Optimal Metric Distortion for Learning-Augmented Matchings on the Line



Jabari Hastings and **Marena Richter**

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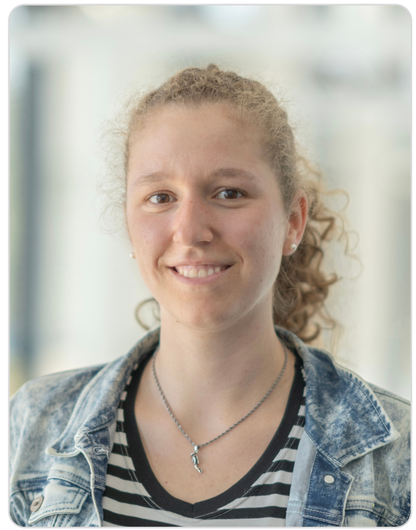


Motivation – School Choice Problem







Motivation – School Choice Problem













Motivation – School Choice Problem





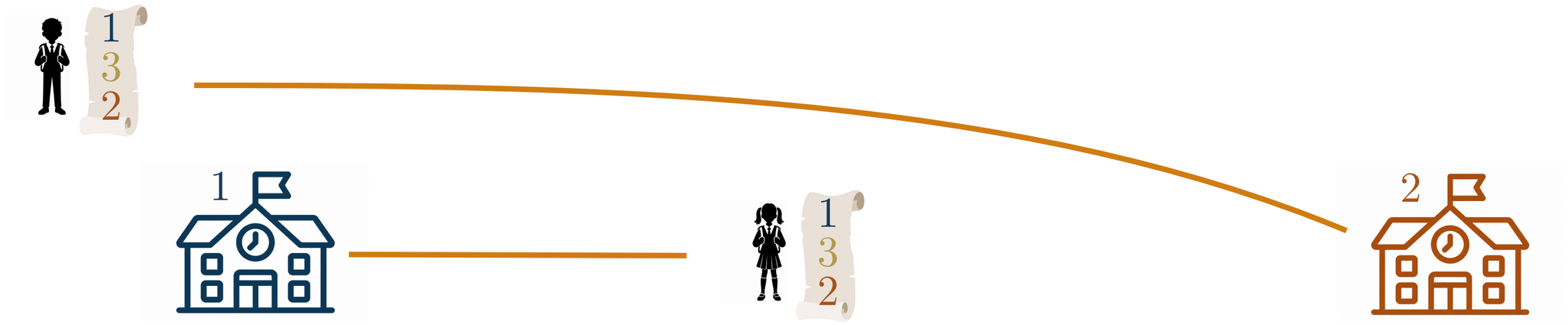








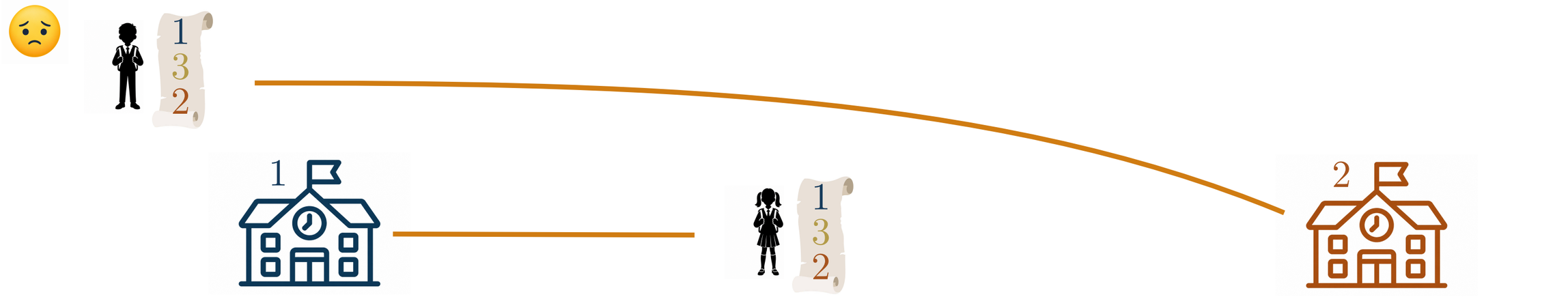
Motivation – School Choice Problem







Motivation – School Choice Problem







Motivation – School Choice Problem









St. Petersburg Times

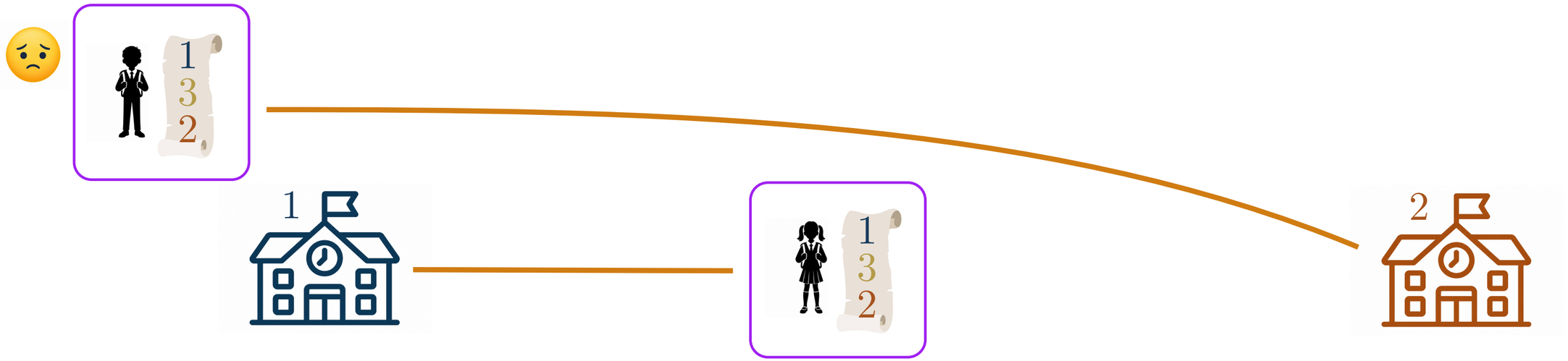
School choice

"This complicated new system requires you to be a savvy parent. Some parents have treated the choice application with a casualness befitting a form for summer camp. Yet it requires the same level of care you would give an IRS tax return."

