PROPERTY RIGHTS AND THE EFFICIENCY OF BARGAINING

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Abstract
We show that efficient bargaining is impossible for a wide class of economic settings and property rights. These settings are characterized by (i) the existence of "adverse efficient opt-out types", whose participation does not change the efficient allocation and who, when they opt out, are the worst type other agents can face, and (ii) non-existence of the "marginal core", and its multivaluedness with a positive probability. We also examine the optimal allocation of property rights within a given class that satisfies (i), such as simple property rights, liability rules, and dual-chooser rules. We characterize property rights that minimize the expected subsidy required to implement efficiency. With two agents, simple property rights that are optimal in this way maximize the expected surplus at the status quo allocation, but this no longer holds with more agents. We also study "second-best" budget-balanced bargaining under a liability rule. The optimal "second-best" liability rule may differ from, but is often close to, the expectation of the victim's harm, which would be optimal if there were no bargaining. However, liability rules that are close to a simple property right result in a lower expected surplus than the simple property right they are near. (JEL: D23, D47, C78, K11)

1. Introduction

Property rights specify an initial default position from which agents may subsequently bargain to determine their ultimate allocation. Following the seminal article of Grossman and Hart (1986), the economics literature discussing the optimal allocation of property rights has largely focused on how they affect ex-ante investments, under

the assumption that bargaining always results in ex-post efficient outcomes. In this paper, we instead examine how property rights affect the efficiency of bargaining and the final allocations that result.[1]

According to the "Coase Theorem" (Coase 1960), in the absence of "transaction costs", parties will reach Pareto efficient agreements regardless of initial property rights. We instead examine settings in which this may not happen due to transaction costs associated with asymmetric information. That property rights may matter for the efficiency of bargaining can be seen by comparing Myerson and Satterthwaite's (1983) conclusion that private information must generate inefficiency in bargaining between a buyer and a seller, with Cramton, Gibbons, and Klemperer's (1987) demonstration that efficient bargaining mechanisms do exist for more evenly distributed (or randomized) property rights. Here we examine more broadly the nature of optimal property rights in such settings.

In addition to simple property rights of the sort considered by Myerson and Satterthwaite (1983) and Cramton, Gibbons, and Klemperer (1987), the legal literature has considered other forms of property rights. Calabresi and Melamed (1972) first highlighted the distinction between "property rules", which correspond to the simple property rights of the economics literature, and "liability rules", in which an agent may harm another agent (e.g., by polluting) but must make a damage payment to the victim. These liability rules may equivalently be thought of as an option-to-own.[2] Calabresi and Melamed (1972) considered such liability rules to be desirable only when bargaining is impractical (in which case they can make the final allocation responsive to values), but subsequent work (Ayres and Talley 1995; Kaplow and Shavell 1995, 1996; Ayres 2005) has suggested the possibility that liability rules may also be desirable when bargaining is possible but imperfect. Ayres (2005) also considered in a two-agent setting "dual-chooser" rules in which both agents can exercise options. In general, liability rules and dual-chooser rules may both be viewed as particular forms of *property rights mechanisms*, in which the default outcome depends on messages sent by the various agents.

This paper advances the literature in two ways. First, we establish a wide class of economic settings and property rights (including both simple property rights and liability rules) in which efficient bargaining is impossible.[3] Second, for these environments in which inefficiency is unavoidable, we examine the optimal allocation of property rights within a given class, such as simple property rights, liability rules, and dual-chooser rules.

Our inefficiency result unifies a number of results in the earlier literature (Myerson and Satterthwaite 1983; Mailath and Postlewaite 1990; Williams 1999; Figueroa and Skreta 2008; Che 2006). In contrast to the earlier literature, our approach to establishing

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1. Matouschek (2004) studies this question as well; we discuss the relation to our paper in what follows.

2. Demski and Sappington (1991) and Noldeke and Schmidt (1995) consider the use of option-to-own contracts to induce efficient ex-ante investments.

3. In Segal and Whinston (2011) we instead establish sufficient conditions for first-best efficiency to be achieved.

inefficiency does not require performing any computations. Instead, it requires only the verification of two simple conditions: (i) existence of "adverse efficient-opt-out types" and (ii) nonemptiness of the core (actually, nonemptiness of a larger set that we call the "marginal core") and its multivaluedness with a positive probability.[4]

We define an "efficient-opt-out type" as a type whose nonparticipation is consistent with efficiency (for any types of the other agents). In addition, for property rights that involve externalities, such as liability rules, we define an "adverse type" as a type who, when he does not participate and behaves noncooperatively (e.g., chooses optimally whether to damage others under a liability rule), minimizes the total expected surplus available to the other agents. (In settings with simple property rights, in which externalities are absent, any type is trivially an adverse type.) Our inefficiency result applies when each agent has a type that is simultaneously an efficient-opt-out type and an adverse type. This assumption is clearly restrictive—for example, it is not satisfied in the presence of intermediate (or randomized) property rights of the kind considered by Cramton, Gibbons, and Klemperer (1987) and Segal and Whinston (2011). Nevertheless, we show that this assumption is satisfied in a number of settings involving simple property rights, liability rules, and dual-chooser rules. (We also allow this assumption to hold in an asymptotic form: for example, a type may become an "almost" adverse efficient-opt-out type as the type goes to $+\infty$.)

In contrast, the nonemptiness and multivaluedness of the core is a typical feature of economic settings. For example, if, under an appropriate definition of "goods", a price equilibrium exists (e.g., a Walrasian equilibrium, or a Lindahl equilibrium), then it will be in the core, and "generically" the core will be multivalued (except for some limiting "competitive" cases with a large number of agents, where the core may converge to a unique Walrasian equilibrium).

Having identified a class of settings in which achieving efficiency is impossible, we then turn to an analysis of the optimal allocation of property rights in those cases. In doing so, we take a mechanism design approach to bargaining, asking what property rights would be optimal if bargaining takes as efficient a form as possible given the allocation of property rights.

We use two different measures of efficiency to identify optimal property rights. In the first, we assume that there is an outside agency who will subsidize the bargaining process in order to achieve efficiency and we examine the effect of property rights on the expected subsidy that is required. One corollary of our impossibility analysis is a simple formula for this expected subsidy. The formula allows us to compare the subsidies required by the various property rights that satisfy (i) and (ii). Among such property rights, we can identify those that minimize the intermediary's expected subsidy.

One interesting benchmark for comparison is the property rights that would maximize the expected surplus were bargaining impossible. With two agents and simple property rights that induce efficient-opt-out types, we show that the intermediary's

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4.   Precursors to this approach can be found in Makowski and Ostroy (1989) and Segal and Whinston (2011).

expected first-best subsidy equals the expected bargaining surplus, and therefore minimizing this expected subsidy is equivalent to maximizing the expected status quo surplus. For example, in the buyer–seller model of Myerson and Satterthwaite (1983), if we can choose who should initially own the object, it is optimal to give it to the agent with the higher expected value for it. We also identify the optimal option-to-own (liability rule) in this same setting, and show that it is exactly the same as the optimal option-to-own when bargaining is impossible, involving an option price (damage payment) that equals the expected value (harm) of the nonchoosing agent (victim). As in the case without bargaining, the optimal option-to-own is strictly better than the best simple property right, but fails to achieve the first-best when there is uncertainty about the value of the nonchoosing agent.

However, the equivalence between what is best for minimizing the expected first-best bargaining subsidy and what is best absent bargaining generally breaks down when there are more than two agents: in such cases, we instead want to raise the values of coalitions including all but one agent (reducing the "hold-out power" of individual agents). We illustrate the difference in two examples, one concerning the optimal ownership of spectrum, and the other examining the optimal liability rule for pollution when there are many victims.

Evaluating property rights by their effects on the expected subsidy required for first-best bargaining may not be the right thing to do, since in most cases a benevolent intermediary willing to subsidize bargaining is not available. Our second efficiency measure is instead the maximal ("second-best") expected surplus that can be achieved in budget-balanced bargaining. Analysis of the second-best problem is complicated by the fact that the optimal allocation rule depends on the identity of the agents least willing to participate (the "critical types"), which in turn depends on the allocation rule. Unfortunately, we are unable to solve for the second-best bargaining procedure at a comparable level of generality to our first-best subsidy calculation. For this reason, we focus on the case of just two agents.

When divided property rights are not possible, we know from Myerson and Satterthwaite (1983) that efficiency is impossible when the budget must be balanced. In such cases, however, use of a liability rule (option-to-own) may offer an improvement over (and can be no worse than) what can be achieved with the best undivided property right.

To identify the optimal liability rule, we begin by characterizing the second-best bargaining mechanism for a given liability rule. Doing so, we identify the critical types for each agent, and the optimal allocation rule.[5] We then use this characterization to study the dependence of the maximal expected surplus on the option price.

We first consider the case in which both agents' values are distributed uniformly. We find that the second-best expected surplus is maximized by setting the option price equal to the expected value of the nonchoosing agent (the "victim"), which is 1/2

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5. Independently, Loertscher and Wasser (2014) analyze the second-best bargaining mechanism for simple intermediate property rights which do not permit first-best efficient bargaining. The two settings share a number of similar technical difficulties to be overcome.

under our normalization. Thus, in the uniform case the optimal option price under second-best bargaining proves to be the same as the price that minimizes the expected first-best subsidy, which is in turn the same as the optimal price in the absence of bargaining.

At the same time, we find a significant difference in how the second-best expected surplus and the expected first-best subsidy vary with the option price. Namely, while in the uniform example the expected first-best subsidy is always lower the closer the price is to 1/2, the second-best expected surplus does not increase monotonically with such changes. Instead, we find that setting the price close to 0 or to 1 yields a lower expected second-best surplus than setting it at exactly 0 or 1 (which corresponds to giving one of the agents a simple property right to the object). In fact, we show that the same conclusion extends to *all* distributions of the two agents' valuations (not just uniform). Thus, contrary to the intuition one might take from the results of Cramton, Gibbons, and Klemperer (1987), less extreme property rights (in the form of a option price that sometimes leads to exercise of the option) may be worse than extreme ones.

Finally, we explore cases in which the two agents' valuations are drawn from different distributions and show that the optimal option price is not generally equal to the victim's expected harm. Nonetheless, an intriguing fact is that in all of the cases we study the optimal option price is very close to the victim's expected harm, and the loss from instead setting the option price equal to that expected harm is small.

In addition to Myerson and Satterthwaite (1983) and Cramton, Gibbons, and Klemperer (1987), a number of other papers examine the effect of property rights on bargaining efficiency. Most, like our previous paper (Segal and Whinston 2011), establish conditions under which the first-best is achievable (for additional references, see Segal and Whinston 2013). Others established impossibility of efficient bargaining in some settings (see Segal and Whinston 2013; Makowski and Ostroy 1989; and Matsushima 2012). Matouschek (2004) was the first paper to consider second-best optimal property rights under asymmetric information bargaining. He studied a model in which asset ownership x is set irrevocably ex ante, and bargaining over other decisions q occurs ex post after agents' types are determined. In contrast to our analysis, bargaining is not allowed to redistribute the initial property rights. He finds that, depending on the parameters, the optimal property rights x will either maximize the total surplus at the disagreement point (as if no renegotiation were possible) or minimize it. Mylovanov and Troger (2014) analyze a two-agent setting like ours, but instead use a specific bargaining protocol in which one agent has the power to make a take-it-or-leave-it offer to the other agent. Finally, in unpublished notes, Che (2006) examines the optimal option-to-own for minimizing the expected first-best subsidy.

The paper is organized as follows. In Section 2 we describe our basic model. Section 3 derives our inefficiency result. In Section 4, we analyze the optimal property rights for minimizing the first-best subsidy. Section 5 analyzes optimal second-best property rights. Section 6 extends our analysis to consider dual-chooser rules. Finally, Section 7 concludes.

2. Set-Up

We consider a general model with N agents, indexed by $i = 1, \ldots, N$, who bargain over a nonmonetary decision $x \in X$, as well as a vector $t \in \mathbb{R}^N$ of monetary transfers. Each agent i privately observes a type $\theta_i \in \Theta_i$, and his resulting payoff is $v_i(x, \theta_i) + t_i$. We assume that the types $(\tilde{\theta}_1, \ldots, \tilde{\theta}_N) \in \Theta_1 \times \cdots \times \Theta_N$ are independent random variables.

We will be interested in examining what is achievable given some initial property rights when the agents engage in the best possible bargaining procedure after their types are realized. To this end, we take a mechanism design approach to bargaining. Appealing to the Revelation Principle, we focus on direct revelation mechanisms $\langle \chi, \tau \rangle$, where $\chi : \Theta \to X$ is the decision rule, and $\tau : \Theta \to \mathbb{R}^N$ is the transfer rule. In particular, we will be interested in implementing an *efficient* decision rule χ^*, which solves

$$\chi^*(\theta) \in \arg\max_{x \in X} \sum_i v_i(x, \theta_i) \quad \text{for all } \theta \in \Theta.$$

We let $V(\theta) \equiv \sum_i v_i(\chi^*(\theta), \theta_i)$ be the maximum total surplus achievable in state θ.

When considering direct revelation mechanisms that correspond to bargaining mechanisms, we restrict them to satisfy *budget balance*

$$\sum_i \tau_i(\theta) = 0 \quad \text{for all } \theta \in \Theta$$

and *(Bayesian) incentive compatibility*

$$\mathbb{E}[v_i(\chi(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i})]$$
$$\geq \mathbb{E}[v_i(\chi(\theta_i', \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i', \tilde{\theta}_{-i})] \quad \text{for all } i, \theta_i, \theta_i' \in \Theta_i.$$

Next we consider participation constraints. For this purpose, we need to describe what outcome each agent i expects when he refuses to participate in the bargaining mechanism. This outcome will depend on the property rights mechanism in place and, in general, on the types of the other agents. For example, the other agents may make some noncooperative choices under a liability rule, and these choices may depend on their types. Alternatively, the other agents may be able to bargain with each other over some parts of the outcome without the participation of agent i, and this bargaining may have externalities on agent i. It is also possible that if agent i refuses to participate, the default will involve a noncooperative game among agents, and the outcome of this game will depend on all the agents' types.

To incorporate all these possibilities, we assume that if agent i refuses to participate and the state of the world is θ, the nonmonetary decision is $\hat{x}_i(\theta)$, and agent i receives a transfer $\hat{t}_i(\theta)$. The resulting reservation utility of agent i is therefore

$$\widehat{V}_i(\theta) \equiv v_i(\hat{x}_i(\theta), \theta_i) + \hat{t}_i(\theta).$$

For example, consider the special case of *simple property rights* that induce a fixed status quo $(\hat{x}, \hat{t}_1, \ldots, \hat{t}_N)$ that cannot be renegotiated at all without all agents' participation. In this case, for each agent i we have $(\hat{x}_i(\theta), \hat{t}_i(\theta)) = (\hat{x}, \hat{t}_i)$, and so the agent's reservation utility takes the form $\widehat{V}_i(\theta) = v_i(\hat{x}, \theta_i) + \hat{t}_i$. Furthermore, the same reservation utility applies when the status quo can be renegotiated by a subset of agents but such renegotiation does not affect the utility of the nonparticipating agent (e.g., because renegotiation can only involve exchange of private goods). In general, the functions $\hat{x}_i(\theta)$ and $\hat{t}_i(\theta)$ depend on both the property rights and assumptions about bargaining.

Given these functions and the resulting reservation utility, the (interim) individual rationality constraints of agent i can be written as

$$\mathbb{E}[v_i(\chi(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i})] \geq \mathbb{E}[\widehat{V}_i(\theta_i, \tilde{\theta}_{-i})] \quad \text{for all } \theta_i. \tag{1}$$

We will say that a property rights mechanism *permits efficient bargaining* if it induces functions $\{\hat{x}_i(\cdot), \hat{t}_i(\cdot)\}_{i=1}^{N}$ such that there exists a budget-balanced, incentive-compatible, and individually rational mechanism implementing an efficient decision rule $\chi^*(\cdot)$.

3. An Inefficiency Theorem

In this section, we provide a set of sufficient conditions ensuring that efficient bargaining is impossible given a set of initial property rights. Our result will have Myerson and Satterthwaite's (1983) result, and several others, as special cases.

