest Nash equilibrium of the truncated game.

Say that an outcome $(x,t) \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_N \times \mathbb{R}^N$ is *worst-case implementable* (*with nondiscrimination*) if there exists a menu profile $(S_i)_{i \in N}$ such that (x,t) constitutes a Nash equilibrium of the induced game with the lowest profits for the principal (and $S_i = S$ for all i). Consider first the case of discrimination. Any uniquely implementable outcome is worst-case implementable by definition, but the converse need not hold. However, if outcome (x,t) is worst-case implemented by some menu profile, truncating the menus at the lowest Nash equilibrium $(\underline{x}, \underline{t})$ uniquely implements $(\underline{x}, \underline{t})$ by Lemma 6, and by construction $(\underline{x}, \underline{t})$ gives the principal at least the same payoff as (x,t). The same argument works with nondiscrimination under Condition I, which ensures that the game's lowest Nash equilibrium $(\underline{x}, \underline{t})$ is symmetric and so truncation preserves the game's symmetry. These arguments establish

Proposition 10. *Under increasing externalities, for any worst-case implementable outcome* [*with nondiscrimination*] *there exists a uniquely implementable outcome* [*with nondiscrimination*] *that gives the principal at least the same payoff* [*provided that Condition I holds*].

The proposition implies that under increasing externalities, a pessimistic principal can restrict attention to unique implementation without loss. A pessimistic principal is assumed, for example, by Da Rin and Hellmann [6]. In their model, the principal is a leading bank, whose objective is to rule out coordination failures in an economy.

Many papers, however, make a more optimistic assumption about equilibrium selection, letting agents coordinate on *their* preferred equilibrium. For example, this assumption is made by Katz and Shapiro [18] in the setting of Network externalities, Segal and Whinston [30] in Exclusive dealing, and Grossman and Hart [8] in Takeovers. We now examine the contracting outcomes under this assumption.

Analysis of the agents' preferred equilibria is particularly straightforward when the externalities are increasing and have a constant sign—either positive or negative. In this case, any menu profile induces a supermodular game with constant-sign externalities. Milgrom and Roberts [22] find that such games have a Nash equilibrium that Pareto dominates all the others. With positive (negative) externalities, it is the highest (lowest) Nash equilibrium. Furthermore, Milgrom and Roberts [23] establish that this equilibrium is selected by a large class of coalitional equilibrium refinements. Specifically, they define the concept of a "coalition-proof" (CP) equilibrium given an "admissible" coalitional communication structure describing the tree of feasible coalitional deviations. An admissible communication structure is one that allows at least individual deviations from any coalition and deviations by the grand coalition.[13] [23] proves that in supermodular

[13] For example, when all coalitional (sub)deviations are allowed, we obtain the "coalition-proof Nash equilibria" of Bernheim et al. [2]. The strongest CP refinement obtains by allowing all coalitional

games with constant-sign externalities, the unique (up to payoff equivalence) CP equilibrium for any admissible communication structure is the agents' Pareto best Nash equilibrium described above.[14] We investigate how our contracting predictions are affected when this equilibrium concept is used. Formally, an outcome $(x, t) \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_N \times \mathbb{R}^N$ is CP implementable (with nondiscrimination) if there exists a menu profile $(S_i)_{i \in N}$ such that (x, t) constitutes a CP equilibrium of the induced game (and $S_i = S$ for all i). (All the results below hold for any admissible communication structure.)

With positive increasing externalities, suppose $(x, t) \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_N \times \mathbb{R}^N$ is a Nash equilibrium of the game induced by some menu profile. If we truncate the menus at (x, t), this outcome becomes the highest Nash equilibrium, and therefore a CP equilibrium. Therefore, (x, t) is CP implementable. The same argument works with nondiscrimination, provided that (x, t) is symmetric (which can be ensured using Lemma 1), so that truncation preserves the symmetry of the game. Thus, we have

Proposition 11. *Suppose we have positive increasing externalities. An outcome is CP implementable if and only if it is simply implementable. If, in addition, Condition I holds and externalities are strictly increasing, then an outcome is CP implementable with nondiscrimination if and only if it is simply implementable with nondiscrimination.*

With negative increasing externalities, however, the CP equilibrium is the lowest Nash equilibrium. This implies that any CP implementable outcome can be uniquely implemented at an arbitrarily small extra cost for the principal. Thus, CP implementation is essentially the same as unique implementation:

Proposition 12. *With negative increasing externalities an outcome is nearly CP implementable if and only if it is nearly uniquely implementable. If, in addition, Condition I holds and externalities are strictly increasing, then an outcome is nearly CP implementable without discrimination if and only if it is nearly uniquely implementable without discrimination.*

Proof. The sufficiency part of both statements follows from [23, Theorem 2(1)]. For the necessity part of the first statement, suppose that (x, t) is a CP equilibrium of the game induced by the tariff profile $(S_i)_{i \in N}$, and let $v_i = u_i(x) - t_i$ denote agent i's equilibrium payoff. By Milgrom and Roberts [23, Theorem 2(3)], any agent i's payoff in any Nash equilibrium of the induced game does not exceed v_i. Consider now the "new" tariffs $\tilde{S}_i = S_i \backslash (x_i, t_i) \cup (x_i, t_i - \varepsilon)$, with $\varepsilon > 0$. Note that if ε is small enough, using finiteness of the game, the set of Nash equilibrium trade profiles of the

(*footnote continued*)
deviations and only individual counter-deviations, which yields the concept called "strongly coalition-proof" equilibrium by Milgrom and Roberts [23] and "semistrong" equilibrium by Kaplan [15].

[14] Milgrom and Roberts [23] allow agents to use correlated mixed strategies, which does not affect the set of coalition-proof equilibria.

"new" game induced by these tariffs is a subset of the set of Nash equilibrium trade profiles of the "old" game. Therefore, at any Nash equilibrium trade profile of the new game at which agent i's trade differs from x_i, his payoff is at most v_i. On the other hand, $(x_i, t_i - \varepsilon)_{i \in N}$ is a Nash equilibrium of the new game at which each agent i receives the payoff of $v_i + \varepsilon$. Therefore, $(x_i, t_i - \varepsilon)_{i \in N}$ is the new game's unique Pareto best Nash equilibrium, and by Milgrom and Roberts [22, Theorem 7] it is the new game's lowest Nash equilibrium. Lemma 6 then implies that the menus $(\tilde{S}_i)_{i \in N}$ truncated at $(x_i, t_i - \varepsilon)_{i \in N}$ uniquely implement $((x_i, t_i - \varepsilon))_{i \in N}$. Since $\varepsilon > 0$ can be taken arbitrarily small, (x, t) is nearly uniquely implementable. This implies the first statement of the proposition. The second statement can be proven with the same argument, using the fact that any outcome that is implementable with nondiscrimination must be symmetric by Lemma 1, therefore truncation at this outcome preserves the game's symmetry. □

4.2. Decreasing externalities

With decreasing externalities, an agent is more willing to accept his offer when other agents reject theirs. Therefore, whenever an outcome (x, t) is implementable as a Nash equilibrium, it can also be implemented in (weakly) dominant strategies by offering menus $S_i = \{(0, 0), (x_i, t_i)\}$ to each agent i. Furthermore, by slightly reducing transfers t_i, the principal can ensure that the choice (0, 0) is a strictly dominated strategy, thus achieving unique implementation:

Proposition 13. *With decreasing externalities, an outcome is nearly uniquely implementable if and only if it is implementable.*

As for unique implementation with nondiscrimination, note that no asymmetric outcome can be uniquely implemented when agents are identical, since a permutation of agents would then yield another Nash equilibrium. However, since the principal is indifferent among such equilibria, the unique implementation requirement seems too restrictive in this case.[15] If instead we assume that agents coordinate on a CP equilibrium, we find that such coordination does not hurt the principal (under an additional restriction on the agents' payoffs):

Proposition 14. *Suppose externalities are strictly decreasing and either positive or negative, Condition I holds, and* $u_i(x_i, x_{-i}) = U(x_i, \sum_{j \neq i} x_j)$. *Then an outcome is implementable if and only if it is CP implementable.*

Proof. The sufficiency part is trivial. We show necessity for the case of positive externalities (the modification to negative externalities is trivial). Suppose that

[15] The truncation trick of Lemma 6 cannot be used to ensure uniqueness here since, among other things, a lowest Nash equilibrium need not exist under decreasing rather than increasing externalities.