Thomas Tobin — St. Petersburg Times, 2003



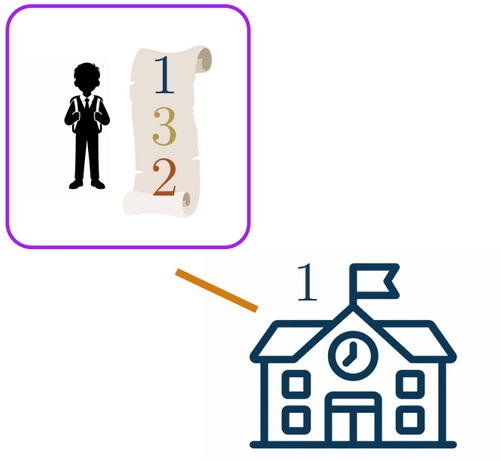
Motivation – School Choice Problem



Rankings: Loose information



Motivation – School Choice Problem





Rankings: Loose information



Motivation – School Choice Problem









Rankings: Loose information

How can we do better?





Motivation – Predictions

Input:

Rankings







+

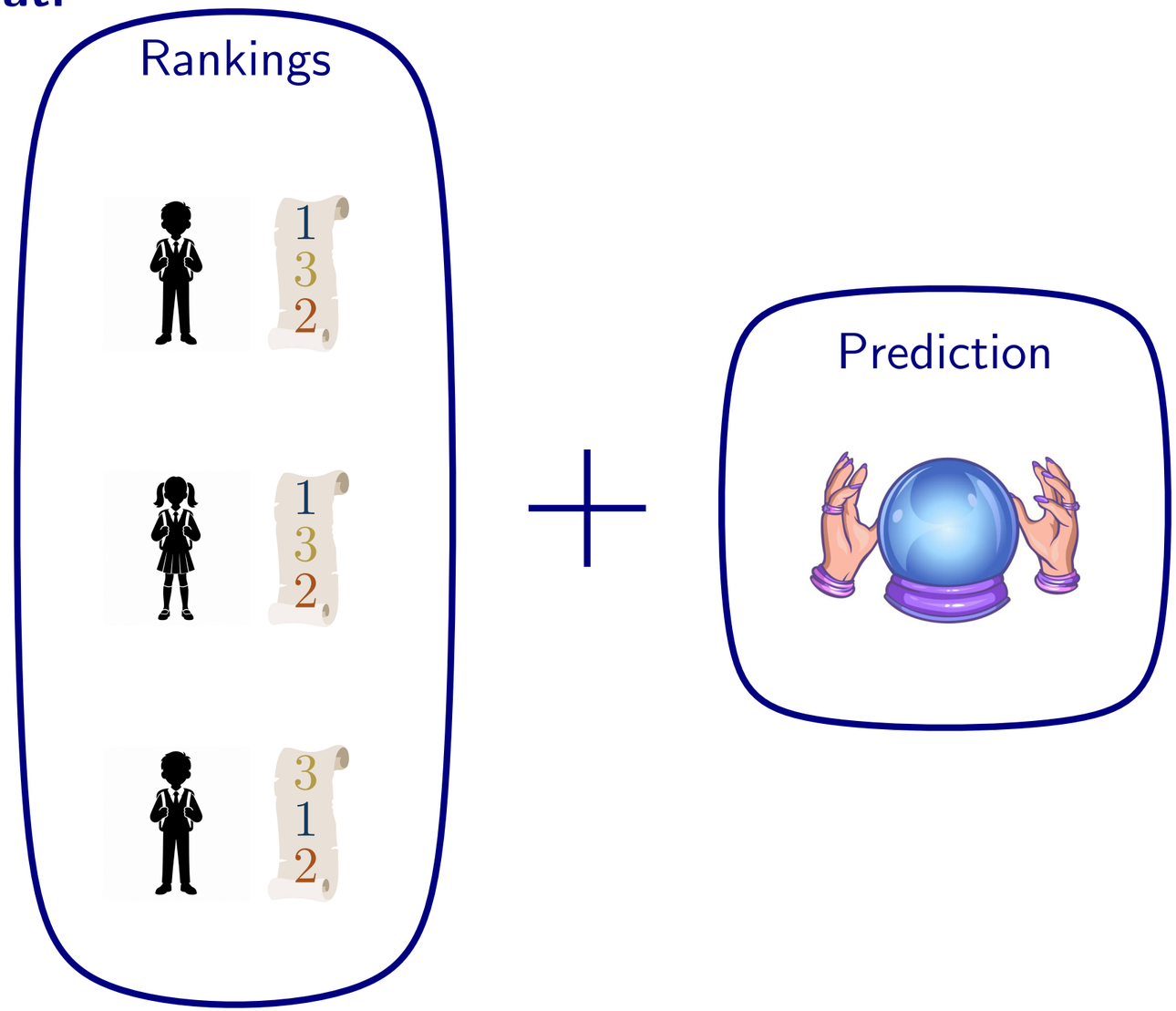
Prediction





Motivation – Predictions

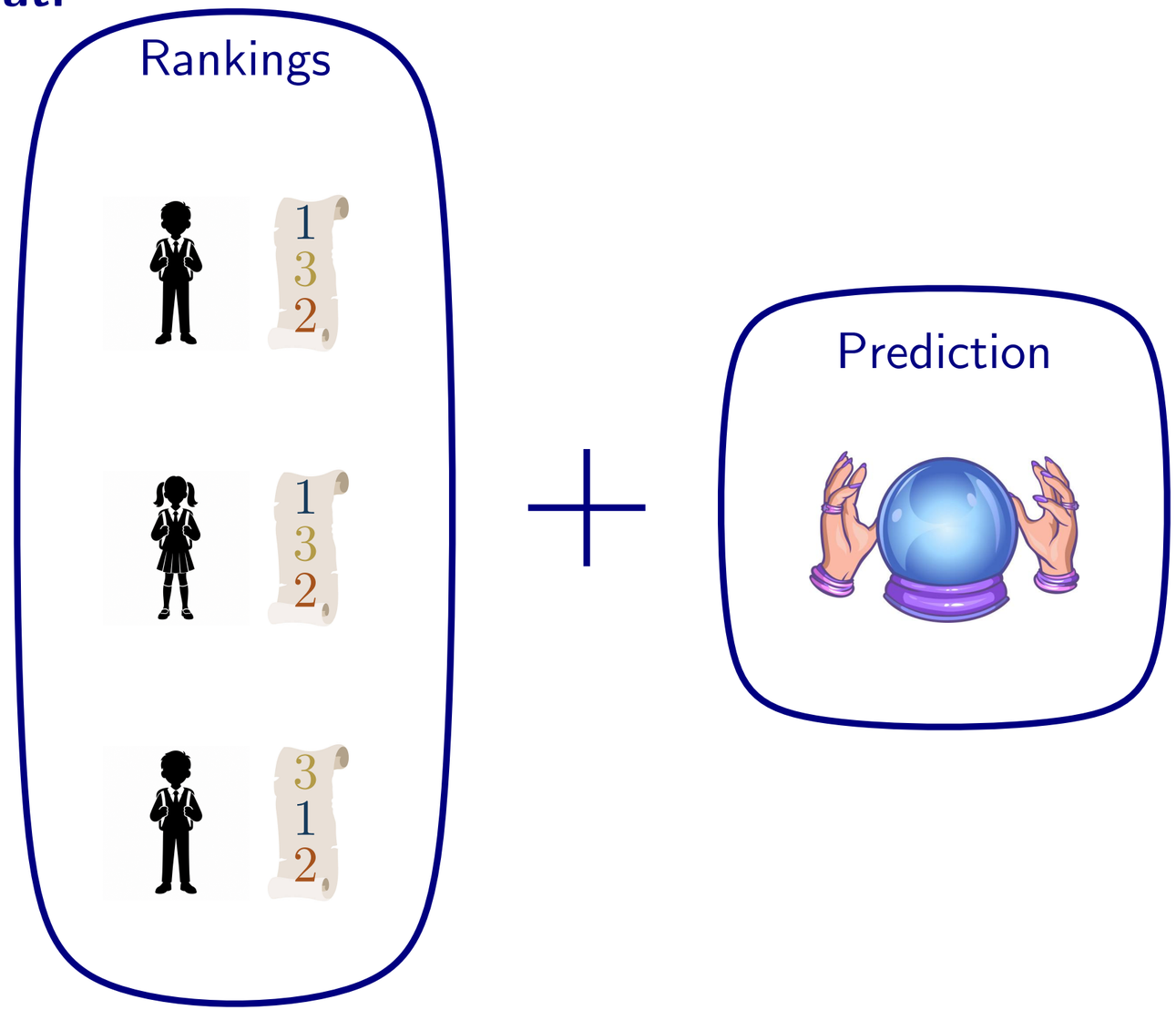
Input:



- Bring predictions to matching world

Motivation – Predictions

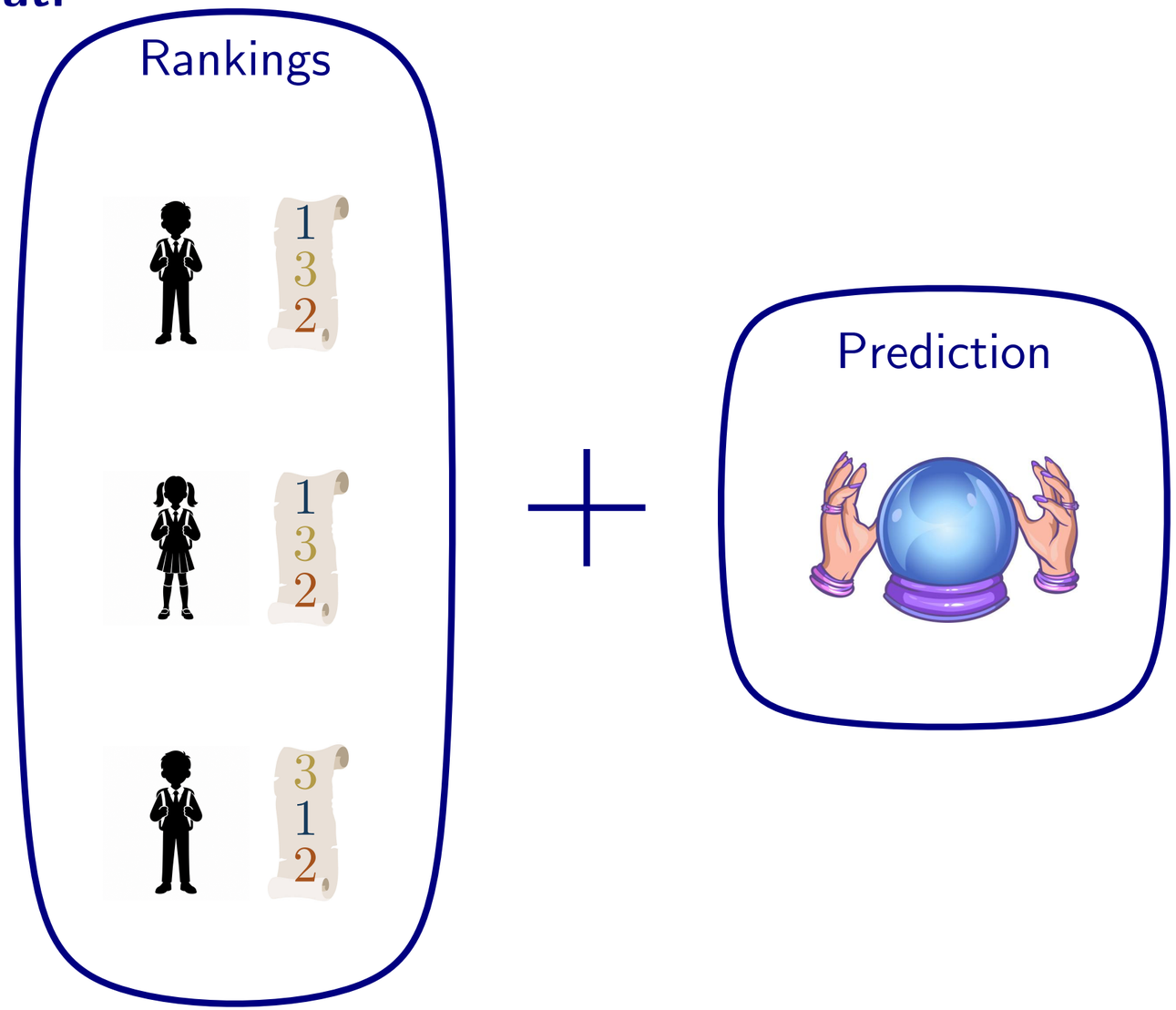
Input:



- Bring predictions to matching world
- Can additional data help?