3.1. Characterization of Intermediary Profits

It will prove convenient to focus on mechanisms in which, for some vector of types $(\hat{\theta}_1, \ldots, \hat{\theta}_N)$, payments as a function of announcements θ take the following form:

$$\tau_i(\theta|\hat{\theta}_i) = \sum_{j \neq i} v_j(\chi^*(\theta), \theta_j) - K_i(\hat{\theta}_i) \tag{2}$$

where

$$K_i(\hat{\theta}_i) = \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})]. \tag{3}$$

Note that these payments describe a Vickey–Clarke–Groves ("VCG") mechanism (see Mas-Colell, Whinston, and Green 1995, Chapter 23). The portion of the payment that depends on the agents' announcements, $\sum_{j \neq i} v_j(\chi^*(\theta), \theta_j)$, causes each agent i to fully internalize his effect on aggregate surplus, thereby inducing him to announce his true type and implementing the efficient allocation rule $\chi^*(\cdot)$. The fixed participation fee $K_i(\hat{\theta}_i)$, however, equals type $\hat{\theta}_i$'s expected gain from participating in

the mechanism absent the fixed charge, so it causes that type's IR constraint to hold with equality. If we imagine that there is an intermediary in charge of this trading process, its expected profit with this mechanism, assuming all agents participate, is given by

$$\pi(\hat{\theta}) = -\mathbb{E}\left[\sum_i \tau_i(\tilde{\theta}|\hat{\theta}_i)\right]$$

$$= \left(\sum_i \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})]\right) - (N-1)\,\mathbb{E}[V(\tilde{\theta})]. \qquad (4)$$

To ensure that all types participate, the participation fee for each agent i can be at most $\inf_{\hat{\theta}_i \in \Theta_i} K_i(\hat{\theta}_i)$, resulting in an expected profit for the intermediary of

$$\bar{\pi} \equiv \inf_{\hat{\theta} \in \Theta} \pi(\hat{\theta}). \qquad (5)$$

If there exists a type $\hat{\theta}_i$ achieving the infimum, namely

$$\hat{\theta}_i \in \arg\min_{\theta_i \in \Theta_i} \mathbb{E}[V(\theta_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i, \tilde{\theta}_{-i})],$$

it will be called agent i's *critical type*. This is a type that has the lowest net expected participation surplus in the mechanism.

The sign of the expected profit (5) determines whether property rights permit efficient bargaining:[6]

LEMMA 1. (i) *Any property rights mechanism at which $\bar{\pi} \geq 0$ permits efficient bargaining.* (ii) *If, moreover, for each agent i, Θ_i is a smoothly connected subset of a Euclidean space, and $v_i(x, \theta_i)$ is differentiable in θ_i with a bounded gradient on $X \times \Theta$, then a property rights mechanism permits efficient bargaining only if $\bar{\pi} \geq 0$.*

3.1.1. Adverse Efficient-Opt-Out Types. For each agent i, let

$$\widehat{V}_{-i}(\theta) \equiv \sum_{j \neq i} v_j(\hat{x}_i(\theta), \theta_j) - \hat{t}_i(\theta)$$

denote the joint payoff of agents other than i when agent i chooses not to participate in the bargaining mechanism. (Observe that we assume there is budget balance in the event of nonparticipation, so that the collective transfer to agents other than i

6. Versions of this result appear, for example, in Makowski and Mezzetti (1994), Krishna and Perry (1998), Neeman (1999), Williams (1999), Che (2006), Schweizer (2006), Figueroa and Skreta (2008), Segal and Whinston (2011), and Segal and Whinston (2013). Part (i) of the lemma can be proven by building a budget-balanced mechanism as suggested by Arrow (1979) and d'Aspremont and Gérard-Varet (1979), and satisfying all agents' participation constraints with appropriate lump-sum transfers. Part (ii) follows from the classical Revenue Equivalence Theorem (e.g., Milgrom and Segal 2002, Section 3.1).

when agent i opts out is $-\hat{t}_i(\theta)$.) Since $V(\theta)$ is the maximal achievable surplus in state θ, we have

$$\widehat{V}_i(\theta) + \widehat{V}_{-i}(\theta) = \sum_j v_j(\hat{x}_i(\theta), \theta_j) \leq \sum_j v_j(\chi^*(\theta), \theta_j)$$

$$= V(\theta) \quad \text{for all } \theta \in \Theta. \tag{6}$$

We now introduce two notions that are central to our analysis: efficient opt-out types and adverse types.

DEFINITION 1. Given a property rights mechanism, type θ_i of agent i is an *efficient-opt-out type* if $\hat{x}_i(\theta_i, \theta_{-i}) = \chi^*(\theta_i, \theta_{-i})$ for all θ_{-i}.

Note that if θ_i is an efficient-opt-out type, then $V(\theta_i, \theta_{-i}) = \widehat{V}_i(\theta_i, \theta_{-i}) + \widehat{V}_{-i}(\theta_i, \theta_{-i})$ for all θ_{-i}. That is, there are never any gains from trade between type θ_i and the other agents, regardless of their types.

DEFINITION 2. Given a property rights mechanism, type θ_i of agent i is an *adverse type* if it minimizes $\mathbb{E}[\widehat{V}_{-i}(\theta_i, \tilde{\theta}_{-i})]$.

Type θ_i is an adverse type if, conditional on agent i opting out, agents other than i are worst off collectively (in expectation) when agent i's type is θ_i. Note in particular that any type θ_i is trivially an adverse type when agent i imposes no externalities on the other agents, so $\widehat{V}_{-i}(\theta)$ does not depend on θ_i. This is the case, for example, with simple property rights.

EXAMPLE 1. Suppose that each of two agents $i = 1, 2$ has a value θ_i for an object, where θ_i is drawn from distribution F_i on $[0, 1]$. On the one hand, agent 1 faces a liability rule and pays agent 2 price $p \in [0, 1]$ if he chooses to take the object. In this case, $\widehat{V}_1(\theta_1, \theta_2) = \max\{\theta_1 - p, 0\}$ and $\widehat{V}_2(\theta_1, \theta_2) = \theta_2 \cdot 1_{\{\theta_1 < p\}} + p \cdot 1_{\{\theta_1 \geq p\}}$. Then agent 2's type $\theta_2 = p$ is an efficient-opt-out type, since when he has this value the outcome of agent 1's exercise decision is efficient regardless of agent 1's type. That type of agent 2 is trivially an adverse type because agent 1's payoff when exercising the option does not depend on agent 2's type.

On the other hand, both $\theta_1 = 1$ and $\theta_1 = 0$ are efficient-opt-out types for agent 1: when $\theta_1 = 1$, agent 1 will always exercise under the liability rule, and it is always efficient for him to do so regardless of agent 2's type; when instead, $\theta_1 = 0$, agent 1 will never exercise under the liability rule, which is also always efficient. Of these two types, $\theta_1 = 0$ is an adverse type for agent 1 when $\mathbb{E}[\tilde{\theta}_2] < p$, since then agent 2 prefers for agent 1 to exercise the option and agent 1 never does, while $\theta_1 = 1$ is an adverse type for agent 1 when $\mathbb{E}[\tilde{\theta}_2] > p$. ∎

The significance of these definitions for our results stems from the following observation.

LEMMA 2. *When agent i has a type θ_i° that is both an adverse type and an efficient-opt-out type, it is a critical type.*

Proof. We can then write for all $\theta_i \in \Theta_i$,

$$
\begin{aligned}
\mathbb{E}[V(\theta_i^\circ, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i^\circ, \tilde{\theta}_{-i})] &= \mathbb{E}[\widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i})] \\
&\leq \mathbb{E}[\widehat{V}_{-i}(\theta_i, \tilde{\theta}_{-i})] \\
&\leq \mathbb{E}[V(\theta_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i, \tilde{\theta}_{-i})]
\end{aligned}
$$

where the equality is because θ_i° is an efficient-opt-out type, the first inequality is because θ_i° is an adverse type, and the second inequality is by (6). $\qquad\square$

Our results will apply not only to settings in which adverse efficient-opt-out types exist, but also to settings in which their existence is only of the following asymptotic form.

DEFINITION 3. *The adverse efficient-opt-out property holds for agent i if there exists a sequence $\{\theta_i^k\}_{k=1}^\infty$ in Θ_i such that as $k \to \infty$,*

$$
\mathbb{E}\left[V\left(\theta_i^k, \tilde{\theta}_{-i}\right) - \widehat{V}_i\left(\theta_i^k, \tilde{\theta}_{-i}\right) - \widehat{V}_{-i}\left(\theta_i^k, \tilde{\theta}_{-i}\right)\right] \to 0, \text{ and}
$$

$$
\mathbb{E}\left[\widehat{V}_{-i}\left(\theta_i^k, \tilde{\theta}_{-i}\right)\right] \to \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}\left[\widehat{V}_{-i}\left(\hat{\theta}_i, \tilde{\theta}_{-i}\right)\right].
$$

Note that this property holds whenever agent i has an adverse efficient-opt-out type θ_i° (in which case we can let $\theta_i^k = \theta_i^\circ$ for all k), but it may also hold in other cases—for example, sometimes we may need to take a sequence with $\theta_i^k \to +\infty$ (in which case we may say informally that $\theta_i = +\infty$ is an adverse efficient-opt-out type). This property allows us to express the intermediary's expected profits as follows.

LEMMA 3. *If the adverse efficient-opt-out property holds for each agent, the intermediary's expected profit (5) can be written as follows:*

$$
\bar{\pi} = \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] - (N-1)\,\mathbb{E}[V(\tilde{\theta})]. \tag{7}
$$

Proof.

$$
\begin{aligned}
\bar{\pi} &= \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})] - (N-1)\,\mathbb{E}[V(\tilde{\theta})] \\
&= \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \{\mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] + \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})]\} \\
&\quad - (N-1)\,\mathbb{E}[V(\tilde{\theta})].
\end{aligned}
$$

On the one hand, equation (6) guarantees that this expression is greater or equal to the right-hand side of equation (7). On the other hand, the adverse efficient-opt-out

property for agent i ensures that

$$\inf_{\hat{\theta}_i \in \Theta_i} \{\mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] + \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})]\}$$

$$\leq \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})].$$

Hence, if this holds for all agents, we obtain (7). ☐

3.2. Inefficiency Result

The adverse efficient-opt-out property is a restrictive property, but it will hold in a number of settings of interest. On the other hand, the second property we require in Proposition 1 below is usually satisfied. It makes use of the following notion.

DEFINITION 4. $w \in \mathbb{R}^N$ is a *marginal core payoff vector* in state θ if

(i) $\sum_{j \neq i} w_j \geq \widehat{V}_{-i}(\theta)$ for all i, and
(ii) $\sum_i w_i = V(\theta)$.

Compared to the usual notion of the core, the marginal core considers only coalitions that include $N - 1$ agents. Condition (i) simply says that the coalition consisting of all agents except agent i does not block (assuming "blocking" yields the coalition the same collective payoff it receives when agent i opts out), while condition (ii) says that the maximal total surplus is achieved. Using (ii), condition (i) can be rewritten as $w_i \leq V(\theta) - \widehat{V}_{-i}(\theta)$—that is, no agent i can receive more than his marginal contribution to the total surplus.

PROPOSITION 1. *Suppose that the assumptions of Lemma* 1(ii) *hold, the adverse efficient-opt-out property holds for each agent, and the set of marginal core payoff vectors is nonempty in all states and multivalued with a positive probability. Then efficient bargaining is impossible.*

Proof. Equation (7) implies that

$$\bar{\pi} \leq \sum_i \mathbb{E}[\widehat{V}_{-i}(\tilde{\theta})] - (N - 1)\, \mathbb{E}[V(\tilde{\theta})]$$

$$= \mathbb{E}\left[V(\tilde{\theta}) - \sum_i [V(\tilde{\theta}) - \widehat{V}_{-i}(\tilde{\theta})] \right]. \tag{8}$$

Now, for a marginal core payoff vector w, for each i we have

$$w_i \leq V(\theta) - \widehat{V}_{-i}(\theta).$$

Moreover, if the marginal core is multivalued, then there exists such a w with the inequality holding strictly for at least one agent. Summing up the inequalities over all

agents yields

$$V(\theta) = \sum_i w_i \leq \sum_i (V(\theta) - \widehat{V}_{-i}(\theta)),$$

with strict inequality when the marginal core is multivalued. If multivaluedness holds with positive probability, (8) implies that $\bar{\pi} < 0$, and so the impossibility of efficient bargaining is implied by Lemma 1(ii). $\square$

   Proposition 1 provides a sufficient condition that can be used to confirm that efficiency is impossible, and that is much easier to check in some settings than is the expected profit calculation required by Lemma 1. Indeed, in many cases the assumptions of the proposition can be checked without making any assumptions about distributions of types.[7] We provide some examples in the next section.

3.3. Some Applications

The assumptions of Proposition 1 cover many classical economic settings. We begin with the setting of Myerson and Satterthwaite (1983), in which the set of agents is $\{s, b\}$ (a seller and a buyer), each agent i's value is $v_i(x, \theta_i) = \theta_i \cdot x_i$, where $x_i \in [0, 1]$ is i's (nonmonetary) consumption, and the set of feasible decisions is $X = \{x \in [0, 1]^2 : x_1 + x_2 = 1\}$. A simple property rights allocation in this case specifies a status quo $((\hat{x}_s, \hat{x}_b), \hat{t}_s, \hat{t}_b) = ((1, 0), 0, 0)$. Let the type space be $\Theta_i = [\underline{\theta}_i, \bar{\theta}_i]$ for $i \in \{s, b\}$ and assume that $(\underline{\theta}_b, \bar{\theta}_b) \cap (\underline{\theta}_s, \bar{\theta}_s) \neq \varnothing$. Then the buyer's lowest type $\underline{\theta}_b$ is an efficient opt-out type if $\underline{\theta}_b \leq \underline{\theta}_s$, and similarly the seller's highest type is an efficient opt-out type if $\bar{\theta}_s \geq \bar{\theta}_b$. Since simple property rights involve no externalities, any type is trivially an adverse type. The core of this cooperative game is multivalued whenever gains from trade are strictly positive (which happens with a positive probability) and single-valued otherwise. Hence, Proposition 1 applies.[8]

   Now we show how this approach extends to settings with more than two agents and property rights that generate no externalities—that is, in which an agent who refuses to participate is guaranteed some consumption and does not care what the other agents do. In such settings, all types are trivially adverse types. Furthermore, the IR constraints (1) make clear that the possibility of efficient bargaining is determined by the reservation utilities $\widehat{V}_i(\cdot)$ of individual agents who refuse to participate but not by the joint surplus $\widehat{V}_{-i}(\cdot)$ of the remaining agents. Thus, changes in property rights $(\hat{x}_i(\cdot), \hat{t}_i(\cdot))$ that affect $\widehat{V}_{-i}(\cdot)$ but do not affect agent i's reservation utility $\widehat{V}_i(\cdot)$ do not affect intermediary

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7. The proposition does not even require type distributions to have positive densities; it is formally correct even if types do not have full support. However, because our definition of "efficient bargaining" requires efficiency even for types with zero density, in such cases the proposition is of less interest.

8.   While the argument assumes that $\underline{\theta}_b \leq \underline{\theta}_s$, and $\bar{\theta}_s \geq \bar{\theta}_b$, these assumptions can be eliminated by noting that any agent of type below $\underline{\theta} \equiv \max\{\underline{\theta}_s, \underline{\theta}_b\}$ receives the object with probability zero, so is therefore indistinguishable from type $\underline{\theta}$, and any agent of type above $\bar{\theta} \equiv \min\{\bar{\theta}_s, \bar{\theta}_b\}$ receives an object with probability one, so is therefore indistinguishable from type $\bar{\theta}$. Therefore, the profit in the mechanism must be the same as if both agents' types were instead distributed on the same interval $[\underline{\theta}, \bar{\theta}]$ (with possible atoms at its endpoints), in which case efficient bargaining is impossible by the argument in the text.

profits (5) and the possibility of efficient bargaining. In the applications which follow, there is a natural way to specify $(\hat{x}_i(\cdot), \hat{t}_i(\cdot))$ for fixed reservation utilities $\widehat{V}_i(\cdot)$ that ensures, on the one hand, the existence of efficient opt-out types, and, on the other hand, nonemptiness of the marginal core, thus enabling the application of Proposition 1 to show impossibility of efficient bargaining in the simplest and most intuitive way. This specification assumes that when an agent i refuses to participate, the remaining agents strike a bargain that maximizes their joint surplus in the absence of agent i. While this specification is seemingly at odds with our emphasis on impossibility of efficient bargaining, for reasons discussed previously it proves a useful device for characterizing intermediary profits and the possibility of efficient bargaining.