outcome (x,t) is a Nash equilibrium of the game induced by offering all agents menu S. Suppose in negation that (x,t) is not a CP equilibrium of the game. Then a coalition $J \subset N$ has a deviation $(x'_J, t'_J) \subset S^J$ that makes each member of J better off given that others play (x_{-J}, t_{-J}), and from which no member of J wants to deviate unilaterally. Let $(x'_{-J}, t'_{-J}) = (x_{-J}, t_{-J})$. Since any agent $i \in J$ is choosing from the same menu S as before, with positive externalities he can be better off at (x', t') than at (x,t) only if $X'_{-i} \equiv \sum_{j \neq i} x'_i > X_{-i} \equiv \sum_{j \neq i} x_i$. But then by the Monotone Selection Theorem, under strictly decreasing externalities agent i's optimal choices from S must satisfy $x'_i \leqslant x_i$. Since this is also trivially true for all $i \notin J$, we must have $X'_{-k} \leqslant X_{-k}$ for all $k \in N$—a contradiction. □

These results establish that with decreasing externalities, agents' coordination does not affect the contracting outcome, both with or without discrimination (though the latter result requires some additional assumptions).

5. Conclusion

We have shown that the effects of coordination and discrimination in contracting with externalities depend crucially on whether externalities are increasing or decreasing. Both features have little or no effect with decreasing externalities, but have important effects with increasing externalities. In the latter case, the principal's inability to coordinate agents reduces the aggregate trade, both when she can discriminate and when she cannot. The principal's inability to discriminate further reduces the aggregate trade (conditional on implementing equal trades with all agents). This occurs because a principal who cannot discriminate is forced to offer equal transfers for equal trades, and thus cannot use a "divide-and-conquer" strategy.

Our results also predict the *welfare* effects of coordination and discrimination under increasing externalities. Consider first the case of positive increasing externalities, where all contracting regimes generate insufficient trade from the social viewpoint. We have shown that in this case, agents' coordination on their preferred equilibrium leads to the same outcome as their coordination on the principal's preferred equilibrium. As for nondiscrimination, its only effect is to force the principal to trade equal amounts with identical agents, and the welfare consequences of this are in general hard to predict.

Consider now the case of negative increasing externalities, in which case all contracting regimes generate excessive trade from the social viewpoint. A reduction in the aggregate trade toward the efficient level is then socially beneficial, provided that total surplus is a quasiconcave function of the aggregate trade. Since agents' coordination on their preferred equilibrium reduces the aggregate trade, it is socially beneficial. A nondiscrimination requirement in this case prevents the principal from using a "divide-and-conquer" strategy and further reduces the aggregate trade (conditional on having equal trades with identical agents), thus it is also socially beneficial.

These results make a case for imposing a nondiscrimination requirement and for facilitating agents' coordination on their preferred equilibrium. Taken together or separately, these two measures reduce excessive trade in the presence of negative increasing externalities, and do not have substantial effects in other cases (except for the fact that nondiscrimination forces equal trades with identical agents). Such legal measures have indeed been adopted in the setting of Takeovers. For example, Burkart et al. [3] report that some U.S. states outlaw discriminatory restricted bids, and others promote shareholders' coordination by requiring that a takeover gain approval by the majority of them. Our analysis suggests a rationale for such rules, which also applies to other settings of contracting with externalities.

Two caveats are in order. First, we have only taken into account the surplus of the contracting parties. However, in some settings, contracting affects the welfare of parties who do not participate in contracting. For example, in vertical contracting, the quantity of intermediate input sold determines the price of the final good, which in turn affects consumer surplus. The surplus of the contracting parties (vertical profit) is maximized by the monopoly outcome. While a nondiscrimination requirement may increase vertical profit by bringing output closer to monopoly output, aggregate social welfare will then be reduced.

The second caveat is that we have only studied the role of a nondiscrimination requirement when agents are ex ante identical. However, one might ponder the effect of such a requirement when agents have exogenous differences. (For example, in the takeover application, some shareholders may enjoy private benefits of control, and others may not.) In the extreme case in which externalities are absent and the exogenous differences are observable, first-degree price discrimination achieves full efficiency, and a nondiscrimination requirement would unambiguously reduce welfare. In the general case with externalities present, however, nondiscrimination could have beneficial as well as detrimental effects. For such cases, a compromise policy could be proposed, which allows the principal to condition on all (substantial) verifiable differences between agents, but forbids other means of discrimination.

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