Motivation – Predictions

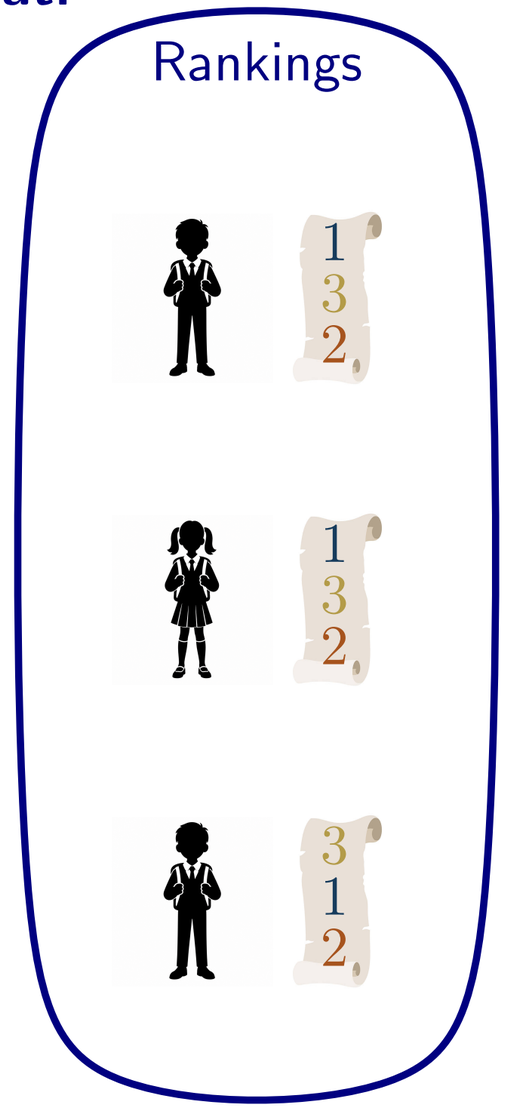
Input:



- Bring predictions to matching world
- Can additional data help?
- Based on past data

Motivation – Predictions

Input:





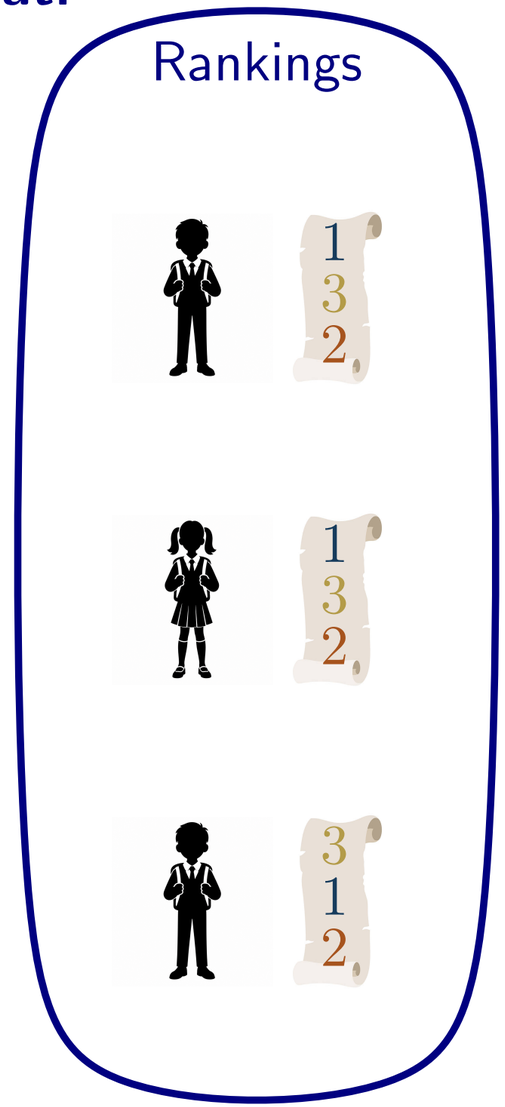


- Bring predictions to matching world
- Can additional data help?
- Based on past data
- Much data, unknown quality



Motivation – Predictions

Input:







- Bring predictions to matching world
- Can additional data help?
- Based on past data
- Much data, unknown quality

Proof of concept for matchings



line instances

Motivation – Predictions

Input:

Rankings







+

Prediction





Two goals for predictions



Consistency

With correct advice:
how close to optimum?

Motivation – Predictions

Input:

Rankings







+

Prediction



Two goals for predictions



Consistency

With correct advice:
how close to optimum?

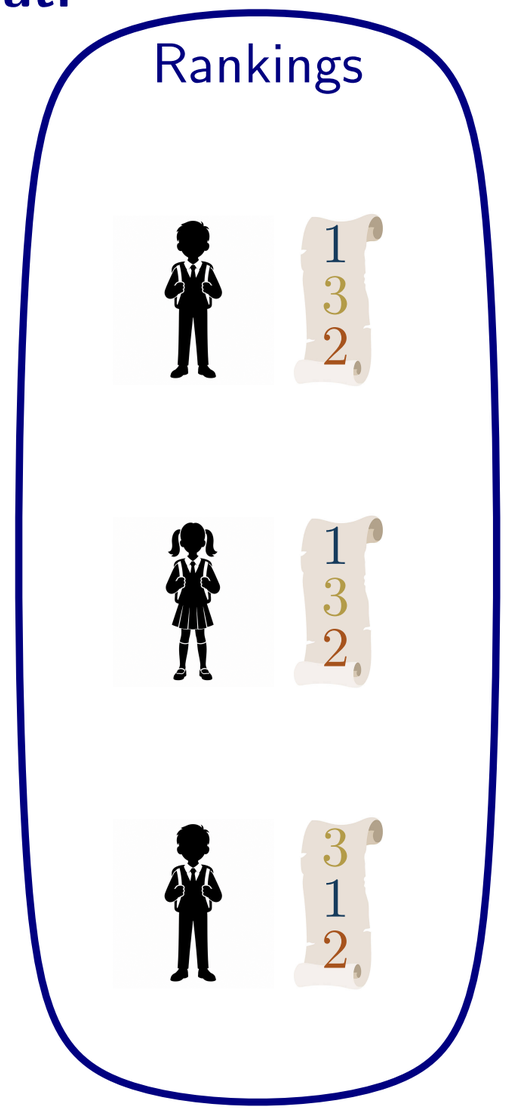


Robustness

With arbitrary advice:
what cost guarantee?

