ONLINE APPENDIX: SOCIAL INTERACTIONS AND LEGISLATIVE ACTIVITY

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Appendix A: Proofs

Proposition 2. The limit equilibrium is defined by equations (2.15)-(2.17).

Proof of Proposition 2. Recall that we have from equations (2.12) and (2.11), from the First Order Conditions, that:

$$c = \frac{\alpha_i}{x_i^*} + \frac{s_i^{*2}}{x_i^{*2}}, \tag{A.1}$$

and

$$\frac{s_i^*}{x_i^*} = \varphi_i \sum_{j \neq i} s_j^* m_{ij}(s^*) x_j^*. \tag{A.2}$$

We also use that:

$$\begin{aligned} x_i^* &= \alpha_i X_{P(i)} && \text{(A.3)} \\ s_i^* &= \alpha_i S_{P(i)}, && \text{(A.4)} \end{aligned}$$

for some $X_{P(i)}, S_{P(i)}$, which comes from the fact that $\frac{s_i^*}{x_i^*}$ and $\frac{x_i^*}{\alpha_i}$ are the same for all agents within a party. Let $P(i) \in \{1, 2\}$ be arbitrary.

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Using (A.3) in (2.12) implies:

$$\begin{aligned} c &= \frac{\alpha_i}{x_i^*} + \frac{s_i^{*2}}{x_i^{*2}} \\ &= \frac{\alpha_i}{\alpha_i X_{P(i)}} + \frac{\alpha_i^2 S_{P(i)}^2}{\alpha_i^2 X_{P(i)}^2} \\ &= \frac{1}{X_{P(i)}} + \frac{S_{P(i)}^2}{X_{P(i)}^2}. \end{aligned}$$

Multiplying both sides by $X_{P(i)}^2$ yields:

$$c X_{P(i)}^2 = X_{P(i)} + S_{P(i)}^2, \tag{A.5}$$

which is (2.17).

Let us now substitute (A.3) in (2.11):

$$\begin{aligned} \frac{\alpha_i S_{P(i)}}{\alpha_i X_{P(i)}} &= \varphi_{P(i)} \sum_{j \neq i} \alpha_j S_{P(j)} m_{ij}(s^*) \alpha_j X_{P(j)} \\ \frac{S_{P(i)}}{X_{P(i)}} &= \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(j)} S_{P(j)} m_{ij}(s^*) \\ &= \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(j)} S_{P(j)} \left(\frac{p(i)p(j)}{\sum_{k \in P(i), k \neq i} p(k) s_k^*} + \frac{(1-p(i))(1-p(j))}{\sum_{k \neq i} (1-p(k)) s_k^*} \right) I_{\{j \in P(i)\}} \\ &\quad + \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(j)} S_{P(j)} \left((1-p(i)) \frac{(1-p(j))}{\sum_{k \neq i} (1-p(k)) s_k^*} \right) I_{\{j \notin P(i)\}}. \end{aligned}$$

Note that for the first two terms, $p(i) = p(j)$ because they are only summed when $j \in P(i)$. For the last, $p(i) \neq p(j)$ as it is summed when $j \notin P(i)$.

Rewriting the above with this implies:

$$\begin{aligned} \frac{S_{P(i)}}{X_{P(i)}} &= \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(i)} S_{P(i)} \left(\frac{p(i)p(i)}{\sum_{k \in P(i), k \neq i} p(i) s_k^*} + \frac{(1-p(i))(1-p(i))}{\sum_{k \neq i} (1-p(k)) s_k^*} \right) I_{\{j \in P(i)\}} \\ &\quad + \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(j)} S_{P(j)} \left((1-p(i)) \frac{(1-p(j))}{\sum_{k \neq i} (1-p(k)) s_k^*} \right) I_{\{j \notin P(i)\}}. \end{aligned}$$

Using that $s_k^* = \alpha_k S_{P(k)}$ leads to:

$$\begin{aligned} \frac{S_{P(i)}}{X_{P(i)}} &= \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(i)} S_{P(i)} \left(\frac{p(i)^2 I_{\{j \in P(i)\}}}{p(i) \sum_{k \in P(i), k \neq i} \alpha_k S_{P(k)}} + \frac{(1-p(i))^2 I_{\{j \in P(i)\}}}{\sum_{k \neq i} (1-p(k)) \alpha_k S_{P(k)}} \right) \\ &\quad + \varphi_{P(i)} \sum_{j \neq i} \alpha_j^2 X_{P(j)} S_{P(j)} \left(\frac{(1-p(i))(1-p(j))}{\sum_{k \neq i} (1-p(k)) \alpha_k S_{P(k)}} \right) I_{\{j \notin P(i)\}}. \end{aligned}$$

Let us focus on the case of $P(i) = 1$, as the other case is symmetric.

$$\frac{S_1}{X_1} = \varphi_1 \sum_{j \neq i} \alpha_j^2 X_1 S_1 \left(\frac{p_1}{\sum_{k \in P(i), k \neq i} \alpha_k S_1} + \frac{(1-p_1)^2}{\sum_{k \neq i}(1-p(k))\alpha_k S_{P(k)}} \right) I_{\{j \in P(i)\}}$$
$$+ \varphi_1 \sum_{j \neq i} \alpha_j^2 X_2 S_2 \left(\frac{(1-p_1)(1-p_2)}{\sum_{k \neq i}(1-p(k))\alpha_k S_{P(k)}} \right) I_{\{j \notin P(i)\}}.$$

Finally, we use that:

$$\begin{aligned}
\sum_{k \neq i}(1-p(k))\alpha_k S_{P(k)} &= \sum_{k \neq i, k \in P(i)} (1-p(k))\alpha_k S_{P(k)} + \sum_{k \neq i, k \notin P(i)} (1-p(k))\alpha_k S_{P(k)} \\
&= \sum_{k \neq i, k \in P(i)} (1-p_1)\alpha_k S_1 + \sum_{k \neq i, k \notin P(i)} (1-p_2)\alpha_k S_2 \\
&= (1-p_1)S_1 \sum_{k \neq i, k \in P(i)} \alpha_k + (1-p_2)S_2 \sum_{k \neq i, k \notin P(i)} \alpha_k \\
&= (1-p_1)S_1 A_1 + (1-p_2)S_2 A_2.
\end{aligned}$$

To finalize the calculations, we use the simplification above for the denominators of the second and third terms.

Note that only α_j is now a function of the summand j itself, in the main expression. We also note that we can now use the indicators of $j \in P(i)$ for the first two terms, and $j \notin P(i)$ of the last term, within sums. These observations lead to the final equation:

$$\begin{aligned}
\frac{S_1}{X_1} &= \varphi_1 X_1 S_1 \sum_{j \neq i} \alpha_j^2 \left(\frac{p_1}{S_1 \sum_{k \in P(i), k \neq i} \alpha_k} + \frac{(1-p_1)^2}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) I_{\{j \in P(i)\}} \\
&\quad + X_2 S_2 \varphi_1 \sum_{j \neq i} \alpha_j^2 \left(\frac{(1-p_1)(1-p_2)}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) I_{\{j \notin P(i)\}} \\
&= \varphi_1 X_1 S_1 \sum_{j \neq i, j \in P(i)} \alpha_j^2 \left(\frac{p_1}{S_1 A_1} + \frac{(1-p_1)^2}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) \\
&\quad + X_2 S_2 \varphi_1 \sum_{j \neq i, j \notin P(i)} \alpha_j^2 \left(\frac{(1-p_1)(1-p_2)}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) \\
&= \varphi_1 X_1 S_1 B_1 \left(\frac{p_1}{S_1 A_1} + \frac{(1-p_1)^2}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) \\
&\quad + X_2 S_2 \varphi_1 B_2 \left(\frac{(1-p_1)(1-p_2)}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) \\
&= \varphi_1 \left(\frac{X_1 B_1 p_1}{A_1} + \frac{X_1 S_1 B_1 (1-p_1)^2}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} + \frac{X_2 S_2 B_2 (1-p_1)(1-p_2)}{(1-p_1)S_1 A_1 + (1-p_2)S_2 A_2} \right) \\
&= \varphi_1 \left(\frac{p_1 X_1 B_1}{A_1} + \frac{(1-p_1)^2 B_1 S_1 X_1 + (1-p_1)(1-p_2) B_2 X_2 S_2}{(1-p_1)A_1 S_1 + (1-p_2)A_2 S_2} \right).
\end{aligned}$$

□

Proof of Proposition 3. Recall that an interior equilibrium is a solution to (2.15) to (2.17).