For one example, consider the double-auction setting of Williams (1999), in which there are N_s sellers with values drawn from a distribution on $[\underline{\theta}_s, \bar{\theta}_s]$ and N_b buyers with values drawn from a distribution on $[\underline{\theta}_b, \bar{\theta}_b]$ with $(\underline{\theta}_b, \bar{\theta}_b) \cap (\underline{\theta}_s, \bar{\theta}_s) \neq \varnothing$. Agent i's value is $v_i(x, \theta_i) = \theta_i \cdot x_i$, where $x_i \in [0, 1]$ is i's consumption in nonmonetary decision x. Each seller i owns one unit of the good (thus, property rights specify his ownership $\hat{x}_i = 1$), and each buyer i owns none of the good (thus, property rights specify his ownership $\hat{x}_i = 0$). The set of feasible decisions is the set $X = \{x \in [0, 1]^N : \sum_i x_i = \sum_i \hat{x}_i\}$. (The setting of Myerson and Satterthwaite described previously is the special case with $N_s = N_b = 1$.) As for the functions $\widehat{V}_{-i}(\cdot)$, as suggested previously, we assume that agents $-i$ trade efficiently among themselves in the event that agent i opts out. If so, then (i) a buyer of type $\underline{\theta}_b$ is an efficient-opt-out type if either $\underline{\theta}_b \leq \underline{\theta}_s$ or $N_b > N_s$, and (ii) a seller of type $\bar{\theta}_s$ is an efficient-opt-out type if either $\bar{\theta}_s \geq \bar{\theta}_b$ or $N_s > N_b$. Moreover, a competitive equilibrium exists in every state and is not unique with a positive probability. Since a competitive equilibrium is always in the core (and, hence, in the marginal core), Proposition 1 applies whenever both (i) and (ii) hold.[9] Note, in contrast, that the calculation of the intermediary's expected profit in this double auction setting would be quite involved (see, for example, Williams 1999).

The same approach also applies to the setting with a public good in which each of N consumers' values is drawn from a distribution on $[0, \bar{\theta}]$, and the cost of provision is $c > 0$ (which could be assumed to be split equally among participating agents in the default outcome). Consider property rights that specify that when an agent opts out, he does not make any payment and is excluded from the public good. (Clearly, if nonparticipants could sometimes enjoy the public good, this would only strengthen participation constraints and make efficient bargaining less likely.) As discussed previously, without loss of generality we can assume that in this case the other agents choose the level of the public good to maximize their joint payoff. In this case, each agent's type 0 is both an efficient-opt-out type and an adverse type. Since a Lindahl

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9.   The argument can also be extended to show impossibility whenever $N_b = N_s$. In this case, note that in an efficient allocation any agent of type below $\underline{\theta} \equiv \max\{\underline{\theta}_s, \underline{\theta}_b\}$ receives an object with probability zero, so is therefore indistinguishable from type $\underline{\theta}$, and any agent of type above $\bar{\theta} \equiv \min\{\bar{\theta}_s, \bar{\theta}_b\}$ receives an object with probability one, so is therefore indistinguishable from type $\bar{\theta}$. Therefore, the profit in the mechanism must be the same as if all agents' types were instead distributed on the same interval $[\underline{\theta}, \bar{\theta}]$ (with possible atoms at its endpoints), in which case efficient bargaining is impossible by the argument in the text.

equilibrium exists in every state and is not unique with a positive probability, and a Lindahl equilibrium is in the core (and the marginal core), Proposition 1 applies.[10]

4. Optimal Property Rights for Minimizing the Expected First-Best Subsidy

When the intermediary's expected profit $\bar{\pi}$ is negative, efficiency is impossible absent a subsidy. The expected value of the subsidy required to achieve efficiency exactly equals $-\bar{\pi}$, so that minimizing the expected subsidy amounts to maximizing the expected profit. Recall from Lemma 3 that the intermediary's expected profit when the adverse efficient-opt-out property holds for all agents can be written as

$$\bar{\pi} = \inf_{\hat{\theta}_1,\dots,\hat{\theta}_N} \mathbb{E}\left[V(\tilde{\theta}) - \sum_i [V(\tilde{\theta}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] \right]. \tag{9}$$

Using this formula we compare property rights possessing this property in terms of this criterion, which in general amounts to maximizing the sum $\sum_i \inf_{\hat{\theta}_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})]$, or $\sum_i \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i^\circ, \tilde{\theta}_{-i})]$ when adverse efficient-opt-out types $\theta_1^\circ, \dots, \theta_N^\circ$ exist for all agents. In the remainder of this section we explore the implications of this prescription.

4.1. Two Agents

We first consider situations with two agents and analyze optimal property rights for an indivisible good. Specifically, as in Myerson and Satterthwaite (1983), each agent i's value θ_i for the good is drawn from a full-support distribution F_i on $[0, 1]$. We first consider which of the two agents should own the good if the goal is to minimize the expected first-best bargaining subsidy. We then investigate whether options to own can improve on simple ownership. This corresponds to the legal literature's question of whether property rules or liability rules are better.[11]

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10. Observe that in the double-auction setting with KN_b buyers and KN_s sellers, the per-agent expected subsidy in an ex-ante optimal mechanism converges to zero as $K \to \infty$. Intuitively, this relates to the fact that the core converges (in probability) to the unique competitive equilibrium of the continuous limit economy. In the competitive limit, the marginal contribution of a buyer who buys equals his value minus the equilibrium price, while the marginal contribution of a seller who sells equals the equilibrium price minus his cost. Hence, in the limit the agents can fully appropriate their marginal contributions while balancing the budget (as in Makowski and Ostroy 1989, 1995, 2001). This relates to the finding of Gresik and Satterthwaite (1989) that the inefficiency in an ex-ante optimal budget-balanced mechanism also shrinks to zero as the number of agents grows. In contrast, in the public-good setting considered in the text, as $N \to \infty$, the core does not shrink. In the limit, each agent is nonpivotal for the provision of the public good, and so his marginal contribution is his whole value, and for him to receive this marginal contribution he should not contribute anything to the provision cost. Thus, in the limit, efficient provision would require the full provision cost to be covered by the intermediary. This is consistent with the finding of Mailath and Postlewaite (1990) that for a public good that is nonexcludable (i.e., must be provided to either everybody or nobody), the probability of providing the public good in any budget-balanced mechanism goes to zero as $N \to \infty$.

11. In Section 6 we also discuss dual-chooser rules in settings with two agents.

4.1.1. Who Should Own? Consider any situation with two agents in which the property rights induce a fixed status quo $(\hat{x}, \hat{t}_1, \ldots, \hat{t}_N)$, and the two agents both have efficient-opt-out types.[12] We then have

$$\widehat{V}_{-1}(\theta_1^\circ, \theta_2) = v_2(\hat{x}, \theta_2) + \hat{t}_2,$$

$$\widehat{V}_{-2}(\theta_1, \theta_2^\circ) = v_1(\hat{x}, \theta_1) + \hat{t}_1,$$

and, using equation (9), the intermediary's expected profit is

$$\bar{\pi}(\hat{x}) = \mathbb{E}\{V(\tilde{\theta}) - [V(\tilde{\theta}) - \widehat{V}_{-1}(\theta_1^\circ, \tilde{\theta}_2)] - [V(\tilde{\theta}) - \widehat{V}_{-2}(\tilde{\theta}_1, \theta_2^\circ)]\}$$

$$= \mathbb{E}\{v_2(\hat{x}, \tilde{\theta}_2) + v_1(\hat{x}, \tilde{\theta}_1) + (\hat{t}_1 + \hat{t}_2) - V(\tilde{\theta})\}$$

$$= \mathbb{E}\{v_2(\hat{x}, \tilde{\theta}_2) + v_1(\hat{x}, \tilde{\theta}_1) - V(\tilde{\theta})\} < 0.$$

In words, a mediator who implements the first-best must subsidize the entire renegotiation surplus. Thus, the status quo $\hat{x}$ that minimizes the expected subsidy (within a class of those that have efficient-opt-out types) must maximize the expected status quo surplus $\mathbb{E}[v_1(\hat{x}, \tilde{\theta}_1) + v_2(\hat{x}, \tilde{\theta}_2)]$. Thus, we have the following proposition.

PROPOSITION 2. *Suppose that the assumptions of Lemma* 1(ii) *hold and there are two agents. Then, among the property rights mechanisms that induce a fixed status quo and cause both agents to have efficient-opt-out types, the one that minimizes the first-best subsidy is the one that maximizes the two agents' joint payoff in the absence of bargaining.*

Since, as we saw in Section 3, both agents have efficient-opt-out types in the setting of Myerson and Satterthwaite, where an agent's ownership $\hat{x}_i$ is either 0 or 1, we have the following corollary.[13]

COROLLARY 1. *Consider the Myerson–Satterthwaite setting in which each of two agents $i = 1, 2$ has value $\theta_i \in [0, 1]$ drawn from a full support distribution F_i. Then assigning ownership to the agent with the higher expected value minimizes the first-best subsidy.*

Thus, to minimize the first-best bargaining subsidy, ownership is best assigned exactly as if bargaining were impossible.

4.1.2. Property Rules vs. Liability Rules.   We now consider the possibility that instead of a simple property right, one agent may be given an option to own. Specifically, imagine that agent 1 can choose to acquire the good from agent 2 at a price p. This

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12.   Recall that with a fixed status quo, any efficient-opt-out type is trivially also an adverse type.

13.   In contrast, efficient-opt-out types do not exist for interior property rights $\hat{x}_i \in (0, 1)$. Indeed, for some such $\hat{x}$ efficiency can be achieved (Cramton, Gibbons, and Klemperer 1987; Segal and Whinston 2011; Loertscher and Wasser 2014), and this $\hat{x}$ would not be optimal in the absence of bargaining, in contrast to the conclusion of Proposition 2.

arrangement may be thought of as a liability rule in which agent 1 can take the good from agent 2, but must then make damage payment p to agent 2. For simplicity, we assume that both agents' type distributions have full support and are overlapping.

As we saw in Example 1, both agents will have adverse efficient-opt-out types in this case. For agent 2 it is his type $\hat{\theta}_2 = p$, while for agent 1 it is his type $\hat{\theta}_1 = 1$ if $\mathbb{E}[\tilde{\theta}_2] < p$, and type $\hat{\theta}_1 = 0$ if $\mathbb{E}[\tilde{\theta}_2] > p$. The marginal core in this case is nonempty and is multivalued in any state θ in which $\widehat{V}_1(\theta_1) + \widehat{V}_2(\theta_1, \theta_2) < V(\theta)$; that is, whenever the exercise decision by type θ_1 is not efficient. Hence, by Proposition 1, efficiency cannot be achieved with a liability rule.

To identify the subsidy-minimizing liability rule, we write the intermediary's profit as

$$\bar{\pi} = \mathbb{E}[\widehat{V}_1(\tilde{\theta}_1) + \widehat{V}_2(\hat{\theta}_1, \tilde{\theta}_2) - V(\tilde{\theta})]$$

$$= \underbrace{\mathbb{E}[\widehat{V}_1(\tilde{\theta}_1) + \widehat{V}_2(\tilde{\theta}_1, \tilde{\theta}_2) - V(\tilde{\theta})]}_{<0 \text{ when option exercise is not first-best}} + \underbrace{\mathbb{E}[\widehat{V}_2(\hat{\theta}_1, \tilde{\theta}_2) - \widehat{V}_2(\tilde{\theta}_1, \tilde{\theta}_2)]}_{\leq 0 \text{ since } \hat{\theta}_1 \text{ is an adverse type}}.$$

Observe that the first term represents the inefficiency resulting from agent 1's exercise of the option. It is negative since agent 1's exercise decision is not always ex-post optimal. However, the inefficiency is uniquely minimized when $p = \mathbb{E}[\tilde{\theta}_2]$, which is the optimal exercise price in the absence of bargaining (since it sets the exercise price equal to the expected externality imposed by the option's exercise; see Kaplow and Shavell 1996). The second term, on the other hand, is nonpositive by definition of $\hat{\theta}_1$ (an adverse type for agent 1) and is zero when $p = \mathbb{E}[\tilde{\theta}_2]$, since then agent 2 is indifferent, on average, about agent 1's exercise decision. Thus, we see that when agent 1 has the option, the option price that minimizes the first-best subsidy is $p = \mathbb{E}[\tilde{\theta}_2]$, and it results in a positive expected subsidy (confirming that achieving the first-best is impossible, as implied by Proposition 1).[14] This option price corresponds exactly to the traditional legal liability rule in which the damage payment equals the victim's expected damage.

Next, consider which agent should have the option. When agent i gets the option and $p = \mathbb{E}[\tilde{\theta}_{-i}]$, the first-best subsidy exactly equals the welfare loss from agent i's exercise of the option in the absence of bargaining. Hence, the agent i should be given the option if and only if he is best assigned the option when bargaining is impossible.

Finally, since the case of a simple property right corresponds to setting $p = 0$ or $p = 1$, the optimal liability rule is strictly better than the best simple property right, exactly as in the case without bargaining.

In summary, we have the following proposition.

PROPOSITION 3. *In the Myerson and Satterthwaite setting in which agents' type distributions have full support, the option-to-own (i.e., liability rule) that minimizes the*

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14. Che (2006) also derives this result and notes the impossibility of two agents achieving efficiency under a liability rule.

expected first-best bargaining subsidy sets the option price equal to the nonchooser's ("victim's") expected value ("harm") and assigns the option to the agent whose optimal exercise in the absence of bargaining results in the greatest expected surplus. The resulting expected subsidy is lower than it is with any simple property right.

4.2. More than Two Agents

When there are more than two agents, choosing the subsidy-minimizing property rights requires that we consider the coalitional, rather than individual, values. For example, shifting the property right to a private good (generating no externalities) from one agent to another is efficiency enhancing in the absence of bargaining if it increases the joint payoff of the two agents. In contrast, this change increases efficiency when bargaining is possible (in the sense of reducing the expected first-best subsidy) if it reduces the sum of these agents' marginal contributions to the total surplus.[15]

We illustrate the new effects through two examples.

4.2.1. Application: Spectrum Licenses.   Consider the following example. Simple property rights to two spectrum licenses, L_1 and L_2, are to be allocated among three firms. Firms 1 and 2 are specialists and each firm $i = 1, 2$ has a value θ_i for license L_i, drawn from a full-support distribution on $\mathbb{R}_+$ with mean μ, and no value for the other license. Firm G is a generalist firm, and has a value θ_G for both licenses, and value $\lambda \theta_G$ for just one of the licenses, where θ_G is drawn from a full-support distribution on $\mathbb{R}_+$ with mean μ_G, and $\lambda \in (0, 1)$. The values θ_1, θ_2, and θ_G are independent random variables. When $\lambda < 1/2$, the licenses are complements for G; when $\lambda > 1/2$, they are substitutes. For example, the licenses might be in two different regions, with firms 1 and 2 being regional firms and firm G being a national firm. In that case, G is likely to find the two licenses complements ($\lambda < 1/2$). Alternatively, the licenses might be to different frequencies, with firms 1 and 2 each having a product that can use one of the frequencies effectively, while firm G may have two products, each of which would use one of the two frequencies. In that case, the frequencies may be substitutes for firm G ($\lambda > 1/2$).

We will compare an allocation of BOTH licenses to G with an allocation of NONE of the licenses to G and license L_i to specialist firm i. Absent bargaining, the expected surplus is larger at BOTH than at NONE if

$$\mu_G - 2\mu > 0 \qquad (10)$$

Note that the best choice between these two allocations of property rights in the absence of bargaining is independent of λ.