Motivation – Predictions

Input:







Two goals for predictions



Consistency

With correct advice:
how close to optimum?



Robustness

With arbitrary advice:
what cost guarantee?

Goal: optimal with perfect advice, safe even when the advice fails.

Problem Definition

Instance:

n agents
n items

ordinal ranking

σ :

a_1	a_2	a_3
b_1	b_3	b_3
b_2	b_2	b_2
b_3	b_1	b_1

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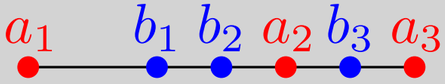
based on a metric instance

d_σ : 

here: a line

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a_1	a_2	a_3
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b_3	b_1	b_1

based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1
a_2 —— b_2
a_3 —— b_3

matching

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based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

Goal: Compute matching M a_1 —— b_1, a_2 —— b_3, a_3 —— b_2 optimizing some objective.

Problem Definition

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a_1	a_2	a_3
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b_2	b_2	b_2
b_3	b_1	b_1

based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

Goal: Compute matching M

a_1 —— b_1
a_2 —— b_3
a_3 —— b_2

optimizing some objective.

top_k cost $\widehat{=}$ sum of k largest distances.

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a_1	a_2	a_3
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based on a line

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Goal: Compute matching M

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a_3 —— b_2

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Theorem [F-R,G,L,R,V,2025]**:** There exists a deterministic **mechanism** that is given **only the rankings** and computes a 3**-approximation** on line instances **for all** k. This is **best possible** for line instances.

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b_3	b_1	b_1

based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

top_k cost $\widehat{=}$ sum of k largest distances.

Goal: Compute matching M

a_1 —— b_1
a_2 —— b_3
a_3 —— b_2

optimizing some objective.

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$$\text{distortion}_k(M,\sigma) := \sup_{d_\sigma} \frac{\text{top}_k(M,d_\sigma)}{\text{OPT}_k(d_\sigma)}$$

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based on a line

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prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

top_k cost $\widehat{=}$ sum of k largest distances.

Goal: Compute matching M

a_1 —— b_1
a_2 —— b_3
a_3 —— b_2

optimizing some objective.

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$$\text{distortion}_k(M,\sigma) := \sup_{d_\sigma} \frac{\text{top}_k(M,d_\sigma)}{\text{OPT}_k(d_\sigma)}$$

all possible line instances agreeing with σ

Problem Definition

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a_1	a_2	a_3
b_1	b_3	b_3
b_2	b_2	b_2
b_3	b_1	b_1

based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

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Goal: Compute matching M a_1 —— b_1, a_2 —— b_3, a_3 —— b_2 with small distortion.

Theorem [F-R,G,L,R,V,2025]**:** There exists a deterministic **mechanism** that is given **only the rankings** and computes a matching M that has **distortion** 3 on line instances **for all** k. This is **best possible** for line instances.

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a_1	a_2	a_3
b_1	b_3	b_3
b_2	b_2	b_2
b_3	b_1	b_1

based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

$$\text{distortion}_k(M,\sigma) := \sup_{d_\sigma} \frac{\text{top}_k(M,d_\sigma)}{\text{OPT}_k(d_\sigma)}$$

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Can we really not do better?



[F-R,G,L,R,V,2025]

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based on a line

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matching

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Can we really not do better?



[F-R,G,L,R,V,2025]



What if we use predictions?

Problem Definition

Instance:

n agents
n items

ordinal ranking

σ :

a_1	a_2	a_3
b_1	b_3	b_3
b_2	b_2	b_2
b_3	b_1	b_1

based on a line

+

prediction

$\widehat{M}$: a_1 —— b_1, a_2 —— b_2, a_3 —— b_3

matching

$$\text{distortion}_k(M,\sigma) := \sup_{d_\sigma} \frac{\text{top}_k(M,d_\sigma)}{\text{OPT}_k(d_\sigma)}$$

Goal: Compute matching M a_1 —— b_1, a_2 —— b_3, a_3 —— b_2 with small distortion.

Theorem: There exists a deterministic **mechanism** that computes a matching M with

$$\textbf{distortion}_k(M,\sigma) = \min\{3, \textbf{distortion}_k(\widehat{M},\sigma)\}$$

on **line instances for all k**.

prediction = 

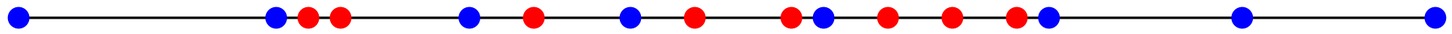


prediction = 



Understanding Optimum Solutions

Lemma [F-R,G,L,R,V]**:** On a known line, left-to-right matching minimizes top$_k$ cost for all k.



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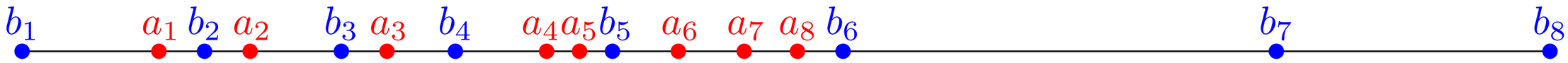
Understanding Optimum Solutions

Lemma [F-R,G,L,R,V]**:** On a known line, left-to-right matching minimizes top_k cost for all k.



Goal: Restore ordering.

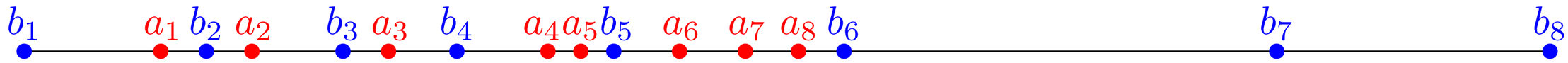
Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
b_7	b_7	b_7	b_7	b_7	b_1	b_1	b_8
b_8	b_8	b_8	b_8	b_8	b_8	b_8	b_1



Restoring Ordering

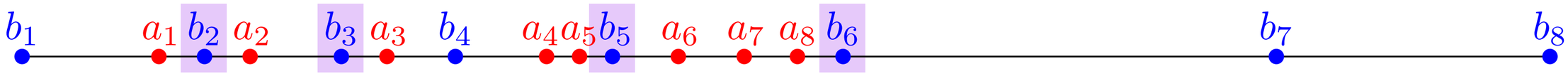


a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
b_7	b_7	b_7	b_7	b_7	b_1	b_1	b_8
b_8	b_8	b_8	b_8	b_8	b_8	b_8	b_1

① Find outermost top items b_ℓ and b_r.