So, rewriting these:

$$S_1 = X_1\varphi_1\left(\frac{p_1 B_1 X_1}{A_1} + \frac{(1-p_1)^2 B_1 S_1 X_1 + (1-p_1)(1-p_2)B_2 S_2 X_2}{(1-p_1)A_1 S_1 + (1-p_2)A_2 S_2}\right). \tag{A.6}$$

$$S_2 = X_2\varphi_2\left(\frac{p_2 B_2 X_2}{A_2} + \frac{(1-p_2)^2 B_2 S_2 X_2 + (1-p_1)(1-p_2)B_1 S_1 X_1}{(1-p_1)A_1 S_1 + (1-p_2)A_2 S_2}\right). \tag{A.7}$$

$$cX_1^2 = X_1 + S_1^2, \qquad cX_2^2 = X_2 + S_2^2. \tag{A.8}$$

Substituting (A.6) into (A.8) leads to

$$cX_1^2 = X_1 + X_1^2\varphi_1^2\left(\frac{p_1 B_1 X_1}{A_1} + \frac{(1-p_1)^2 B_1 S_1 X_1 + (1-p_1)(1-p_2)B_2 S_2 X_2}{(1-p_1)A_1 S_1 + (1-p_2)A_2 S_2}\right)^2.$$

or

$$cX_1 = 1 + X_1\varphi_1^2\left(\frac{p_1 B_1 X_1}{A_1} + \frac{(1-p_1)^2 B_1 S_1 X_1 + (1-p_1)(1-p_2)B_2 S_2 X_2}{(1-p_1)A_1 S_1 + (1-p_2)A_2 S_2}\right)^2. \tag{A.9}$$

There is a similar expression for S_2, X_2. Note that the right hand side of (A.9) lies above the left hand side as we approach $X_1 = 0$ (same for X_2). To have an interior solution, we need the right hand side to sometimes fall at or below the left hand side for positive X_1.

Suppose that the equilibrium (when it exists) is such that $X_1 \geq X_2$, and the other case is analogous just reversing subscripts everywhere. Then the right hand side is less than what we get by replacing X_2 by X_1, and so we want

$$cX_1 \geq 1 + X_1^3\varphi_1^2\left(\frac{p_1 B_1}{A_1} + \frac{(1-p_1)^2 B_1 S_1 + (1-p_1)(1-p_2)B_2 S_2}{(1-p_1)A_1 S_1 + (1-p_2)A_2 S_2}\right)^2. \tag{A.10}$$

for some interior X_1. Rewriting

$$cX_1 \geq 1 + X_1^3\varphi_1^2\left(\frac{p_1 B_1}{A_1} + \frac{(1-p_1)^2 B_1 + (1-p_1)(1-p_2)B_2\frac{S_2}{S_1}}{(1-p_1)A_1 + (1-p_2)A_2\frac{S_2}{S_1}}\right)^2. \tag{A.11}$$

The right hand side is maximized either at $\frac{S_2}{S_1} = 0$ or $\frac{S_2}{S_1} = \infty$, and so it is sufficient to have

$$cX_1 \geq 1 + X_1^3\varphi_1^2\left(p_1\frac{B_1}{A_1} + (1-p_1)\max\left[\frac{B_1}{A_1}, \frac{B_2}{A_2}\right]\right)^2. \tag{A.12}$$

Let

$$D_1 = p_1\frac{B_1}{A_1} + (1-p_1)\max\left[\frac{B_1}{A_1}, \frac{B_2}{A_2}\right]$$

Then (A.12) can be rewritten as

$$cX_1 \geq 1 + X_1^3 \varphi_1^2 D_1^2. \tag{A.13}$$

for some positive X_1. Note that

$$D_1 \leq D = \max\left[\frac{B_1}{A_1}, \frac{B_2}{A_2}\right]$$

So, it is sufficient to have

$$cX_1 \geq 1 + X_1^3 \varphi_1^2 D^2. \tag{A.14}$$

for some positive X_1.

It is necessary and sufficient to check that the left hand side and right hand side are tangent at the point at which the slope of the right hand side is c. This happens at $X_1 = \sqrt{\frac{c}{3\varphi_1^2 D^2}}$ and then the corresponding sufficient condition becomes:

$$c\left(\frac{c}{3\varphi_1^2 D^2}\right)^{1/2} \geq 1 + \left(\frac{c}{3\varphi_1^2 D^2}\right)^{3/2} \varphi_1^2 D^2, \tag{A.15}$$

or

$$\frac{2c^{3/2}}{3\sqrt{3}} \geq \varphi_1 D. \tag{A.16}$$

Having this hold also for the other case, leads to the claimed expression. □

Appendix B: Additional Aspects of the Theory

B.1. Best Response Dynamics

Best response dynamics are described as follows. Consider starting at some vectors s^0, x^0. Then the best response dynamics are described by:

$$s_i^t = x_i^{t-1} \varphi_i \sum_{j \neq i} m_{ij}(s^{t-1}) s_j^{t-1} x_j^{t-1}, \tag{B.1}$$

and

$$x_i^t = \frac{\alpha_i}{c} + s_i^{t-1} \frac{\varphi_i}{c} \sum_{j \neq i} m_{ij}(s^{t-1}) s_j^{t-1} x_j^{t-1}. \tag{B.2}$$

It follows that if $s^0 = \mathbf{0}$, then $m_{ij}(s^{t-1}) = 0$ for all ij (recall Footnote 11) and we get immediate convergence to $s_i^t = 0, x_i^t = \frac{\alpha_i}{c}$ for all t. Otherwise, s^t, x^t will be positive for all t.

To see how these best response dynamics work for a special case, let us consider the situation in which there is some S^0, X^0 such that $s_i^0 = \alpha_i S^0$ and $x_i^0 = \alpha_i X^0$ (which has to eventually hold at any limit point) - i.e. when we can use Proposition 2.[1]

1. This is also useful in determining the instability of equilibria.

In that case, working with the limiting or continuum case, in which the matching function is symmetric within a party, and presuming that $S_k^{t-1} > 0$ for each party (which happens after the first period if some $s_j^0 > 0$ and otherwise the solution is already described above), we end up with the following dynamics. For party k (letting k' denote the other party):

$$S_k^t = X_k^{t-1}\varphi_k \left(m_{kk}(S^{t-1})B_k S_k^{t-1}X_k^{t-1} + m_{kk'}(S^{t-1})B_{k'}S_{k'}^{t-1}X_{k'}^{t-1}\right), \qquad \text{(B.3)}$$

and

$$X_k^t = \frac{1}{c} + S_k^{t-1}\frac{\varphi_k}{c}\left(m_{kk}(S^{t-1})B_k S_k^{t-1}X_k^{t-1} + m_{kk'}(S^{t-1})B_{k'}S_{k'}^{t-1}X_{k'}^{t-1}\right). \qquad \text{(B.4)}$$

where

$$m_{kk}(S^{t-1}) = \frac{p_k}{S_k A_k} + \frac{(1-p_k)^2}{(1-p_1)S_1A_1 + (1-p_2)S_2A_2},$$

and

$$m_{kk'}(S^{t-1}) = \frac{(1-p_1)(1-p_2)}{(1-p_1)S_1A_1 + (1-p_2)S_2A_2}.$$

B.2. Discussion of the Model

The extensive literature on network formation, starting from its early incarnation in Jackson and Wolinsky (1996); Dutta and Mutuswami (1997); Bala and Goyal (2000); Currarini and Morelli (2000); Jackson and Watts (2002); Jackson (2005); Herings, Mauleon, and Vannetelbosch (2009), provides insight into how networks form, when inefficient networks form, and how that depends on the setting. More recently, the literature has also begun to develop models that incorporate some heterogeneity and are still tractable enough to allow for fitting the models to data, as in Leung (2015); Sheng (2020); Chandrasekhar and Jackson (2016); Mele (2017); Graham (2017); de Paula, Richards-Shubik, and Tamer (2018); Leung (2019); and some of that literature also allows for homophily, such as Currarini, Jackson, and Pin (2009, 2010); Banerjee, Chandrasekhar, Duflo, and Jackson (2018); Mele (2018). The models that are tractable enough to fit to data require a structure that limits the multiplicity of stable (equilibrium) networks, and such that those can be estimated with a practical number of calculations.