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15.   This effect can therefore be interpreted as the effect on the two agents' joint payoff in bargaining, if this bargaining permits each agent to extract his marginal contribution to the grand coalition. This relates to the analysis of Segal (2003); however, the latter considers the Shapley value, in which agents receive weighted combinations of their marginal contributions to different coalitions, hence the results are not directly comparable.

Now consider the subsidy-minimizing property rights when there is bargaining. The following table summarizes the coalitional values, under the assumption that each two-agent coalition maximizes its joint payoff:

Property Rights Allocation:	$\widehat{V}_{-G}$	$\widehat{V}_{-1}$	$\widehat{V}_{-2}$
BOTH	0	$\max\{\theta_G, \lambda\theta_G + \theta_2\}$	$\max\{\theta_G, \lambda\theta_G + \theta_1\}$
NONE	$\theta_1 + \theta_2$	$\max\{\lambda\theta_G, \theta_2\}$	$\max\{\lambda\theta_G, \theta_1\}$

Under both of these property rights allocations, the adverse efficient-opt-out property holds, and efficient bargaining is impossible.[16,17] Using expression (7), BOTH is better than NONE if

$$\sum_{i=1,2} \mathbb{E}[\max\{\tilde{\theta}_G, \lambda\tilde{\theta}_G + \tilde{\theta}_i\}] > 2\mu + \sum_{i=1,2} \mathbb{E}[\max\{\lambda\tilde{\theta}_G, \tilde{\theta}_i\}],$$

which can be rewritten as

$$\mu_G - 2\mu > (2\lambda - 1)\mu_G + \sum_{i=1,2} (\mathbb{E}[\max\{0, \tilde{\theta}_i - \lambda\tilde{\theta}_G\}]$$
$$- \mathbb{E}[\max\{0, \tilde{\theta}_i - (1-\lambda)\tilde{\theta}_G\}]) \tag{11}$$

When $\lambda = 1/2$, so that the licenses are neither substitutes nor complements for firm G, this reduces to the condition that $\mu_G - 2\mu \geq 0$. In that case, the best property-rights allocation is the same as in the absence of bargaining. When $\lambda = 1$, so that the licenses are perfect substitutes for firm G, condition (11) reduces to

$$0 > \sum_{i=1,2} \mathbb{E}[\max\{\tilde{\theta}_G, \tilde{\theta}_i\}],$$

which never holds. Thus, NONE is always optimal when λ is close enough to 1 (by continuity). Finally, when $\lambda = 0$, so that the licenses are perfect complements for firm G, condition (11) reduces to $2(\mu_G - 2\mu) > -\sum_{i=1,2} \mathbb{E}[\max\{0, \tilde{\theta}_i - \tilde{\theta}_G\}]$. Because the right-hand side is negative, bargaining pushes the optimal allocation toward

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16. For the adverse efficient opt-out property to hold, it is essential that types are unbounded above. If we imposed upper bounds on types, the qualitative results would still hold provided that the bounds are high enough, but these results could no longer be established using our methods.

17. Under either property rights allocation there is a competitive equilibrium in which the price of each license i is θ_i, and there exist additional competitive equilibria with nearby prices as long as the efficient allocation is unique. Since all competitive equilibria yield core payoff profiles, the core is always nonempty and is multivalued with probability 1, so by Proposition 1 efficient bargaining is impossible under either property rights allocation.

BOTH relative to what would be optimal without bargaining when λ is close enough to 0.

4.2.2. Application: Liability Rule for Pollution.   Consider a setting in which agent 0 (the "firm") chooses whether to pollute, labeled by $x \in \{0, 1\}$. The firm's utility is $v_0(x, \theta_0) = \theta_0 x$, where $\theta_0 \in \mathbb{R}_+$ denotes its value for polluting. Agents $i = 1, \ldots, N$ are consumers, whose utilities are given by $v_i(x, \theta_i) = (1 - x)\theta_i$ with $\theta_i \in \mathbb{R}_+$. Efficient pollution is therefore given by

$$\chi^*(\theta) = 1 \quad \text{if and only if } \theta_0 \geq \sum_{i \geq 1} \theta_i.$$

We assume that, for all i, $\tilde{\theta}_i$ has a full-support distribution on $\mathbb{R}_+$ that has a density.

The property rights are given by a liability rule: the firm can choose to pollute, in which case it must pay pre-specified "damages" $p_i \geq 0$ to each consumer $i \geq 1$. Thus, if the firm does not participate in bargaining, it optimally chooses $\hat{x}_0(\theta) = \chi^*(\theta_0, p_1, \ldots, p_N)$, and its transfer is given by $\hat{\tau}_0(\theta) = -(\sum_i p_i)\hat{x}_0(\theta)$.

We must also specify what happens if agent $i \geq 1$ does not participate. To obtain the results in the simplest possible way, we assume for now that all the other agents then bargain efficiently among each other, given that agent i must be paid compensation p_i if pollution is chosen. Thus, they optimally choose pollution level $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$, and agent i's compensation is $\hat{\tau}_i(\theta) = p_i \hat{x}_i(\theta)$. (We discuss the role of this assumption in Remark 1.)

Given these assumptions, each agent $i \geq 1$ has an efficient-opt-out type $\theta_i^\circ = p_i$. This type is also trivially adverse, since the agent imposes no externalities on the others. Hence, by Lemma 2, it is agent i's critical type.

The firm, on the other hand, has two efficient-opt-out types: $\theta_0 = 0$ (which never pollutes in the first-best and does not pollute when it does not participate) and $\theta_0 = +\infty$ (which always pollutes in the first-best and pollutes when it does not participate). Furthermore, $\theta_0 = 0$ is an adverse type if $\sum_{i \geq 1} p_i \geq \mathbb{E}[\sum_{i \geq 1} \tilde{\theta}_i]$ while $\theta_0 = +\infty$ is an adverse type if the inequality is reversed. (Of course, formally speaking $\theta_0 = +\infty$ is not a "type", but taking a sequence $\theta_0^k \to +\infty$ shows that the firm does satisfy the adverse efficient-opt-out property.)

REMARK 1.   Observe that if the intermediary can choose $\hat{x}_i(\theta)$ following nonparticipation of agent $i \geq 1$ to minimize the expected first-best subsidy, then it can do no better than setting $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$, as we assumed previously. Indeed, since the intermediary has to satisfy the participation constraint of type $\theta_i = p_i$ regardless of $\hat{x}_i(\theta)$, formula (7) bounds below the intermediary's expected subsidy. However, by choosing $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$ the intermediary ensures that type $\theta_i = p_i$ is a critical type, and therefore its participation constraints imply all the other types' participation constraints, so the lower bound on the expected subsidy is actually achieved. Therefore, the following analysis of optimal damages p applies to the situation where the intermediary can choose $\hat{x}_i(\theta)$ optimally.

Now we identify the vector of damages $p = (p_1, \ldots, p_N)$ that minimizes the expected first-best subsidy. Using equation (7), the problem can be written as

$$\max_{p_1, \ldots, p_N \geq 0} \sum_{i=0}^{N} \mathbb{E}\big[\widehat{V}_{-i}(\theta_i^{\circ}, \tilde{\theta}_{-i})\big],$$

where

$$\mathbb{E}\big[\widehat{V}_{-0}(\theta_0^{\circ}, \tilde{\theta}_{-0})\big] = \min\left\{\sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i], \sum_{i \geq 1} p_i\right\}$$

and

$$\mathbb{E}\big[\widehat{V}_{-i}(\theta_i^{\circ}, \tilde{\theta}_{-i})\big] = \mathbb{E}\left[\max\left\{\tilde{\theta}_0 - p_i, \sum_{j \neq i, \, j \geq 1} \tilde{\theta}_j\right\}\right] \quad \text{for } i \geq 1.$$

Note that using the Envelope theorem, for $i \geq 1$,

$$\frac{\partial \mathbb{E}\big[\widehat{V}_{-i}(\theta_i^{\circ}, \tilde{\theta}_{-i})\big]}{\partial p_i} = -\Pr\left\{\tilde{\theta}_0 - p_i > \sum_{j \neq i, \, j \geq 1} \tilde{\theta}_j\right\} \in (-1, 0), \qquad (12)$$

while

$$\frac{\partial \mathbb{E}\big[\widehat{V}_{-0}(\theta_0^{\circ}, \tilde{\theta}_{-0})\big]}{\partial p_i} = \begin{cases} 1 & \text{if } \sum_{j \geq 1} \mathbb{E}[\tilde{\theta}_j] > \sum_{j \geq 1} p_j, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, at the optimum we must have $\sum_{i \geq 1} p_i = \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i]$—that is, the total damages paid by the firm should equal the total expectation damages for the affected parties. This would also be optimal in a setting where bargaining is impossible.

However, in contrast to the setting without bargaining, it now matters how the damages are allocated among consumers. The problem of optimal allocation of damages can be formulated as

$$\max_{p \in \mathbb{R}_+^N} \sum_{i \geq 1} \mathbb{E}\big[\widehat{V}_{-i}(p_i, \tilde{\theta}_{-i})\big] \quad \text{s.t.} \sum_{i \geq 1} p_i = \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i].$$

Note that by (12), $\partial \mathbb{E}[\widehat{V}_{-i}(p_i, \tilde{\theta}_{-i})]/\partial p_i$ is nondecreasing in p_i, so the objective function is convex, and is therefore maximized at a vertex of the feasible set—that is, a point p such that

$$p_i = \begin{cases} \sum_{j \geq 1} \mathbb{E}[\tilde{\theta}_j] & \text{for } i = i^*, \\ 0 & \text{for } i \neq i^*, \end{cases}$$

where

$$i^* \in \arg\max_{i \geq 1}\left\{\mathbb{E}\left[\widehat{V}_{-i}\left(\sum_{j \geq 1} \mathbb{E}[\tilde{\theta}_j], \tilde{\theta}_{-i}\right)\right] + \sum_{j \geq 1, \, j \neq i} \mathbb{E}[\widehat{V}_{-j}(0, \tilde{\theta}_{-j})]\right\}.$$

Thus, all of the damages should be paid to a single consumer, with the consumer selected to maximize the total expected surplus of the N coalitions consisting of the firm and $N - 1$ affected parties.[18]

5. Optimal Property Rights with Second-Best Bargaining

In many circumstances, there is no intermediary available to subsidize trade. In that case, a more appropriate approach to determining optimal property rights involves looking at second-best mechanisms that maximize expected surplus subject to a budget balance constraint. Analyzing that problem, however, is complicated by the interplay between the mechanism chosen and the agents' critical types: those critical types depend on the mechanism being employed, but the best mechanism depends on the agent's critical types (because they determine which IR constraints bind).[19] In this section, we analyze this problem. As this is a much harder problem than the first-best problem studied earlier, we restrict attention to the case of two agents trading a single indivisible good, where those agents' types are independently distributed on $[0, 1]$.[20] We consider the general case in which the c.d.f.s of the two agents' types are F^1 and F^2 respectively, with strictly positive densities f^1 and f^2.

Myerson and Satterthwaite (1983) characterized the optimal second-best mechanism for the case of simple property rights, where one agent is a seller (the initial owner) and the other agent is the buyer. The optimal mechanism when agent 2 is the seller and agents' types are uniformly distributed is shown in Figure 1 [the shaded region shows the type profiles (θ_1, θ_2) at which agent 1 ends up getting the good], which leads to a surplus loss of 7/64 (from the first-best surplus of 3/4). It involves a trading "gap" $l = 1/4$, which represents the amount by which the buyer's value must exceed the seller's value for trade to occur. Cramton, Gibbons, and Klemperer (1987) showed that the first-best is achievable for a convex set of intermediate property rights if dividing or randomizing property rights is possible (see also Segal and Whinston 2011). However, when divided or randomized property rights are not possible, it may be possible to improve upon simple undivided property rights with more complex property rights mechanisms. In the remainder of this section, we examine this possibility for liability rules.

5.1. Characterization of the Second-Best Bargaining Mechanism for a Given Liability Rule

Without loss of generality, we will take the agent who possesses the option to have the good to be agent 1 (the "active agent"); agent 2 is the "passive agent" or "victim".

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18.   As we have seen, when $N = 1$ efficiency is impossible. However, when $N > 1$, efficiency may be possible at the optimal property rights. For example, this is the case when the distribution of each victim i's value θ_i is concentrated around its mean, $\mathbb{E}[\theta_i]$, while the distribution of θ_0 is disperse. This can be verified by checking that the intermediary's first-best profit given by (7) is positive.

19.   For any mechanism, not just VCG mechanisms, we now refer to an agent's "critical type" as a type that has the smallest participation surplus among all of the agent's types.

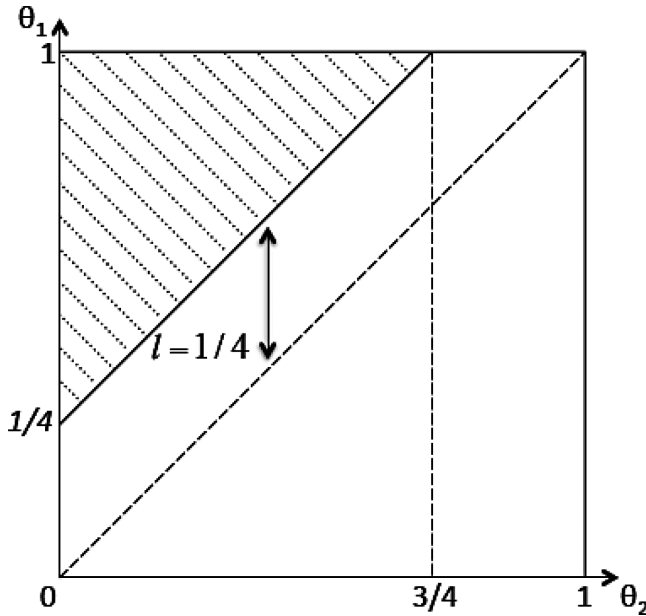20.   The restriction to $[0, 1]$ is just a normalization.

FIGURE 1. The second-best optimal (Myerson–Satterthwaite) mechanism when agent 1 (the buyer) and agent 2 (the seller) both have values that are uniformly distributed on the interval [0, 1].

Note that if $p = 0$ then agent 1 will always exercise his option in the default, so this liability rule is equivalent to agent 1 being the owner with a simple property right. If, instead, $p = 1$, then agent 1 will never exercise his option, so the liability rule is then equivalent to agent 2 being the owner with a simple property right. Hence, the *optimal* liability rule cannot be worse than the optimal simple property right. However, we will see that there are always some liability rules that are worse than the best simple property right.

Our analysis hinges on identifying critical types. For the passive agent 2, any type θ_2^p whose probability of receiving the good in the mechanism is $F_1(p)$, equal to the probability of receiving the good in the default, is a critical type. To see this fact, observe that type θ_2^p's participation surplus simply equals the difference in its expected transfer when participating in the mechanism and when opting out. Any other type θ_2' can guarantee the same participation surplus by pretending to be type θ_2^p when participating in the mechanism. Thus, if type θ_2^p is willing to participate, then so is *every* type. In general, as we will see, there will be an interval of types $[\underline{\theta}_2, \bar{\theta}_2]$ who will receive the good with probability $F_1(p)$ in the mechanism, all of whom will be critical.

As for the active agent 1, we observe that this agent's critical types always include either $\hat{\theta}_1 = 0$, or $\hat{\theta}_1 = 1$, or both. To see this, observe that in the default outcome this agent's payoff is $\widehat{V}_1(\theta_1) = \max\{\theta_1 - p, 0\}$, which is a convex function whose derivative is 0 below p and 1 above p. This agent's expected payoff $U_1(\theta_1)$

in any mechanism, on the other hand, has a derivative that equals that type's expected probability of receiving the good in the mechanism, so $U_1'(\theta_1) \in [0, 1]$ for all θ_1. Thus, $U_1(0) - \widehat{V}_1(0) \le U_1(\theta_1) - \widehat{V}_1(\theta_1)$ for all $\theta_1 \le p$, and $U_1(1) - \widehat{V}_1(1) \le U_1(\theta_1) - \widehat{V}_1(\theta_1)$ for all $\theta_1 \ge p$, and hence $\min\{U_1(0) - \widehat{V}_1(0), U_1(1) - \widehat{V}_1(1)\} \le U_1(\theta_1) - \widehat{V}_1(\theta_1)$ for all θ_1.