Restoring Ordering



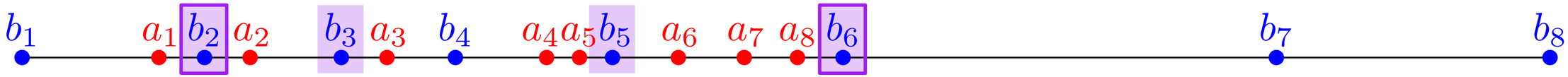
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
b_7	b_7	b_7	b_7	b_7	b_1	b_1	b_8
b_8	b_8	b_8	b_8	b_8	b_8	b_8	b_1

① Find outermost top items b_ℓ and b_r.

Consider all top items.



Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
b_7	b_7	b_7	b_7	b_7	b_1	b_1	b_8
b_8	b_8	b_8	b_8	b_8	b_8	b_8	b_1

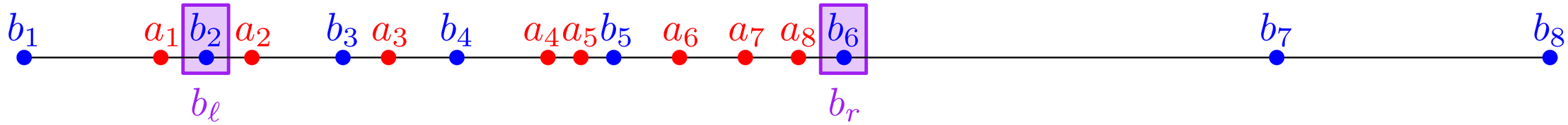
① Find outermost top items b_ℓ and b_r.

Consider all top items.

outermost $\widehat{=}$ bottom row of top items



Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
b_7	b_7	b_7	b_7	b_7	b_1	b_1	b_8
b_8	b_8	b_8	b_8	b_8	b_8	b_8	b_1

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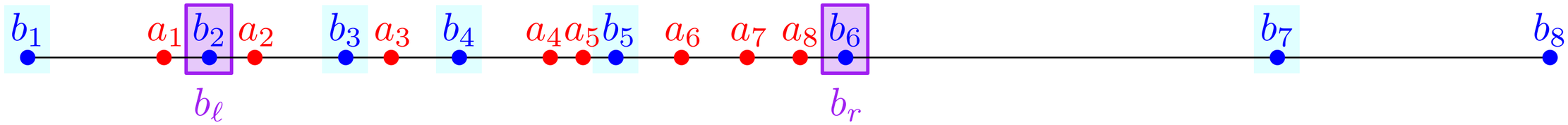
Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
b_7	b_7	b_7	b_7	b_7	b_1	b_1	b_8
b_8	b_8	b_8	b_8	b_8	b_8	b_8	b_1

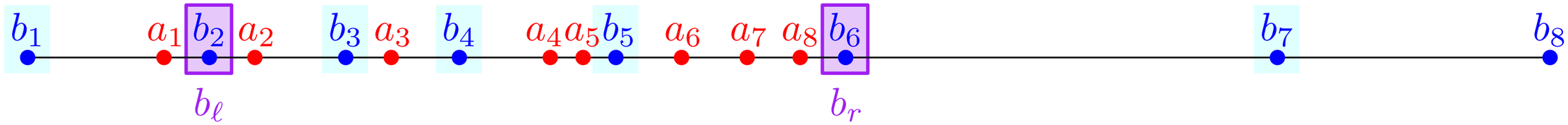
① Find outermost top items b_ℓ and b_r.

② Find sortable items.

$$B_{\text{in}} = \{b \in B : b \succeq_a b_\ell \text{ or } b \succeq_a b_r \text{ for some } a \in A\}$$



Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
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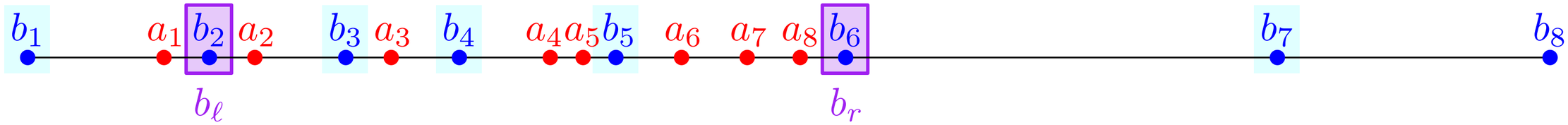
Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
b_1	b_3	b_4	b_4	b_4	b_6	b_5	b_5
b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
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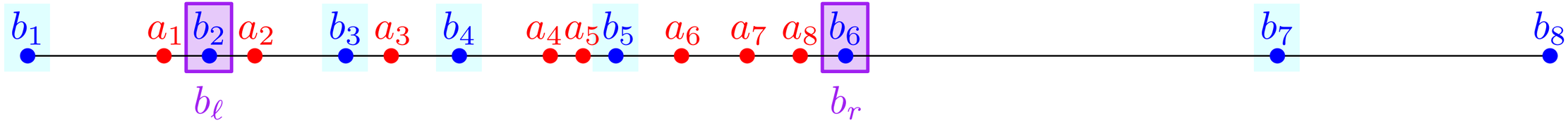
③ Find weak agent ordering.



Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
b_2	b_2	b_3	b_5	b_5	b_5	b_6	b_6
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b_3	b_4	b_2	b_3	b_3	b_4	b_4	b_4
b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
b_6	b_6	b_6	b_1	b_1	b_7	b_2	b_2
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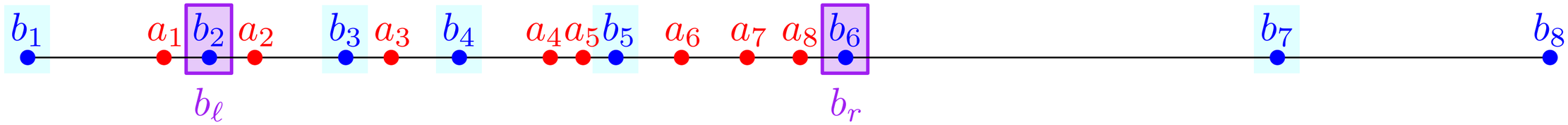

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b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
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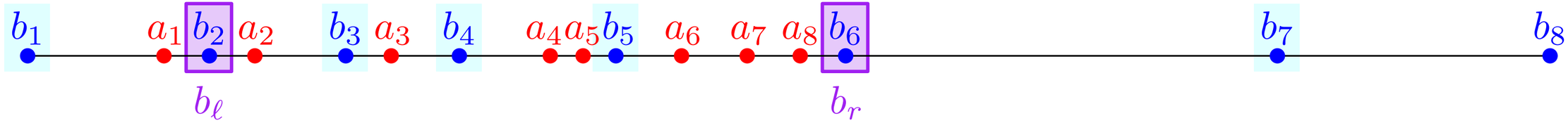
Not sorted: **Outliers:** $B_{\text{out}} = B \setminus B_{\text{in}}$



Restoring Ordering



a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
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b_4	b_1	b_5	b_6	b_6	b_3	b_3	b_3
b_5	b_5	b_1	b_2	b_2	b_2	b_7	b_7
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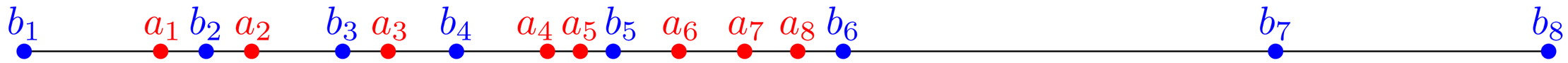
Known reconstruction [F-R,G,L,R,V,2025].

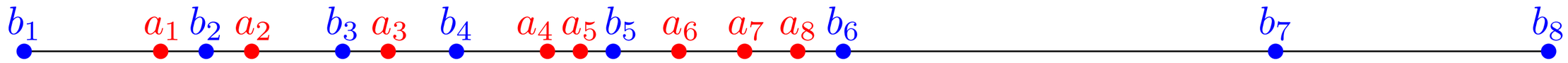
Matching with Reserved Outliers



Algorithm: Given: Preferences σ.



Matching with Reserved Outliers



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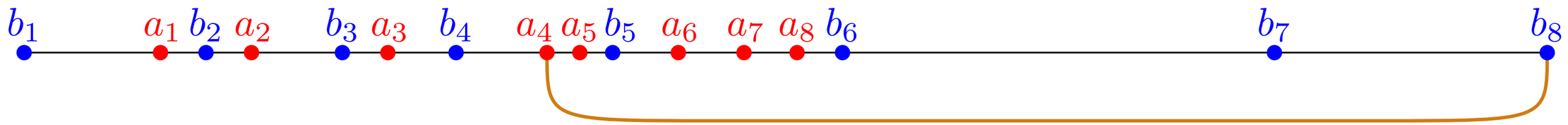

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② Assign outliers arbitrarily.



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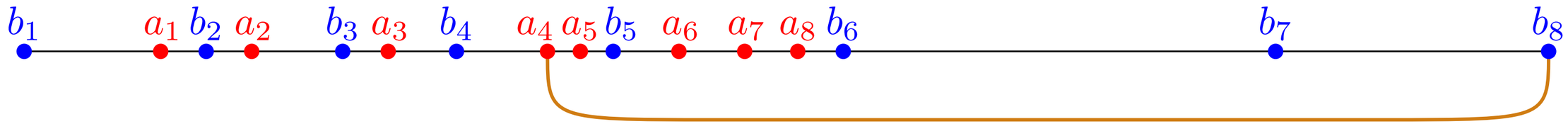

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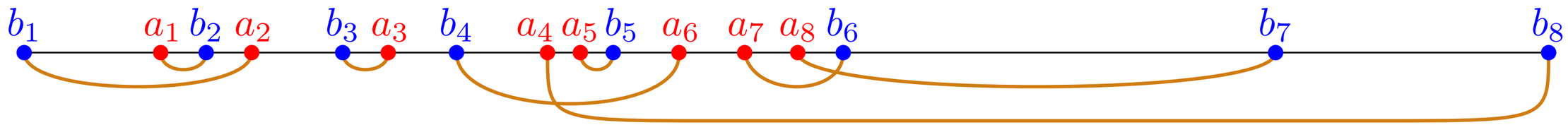
② Assign outliers arbitrarily.

③ Weakly order remaining agents

Agents with same top item: Break ties arbitrarily.



Matching with Reserved Outliers



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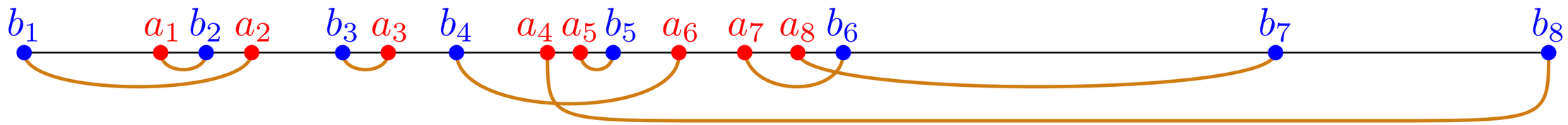
Matching with Reserved Outliers



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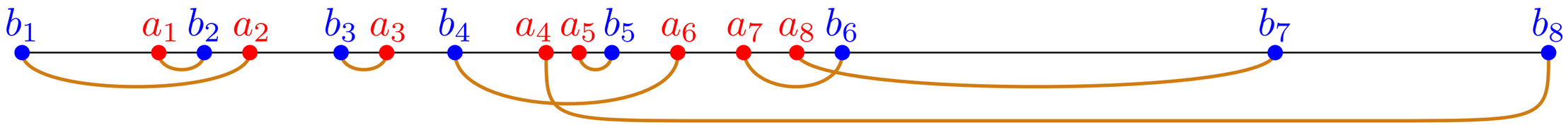
Lemma: $\text{distortion}_k(M, \sigma) \leq 3$.

Matching with Reserved Outliers



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Any choice works!