We only have a handful of such estimable models that involve non-trivial interaction effects; and generally those models are stylized in some way. For instance, as shown in Sheng (2020), the choice of the specific model can be important since models with indirect network effects (utility from friends-of-friends) lead to (i) a lack of identification (multiple configurations of parameters leading to the same outcomes) and, (ii) computational intractability with as few as 20 players, due to a curse of dimensionality. To make progress, she proposes a model with endogenous links that have “dependence [that] has a particular structure such that conditional on some network heterogeneity and individual heterogeneity, the links become independent.” An alternative approach is that of Mele (2017). He proposes an empirical model of network formation that allows for homophily in network formation. Again, he shows that there is a curse of dimensionality in using standard estimation methods unless some strong asymptotic independence conditions are satisfied. Other approaches are to have certain subgraphs generate value and then model the formation of those subgraphs directly (Chandrasekhar and Jackson 2016), or to have payoffs based on combinations of individual characteristics, geography, or assortativity (e.g., Currarini et al. 2009; Leung 2015; Graham 2017; Leung 2019).

Here we want a model in which the value to a given pairing depends on their subsequent mutual (legislative) efforts, and so we need a model in which expected values of links can be calculated conditional upon future efforts, and those efforts can also be characterized as a function of the pairings. Using random meeting probabilities to derive link formation does exactly this by reducing the dimension of choices while allowing for rich interdependencies, homophily, and still yielding a clean characterization of both types of efforts. The model we work with is the only one we have found in the literature that fits all of these criteria, and which are needed for this application.

In summary, one has to be judicious in modeling network formation to obtain a formulation that also allows for homophily, as well as choices that affect network positions, and remains both well-identified and estimable. Meanwhile, existing empirical models of games on networks that are well-identified (e.g., de Paula et al. 2019, advancing the work of Bramoullé et al. 2009) do not allow for endogenous networks - they assume that the network is fixed and exogenous, and require a different data set-up than ours.[2]

Thus one can see why models that incorporate both behavior and network formation are few: Cabrales, Calvo-Armengol, and Zenou (2011); König, Tessone, and Zenou (2009); Goldsmith-Pinkham and Imbens (2013); Hiller (2017); Badev (2017, 2020); Hsieh, Lee, and Boucher (2019); Hsieh, König, and Liu (2020). These models necessarily sacrifice some richness in order to incorporate both network formation and endogenous behaviors and to allow for an interaction between them. Nonetheless, they can still be quite rich and, as we show here, can still fit data well. Of this class, in order to work with a tractable model that we can extend to have yet a third dimension of group identity and homophily, and still take to (static) data, we build upon the model of Cabrales, Calvo-Armengol, and Zenou (2011).[3] This introduces another dimension to the estimation, of group interaction rates, and thus requires that the model be tractable

2. For example, to recover an unobserved exogenous network as they set out to, de Paula et al. (2019) assumes (i) an exogenous network that is sufficiently sparse, (ii) the network does not change over time, (iii) a panel data structure, with large enough time-series dimension, and (iv) a linear in means model. Our set-up and data structure do not have any of these 4 properties, as alluded to previously. Furthermore, the estimation of this set-up must involve shrinkage estimators and their resulting bias due to the size of the parameter space (N^2 parameters to recover from just the network itself).

3. Recent alternatives that accommodate all three dimensions include Badev (2020), Hsieh et al. (2019) and Hsieh et al. (2020). While they constitute important advances to the literature, their solutions are not applicable to our problem. For instance, all of them assume observable networks and unidimensional actions (in the case of Badev 2020, actions are binary). In terms of estimation, we complement their

enough to still solve with a third dimension of endogeneity. Finally, a close look at the fit of the data provides support for our modeling choices.[4] In Section 6.1, we show that the predicted links from the model are highly correlated with measures that use disaggregate data between Congress members (e.g. Fowler 2006), even if we do not use the latter in estimation. Second, strategic outcomes on this network, such as the probability of bill approval, are fit well within and across parties/politicians. Third, homophily is quantitatively important: a model with homophily fits the data significantly better than one without it. Fourth, our model is shown to outperform alternative approaches to the characterization of G in terms of in-sample mean squared error.

B.3. Examples of Equilibrium G Across Parameter Configurations

In our model, $\{\alpha_i, c, \varphi_i, p_1, p_2\}$ parametrize the incentives for social and legislative effort. The network G is then generated by agents' strategic decisions, taking those incentives into account. Here, we showcase the rich class of equilibrium networks G that can arise in as the parameters vary.

The parameter values are purposefully kept similar to those in Figure 1 for comparability, although we set $n_1 = n_2 = 2$ to visualize G (a 4 x 4 matrix). For convenience, we let politicians 1 and 2 be in Party 1, and politicians 3 and 4 in Party 2, so that the first two rows/columns of G below represent connections among Party 1 members. We set $c = 2.25$ and we keep φ_i constant within parties. For Party 1, we set $\varphi_1 = 1$. We set $\alpha_i = 1$ for one member in each party (politicians 1 and 3). Our examples below vary the remaining five parameters $\{p_1, p_2, \varphi_2, \alpha_2, \alpha_4\}$.

Example 1: Complete Bipartisanship

Parameters: $p_1 = p_2 = 0, \alpha_2 = \alpha_4 = 1, \varphi_2 = 1.$

$$G = \begin{bmatrix} 0 & 0.063 & 0.063 & 0.063 \\ 0.063 & 0 & 0.063 & 0.063 \\ 0.063 & 0.063 & 0 & 0.063 \\ 0.063 & 0.063 & 0.063 & 0 \end{bmatrix}$$

There are no partisan biases in relative rates of meeting potential partners, and all members end up equally connected. This is because all politicians are identical, have high enough types for social interaction to occur, there is no homophily to bias social interactions ($p_1 = p_2 = 0$), and there are no differential incentives to socialize by party ($\varphi_1 = \varphi_2$). Here, all politicians exert the same social efforts s_i.

Bayesian methods with a frequentist approach which shows identification of our parameters and provides a less computationally intensive estimation procedure.

4. Most notably, we see the value of (i) a (biased) random socialization protocol, (ii) choices made on effort levels, and (iii) mean-zero i.i.d. measurement errors on our observed proxies.

Example 2: Bias in Mixing

Parameters: $p_1 = p_2 = 0.5, \alpha_2 = \alpha_4 = 1, \varphi_2 = 1$.

$$G = \begin{bmatrix} 0 & 0.094 & 0.031 & 0.031 \\ 0.094 & 0 & 0.031 & 0.031 \\ 0.031 & 0.031 & 0 & 0.094 \\ 0.031 & 0.031 & 0.094 & 0 \end{bmatrix}$$

Relative to Example 1, the introduction of partisanship (structural homophily $p_1 = p_2 = 0.5$) biases social interactions along party lines, despite politicians having identical types (α_is) and identical party-level incentives to socialize (φs). In this example, politicians all exert the same social efforts s_i.