For $i = 1, 2$, and for $\lambda > 0$ let

$$\underline{\omega}_i(\theta_i|\lambda) \equiv \theta_i - \frac{\lambda}{1+\lambda} \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \quad \text{and} \quad \bar{\omega}_i(\theta_i|\lambda) \equiv \theta_i + \frac{\lambda}{1+\lambda} \frac{F_i(\theta_i)}{f_i(\theta_i)}$$

denote agent i's virtual values when his downward/upward ICs bind. We assume that both $\underline{\omega}_i(\cdot|1)$ and $\bar{\omega}_i(\cdot|1)$ are strictly increasing and continuous functions in θ_i (this implies the same properties for any $\lambda > 0$. Note that

$$\underline{\omega}_i(\theta_i|\lambda) \le \theta_i \le \bar{\omega}_i(\theta_i|\lambda),$$

and that the inequalities are strict for $\theta_i \in (0, 1)$, provided that $\lambda > 0$.

Also, for $\gamma \in [0, 1]$, let agent 1's weighted virtual value be

$$\omega_1(\theta_1|\lambda, \gamma) \equiv (1 - \gamma)\underline{\omega}_1(\theta_1|\lambda) + \gamma\bar{\omega}_1(\theta_1|\lambda).$$

In our characterization, the weight γ will equal 0 when $\theta_1 = 0$ is a unique critical type for agent 1 (so that his downward IC constraints bind), and will equal 1 when $\theta_1 = 1$ is a unique critical type (so that agent 1's upward incentive constraints bind). We can have $\gamma \in (0, 1)$ when both of these types are critical for agent 1.

Finally, for $\lambda > 0$ and $\gamma \in [0, 1]$, define $\bar{\theta}_1(\theta_2|\lambda, \gamma) \equiv \omega_1^{-1}(\bar{\omega}_2(\theta_2|\lambda)|\lambda, \gamma)$ and $\underline{\theta}_1(\theta_2|\lambda, \gamma) \equiv \omega_1^{-1}(\underline{\omega}_2(\theta_2|\lambda)|\lambda, \gamma)$. Given agent 2's type θ_2, and values λ and γ, these are the types of agent 1 at which agent 1's weighted virtual valuation equals agent 2's upward and downward virtual values, respectively. They will form part of the boundary of the set of type profiles at which agent 1 gets the good in the optimal mechanism, and are depicted in Figure 2. Under our assumptions they are both continuous increasing functions and $\bar{\theta}_1(\theta_2|\lambda, \gamma) \ge \underline{\theta}_1(\theta_2|\lambda, \gamma)$ for all θ_2.

We begin with the following characterization result (all proofs for results in this section are in the Appendix).

LEMMA 4. *When there is a liability rule in which agent* 1 *has the option to own in return for a payment of $p \in [0, 1]$, the second-best allocation rule takes the following form (with probability 1): For some fixed $\lambda > 0$ and $\gamma \in [0, 1]$,*

$$x_1(\theta_1, \theta_2) = \begin{cases} 1 & \text{for} \quad \theta_1 > \hat{\theta}_1(\theta_2), \\ 0 & \text{for} \quad \theta_1 < \hat{\theta}_1(\theta_2), \end{cases} \tag{13}$$

where

$$\hat{\theta}_1(\theta_2) \equiv \max\left\{\underline{\theta}_1(\theta_2|\lambda, \gamma), \min\{p, \bar{\theta}_1(\theta_2|\lambda, \gamma)\}\right\}. \tag{14}$$
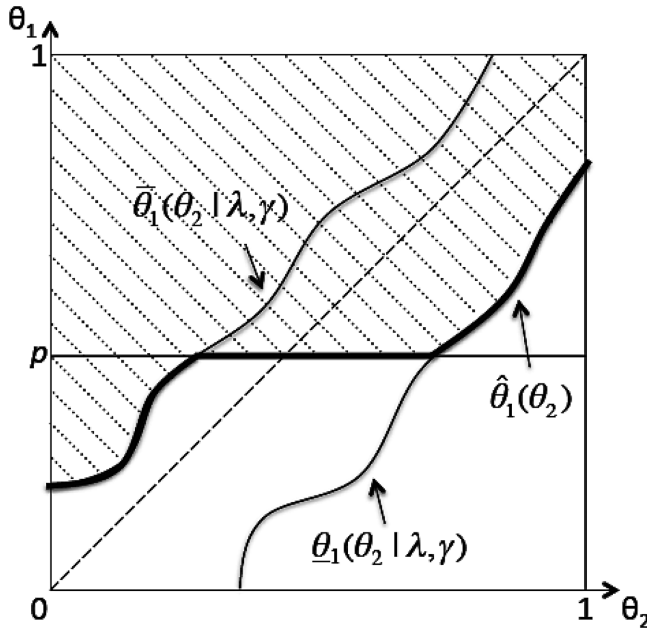
FIGURE 2. The types $\bar{\theta}_1(\theta_2|\lambda, \gamma)$ and $\underline{\theta}_1(\theta_2|\lambda, \gamma)$, and the boundary $\hat{\theta}_1(\theta_2)$ of Lemma 4.

Furthermore,

$$\gamma \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2) - p] \geq 0 \quad and \quad (1-\gamma)\,\mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2) - p] \leq 0. \qquad (15)$$

In Figure 2, the function $\hat{\theta}_1(\theta_2)$ defined in (14), which forms the boundary of the region in which agent 1 gets the good in the mechanism, is shown in heavy trace.

Condition (15) reflects agent 1's IR constraint. Under the liability rule, agent 1's utility is exactly $1 - p$ larger when he is type 1 than when he is type 0. On the other hand, the difference in expected payoffs for types $\theta_1 = 1$ and $\theta_1 = 0$ in the mechanism equals $1 - \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)]$.[21] Thus, type $\theta_1 = 0$ must be the unique critical type for agent 1 when $\mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)] < p$ (and so $\gamma = 0$), while type $\theta_1 = 1$ must be his unique critical type when the reverse strict inequality holds (and so $\gamma = 1$ in this case).

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21.   To see this, observe that by the Envelope theorem, this difference in expected payoffs in the mechanism equals

$$\int_0^1 \left(\int_0^{\hat{\theta}_2(\theta_1)} f_2(\theta_2)d\theta_2 \right) d\theta_1,$$

where $\hat{\theta}_2(\cdot) = \min\{\theta_2 : \hat{\theta}_1(\theta_2) = \theta_1\}$. But, reversing the order of integration, this can be rewritten as

$$\int_0^1 \left(\int_{\hat{\theta}_1(\theta_2)}^1 d\theta_1 \right) f_2(\theta_2)d\theta_2 = 1 - \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)].$$
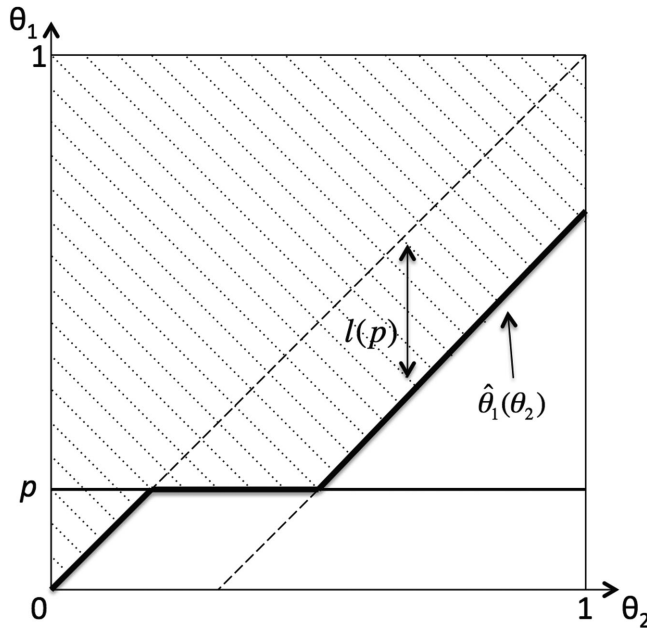
FIGURE 3. The second-best mechanism for $p < 3/8$ when both agents' values are uniformly distributed on the interval $[0, 1]$.

5.2. Second-Best Surplus Given Option Price p

5.2.1. Uniformly Distributed Types.   For the specific case in which both agents' types are drawn from the uniform distribution, Lemma 4 implies that the second-best mechanism takes the following form.

PROPOSITION 4.   *When both agents' types are drawn from the uniform distribution and there is a liability rule in which agent 1 has the option to own in return for a payment of $p \in [0, 1]$, the optimal second-best allocation rule takes the following form, for some function $l(p)$:*[22]

- *for $p < 3/8$: $x_1(\theta_1, \theta_2) = 1$ if and only if (i) $\min\{\theta_1, p\} \geq \theta_2$, (ii) $\theta_1 \geq p$ and $\theta_2 \in [p, p + l(p)]$, or (iii) $\theta_1 \geq \theta_2 - l(p)$ and $\theta_2 > p + l(p)$;*
- *for $p \in [3/8, 5/8]$: $x_1(\theta_1, \theta_2) = 1$ if and only if (i) $\theta_1 \geq \theta_2 + p - 3/8$ and $\theta_2 < 3/8$, (ii) $\theta_1 \geq p$ and $\theta_2 \in [3/8, 5/8]$, or (iii) $\theta_1 \geq \theta_2 + p - 5/8$ and $\theta_2 > 5/8$;*
- *for $p > 5/8$: $x_1(\theta_1, \theta_2) = 1$ if and only if (i) $\theta_1 \geq \theta_2 \geq p$, (ii) $\theta_1 \geq p$ and $\theta_2 \in [p, p - l(p)]$, or (iii) $\theta_1 \geq \theta_2 + l(p)$ and $\theta_2 < p - l(p)$.*

Figures 3–5 show the sets of types for which agent 1 receives the good for the three cases identified in Proposition 4. The three cases correspond to situations in which

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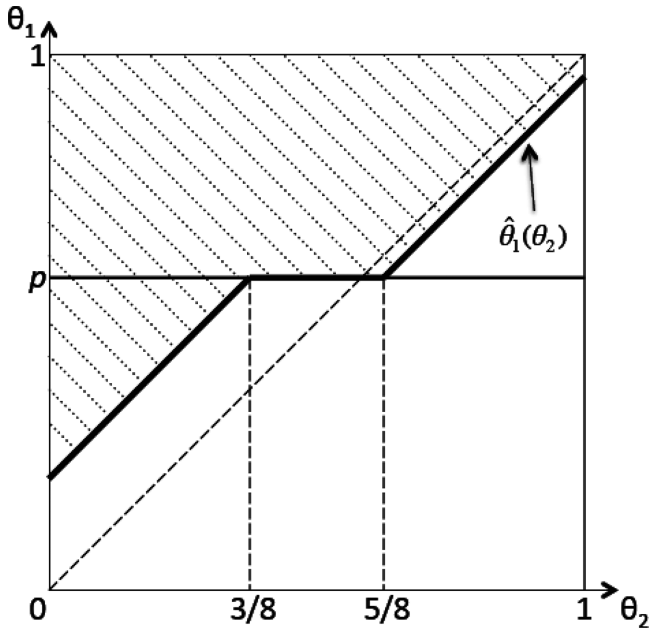22.   We describe the function $l(p)$ in the proof of the proposition, contained in the Appendix.

FIGURE 4. The second-best mechanism for $3/8 < p < 5/8$ when both agents' values are uniformly distributed on the interval $[0, 1]$.
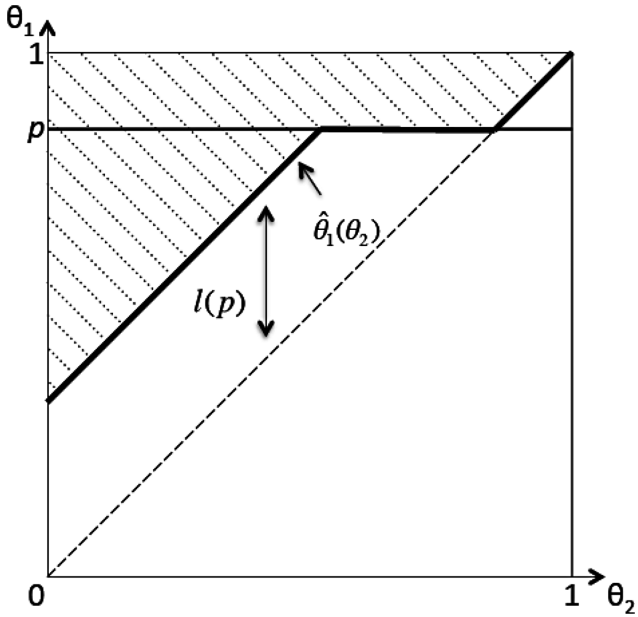

FIGURE 5. The second-best mechanism for $p > 5/8$ when both agents' values are uniformly distributed on the interval $[0, 1]$.
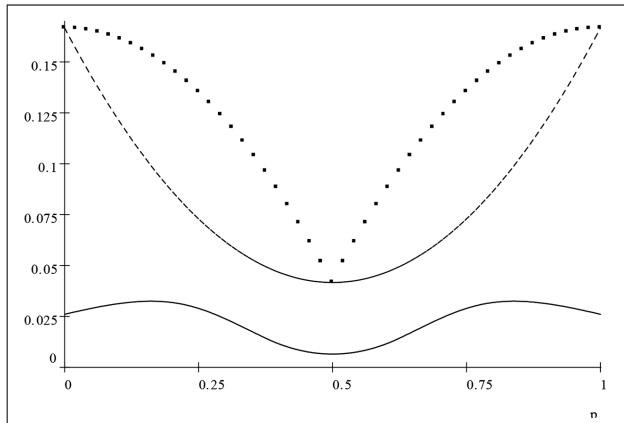
FIGURE 6. Expected subsidy needed for first-best (dotted curve). Expected inefficiency if there were no bargaining (dashed curve). Expected inefficiency in second-best bargaining (solid curve).

agent 1's critical type is $\hat{\theta}_1 = 1$ (for $p < 3/8$), $\hat{\theta}_1 = 0$ (for $p > 5/8$), and both types 0 and 1 are critical types (for $p \in [3/8, 5/8]$). Note that the critical type is 1 (resp. 0) for low (resp. high) p, which are cases where the property right is relatively close to agent 1 (resp. 2) having a simple ownership right.[23] The "gap" function $l(p)$ is similar to the Myerson–Satterthwaite gap seen in Figure 1, and, like that gap, its size is set to achieve budget balance.

The second-best expected surplus as a function of p can be derived analytically in the case in which both agents' values are uniformly distributed (see the Appendix for its derivation). Figure 6 graphs the resulting second-best inefficiency as a function of p. For comparison, the figure also shows the inefficiency with no bargaining and the expected subsidy for a planner who would subsidize trade to achieve the first-best. As can be seen in the figure, the optimal property right has $p = 1/2$—equal to the expected value of the passive agent—in all three cases.

5.2.2. General Distributions.   While the optimal second-best p with uniformly distributed types for both agents is the same as when bargaining is not possible and as when there is an intermediary willing to subsidize trade, Figure 6 shows one significant difference between the second-best case and these others: the surplus achievable with a liability rule is not monotone increasing as p moves toward $1/2$, and is in fact lower for p close to 0 (resp. 1) than at $p = 0$ (resp. 1). That is, a slightly interior p is worse than the simple property right it is near.[24] The fact that liability rules which induce default

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23.   To compare these figures to Figure 2, note that in the special case of the uniform distribution, $\bar{\theta}_1(\theta_2|\lambda, 1) = \theta_2$ and $\underline{\theta}_1(\theta_2|\lambda, 0) = \theta_2$ for all θ_2.

24.   In contrast, it can be shown that in the case of randomized simple property rights the set of status quos for which first-best is achieved is convex (the set is nonempty by Segal and Whinston 2011), and that a move in the direction of this set raises the second-best expected surplus. Loertscher and Wasser (2014)

allocations close to but different from a simple property right are worse than that simple property right does not depend on our assumption of a uniform distribution. As the following proposition shows, it is true for *any* distributions of values for the two agents.