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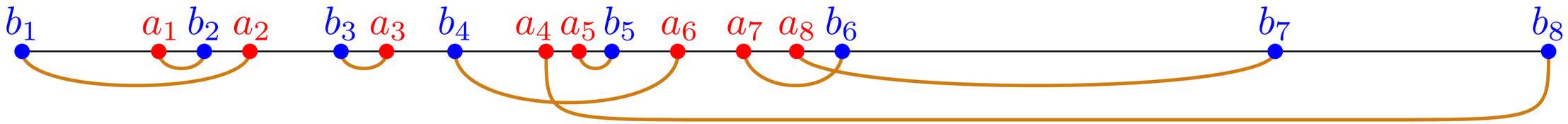

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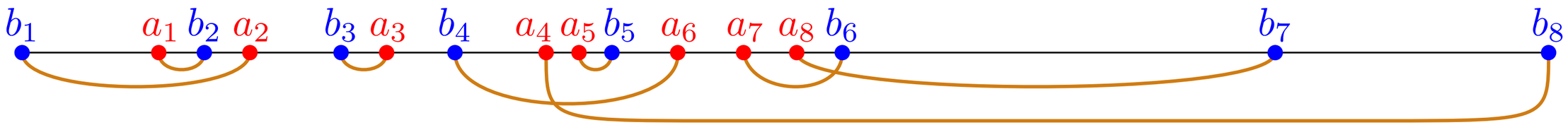

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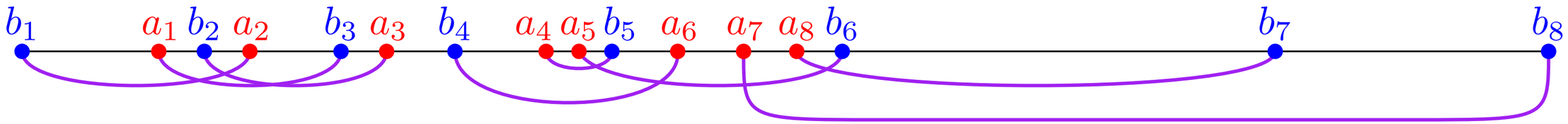
Matching with Reserved Outliers



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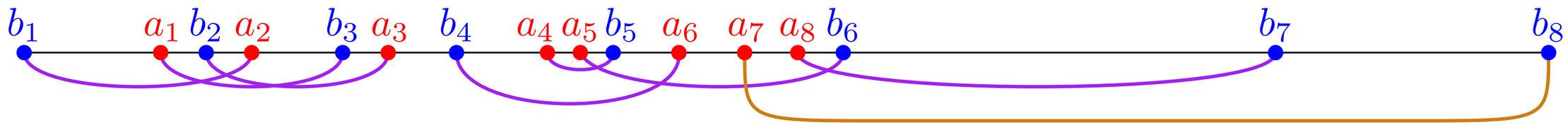

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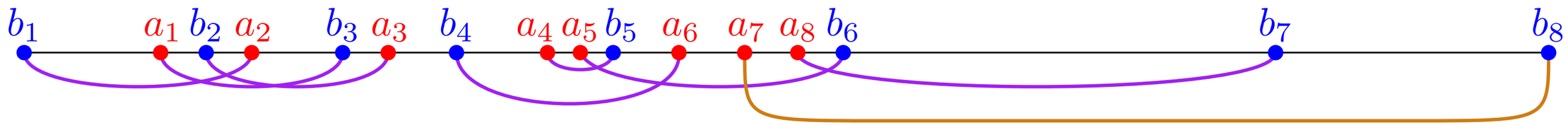

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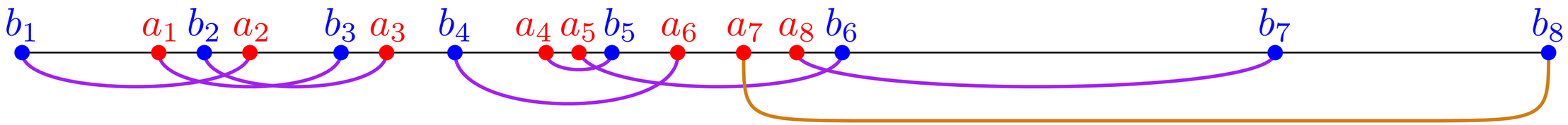

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③ Strongly order remaining agents

Break remaining ties using the predicted item order.

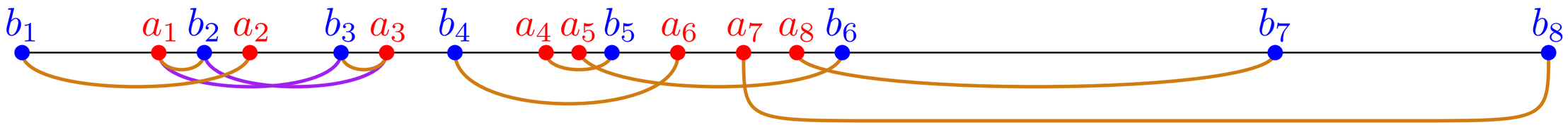
④ Match greedily left to right $\to M$

Lemma: $\text{distortion}_k(M, \sigma) \leq 3$.



Let's use the prediction!

Matching with Reserved Outliers



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Let's use the prediction!

Take Home Message



Predictions for matching

A proof of concept
for ordinal matching
on the line.

Take Home Message



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New family of matchings

Use prediction for:

- Assignment of Outliers
- order remaining agents

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1-consistency

3-robustness

Perfect advice: optimal.

Any advice: distortion ≤ 3.

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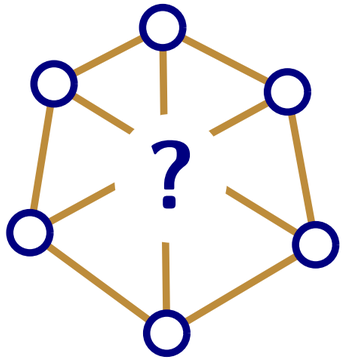

Open questions

- general metrics
- other forms of prediction
- other cost objectives

Take Home Message



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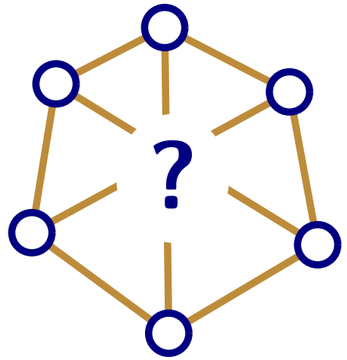
Perfect advice: optimal.

Any advice: distortion ≤ 3.



Open questions

- general metrics
- other forms of prediction
- other cost objectives

Thank you, questions?