Example 3: Full Partisanship

Parameters: $p_1 = 1, p_2 = 0.5, \alpha_2 = \alpha_4 = 1, \varphi_2 = 1$.

$$G = \begin{bmatrix} 0 & 0.125 & 0 & 0 \\ 0.125 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.125 \\ 0 & 0 & 0.125 & 0 \end{bmatrix}$$

Party 1 is fully partisan, so its members can never meet those in Party 2. This induces full sorting along party lines. Even though party 2 would be willing to mix with party 1, they do not manage to, given that party 1 does not mix.

Example 4: Nuanced Biased Socialization

Parameters: $p_1 = p_2 = 0.5, \alpha_2 = \alpha_4 = 0.5, \varphi_2 = 1$.

$$G = \begin{bmatrix} 0 & 0.048 & 0.032 & 0.016 \\ 0.048 & 0 & 0.016 & 0.008 \\ 0.032 & 0.016 & 0 & 0.048 \\ 0.016 & 0.008 & 0.048 & 0 \end{bmatrix}$$

We revisit Example 2, but decrease the type of one politician in each party ($\alpha_2 < 1, \alpha_4 < 1$). As a result, socialization is still biased along parties, but in a heterogeneous way. Politicians socialize more often within parties, but the higher type politicians (politicians 1 and 3) have greater incentives to legislate, and that induces them to socialize more, and they are more likely to meet across party lines than the low type ones.

Example 5: Nuanced Biased Socialization II

Parameters: $p_1 = p_2 = 0.5, \alpha_2 = \alpha_4 = 0.5, \varphi_2 = 1.3$.

$$G = \begin{bmatrix} 0 & 0.046 & 0.039 & 0.019 \\ 0.046 & 0 & 0.019 & 0.010 \\ 0.039 & 0.019 & 0 & 0.075 \\ 0.019 & 0.010 & 0.075 & 0 \end{bmatrix}$$

We now increase φ_2 relative to Example 4. This increases the incentives to socialize for politicians in Party 2, yielding stronger equilibrium connections among them. However, politicians in Party 1 understand this and increase their social efforts as well. This allows them meet those Party 2 members more often (since the latter's externalities are now higher). Compared to Example 4, this yields stronger connections between Politician 1 and those in the opposing party, but weaker connections within Party 1 *despite homophily*.

Example 6: Reversal of Partisanship Despite Homophily

Parameters: $p_1 = 0.3, p_2 = 0.5, \alpha_2 = 0.5, \alpha_4 = 1, \varphi_2 = 1.3$.

$$G = \begin{bmatrix} 0 & 0.045 & 0.051 & 0.051 \\ 0.045 & 0 & 0.026 & 0.026 \\ 0.051 & 0.026 & 0 & 0.160 \\ 0.051 & 0.026 & 0.160 & 0 \end{bmatrix}$$

We now begin with the parameters in Example 5, and then decrease p_1 and increase α_4. The high type politician in Party 1 now has stronger connections with opposing party politicians than with 1's own party member, *despite Party 2's strong homophily*. Politicians in Party 1 internalize the stronger types and incentives to socialize in Party 2, and choose effort that is large enough to overcome such homophily.

B.4. Endogenous Partisanship

A natural extension of our model would be to endogenize the p_i's. We comment here on potential directions and issues that arise.

First, it is easy to see that if one simply endogenized the p_i's within the current model without introducing any costs of affecting p_i, then the solutions would be corner solutions. If a group can choose its p_i without having any costs of selecting p_i, then (generically in the parameters) one of the two groups would want to be entirely partisan, since one of the two groups would find interacting with itself more beneficial than interacting across the aisle. Such a corner solution is clearly of little interest, and is incompatible with our empirical estimates.

More generally, there are interactions, both within and across parties, that happen naturally due to committee membership among other things and would be difficult to prevent, and others that might be costly to encourage. This suggests that there would minimum and maximum levels of partisanship that could be attained and also that one would need to model a nonlinear cost of partisanship. Once one provided a nonlinear cost to capture the high cost of going to either extreme of $p_i = 0$ or $p_i = 1$, one would end up with an interior equilibrium. A challenge would be that this could be dependent upon the cost formulation, and so one would need to work with a flexible enough cost function to allow the model to fit the data.

Having three endogenous choices for each of the two parties - partisanship, social effort, and legislative effort - would then end up producing a model for which analytic characterizations of the equilibrium would no longer be possible, and for which the multiplicity of equilibria would more difficult to ascertain. There would be two approaches. One would be to work entirely with numerical simulations. Since the interest in endogenizing partisanship levels would presumably be to understand how they interact with other variables and change incentives, this would require a very rich and complex set of simulations, especially as they would be sensitive to the choice of the cost function.

Another approach, and perhaps the most fruitful, would be to fix one of the other effort variables and return to a model in which there are just two different action variables that agents/parties are making. Given the importance of partisanship on the endogeneity of the network, a starting point might be to fix the x_i's and then work with the other variables. This could be an interesting approach for further research. We chose to work with endogenizing the network and legislative effort, holding partisanship constant, as these seem to be the first-order questions, but understanding partisanship is also a very interesting topic.

Appendix C: Formal Arguments for Identification

Recall that in our extension in Section 2.3, preferences are given by:

$$\tilde{u}_i(x_i, x_{-i}) = \alpha_i x_i + \varphi_i \sum_j g_{ij} x_i x_j - \frac{1}{2} c x_i^2 - \frac{1}{2} s_i^2, \tag{C.1}$$

where α_i is now interpreted as the heterogeneous marginal cost of legislative effort for i and $\varphi_i \equiv \gamma_{P(i)} e^{-\lambda \rho V_{i,0}} (1 - e^{-\lambda \zeta_{P(i)}})$, where $\zeta_{P(i)}$ was the electoral return to passing a bill (in the reelection equation), $\gamma_{P(i)}$ was the scale parameter in the shock for passing the bill, λ was the parameter from the exponentially distributed reelection shock, and $V_{i,0}$ is the winning margin for i in the previous election.

We now prove (point) identification of the following parameters from this model: $\{\{\alpha_i\}_{i=1}^n, \lambda\rho, \{\lambda\zeta_{P(i)}\}_{P(i)=1,2}, c, \tilde{A}_1\gamma_1, \tilde{A}_2\gamma_2\}$. To prove identification, we make use of the equilibrium conditions of s_i^* and x_i^* derived from the first order conditions. These are given in equations (2.7) and (2.8) in the main text. We also use equation (2.4) on the probability of passing a bill. It will also prove useful to work with the equation combining (2.8) into (2.7):

$$x_i^* = \frac{1}{c} \left(\alpha_i + \frac{s_i^{*2}}{x_i^*} \right) \tag{C.2}$$

Finally, recall that we impose a normalization to pin down the location of the distribution of α_i in the first Congress in the sample. Below, we simply assume that there is a legislator 0 with α_0 known, although in the empirical specifications, we simply omit the constant from z_i (which implies that we know α_i for an i with $z_i = 0$) since we parametrize α_i. Note that the arguments below do not rely on having measurement errors or on the parametrization of α_i. We drop the notation τ as our identification arguments are valid within each Congress.[5]

Dividing both sides of (2.7) by x_i^* for an arbitrary politician i yields:

$$\frac{s_i^*}{x_i^*} = \gamma_{P(i)}(1 - e^{-\lambda\zeta_{P(i)}})e^{-\lambda\rho V_{i,0}} \sum_{j \neq i} s_j^* m_{ij}(s^*) x_j^*. \tag{C.3}$$