PROPOSITION 5. *There exists a $\delta > 0$ such that any liability rule with $p \in [1, 1 - \delta]$ (resp, $p \in [0, \delta]$) has a lower second-best expected surplus than $p = 1$ (resp, $p = 0$), which is equivalent to simple ownership by agent 2 (resp. agent 1).*

To understand Proposition 5, note that starting at $p = 1$ a small reduction in p weakly increases the default payoff of every type of the active agent 1. At the same time, it increases the expected payoff in the default to essentially all types of the passive agent 2 (whose default payoff when $p = 1$ simply equals his type), since he then gains $(p - \theta_2)[1 - F_1(p)]$.[25] Thus, this change tightens IR constraints, reducing the achievable second-best surplus.

We have also explored computationally (using Lemma 4) a range of cases in which the two agents' values are drawn from differing distributions. In general, it is not the case that the optimal option price p equals the victim's expected harm, $\mathbb{E}[\tilde{\theta}_2]$. For example, when the active agent's (agent 1) value has density $f_1(\theta_1) = 0.2 + 1.6(\theta_1)$ while the passive agent's (agent 2) value is uniformly distributed, searching over $p \in [0.45, 0.55]$ in increments of 0.001 yields the optimal option price $p = 0.516$. The expected surplus at this p is 0.726996, compared to the expected surplus of 0.726881 when $p = 0.5 = \mathbb{E}[\tilde{\theta}_2]$.[26] Interestingly, just as in this example, in all of the cases we have computed, the optimal option price p is extremely close to the victim's expected harm and the loss from setting instead $p = \mathbb{E}[\tilde{\theta}_2]$ is very small.[27]

6. Dual-Chooser Rules

For another application, in this section we consider whether "dual-chooser" rules, as described by Ayres (2005), can improve upon simple property rights or liability rules when there are two agents. In a dual-chooser rule, one agent (we will assume agent 2) is the initial owner of the good, but the other agent can get it if both agents agree to this at a prespecified price p. We assume that each agent uses his dominant strategy of agreeing to trade if and only if it yields nonnegative profits.

We assume that both agents' values for the good are independently drawn from the same interval, which we normalize to be $[0, 1]$. Our first observation is that with

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characterize the second-best optimal bargaining mechanism with simple randomized property rights for which the first-best is not attainable, including for cases with more than two agents.

25.   The complication in the proof is that this is not true for type $\theta_2 = 1$ (or types near it for $p < 1$).

26.   As a check that this finding is not a result of computational imprecisions, we also examined the case in which we switched the two agents value distributions, as well as other cases with a uniformly distributed value for agent 1. For such cases, we know that the optimal p in fact equals $\mathbb{E}[\tilde{\theta}_2]$ (see Lemma A.1 in the Appendix). Our computational algorithm yields an optimal p exactly equal to $\mathbb{E}[\tilde{\theta}_2]$ in all of these cases.

27.   We have computed solutions for a set of cases in which each agent i's value distribution has density $f_i(\theta_i) = 1 - \Delta_i + 2\Delta_i \theta_i$ on $\theta_i \in [0, 1]$, for a grid where $\Delta_i \in \{-0.8, -0.4, 0, 0.4, 0.8\}$ for $i = 1, 2$. Unfortunately, we have been unable to produce an analytical result showing that this is generally true.

this property rights mechanism, agent 2's type $\hat{\theta}_2 = 1$ is an adverse efficient-opt-out type, while agent 1's type $\hat{\theta}_1 = 0$ is an adverse efficient-opt-out type (these types never trade, either in the default mechanism or in the efficient mechanism). Since these types have the same reservation utilities as in the standard Myerson–Satterthwaite setting in which agent 2 is the owner, we see immediately that the expected first-best subsidy is the same as in the Myerson–Satterthwaite setting, regardless of p.

As for the second-best expected surplus, observe that for no p can it exceed that in the Myerson–Satterthwaite setting where agent 2 has a simple property right because, for any p, the participation constraints of types $\hat{\theta}_2 = 1$ and $\hat{\theta}_1 = 0$ must still be satisfied, and the reservation utilities of these types are the same as in the Myerson–Satterthwaite setting. On the other hand, the second-best expected surplus can be *strictly lower* than in the Myerson–Satterthwaite setting. For example, we have the following proposition.

PROPOSITION 6. *If $\tilde{\theta}_1, \tilde{\theta}_2 \sim U[0, 1]$, then the Myerson–Satterthwaite mechanism fails to satisfy IR for any dual-chooser rule with posted price $p \in (0, 1/2) \cup (1/2, 1)$.*

Proof. Consider first agent 1. His expected utility in the dual-chooser rule is $\widehat{V}_1(\theta_1) = \max\{0, (\theta_1 - p)p\}$, while his expected utility in the Myerson–Satterthwaite mechanism is $U_1^{MS}(\theta_1) = (\max\{0, \theta_1 - 1/4\})^2/2$ (this can be calculated either from the dominant-strategy "pricing" implementation of this allocation rule, or by the integral formula using the allocation rule). So clearly for $p < 1/4$, the IR of $\theta_1 = 1/4$ will fail. For $1/4 \leq p < 1/2$, consider instead type $\theta_1 = p + 1/4$ (this will actually be the "critical type"). For this type the participation surplus is $p^2/2 - p/4 = p(p - 1/2)/2 < 0$.

Now consider agent 2. His expected utility in the dual-chooser rule is $\widehat{V}_2(\theta_2) = \theta_2 + \max\{0, (1 - p)(p - \theta_2)\}$, while his expected utility in the Myerson–Satterthwaite mechanism is $U_2^{MS}(\theta_2) = \theta_2 + (\max\{0, 3/4 - \theta_2\})^2/2$. So clearly for $p > 3/4$, the IR of $\theta_2 = 3/4$ will fail. For $1/2 < p \leq 3/4$, consider type $\theta_2 = p - 1/4$ (this will actually be the "critical type"). For this type the participation surplus is $(1 - p)^2/2 - (1 - p)/4 = (1 - p)(1/2 - p)/2 < 0$. $\square$

We conclude then that when bargaining under asymmetric information will take place (of the form we have considered), dual-chooser rules cannot improve upon simple property rights, and can be worse. (They must therefore also be weakly worse than the best liability rule.) This can be contrasted with the case in which bargaining is impossible, where for any $p \in (0, 1)$ the surplus is *strictly higher* under a dual-chooser rule than under a simple property right.

7. Conclusion

The critical role of property rights for economic efficiency has long been recognized. In this paper, we shed new light on this role by examining how property rights affect efficiency when agents will bargain under conditions of asymmetric information.

Our results have implications for several literatures. Relative to the theoretical mechanism design literature, we provide a new set of sufficient conditions characterizing when efficiency through bargaining is impossible, which applies not only to the traditional case of simple property rights, but also to more general property rights mechanisms. We then show how efficiency is affected by the property rights allocation in such cases.

In organizational economics, losses due to ex-post bargaining inefficiencies were a central theme of Williamson's Transaction Cost Economics approach to the firm. One can view our analysis, in which we study how property rights can affect those losses, as taking the Grossman–Hart–Moore (Grossman and Hart 1986; Hart and Moore 1990; Hart 1995) Property Rights Theory approach of asking how asset ownership affects efficiency, but doing so focusing instead on Williamson's costs of haggling, rather than on inefficiencies in ex-ante investments. Like the Property Rights Theory, our approach has implications not only for asset ownership, but also for allocation of decision rights within firms.

Finally, in the legal literature, ever since Calabresi and Melamed (1972), scholars have been interested in the performance of different property rights regimes. Our results shed new light on this issue when bargaining is imperfect due to the presence of asymmetric information. In particular, we have highlighted the effect that bargaining has on the choice among property rights regimes, relative to the case in which bargaining is impossible.

Appendix A: Proofs

A.1. Proof of Lemma 4

Note than in any Bayesian incentive compatible mechanism, agent 2's expected consumption $1 - \mathbb{E}[x_1(\tilde{\theta}_1, \theta_2)]$ must be nondecreasing in θ_2, therefore there will be a type $\hat{\theta}_2$ such that

$$\{1 - F_1(p) - \mathbb{E}[x_1(\tilde{\theta}_1, \theta_2)]\}\text{sign}(\theta_2 - \hat{\theta}_2) \geq 0 \quad \text{for all } \theta_2. \tag{A.1}$$

Consider the designer's "relaxed problem" in which she chooses $\hat{\theta}_2$, the allocation rule $x(\cdot)$, and interim expected utilities $U_1(\cdot)$, $U_2(\cdot)$ to maximize expected surplus subject to (A.1), expected budget balance, first-order incentive compatibility (ICFOC), agent 1's participation constraints $\text{IR}_1(0)$ and $\text{IR}_1(1)$, and agent 2's participation constraint $\text{IR}_2(\hat{\theta}_2)$.[28] The Lagrangian for this problem (leaving ICFOC

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28.   First-order incentive compatibility (ICFOC) requires that for each $i = 1, 2$, each θ_i,

$$U_i(\theta_i) = U_i(0) + \mathbb{E}_{\tilde{\theta}_{-i}}\left[\int_0^{\theta_i} x_i(\tau, \tilde{\theta}_{-i})d\tau\right].$$

By standard arguments, ICFOC and $\mathbb{E}_{\tilde{\theta}_{-i}}[x_i(\theta_i, \tilde{\theta}_{-i})]$ being nondecreasing in θ_i for each i are jointly necessary and sufficient for incentive compatibility. Moreover, by standard arguments that adjust the

as constraints) is

$$\mathbb{E}[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))] + \mathbb{E}[\delta(\tilde{\theta}_2)(1 - F_1(p) - x_1(\tilde{\theta}))\text{sign}(\tilde{\theta}_2 - \hat{\theta}_2)]$$

$$+ \lambda\{\mathbb{E}[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))] - \mathbb{E}[U_1(\tilde{\theta}_1)] - \mathbb{E}[U_2(\tilde{\theta}_2)]\}$$

$$+ \mu_0 U_1(0) + \mu_1(U_1(1) - (1 - p)) + \nu[U_2(\hat{\theta}_2) - p[1 - F_1(p)] - \hat{\theta}_2 F_1(p)]$$

subj. to ICFOC.

It is easy to see that $\lambda > 0$ (since the first-best is impossible), while $\mu_0, \mu_1 \geq 0$ must satisfy the complementary slackness conditions

$$\mu_0 U_1(0) = 0 \quad \text{and} \quad \mu_1[U_1(1) - (1 - p)] = 0, \tag{A.2}$$

and $\delta(\cdot) \geq 0$ must satisfy the complementary slackness conditions

$$\delta(\theta_2)\{\mathbb{E}[x_1(\tilde{\theta}_1, \theta_2)] - (1 - F_1(p))\} = 0 \quad \text{for all } \theta_2 \neq \hat{\theta}_2. \tag{A.3}$$

Note that the solution must have $\nu = \lambda$ (otherwise we could raise the Lagrangian by adding a constant to $U_2(\cdot)$ without affecting ICFOC), and $\mu_0 + \mu_1 = \lambda$ (otherwise we could raise the Lagrangian by adding a constant to $U_1(\cdot)$ without affecting ICFOC). Hence, we can rewrite the Lagrangian as

$$(1 + \lambda)\mathbb{E}[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))] + \mathbb{E}[\delta(\tilde{\theta}_2)(1 - F_1(p) - x_1(\tilde{\theta}))$$

$$\times \text{sign}(\tilde{\theta}_2 - \hat{\theta}_2)] - \lambda\left(\frac{\lambda - \mu_1}{\lambda}\right)\{\mathbb{E}[U_1(\tilde{\theta}_1)] - U_1(0)\}$$

$$- \lambda\left(\frac{\mu_1}{\lambda}\right)\{\mathbb{E}[U_1(\tilde{\theta}_1)] - U_1(1) + 1 - p\}$$

$$- \lambda\{\mathbb{E}[U_2(\tilde{\theta}_2)] - U_2(\hat{\theta}_2)\} - \lambda\{p[1 - F_1(p)] + \hat{\theta}_2 F_1(p)\}.$$

Note also that we can always satisfy one of the complementary slackness conditions (A.2) by adding a constant to $U_1(\cdot)$. To be able to satisfy both of them at the same time while satisfying $\text{IR}_1(1)$ and $\text{IR}_1(0)$, we need to be in one of the following three cases: (i) $\mu_0, \mu_1 > 0$ implies $U_1(1) - U_1(0) = 1 - p$, (ii) $\mu_1 = 0$ implies $\mu_0 = \lambda > 0$ hence $U_1(0) = 0$ and $U_1(1) - U_1(0) = U_1(1) \geq 1 - p$, or (iii) $\mu_0 = 0$ implies $\mu_1 = \lambda$ hence $U_1(1) = 1 - p$ and $U_1(1) - U_1(0) = 1 - p - U_1(0) \leq 1 - p$.

For now, fix $\hat{\theta}_2$ and consider maximizing with respect to the allocation rule $x_1(\cdot)$ the simplified Lagrangian (which drops all terms not involving the allocation

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transfer rule (e.g., Lemma 1 in Segal and Whinston 2011), ex-ante budget balance can be strengthened to ex-post budget balance without affecting expected surplus or any of the other constraints.

rule)

$$(1 + \lambda)\,\mathbb{E}[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))] - \mathbb{E}[\delta(\tilde{\theta}_2) x_1(\tilde{\theta}) \mathrm{sign}\,(\tilde{\theta}_2 - \hat{\theta}_2)]$$

$$- \lambda \left(\frac{\lambda - \mu_1}{\lambda} \right) \{\mathbb{E}[U_1(\tilde{\theta}_1)] - U_1(0)\} - \lambda \left(\frac{\mu_1}{\lambda} \right) \{\mathbb{E}[U_1(\tilde{\theta}_1)] - U_1(1) + 1 - p\}$$

$$- \lambda \{\mathbb{E}[U_2(\tilde{\theta}_2)] - U_2(\hat{\theta}_2)\}.$$

Now using ICFOC and integration by parts, dividing by $1 + \lambda$, and letting $\hat{\delta}(\theta_2) \equiv \delta(\theta_2)/(1 + \lambda)$ and $\gamma \equiv \mu_1/\lambda$, this simplified Lagrangian can be rewritten as

$$\mathbb{E}\left[\begin{array}{c} \omega_1(\tilde{\theta}_1 | \lambda, \gamma) x_1(\tilde{\theta}) + \left\{ 1\{\tilde{\theta}_2 \geq \hat{\theta}_2\} \underline{\omega}_2(\tilde{\theta}_2 | \lambda) + 1\{\theta_2 < \hat{\theta}_2\} \bar{\omega}_2(\tilde{\theta}_2 | \lambda) \right\} (1 - x_1(\tilde{\theta})) \\ - \mathrm{sign}\,(\theta_2 - \hat{\theta}_2) \hat{\delta}(\tilde{\theta}_2) x_1(\tilde{\theta}) \end{array} \right].$$

It is maximized pointwise by a solution of the form (13), where

$$\hat{\theta}_1(\theta_2) = \begin{cases} \omega_1^{-1}(\bar{\omega}_2(\theta_2 | \lambda) - \hat{\delta}(\theta_2) | \lambda, \gamma) & \text{for } \theta_2 < \hat{\theta}_2, \\ \omega_1^{-1}(\underline{\omega}_2(\theta_2 | \lambda) + \hat{\delta}(\theta_2) | \lambda, \gamma) & \text{for } \theta_2 > \hat{\theta}_2. \end{cases} \qquad (\text{A.4})$$

Let $\bar{\theta}_2(\theta_1 | \lambda, \gamma) \equiv \{\theta_2 : \underline{\theta}_1(\theta_2 | \lambda, \gamma) = \theta_1\}$ and $\underline{\theta}_2(\theta_1 | \lambda, \gamma) \equiv \{\theta_2 : \bar{\theta}_1(\theta_2 | \lambda, \gamma) = \theta_1\}$, and define $\bar{\theta}_2 \equiv \bar{\theta}_2(p | \lambda, \gamma)$ and $\underline{\theta}_2 \equiv \underline{\theta}_2(p | \lambda, \gamma)$. Note that under our assumptions we have $\underline{\theta}_2 \leq \bar{\theta}_2$. Observe that by (A.1) and the definition of $\underline{\theta}_2, \bar{\theta}_2$, we have $\hat{\theta}_1(\theta_2) \leq p < \bar{\theta}_1(\theta_2 | \lambda, \gamma)$ for all $\theta_2 \in (\underline{\theta}_2, \hat{\theta}_2)$ and $\hat{\theta}_1(\theta_2) \geq p > \underline{\theta}_1(\theta_2 | \lambda, \gamma)$ for all $\theta_2 \in (\hat{\theta}_2, \bar{\theta}_2)$. This in turn implies, using (A.4), that for all $\theta_2 \in (\min\{\underline{\theta}_2, \hat{\theta}_2\}, \max\{\hat{\theta}_2, \bar{\theta}_2\})$ we have $\hat{\delta}(\theta_2) > 0$ and therefore for all such θ_2, by (A.3), $\mathbb{E}[x_1(\tilde{\theta}_1, \theta_2)] = 1 - F_1(p)$, which implies $\hat{\theta}_1(\theta_2) = p$.

