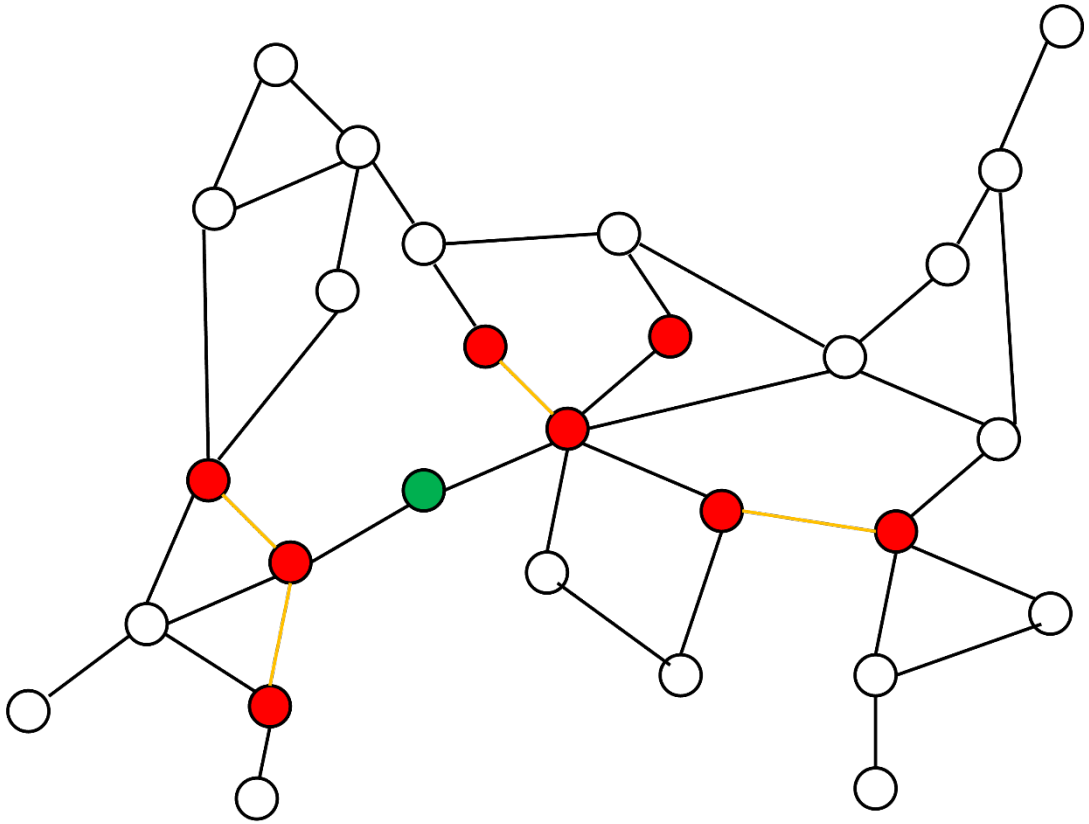
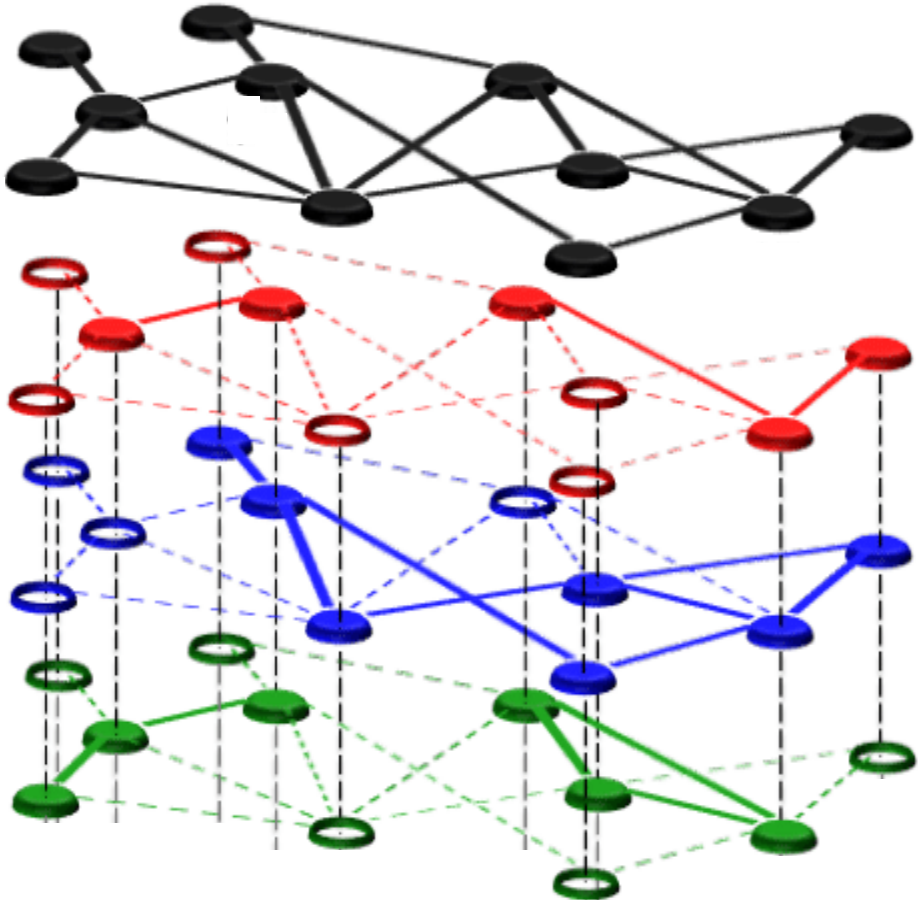
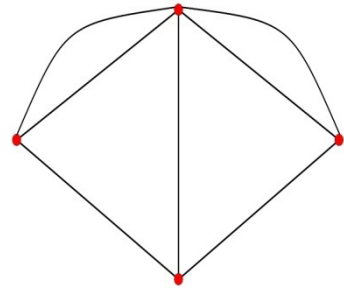
Multiplexing and Diffusion