Now, dividing (C.3) by its analogue for a politician j in the same party as i for whom $V_{j,0} \neq V_{i,0}$ yields:

$$\frac{s_i^*/x_i^*}{s_j^*/x_j^*} \quad = \quad e^{-\lambda\rho(V_{i,0}-V_{j,0})}, \tag{C.4}$$

where we have used that $P(i) = P(j)$ and that $\tilde{A}_{P(i)} = \sum_{j \neq i} s_j^* m_{ij}(\mathbf{s}^*) x_j^*$ is constant across politicians. It follows that we identify the product $\lambda\rho$.[6]

We can identify $\zeta_{P(i)}$ across parties by rewriting (2.4) using (2.7):

$$\begin{aligned} P(y_i = 1) \quad &= \quad \gamma_{P(i)} \sum_{j \neq i} g_{ij}(s^*) x_i^* x_j^* \\ &= \quad \gamma_{P(i)} s_i^* \sum_{j \neq i} s_j^* m_{ij}(s^*) x_i^* x_j^* \\ &= \quad \frac{\gamma_{P(i)}}{\varphi_i} s_i^{*2} \\ &= \quad \frac{1}{e^{-\lambda\rho V_{i,0}}(1 - e^{-\lambda\zeta_{P(i)}})} s_i^{*2}. \end{aligned} \tag{C.6}$$

where the third line uses (2.7) and the last line uses the definition of φ_i. The only unknown in the last line is $\lambda\zeta_{P(i)}$. When accounting for measurement error, it suffices to note that $\log(P(y_i = 1)) = \log\left(\frac{1}{1-e^{-\lambda\zeta_{P(i)}}} e^{\lambda\rho V_{i,0}} s_i^2\right) + 2\varepsilon_i$, where ε_i is mean 0. Hence, $\lambda\zeta_{P(i)}$ is identified for both parties by the average probability of passing a bill for politicians in $P(i)$ given their observed effort levels.

Now, let us return to (C.3). The product $\tilde{A}_{P(i)}\gamma_{P(i)}$ is the only unknown on the right hand side, so it is identified for the arbitrary party $P(i)$. As a result, the ratio

5. The normalization assumption can be imposed only in one Congress, as we can rely on the overlap of politicians across Congresses to maintain the assumption in later periods.

6. For completeness, when s_i^*, x_i^* are observed with measurement error as in (5.1), we find that:

$$\frac{s_i^*/x_i^*}{s_j^*/x_j^*} = \frac{s_i/x_i}{s_j/x_j} e^{(\varepsilon_i - \varepsilon_j)+(v_j - v_i)}. \tag{C.5}$$

Since the measurement errors are independent and mean 0, we can apply a log operator and then the expectation operator on both sides of (C.5). Hence, $\lambda\rho$ is still identified.

$\tilde{A}_1\gamma_1/\tilde{A}_2\gamma_2$ can be identified. The intuition is easily seen by dividing (C.3) for i and k for different parties.

$$\frac{s_i^*/x_i^*}{s_k^*/x_k^*} = \frac{\tilde{A}_{P(i)}\gamma_{P(i)}(1-e^{-\lambda\xi_{P(i)}})}{\tilde{A}_{P(k)}\gamma_{P(k)}(1-e^{-\lambda\xi_{P(k)}})}e^{-\lambda\rho(V_{i,0}-V_{k,0})}, \tag{C.7}$$

so that this ratio is identified by the systematic variation in relative choices of social and legislative effort across members of opposite parties.

We now proceed with identification of α_i for all i. To do so, we rewrite (C.2) as:

$$x_i^* = \frac{\alpha_i}{c - \left(\varphi_i \sum_{j\neq i} s_j^* m_{ij}(\mathbf{s}^*)x_j^*\right)^2} \tag{C.8}$$

Taking logs and using (5.1) implies[7]:

$$\begin{aligned} \log(x_i) + v_i &= \log(\alpha_i) - \log(c - \tilde{A}^2_{P(i)}\varphi_i^2) \\ \log(x_i) &= \log(\alpha_i) - \log(c - \tilde{A}^2_{P(i)}\varphi_i^2) - v_i \\ &= \log(\alpha_i) - \log\left(c - (\tilde{A}_{P(i)}\gamma_{P(i)}e^{-\lambda\rho V_{i,0}}(1-e^{-\lambda\xi_{P(i)}}))^2\right) - v_i. \end{aligned} \tag{C.9}$$

Recall that the term $((\tilde{A}_{P(i)}\gamma_{P(i)})e^{-\lambda\rho V_{i,0}}(1-e^{-\lambda\xi_{P(i)}}))^2$ has already been identified, as each of its 3 components are identified. Hence, (C.9) has only 2 unknowns: c and α_i. Since this equation is valid for every i, it is also valid for the normalizer politician "0" whose type α_0 is known by assumption. Hence, c is pinned down by equation (C.9) for the normalizer, as c is the only unknown in that case and $\log(\cdot)$ is a strictly monotonic function. With measurement error, we also use that v_i is mean 0 and i.i.d.

Once c is pinned down, then α_i is identified for every i from equation (C.9) under the analogous argument, as α_i is the only unknown.

Appendix D: Rewriting the Model in terms of Moment conditions over i

In this Section, we provide the derivation for transforming the model's equilibrium outcomes to the moment equations described in Section 5. Let us begin with (C.3):

$$\frac{s_i}{x_i}e^{\varepsilon_i - v_i} = \tilde{A}_{P(i)}\gamma_{P(i)}(1-e^{-\lambda\xi_{P(i)}})e^{-\lambda\rho V_{i,0}}.$$

We can rewrite this expression as:

$$\log\left(\frac{s_i}{x_i}\right) = \log(\tilde{A}_{P(i)}\gamma_{P(i)}) + \log(1-e^{-\lambda\xi_{P(i)}}) - \lambda\rho V_{i,0} + (v_i - \varepsilon_i).$$

7. In the absence of measurement error, simply replace $v_i = \mathbf{0}$ below and the same arguments stand.

Applying expectations over the measurement errors (which are mean zero) on both sides of the expression yields the first set of equations above. They are analogous to moment conditions which coincide with the OLS estimator with party-specific intercept parameters.[8]

For the second set of equations, we use the parametrization (5.3) in (C.9) to obtain:

$$\log(x_i) \quad = \quad z_i'\beta - \log\left(c - (\tilde{A}_{P(i)}\gamma_{P(i)}e^{-\lambda\rho V_{i,0}}(1-e^{-\lambda\xi_{P(i)}}))^2\right) - v_i\,.$$

Exploiting the orthogonality conditions on v_i yields the second set of moment conditions. We note that the location normalization is important here: otherwise, c could not be separately identified from the constant in $z_i'\beta$. In fact, the cost of legislative effort c is pinned down by the average legislative behavior of politicians, conditional on individual characteristics and electoral returns. But it could be increased if all types are similarly increased.

For the final equation, we rewrite (C.6) using (5.1):

$$P(y_i = 1) \quad = \quad \frac{1}{e^{-\lambda\rho V_{i,0}}(1-e^{-\lambda\xi_{P(i)}})} s_i^2 e^{2\varepsilon_i}\,,$$

which implies that:

$$\log(P(y_i = 1)) \quad = \quad \lambda\rho V_{i,0} - \log(1-e^{-\lambda\xi_{P(i)}}) + 2\log(s_i) + 2\varepsilon_i\,.$$

D.1. Details on Estimation

We now provide further details on how the estimation procedure was implemented, including the starting values for the numerical solution to the GMM optimizer and numerical details on the computation of standard errors.

D.1.1. OLS and plug-in Approach as Starting Values for Optimization. For the starting values for GMM optimization, we use simple closed form estimates for most parameters of interest, borne out of the separability of the moment equations. We then use different starting points for the remaining parameter, c.

8. We note that $\log(1-e^{-\lambda\xi_{P(i)}})$ can be split from the term $\log(\tilde{A}_{P(i)}\gamma_{P(i)})$ since it is identified from another equation.