Next, for $\theta_2 < \min\{\underline{\theta}_2, \hat{\theta}_2\}$, $\hat{\delta}(\theta_2) \geq 0$ implies that $\hat{\theta}_1(\theta_2) \leq \bar{\theta}_1(\theta_2 | \lambda, \gamma) < p$, therefore $\mathbb{E}[x_1(\tilde{\theta}_1, \theta_2)] > 1 - F_1(p)$, and so by (A.3) $\hat{\delta}(\theta_2) = 0$, implying $\hat{\theta}_1(\theta_2) = \bar{\theta}_1(\theta_2 | \lambda, \gamma)$. Similarly for $\theta_2 > \max\{\hat{\theta}_2, \bar{\theta}_2\}$, $\hat{\delta}(\theta_2) \geq 0$ implies that $\hat{\theta}_1(\theta_2) \geq \underline{\theta}_1(\theta_2 | \lambda, \gamma) > p$, therefore $\mathbb{E}[x_1(\tilde{\theta}_1, \theta_2)] < 1 - F_1(p)$, and so by (A.3) $\hat{\delta}(\theta_2) = 0$, implying $\hat{\theta}_1(\theta_2) = \underline{\theta}_1(\theta_2 | \lambda, \gamma)$. Thus, the solution takes the form

$$\hat{\theta}_1(\theta_2) = \begin{cases} \bar{\theta}_1(\theta_2 | \lambda, \gamma) & \text{for } \theta_2 < \min\{\underline{\theta}_2, \hat{\theta}_2\}, \\ p & \text{for } \theta_2 \in (\min\{\underline{\theta}_2, \hat{\theta}_2\}, \max\{\hat{\theta}_2, \bar{\theta}_2\}), \\ \underline{\theta}_1(\theta_2 | \lambda, \gamma) & \text{for } \theta_2 > \max\{\hat{\theta}_2, \bar{\theta}_2\}. \end{cases} \qquad (\text{A.5})$$

This characterizes the solution of the relaxed problem subject to (A.1) for a fixed $\hat{\theta}_2$. Now consider the optimal choice of $\hat{\theta}_2$: since for $\hat{\theta}_2 < \underline{\theta}_2$ or $\hat{\theta}_2 > \bar{\theta}_2$ the solution given by (A.5) would also satisfy (A.1) for $\hat{\theta}_2 \in [\underline{\theta}_2, \bar{\theta}_2]$ (while the converse is not true), it is optimal to choose $\hat{\theta}_2 \in [\underline{\theta}_2, \bar{\theta}_2]$, in which case the function $\hat{\theta}_1(\theta_2)$ is given by equation (14).

The complementary slackness conditions (A.2) are given by equation (15), since by ICFOC and Fubini's theorem,

$$U_1(1) - U_1(0) = \int_0^1 \mathbb{E}[x_1(\theta_1, \tilde{\theta}_2)]d\theta_1 = \mathbb{E}\left[\int_0^1 x_1(\theta_1, \tilde{\theta}_2)d\theta_1\right] = 1 - \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)].$$

(A.6)

Finally, note that the constructed solution actually satisfies all the incentive constraints (since for each i, $\mathbb{E}_{\tilde{\theta}_{-i}} x_i[(\theta_i, \tilde{\theta}_{-i})]$ is nondecreasing in θ_i) and all of the participation constraints (by (A.1) and the argument in the main text before the lemma, type $\hat{\theta}_2$ is a critical type for agent 2, and either $\theta_1 = 0$ or type $\theta_1 = 1$ is a critical type for agent 1). Thus, by the classical characterization of solutions to a convex optimization program as saddle points of the Lagrangian (e.g., Luenberger, 1969, Theorem 1,2 of Chapter 8), an allocation rule is part of a constrained-optimal budget-balanced mechanism if and only if it satisfies conditions (13)–(15) for some $\lambda > 0$ and some $\gamma \in [0, 1]$.

We now describe a transfer rule that implements the allocation rule characterized in the lemma in a dominant strategy IC mechanism that has the right participation constraints binding. When $\gamma < 1$, that is $\text{IR}_1(0)$ binds, we let $t_1(\theta_1, \theta_2) = -\hat{\theta}_1(\theta_2)x_1(\theta_1, \theta_2)$—that is, agent 1 pays $\hat{\theta}_1(\theta_2)$ when he consumes the object.[29] When $\gamma > 0$, that is $\text{IR}_1(1)$ binds, we let $t_1(\theta_1, \theta_2) = -p + (1 - x_1(\theta_1, \theta_2))\hat{\theta}_1(\theta_2)$—i.e., agent 1 first takes the object for price p and then is paid $\hat{\theta}_1(\theta_2)$ when he gives it up.[30] For $\gamma \in (0, 1)$, by (15) the two payments have the same expectation over θ_2 for every θ_1. In particular, in that case we can elect the first option for t_1 when $\theta_1 < p$ and the second option when $\theta_1 > p$, yielding transfer rule

$$t_1(\theta_1, \theta_2) = \begin{cases} -\hat{\theta}_1(\theta_2)x_1(\theta_1, \theta_2) & \text{if } \theta_1 < p \\ -p + (1 - x_1(\theta_1, \theta_2))\hat{\theta}_1(\theta_2) & \text{if } \theta_1 > p \end{cases}.$$

(A.7)

As for agent 2, we let

$$t_2(\theta_1, \theta_2) = \begin{cases} \underline{\theta}_2(\theta_1|\lambda, \gamma)x_1(\theta_1, \theta_2) & \text{if } \theta_1 < p \\ p - \bar{\theta}_2(\theta_1|\lambda, \gamma)(1 - x_1(\theta_1, \theta_2)) & \text{if } \theta_1 > p \end{cases}.$$

(A.8)

That is, if agent 1 would not exercise his option at the default, then agent 2 receives $\underline{\theta}_2(\theta_1|\lambda, \gamma)$ whenever he sells the object, while if agent 1 would exercise his option at the default, then agent 2 receives p but pays back $\bar{\theta}_2(\theta_1|\lambda, \gamma)$ whenever he ends up keeping the object.[31]

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29.   Since $x_1(0, \theta_2) = 0$ for all θ_2 when $\gamma < 1$, type 0 of agent 1 has an expected payoff of 0.

30.   Since $x_1(1, \theta_2) = 1$ for all θ_2 when $\gamma > 0$, type 1 of agent 1 has an expected payoff of $1 - p$.

31.   Observe that with this payment rule, type $\hat{\theta}_2$ of agent 2 has expected payoff $\hat{\theta}_2 F_1(p) + p[1 - F_1(p)]$, so $\text{IR}_2(\hat{\theta}_2)$ holds with equality.

Adding the two transfer rules (A.7) and (A.8) yields a budget deficit for the case where $\gamma \in (0, 1)$ of

$$[\underline{\theta}_2(\theta_1|\lambda, \gamma) - \hat{\theta}_1(\theta_2)]x_1(\theta_1, \theta_2) \quad \text{when } \theta_1 < p, \tag{A.9}$$

$$[\hat{\theta}_1(\theta_2) - \bar{\theta}_2(\theta_1|\lambda, \gamma)](1 - x_1(\theta_1, \theta_2)) \quad \text{when } \theta_1 > p.$$

A.2. Proof of Proposition 4

When F_1, F_2 are uniform distributions on $[0, 1]$,

$$\underline{\omega}_i(\theta_i|\lambda) = \frac{1 + 2\lambda}{1 + \lambda}\theta_i - \frac{\lambda}{1 + \lambda} \quad \text{and} \quad \bar{\omega}_i(\theta_i|\lambda) = \frac{1 + 2\lambda}{1 + \lambda}\theta_i. \tag{A.10}$$

Then, letting $\bar{\theta}_2(\theta_1) \equiv \{\theta_2 : \underline{\theta}_1(\theta_2|\lambda, \gamma) = \theta_1\}$ and $\underline{\theta}_2(\theta_1) \equiv \{\theta_2 : \bar{\theta}_1(\theta_2|\lambda, \gamma) = \theta_1\}$, we have

$$\underline{\theta}_2(\theta_1) = \theta_1 - \underline{l}, \bar{\theta}_2(\theta_1) = \theta_1 + \bar{l}, \bar{\theta}_1(\theta_2) = \theta_2 + \underline{l}, \underline{\theta}_1(\theta_2) = \theta_2 - \bar{l},$$

and

$$\hat{\theta}_1(\theta_2) = \min\{\max\{\theta_2 - \bar{l}, p\}, \theta_2 + \underline{l}\},$$

where $\underline{l} = \lambda(1 - \gamma)/(1 + 2\lambda)$ and $\bar{l} = \lambda\gamma/(1 + 2\lambda)$.

A.2.1. Only $IR_1(0)$ Binds.
Now consider the relaxed problem in which we ignore $IR_1(1)$. The solution to that problem corresponds to the case in which $\gamma = 0$, hence $\bar{l} = 0$. Let $\underline{l} = l = \lambda/(1 + 2\lambda)$. We can use the following transfer for agent 1:

$$t_1(\theta_1, \theta_2) = -\hat{\theta}_1(\theta_2)x_1(\theta_1, \theta_2) \qquad \text{when } \theta_1 < p,$$

$$t_1(\theta_1, \theta_2) = -\mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)] + (1 - x_1(\theta_1, \theta_2))\hat{\theta}_1(\theta_2) \quad \text{when } \theta_1 > p,$$

since in both cases $\mathbb{E}[t_1(\theta_1, \tilde{\theta}_2)] = \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)x_1(\theta_1, \tilde{\theta}_2)]$. Given these transfers, and the transfer (A.8) for agent 2, the budget deficit is

$$\left[\underline{\theta}_2(\theta_1) - \hat{\theta}_1(\theta_2)\right]x_1(\theta_1, \theta_2) \qquad \text{when } \theta_1 < p,$$

$$\left[\hat{\theta}_1(\theta_2) - \bar{\theta}_2(\theta_1)\right](1 - x_1(\theta_1, \theta_2)) + p - \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)] \quad \text{when } \theta_1 > p.$$

Focusing first on the region where $p < \theta_1 < \theta_2$, the subsidy there is the whole gains from trade $\theta_2 - \theta_1$. Since the efficient expected gains from trade in the Myerson–Satterthwaite model with $U[0, 1]^2$ distribution is $1/6$, the expected gains from trade on the region $p < \theta_1 < \theta_2$ is $(1 - p)^3/6$ (the probabilities and the gains themselves
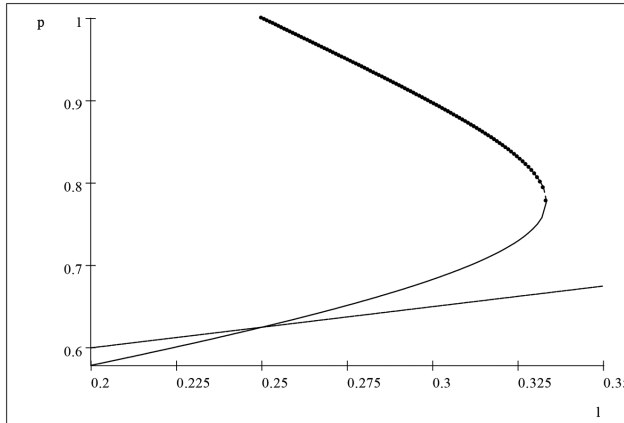
FIGURE A.1. Gap as a function of exercise price when only $\mathrm{IR}_1(0)$ binds.

are scaled by $1 - p$.[32] Now, in the region $\theta_2 + l < \theta_1 < p$, the subsidy $\theta_1 - \theta_2 - 2l$ can be interpreted as (a) paying the gains from trade as if agent 1's value were $\theta_1 - l$ and trade were efficient for that value, and then (b) getting back l on every trade that happened. In expectation, (a) costs $(p - l)^3/6$, and (b) yields $l(p - l)^2/2$.[33] Finally, we have the term $p - \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)]$, which has to be paid when $\theta_1 > p$, which in expectation costs $(1 - p)[(p - l)^2/2 - (1 - p)^2/2]$. Adding all the terms yields

$$(1 - p)^3/6 + (p - l)^3/6 - l(p - l)^2/2 + ((p - l)^2/2 - (1 - p)^2/2)(1 - p)$$

$$= p - lp + \frac{1}{2}l^2 - \frac{2}{3}l^3 - \frac{1}{2}p^2 + l^2 p - \frac{1}{3}.$$

Requiring ex-ante budget balance sets this expression to 0. We want to express l as a function of p, but it is easier to do the reverse. The two roots of the quadratic equation in p are

$$p_\pm(l) = 1 - l(1 - l) \pm \frac{1}{3}\sqrt{3(1 - l)^3(1 - 3l)}. \qquad (A.11)$$

In Figure A.1, the two solutions are graphed as the dotted and the solid curve, respectively. Combining the two curves yields the graph of function $l(p)$, which is inverse U-shaped.

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32.   In general, the Myerson–Satterthwaite deficit with uniform distributions on $[0, 1]$ and a "gap" equal to l is $(1 - 4l)(1 - l)^2/6$ (see Myerson and Satterthwaite 1983, p. 277). So when $l = 0$, the deficit is $1/6$. We get $(1 - p)^3/6$ because the probability of being in the region $p < \theta_1 < \theta_2$ is $(1 - p)^2$ and the region is $[0, 1]^2$ scaled down by $(1 - p)$.

33.   Alternatively, the region $\max\{\theta_1, \theta_2\} < p$ is a scaled-down version of a Myerson–Satterthwaite $[0, 1]^2$ trading box with a "gap" equal to l/p. Using the formula in footnote 32, the Myerson–Satterthwaite deficit would be $(1 - 4l/p)(1 - l/p)/6$. The probability of an outcome in this region is p^2 and the deficit is scaled down by p, so this region contributes $(p - 4l)(p - l)/6$ to the expected deficit.
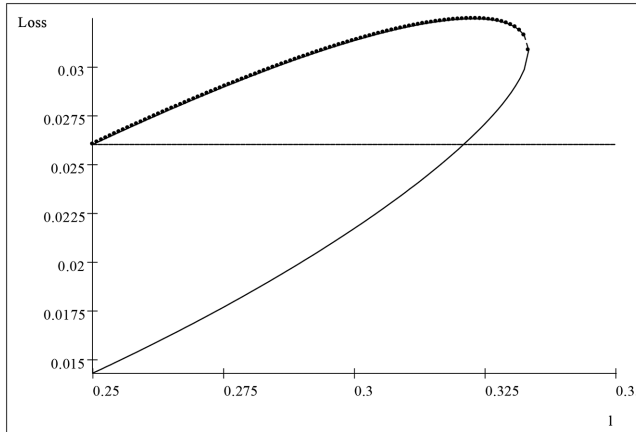
FIGURE A.2. Expected welfare loss when only $\text{IR}_1(0)$ binds.