More precisely, recall that the estimating equations are the empirical counterparts to equations (5.4)-(5.10) and are given by:

$$\frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{s_i}{x_i}\right)-\log(\tilde{A}_1\gamma_1)-\tilde{\xi}_1+\lambda\rho V_{i,0}\right)I_{\{i\in P_1\}}=0 \tag{D.1}$$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{s_i}{x_i}\right)-\log(\tilde{A}_2\gamma_2)-\tilde{\xi}_2+\lambda\rho V_{i,0}\right)I_{\{i\in P_2\}}=0 \tag{D.2}$$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{s_i}{x_i}\right)-\log(\tilde{A}_{P(i)}\gamma_{P(i)})-\tilde{\xi}_{P(i)}+\lambda\rho V_{i,0}\right)V_{i,0}=0 \tag{D.3}$$

$$\frac{1}{n}\sum_{i=1}^{n}(\log(x_i)-z_i'\beta+\log\left(c-(\tilde{A}_{P(i)}\gamma_{P(i)}e^{-\lambda\rho V_i}e^{\xi_{P(i)}})^2\right))=0 \tag{D.4}$$

$$\frac{1}{n}\sum_{i=1}^{n}z_i(\log(x_i)-z_i'\beta+\log\left(c-(\tilde{A}_{P(i)}\gamma_{P(i)}e^{-\lambda\rho V_i}e^{\tilde{\xi}_{P(i)}})^2\right))=0 \tag{D.5}$$

$$\frac{1}{n}\sum_{i=1}^{n}(\log(P(y_i=1))-\lambda\rho V_{i,0}+\tilde{\xi}_1-2\log(s_i))I_{i\in P_1}=0 \tag{D.6}$$

$$\frac{1}{n}\sum_{i=1}^{n}(\log(P(y_i=1))-\lambda\rho V_{i,0}+\tilde{\xi}_2-2\log(s_i))I_{i\in P_2}=0, \tag{D.7}$$

Careful inspection of equations (D.1) - (D.7) shows how to come up with appropriate starting points.

First, it is immediate that the only parameters in (D.1)-(D.3) are $\log(\tilde{A}_1\gamma_1)$, $\log(\tilde{A}_2\gamma_2)$ and $\lambda\rho$. Furthermore, those three equations are exactly the moment conditions implied by OLS estimation of $\log\left(\frac{s_i}{x_i}\right)$ on $I_{i\in P_1}, I_{i\in P_2}, V_{i,0}$. The OLS coefficients of this regression set equations (D.1) - (D.3) to exactly 0. Hence, we use these OLS estimates as starting values for $\log(A_1\gamma_1), \log(A_2\gamma_2), \lambda\rho$. We use an analogous argument on equations (D.6) and (D.7). In this second "regression", we use the OLS estimates of the outcome $\log(P(y_i=1))-(\lambda\rho)^{start}V_{i,0}-2\log(s_i)$ on $I_{i\in P_1}, I_{i\in P_2}$, where $(\lambda\rho)^{start}$ are the starting values for $\lambda\rho$. This second regression results in estimates for $(\tilde{\xi}_1,\tilde{\xi}_2)=(\log(1-e^{-\lambda\xi_1}),\log(1-e^{-\lambda\xi_2}))$ which set (D.6)-(D.7) to 0, which we use as starting values. Finally, equations (D.4)-(D.5) also come from a separate OLS regression where the outcome is $\log(x_i)$, the independent variables are a constant and z_i. The OLS coefficients on z_i is the starting value for β and set equations (D.5) to 0. The normalization assumption - not including a constant in z_i - guarantees that only one c satisifies (D.4). Hence, the outlined procedure delivers us starting values (and consistent estimators) for all parameters except c.

Our GMM estimator are the set of parameters that minimize the GMM objective function given moments (D.1-(D.7), given starting values for all parameters except c as described above, and across different starting values for c.

D.1.2. Computation of Standard Errors. To compute the standard errors for our GMM estimates, we use a consistent estimator based on its asymptotic value. Given the model is exactly identified, we can use the identity matrix as a weighting matrix.

As it is well known, the asymptotic variance matrix (of $\sqrt{n}$ times) our parameters of interest is then given by $(\Gamma'\Omega^{-1}\Gamma)^{-1}$, where $\Gamma = \mathbb{E}\frac{\partial g(\tilde{s}_i, \tilde{x}_i, \theta)}{\partial \theta'}$ and $\Omega = \mathbb{E}(g(\tilde{s}_i, \tilde{x}_i, y_i, z_i, \theta) g(\tilde{s}_i, \tilde{x}_i, y_i, z_i, \theta)')$.

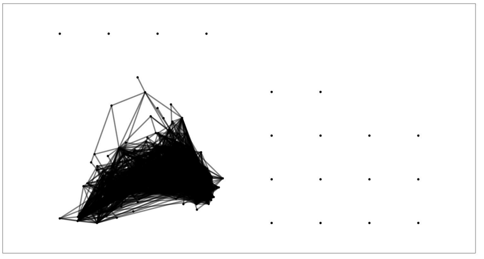
We compute Γ analytically, by taking derivatives of each moment equation in relation to each parameter. We then replace the expectation by its empirical counterpart (the mean across all politicians).

D.1.3. Finite Sample Corrections for the Standard Errors. In finite samples, Ω can be close to singular. This appears to be the case in some of the specifications in our paper. To improve the finite sample performance, we implement the correction used in Cameron et al. (2011). This involves increasing the standard errors in Ω by adding a small perturbation to its eigenvalues. This perturbation is sufficient to remove singularity.

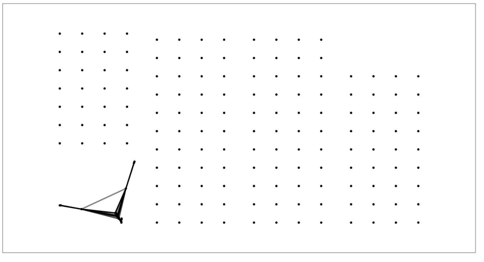
Such a procedure uses the spectral decomposition of $\Omega = D \Lambda D'$, where Λ is a diagonal matrix of eigenvalues. We then add a small $\delta_\Omega > 0$ to the diagonal of $\hat{\Lambda}$, therefore increasing the eigenvalues of $\hat{\Omega}$. Since this procedure increases standard errors, the new standard errors are still valid for our parameters.

In practice, we pick $\delta_\Omega = 0.00001$, and use it on the eigenvalues that are smaller than 10^{-7}. This is typically 1 or 2 of the eigenvalues of our estimated Ω.

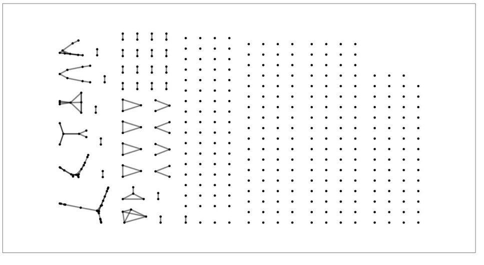
Appendix E: Additional Tables and Figures



(a) More than 3 Directed Cosponsorships



(b) Committee Network



(c) Alumni Network

FIGURE E.1. Examples of Alternative Congress 110 Networks. We show illustrations of alternative networks used in the literature that we use to compare our model against. A link in the committee network exists if two legislators sit in one of the 7 main committees together (see the Data section). A link in the alumni network exists if two legislators attended the same university within 8 years of one another. While in the empirical specification of equation (6.2) the cosponsorship network is taken as the amount of directed cosponsorships, we illustrate it here by plotting the upper triangular matrix of directed cosponsorships, with a link formed if a legislator cosponsors more than 3 bills by another.

TABLE E.1. Main results, Specification 2.

	Congress					
	105	106	107	108	109	110
c	0.263 (0.001)	0.277 (0.001)	0.292 (0.001)	0.291 (0.001)	0.293 (0.001)	0.271 (0.001)
$\tilde{\zeta}_{Dem}$	5.890 (0.160)	5.507 (0.179)	5.720 (0.174)	5.554 (0.176)	5.546 (0.174)	3.125 (0.119)
$\tilde{\zeta}_{Rep}$	3.404 (0.178)	2.695 (0.146)	3.117 (0.170)	2.880 (0.172)	2.935 (0.158)	4.311 (0.181)
$\lambda\rho$	0.101 (0.072)	0.138 (0.056)	0.015 (0.068)	0.010 (0.073)	0.038 (0.080)	0.022 (0.070)
Rep	0.078 (0.053)	0.143 (0.049)	0.212 (0.059)	0.098 (0.062)	0.108 (0.057)	0.047 (0.084)
$Ideology$	-0.387 (0.053)	-0.293 (0.067)	-0.368 (0.065)	-0.400 (0.105)	-0.382 (0.112)	-0.322 (0.048)
$Tenure$	0.007 (0.003)	0.009 (0.003)	0.007 (0.003)	0.010 (0.003)	0.009 (0.003)	0.004 (0.002)
$Appropriations$	-0.007 (0.042)	0.035 (0.033)	-0.041 (0.038)	-0.076 (0.040)	-0.046 (0.044)	-0.062 (0.032)
$Energy\ and\ Commerce$	-0.024 (0.039)	-0.011 (0.039)	-0.065 (0.046)	-0.143 (0.135)	0.016 (0.046)	-0.003 (0.026)
$Oversight$	0.014 (0.034)	0.069 (0.038)	0.029 (0.058)	0.022 (0.053)	0.018 (0.051)	0.023 (0.033)
$Rules$	0.122 (0.023)	0.128 (0.025)	0.098 (0.035)	0.160 (0.041)	0.210 (0.057)	0.149 (0.020)
$Leadership$	0.124 (0.040)	0.110 (0.034)	0.140 (0.066)	0.123 (0.108)	0.280 (0.033)	0.109 (0.117)
$Transportation$	-0.052 (0.037)	-0.020 (0.047)	0.011 (0.030)	-0.123 (0.106)	0.001 (0.044)	-0.002 (0.025)
$WaysAndMeans$	-0.087 (0.043)	-0.018 (0.036)	-0.015 (0.038)	-0.033 (0.049)	0.010 (0.056)	-0.023 (0.037)
$Ideology \times Rep$	0.541 (0.094)	0.419 (0.095)	0.409 (0.104)	0.622 (0.131)	0.610 (0.136)	0.591 (0.120)
$Tenure \times Rep$	0.010 (0.004)	0.008 (0.004)	0.006 (0.005)	0.008 (0.005)	0.004 (0.004)	0.005 (0.005)
$Appropriations \times Rep$	0.036 (0.052)	-0.044 (0.042)	-0.067 (0.052)	-0.002 (0.056)	-0.016 (0.061)	-0.127 (0.075)
$Energy\ and\ Commerce \times Rep$	-0.023 (0.053)	-0.003 (0.052)	0.010 (0.060)	0.052 (0.139)	-0.064 (0.061)	-0.084 (0.064)
$Oversight \times Rep$	0.062 (0.047)	0.046 (0.045)	0.003 (0.066)	-0.007 (0.065)	0.048 (0.063)	-0.005 (0.055)
$Rules \times Rep$	0.021 (0.045)	-0.003 (0.036)	0.008 (0.044)	-0.050 (0.053)	-0.005 (0.069)	0.058 (0.037)
$Leadership \times Rep$	0.000 (0.045)	-0.111 (0.057)	-0.059 (0.094)	0.006 (0.128)	-0.207 (0.078)	0.059 (0.146)
$Transportation \times Rep$	0.015 (0.049)	-0.012 (0.056)	-0.051 (0.043)	0.114 (0.113)	-0.015 (0.057)	-0.062 (0.048)
$WaysAndMeans \times Rep$	0.035 (0.056)	-0.011 (0.055)	0.016 (0.057)	0.008 (0.062)	-0.077 (0.072)	-0.157 (0.075)
γ_{Dem}	[0.0001,0.0001]	[0.0002,0.0002]	[0.0002,0.0002]	[0.0002,0.0002]	[0.0002,0.0002]	[0.0020,0.0020]
γ_{Rep}	[0.0011,0.0011]	[0.0023,0.0023]	[0.0015,0.0015]	[0.0018,0.0019]	[0.0018,0.0018]	[0.0005,0.0005]
N	424	427	426	431	429	426

Notes: Standard errors in parentheses. The table presents the results from the GMM estimation under the second specification. That is, we replace the Grosewart measure by dummy variables for the most important committees. The variable Leadership represents a dummy of whether the politician was the Speaker, the Majority or Minority Leader, or the Majority or Minority Whip. Rep is a dummy variable for belonging to the Republican Party. The estimates of γ_{Dem} and γ_{Rep} are their estimated sets. Standard errors are estimated as discussed in Appendix D. All other notes follow those in Table 3.

Appendix F: Additional Reduced Form Evidence on The Effect of Cosponsors on Bill Passage

We begin by looking at specifications which show the correlation between cosponsors of a bill and whether the bill is approved or not. In our model, cosponsorships can only help bill approval through extending the (endogenously formed) network.

F.1. Data

We use data from the 93rd (1973-1975) to the 110th Congress (2007-2009), originally from the Library of Congress, and used in Fowler (2006). The data includes all bills (both House and Senate) in these periods, with data for the politicians in each Congress (such as tenure, party, ideology measure), the cosponsoring decisions for each bill in each Congress and Senate (i.e. who sponsored and cosponsored each one) and the outcomes for each (passed house, passed Senate, was vetoed or not, and so forth).

With this data, it is possible to construct network variables such as: the number of cosponsors for each bill, average number of cosponsors for a politician's own bills, a network graph using cosponsorship decisions as links. The focus is on House bills. Table F.1 presents the summary statistics.

TABLE F.1. Summary statistics for Appendix F.

Variable	Obs	Mean	Std. Dev.	Min	Max
Pass	139021	0.077	0.267	0	1
Party	138986	60.32% Democrat			
		39.37 % Republican			
Ideology	137426	-0.069	0.388	-0.757	1.685
Tenure	138986	5.974	4.096	1	27
Number of cosponsors	139021	10.311	27.084	0	406
Avg. cosponsors of cosp.	139021	6.239	8.55	0	175

F.2. Empirical Specifications

A first approach to this problem is to test whether networks do impact bill approval in Congress. To do so, we can check whether the number of cosponsors of a bill and the extended network of those cosponsors are positively correlated with passing rates in Congress (as in our structural model). To do so, consider the following regression:

$$pass_{i,k} = \beta_1 cosponsors_{i,k} + \beta_2 average\ cosponsors\ of\ cosponsors_{i,k} + X_i'\gamma + \varepsilon_{i,k} \tag{F.1}$$

where *cosponsors* represents the number of cosponsors of bill k (proposed by sponsor i); and *average cosponsors of cosponsors* represents the average number of cosponsors that cosponsors of this bill have (in their own bills). The latter captures the influence, or additional order network effects of those agents. X_i represent a series of politician level controls, such as the sponsor's ideology, tenure, party.

Equation (F.1) implies that having additional cosponsors (captured by β_1) and those cosponsors being more influential/with larger networks (β_2) are associated with the approval of legislation.

One may expect the OLS estimates of (F.1) to be inconsistent. First, it is possible that certain sponsors/politicians are more politically able and/or have better bills, and so would attract more cosponsors and better networks. In our model, higher types/returns α_i socialize more and have larger and more influential networks, and hence would be observed to cosponsor more on average.