$\text{IR}_1(1)$ is satisfied when $\mathbb{E}[\hat{\theta}_1(\theta_2)] \leq p$; that is, $p - l \geq 1 - p$, or $p \geq (1 + l)/2$. In Figure A.1, the lower boundary of the region in which $\text{IR}_1(1)$ is satisfied is described by the dashed line. The intersection of the dashed line with the solid curve solves the equation $p_-(l) = (1 + l)/2$, which yields $l = 1/4$ and $p = p_-(l) = 5/8$. Thus, the solution to the relaxed problem in which only $\text{IR}_1(0)$ is imposed satisfies $\text{IR}_1(1)$, and is therefore the solution of the true problem, if and only if $p \geq 5/8$. This describes the third case listed in Proposition 4.

Expected Welfare Loss. The expected welfare loss when $p \geq 5/8$ can be calculated as

$$p^3/6 - (p - l)^3/6 - l\,(p - l)^2/2 = \frac{1}{6}l^2\,(3p - 2l)\,.$$

(The first term is if there were no trade at all for values below p, the second term is expected gains from trade on the triangle below p assuming that l is wasted each time, and the third term accounts for l not being wasted.)

Substituting p from (A.11), which describes the dotted and solid curves in Figure A.1, yields the welfare loss $l^2(3p_\pm(l) - 2l)/6$, which is plotted in Figure A.2.

Reducing p from 1 corresponds to moving along the curve clockwise, starting at $l = 1/4$ and $p = p_+(1/4) = 1$ and moving on the dotted curve, then shifting to the solid curve and ending at $l = 1/4$ and $p = p_-(1/4) = 5/8$. Note that the loss is increasing as we reduce p from 1 for most of the dotted curve, and the point at which it is maximized is obtained by solving the first-order condition

$$0 = \frac{d}{dl}(l^2(3p_+(l) - 2l)) \quad \text{on } l \in [1/4, 1/2]\,.$$

The solution is $l \approx 0.323$, which corresponds to $p \approx 0.839$.

The horizontal line in Figure A.2 depicts the Myerson–Satterthwaite welfare loss (i.e., that obtained when $p = 1$ and $l = 1/4$), which is $5/192$. The horizontal line intersects the solid curve at $\hat{l} \approx 0.321$, which corresponds to $\hat{p} = p_-(\hat{l}) \approx 0.720$. Thus, the welfare loss exceeds the Myerson–Satterthwaite loss when $p \geq \hat{p}$, and it is below the Myerson–Satterthwaite loss when $p \in [5/8, \hat{p})$.

A.2.2. Only $IR_1(1)$ *Binds.*   By symmetry, when $p \leq 3/8$, the solution has only $IR_1(1)$ bind, and we obtain the first case of Proposition 4 with the gap function $l(p) = l(1 - p)$, and the same welfare loss as for option price $1 - p$.

A.2.3. Both $IR_1(0)$ *and* $IR_1(1)$ *Bind.*   When $p \in (3/8, 5/8)$ both $IR_1(0)$ and $IR_1(1)$ must bind. Applying (A.9), the budget deficit in this case is

$$
\begin{array}{ll}
\theta_1 - \theta_2 - 2\underline{l} & \text{when } \theta_2 + \underline{l} < \theta_1 < p, \\
\theta_2 - \theta_1 - 2\bar{l} & \text{when } p < \theta_1 < \theta_2 - \bar{l}, \\
0 & \text{otherwise.}
\end{array}
$$

By (15), $\mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)] = p$. This implies that the probabilities of the two regions $A \equiv \{\theta : \theta_2 + \underline{l} < \theta_1 < p\}$ and $B \equiv \{\theta : p < \theta_1 < \theta_2 - \bar{l}\}$ are equal (these are the two regions in which the final allocation differs from what would happen if agent 1 simply exercised his option optimally). This involves having $\bar{l} = \underline{l} + (1 - 2p)$.

Consider first the case of $p = 1/2$. In this case, regions A and B have equal probability when $\underline{l} = \bar{l}$. The optimal allocation in this case can be interpreted as separate Myerson–Satterthwaite mechanisms for the cases $\theta_1, \theta_2 < 1/2$ (in which agent 1 is the buyer) and $\theta_1, \theta_2 > 1/2$ (in which agent 1 is the seller), with no cross-subsidization between the two cases. The unique gap that achieves budget balance and maximizes expected surplus is half of the Myerson–Satterthwaite gap: $\underline{l} = \bar{l} = 1/2 \cdot 1/4 = 1/8$. For other $p \in (3/8, 5/8)$, we characterize the solution with the help of the following lemma (which holds for an arbitrary distribution of agent 2's type, as long as agent 1's type is uniformly distributed).

LEMMA A.1.   *Suppose that agent 1's type* $\tilde{\theta}_1$ *is uniformly distributed and let* $\hat{\theta}_1(\theta_2|p)$ *describe the second-best allocation rule given* p *(as specified in equation (14)). Then, if for any* p *and* p' *both* $IR_1(0)$ *and* $IR_1(1)$ *bind in the optimal second-best mechanism, then* $\hat{\theta}_1(\theta_2|p') = \hat{\theta}_1(\theta_2|p) + (p' - p)$.

Proof.   When $\tilde{\theta}_1$ is uniformly distributed on $[0, 1]$, we have

$$
\omega_1(\theta_1|\lambda, \gamma) = \frac{(1 + 2\lambda)}{1 + \lambda}\theta_1 - (1 - \gamma)\frac{\lambda}{1 + \lambda},
$$

and therefore

$$\omega_1^{-1}(\omega|\lambda, \gamma) = \frac{(1 + \lambda)\omega + \lambda(1 - \gamma)}{1 + 2\lambda}.$$

Thus,

$$\bar{\theta}_1(\theta_2|\lambda, \gamma) = \frac{(1 + \lambda)\bar{\omega}_2(\theta_2|\lambda) + \lambda(1 - \gamma)}{1 + 2\lambda},$$

$$\underline{\theta}_1(\theta_2|\lambda, \gamma) = \frac{(1 + \lambda)\underline{\omega}_2(\theta_2|\lambda) + \lambda(1 - \gamma)}{1 + 2\lambda}.$$

Start with the second-best optimal mechanism for option price p, described by Lemma 4, with cutoff function $\hat{\theta}_1(\cdot|p')$ and parameters $\lambda > 0$ and $\gamma \in (0, 1)$ (as both $\mathrm{IR}_1(1)$ and $\mathrm{IR}_1(0)$ bind for p). To show the second-best optimality of the allocation rule described by $\hat{\theta}_1(\theta_2|p') = \hat{\theta}_1(\theta_2|p) + p' - p$ given option price p', we argue that it satisfies all the conditions in Lemma 4 with parameters $\lambda' = \lambda$ and $\gamma' = \gamma - ((1 + 2\lambda)/\lambda)(p' - p)$. Indeed, with these parameters, (14) is verified by construction, and (15) holds as since $\mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2|p')] - p' = \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2|p)] - p = 0$. It remains to show that the mechanism described by $\hat{\theta}_1(\cdot|p')$ and accompanying payments (A.7) and (A.8) satisfies budget balance given option price p', provided that the original mechanism described by $\hat{\theta}_1(\cdot|p)$ does so given option price p.

To see budget balance, given option price p and the allocation rule described by $\hat{\theta}_1(\cdot|p)$ define the two regions $A_p \equiv \{\theta \in [0, 1]^2 : \theta_1 \in (\hat{\theta}_1(\theta_2|p), p)\}$ and $B_p \equiv \{\theta \in [0, 1]^2 : \theta_1 \in (p, \hat{\theta}_1(\theta_2|p))\}$. These regions include all the states in which the final allocation differs from what would happen if agent 1 simply exercised his option optimally, and in which subsidy (A.9) may differ from zero. Using condition (15) and agent 1's uniform type distribution, we have $\Pr\{A_p\} = \Pr\{B_p\} = \Pr\{A_{p'}\} = \Pr\{B_{p'}\}$. Now observe that, letting $\delta = p' - p$, the budget deficit (A.9) in every state $(\theta_1 + \delta, \theta_2) \in A_{p'}$ in the mechanism described by $\hat{\theta}_1(\cdot|p') = \hat{\theta}_1(\theta_2|p) + \delta$ given option price p' is exactly δ smaller than the budget deficit in the corresponding state $(\theta_1, \theta_2) \in A_p$ in the mechanism described by $\hat{\theta}_1(\cdot|p)$ given option price p, while the budget deficit (A.9) in every state $(\theta_1 + \delta, \theta_2) \in B_{p'}$ in the mechanism described by $\hat{\theta}_1(\cdot|p')$ given option price p' is exactly δ greater than the budget deficit in the corresponding state $(\theta_1, \theta_2) \in B_p$ in the mechanism described by $\hat{\theta}_1(\cdot|p)$ given option price p. Therefore, the expected budget deficit in the mechanism described by $\hat{\theta}_1(\cdot|p')$ given option price p' is the same as the expected budget deficit in the mechanism described by $\hat{\theta}_1(\cdot|p)$ given option price p, which is zero by assumption. $\qquad\square$

The lemma yields the optimal solution for the second case listed in Proposition 4. The lemma also implies that the optimal mechanism in the region in which both $\mathrm{IR}_1(0)$ and $\mathrm{IR}_1(1)$ bind has a constant improvement in expected surplus over the expected surplus arising at the default, when agent 1 exercises his option optimally. Since the

latter expected surplus is maximized at $p = \mathbb{E}[\tilde{\theta}_2]$, so is the former (note that this holds for an arbitrary distribution of $\tilde{\theta}_2$).

A.3. Proof of Proposition 5

Consider the Myerson–Satterthwaite solution, which corresponds to the case of $p = 1$ and $\gamma = 0$ of Lemma 4. Let λ_1 denote the Lagrange multiplier on expected budget balance in this solution.

Now, fix the option price $\bar{p}$ and a type $\hat{\theta}_2$ and consider the program $R(\bar{p}, \hat{\theta}_2)$ of choosing the allocation rule, the utility mappings $U_1(\cdot)$ and $U_2(\cdot)$, and the "ironing point p" (which affects the solution through constraint (A.1)) to maximize expected surplus plus λ_1 times expected revenue subject to only $\text{IR}_2(\hat{\theta}_2)$, $\text{IR}_1(0)$, ICFOC, and constraint (A.1), with optimization being over the allocation rule $x_1(\cdot)$ and $p \in [\hat{\theta}_2, 1]$. This program can be written as

$$\max_{\substack{x_1(\cdot),U_1(\cdot),U_2(\cdot) \\ p\in[\hat{\theta}_2,1]}} (1+\lambda_1)\mathbb{E}[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1-x_1(\tilde{\theta}))] - \lambda_1\{\mathbb{E}[U_1(\tilde{\theta}_1)] + \mathbb{E}[U_2(\tilde{\theta}_2)]\}$$

$$R(\bar{p},\hat{\theta}_2))$$

subject to (A.1), (ICFOC), and

$$U_1(0) \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad \text{IR}_1(0)$$
$$U_2(\hat{\theta}_2) \geq \bar{p}[1 - F_1(\bar{p})] + \hat{\theta}_2 F_1(\bar{p}). \qquad\qquad \text{IR}_2(\hat{\theta}_2)$$

We will first show that there is a type $\hat{\theta}_2 \in (\underline{\theta}_2(1), 1)$ such that the solution has $p = 1$. To see this, observe that by arguments in the proof of Lemma 4, the solution takes the form described by (13) and (A.5) [note that we are in the case $\underline{\theta}_2(p) \leq \underline{\theta}_2(1) < \hat{\theta}_2$]. Now maximization of the Lagrangian over p takes the form

$$p \in \arg\max_{p'\in[\hat{\theta}_2,1]}(1 - F_1(p'))\mathbb{E}[\hat{\delta}(\tilde{\theta}_2)\text{sign}\,(\tilde{\theta}_2 - \hat{\theta}_2)]. \qquad (\text{A.12})$$

By (A.4) and the fact that $\underline{\omega}_i(\theta_i|\lambda_1) \leq \theta_i \leq 1$ for each $i = 1, 2$, $\theta_i \in [0, 1]$, we must have

$$\hat{\delta}(\theta_2) = \max\{\bar{\omega}_2(\theta_2|\lambda_1) - \underline{\omega}_1(p|\lambda_1), 0\} \geq \max\{\bar{\omega}_2(\theta_2|\lambda_1) - 1, 0\} \quad \text{for } \theta_2 < \hat{\theta}_2,$$
$$\hat{\delta}(\theta_2) = \max\{\underline{\omega}_1(p|\lambda_1) - \underline{\omega}_2(\theta_2|\lambda_1), 0\} \leq 1 - \underline{\omega}_2(\theta_2|\lambda_1) \quad\qquad \text{for } \theta_2 > \hat{\theta}_2,$$

and therefore

$$\mathbb{E}[\hat{\delta}(\tilde{\theta}_2)\text{sign}\,(\tilde{\theta}_2 - \hat{\theta}_2)]$$

$$\leq \int_{\underline{\theta}_2(p|\lambda_1,0)}^{\underline{\theta}_2(1|\lambda_1,0)} [1 - \bar{\omega}_2(\theta_2|\lambda_1)]dF_2(\theta_2) + \int_{\underline{\theta}_2(1|\lambda_1,0)}^{\hat{\theta}_2} [1 - \bar{\omega}_2(\theta_2|\lambda_1)]dF_2(\theta_2)$$

$$+ \int_{\hat{\theta}_2}^{1} [1 - \underline{\omega}_2(\theta_2|\lambda_1)]dF_2(\theta_2)$$

$$= \int_{\underline{\theta}_2(p|\lambda_1,0)}^{\underline{\theta}_2(1|\lambda_1,0)} [1 - \bar{\omega}_2(\theta_2|\lambda_1)]dF_2(\theta_2) + \int_{\underline{\theta}_2(1|\lambda_1,0)}^{1} [1 - \bar{\omega}_2(\theta_2|\lambda_1)]dF_2(\theta_2)$$

$$+ \int_{\hat{\theta}_2}^{1} [\bar{\omega}_2(\theta_2|\lambda_1) - \underline{\omega}_2(\theta_2|\lambda_1)]dF_2(\theta_2).$$

The second integral is strictly negative (since $\bar{\omega}_2(\theta_2|\lambda_1) > \bar{\omega}_2(\underline{\theta}_2(1)|\lambda_1) = 1$ for all $\theta_2 > \underline{\theta}_2(1)$), while the first and third approach zero as $\hat{\theta}_2 \to 1$ (the third integral equals $\lambda_1(1 - \hat{\theta}_2)$, while $\underline{\theta}_2(p|\lambda_1,0) \to \underline{\theta}_2(1|\lambda_1,0)$ as $\hat{\theta}_2$, and hence p, approaches 1). Hence, their sum is negative for $\hat{\theta}_2 \in (\underline{\theta}_2(1), 1)$ close enough to 1. Then (A.12) implies that for such a value of $\hat{\theta}_2$, the solution to program $R(1, \hat{\theta}_2)$ has ironing level $p = 1$. At the solution, both type $\hat{\theta}_2 > \underline{\theta}_2(1)$ and type $\theta_2 = 1$ receive the good with probability 1, just as in the default, and so $\text{IR}_2(\hat{\theta}_2)$ implies $\text{IR}_2(1)$. Because the solution to program $R(1, 1)$ also satisfies $\text{IR}_2(\hat{\theta}_2)$, the solutions to programs $R(1, \hat{\theta}_2)$ and $R(1, 1)$ coincide. Therefore, the value of $R(1, \hat{\theta}_2)$ equals the value of $R(1, 1)$, which is the expected surplus in the Myerson–Satterthwaite solution (the second-best budget-balanced mechanism for agent 2's simple property right).

However, the value of program $R(1, \hat{\theta}_2)$ is strictly higher than that of program $R(\bar{p}, \hat{\theta}_2)$ for $\bar{p} \in (\hat{\theta}_2, 1)$, since the latter program's $\text{IR}_2(\hat{\theta}_2)$ is tighter by $[1 - F_1(\bar{p})](\bar{p} - \hat{\theta}_2) > 0$, while all the other constraints in the two programs coincide. In turn, the value of $R(\bar{p}, \hat{\theta}_2)$ weakly exceeds the expected surplus in the second-best optimal mechanism for option price $\bar{p}$, since that mechanism is budget-balanced and satisfies all of the constraints in program $R(\bar{p}, \hat{\theta}_2)$ (in particular, by (A.5), the ironing level in that mechanism is $p = \bar{p} > \hat{\theta}_2$).

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