To control for that, consider the fixed effects regression:

$$pass_{i,k} = \alpha_i + \beta_1 cosponsors_{i,k} + \beta_2 average\ cosponsors\ of\ cosponsors_{i,k} + \varepsilon_{i,k} \tag{F.2}$$

where α_i is a fixed effect for the politician who sponsors the bill. This effect captures the above problem, and would use the following variation: different bills by the same sponsor can have different number of cosponsors/extended network. The differences in their outcomes in Congress would then be attributed to the different (observed proxies for) networks.

A threat to identification in (F.2) is that we are not controlling for bill quality. The same sponsor can have some bills which are better than others, which by themselves attract more cosponsors. To deal with this issue, one can increase the set of controls, for instance focus on the specific characteristics of the Senate sponsor of the House bill.

This is done using the following specification:

$$\begin{aligned} pass_{i,j,k} = {} & \alpha_i + \gamma_j + \beta_1 cosponsors_{i,j,k} \\ & + \beta_2 average\ cosponsors\ of\ cosponsors_{i,j,k} + \varepsilon_{i,j,k} \end{aligned} \tag{F.3}$$

where α_i, γ_j represents a fixed effect for the House sponsor (i) and Senate sponsor (j) pair. The bills studied here are those present in both chambers.

Our preferred specification further controls for bill type. Although the above intuitively should do so, there is still a threat that part of the bill quality is not being captured by having the same sponsors in both chambers.

For that reason, consider the within bill variation model:

$$\begin{aligned} pass_{i,j,k,h} = {} & \delta_k + \beta_1 cosponsors_{i,j,k,h} \\ & + \beta_2 average\ cosponsors\ of\ cosponsors_{i,j,k,h} + \varepsilon_{i,j,k,h} \end{aligned} \tag{F.4}$$

$$\begin{aligned} pass_{i,j,k,s} = {} & \delta_k + \beta_1 cosponsors_{i,j,k,s} \\ & + \beta_2 average\ cosponsors\ of\ cosponsors_{i,j,k,s} + \varepsilon_{i,j,k,s} \end{aligned} \tag{F.5}$$

In this version, we are using variation in outcomes for the identical bills across chambers (h for House, s for Senate). We posit that the same bill, if it faces different results in separate chambers, must have that due to differential (networks) supporting it. It cannot be coming from bill quality, as it is the same bill in both scenarios. It cannot be coming from different politician abilities, as these are spanned by δ_k. The difference in outcomes is due to network effects.

Identification in (F.4)-(F.5) is due to the availability of bills that switch status across chambers.

We also use the definitions of identical bills in the Senate, as defined by the Library of Congress. This is done by checking for identical bills in the Senate (under related bills) for all house bills in Congresses 93-110. Table F.2 shows that there are bills that switch status across chambers, which is key to our identification. These constitute around 20% of the sample.

TABLE F.2. Bills with "Switching" outcomes.

	(1) Outcome $\Delta_{h-s} Billpass$	(2) Frequency	(3) Percent
Panel A: All identical bills			
	Pass Senate, Not Pass in House	1,073	8.30
	Same Outcome in Both	10,473	81.02
	Pass House, Not Pass in Senate	1,380	10.68
N: 12926			
Panel B: $Cosponsors > 0$ in both			
	Pass Senate, Not Pass in House	524	8.13
	Same Outcome in Both	5120	79.43
	Pass House, Not Pass in Senate	802	12.44
N: 6446			

Notes: Panel A: All bills with paired observations.
Panel B: Only those with number of cosponsors bigger than zero in both the House and Senate.

F.3. Results

Table F.3 presents the results across our various specifications (F.1), (F.2), (F.3) and (F.4)-(F.5).

As can be seen, the estimates of β_1 and β_2 are positive and very significant across specifications (Linear, Linear with controls, House Sponsor Fixed Effects, House and Senate Sponsor Fixed Effects and within bill variation). The number of cosponsors is positively associated with the approval of bills, as is their influence along the congressional network.

The estimate of β_1 is between 0.0003 and 0.0005. This represents that an additional cosponsor correlates with a (directly) increased probability of approval by 0.05%. This is a small, but non negligible amount, as bills usually have many cosponsors. The coefficient for β_2 amplifies this effect, and is estimated to be around 3 times as large as β_1 (in Columns (1)-(4)). This implies that adding a cosponsor who has on average 10 cosponsors on his own bill, is associated with an average increase

TABLE F.3. (Appendix F.) Main results.

	(1) Linear	(2) Linear w/ Controls	(3) House Sp. FE	(4) House & Sen. Sp. FE	(5) Within bills
Cosponsors	0.000589***	0.000554***	0.000594***	0.000584***	0.00107***
	(0.0000540)	(0.0000547)	(0.0000527)	(0.000107)	(0.0000713)
Average Cosp. of cosp.	0.00275***	0.00140***	0.00227***	0.00162***	0.00104***
	(0.000214)	(0.000209)	(0.000208)	(0.000543)	(0.000369)
Constant	0.0536***	-0.0742***	0.0566***	0.102***	0.0854***
	(0.00250)	(0.00772)	(0.00135)	(0.0252)	(0.00447)
N	137703	137426	137703	12852	12926
R^2	0.015	0.035	0.010	0.044	0.014

Notes: Standard errors in parentheses, clustered at the House Sponsor level (first 4 columns) and Senate Sponsor (Column (5), due to lack of data to cluster at the House sponsor). Individual controls in Column (2) include Tenure, Party, Ideology and Congress.$^{*}p < .1$, $^{**}p < .05$, $^{***}p < .01$. The first column is the OLS regression, the second puts controls (described above), the third is fixed effects at the House Sponsor level, the fourth has fixed effects of both House and Senate sponsor. Column (5) is the specification with within bill variation. N for Column (5) is the number of bills we have pairs of observations. It is larger than (4) because it does not use information on the id code of the sponsor in the House.

of $0.000541 + 10 \times 0.00162 = 0.0167$, or a 1.67 point increase in the percentage probability of approval.

Table F.4 allows for heterogeneity in the effects for the House and the Senate, for the specification of (F.4)-(F.5). The results confirm the positive and significant effects in the House, and shows that the influence term β_2 is really important in the House, although not so much from the Senate, which presents noisy estimates.

Our results indicate that it is seemingly advantageous to have additional cosponsors. In the context of the structural model, this means there are gains in having larger networks and more connections. We should hence, observe denser networks in Congress. This seems to be the case in our structural model. It also seems to hold in evidence in Fowler (2006) and Cho and Fowler (2010). This suggests that models with sparse equilibrium interconnections would not provide a good fit for Congressional activity.

TABLE F.4. Effect heterogeneity.

	(1) pass	(2) pass	(3) pass	(4) pass
House Outcome (Indicator)	0.0386***	0.00273	0.0529***	-0.0323*
	(0.00512)	(0.00804)	(0.00651)	(0.0182)
Cosponsors	0.00111***	0.00102***	0.000971***	0.000813***
	(0.000112)	(0.000136)	(0.000124)	(0.000152)
Average cosp. of cosp.	0.00204***	0.000850*	0.00102	-0.00127
	(0.000374)	(0.000433)	(0.000696)	(0.000874)
Interaction: House × cosponsors		0.000126		0.000226
		(0.000165)		(0.000207)
Interaction: House × avg. cosp. of cosp.		0.00333***		0.00478***
		(0.000599)		(0.00103)
Constant	0.0544***	0.0717***	0.0543***	0.103***
	(0.00543)	(0.00620)	(0.0133)	(0.0168)
N	12926	12926	6446	6446
R^2	0.021	0.025	0.021	0.026

Notes: Standard errors in parentheses, clustered at the senate sponsor level. Tests reject the hypothesis that the coefficients of the interactions are the same as those without. Columns (1) and (2) focus on all bills with paired observations. Columns (3) and (4) only on bills with positive number of cosponsors in both the House and the Senate. N is the number of bills (each bill has 2 observations).$^{*}\, p < .1$, $^{**}\, p < .05$, $^{***}\, p < .01$.

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