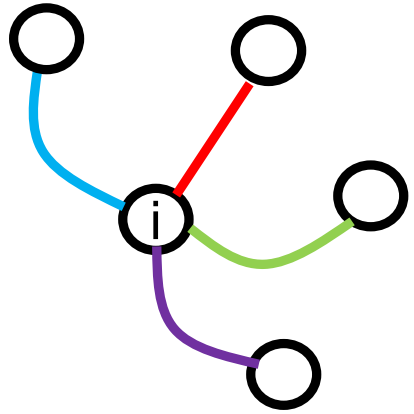


Chandrasekhar, Chaudhary, Golub, Jackson

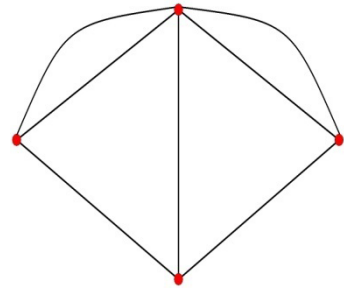
``Multiplexed'' Networks



- Compare two people:
  - One has different friends for different purposes
    - friends from work
    - friends they go to dinner with
    - friends they borrow money from
    - friends they give advice to…



``Multiplexed'' Networks



- Compare two people:
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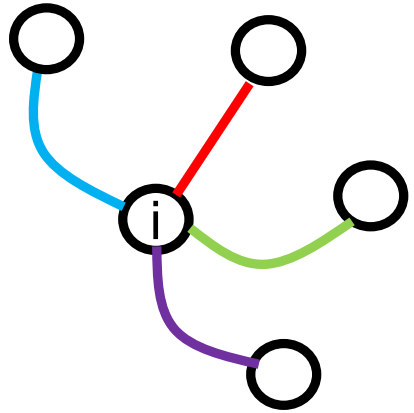
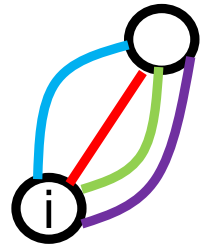
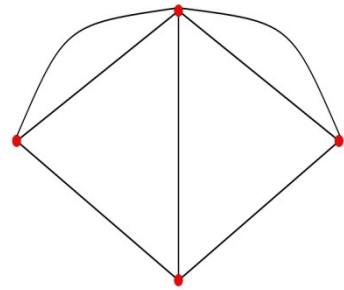
    - friends they give advice to…
  - Another person does everything with the same friend





``Multiplexed'' Networks



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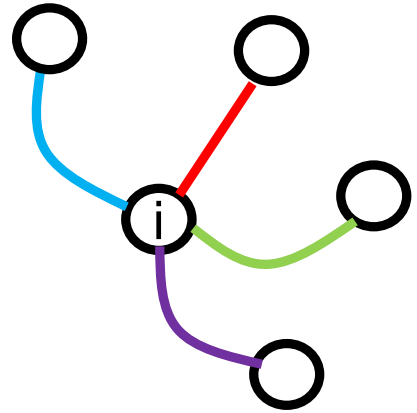
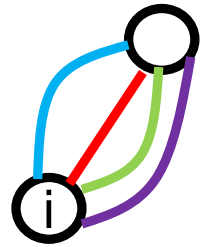
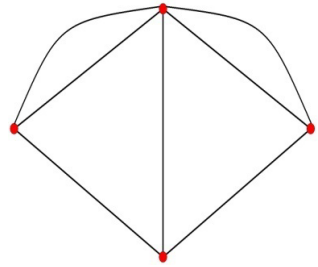




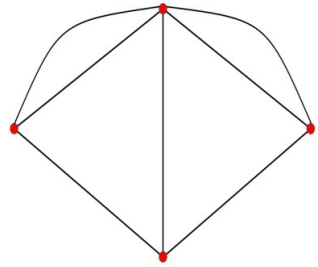
- Which one is more likely to learn/diffuse information?
- Which one is more likely to adopt a new technology, adopt a new behavior?

Multiple Layers of Relationships



- Different types of relationships are correlated, but distinct
- Interacting with more people can help diversify access to information, new technologies
- But, more likely to interact with someone you do many things with, and can get more pressure from them…

Questions

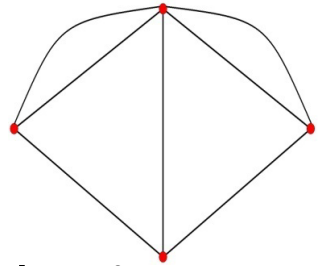


- Are there systematic patterns in how different layers of relationships overlap - ``multiplexing''?

- How does multiplexing affect diffusion?

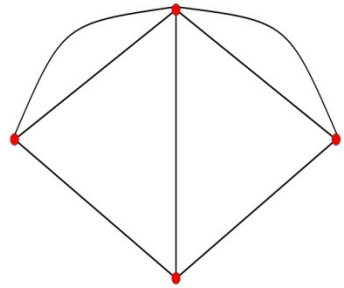
- How does the effect of multiplexing depend on what is being diffused?
  - simple contagion: spread of idea or disease
  - complex contagion: technology adoption, norms of behavior

Multiplex Literature



- Multiplexing in data, surveys: Simmel 1908, Wasserman Faust 1994, Boccaletti etal. 2014, Kivella etal. 2014, Dickison etal. 2016
- Formation: Billand, Bravard, Joshi, Mahmud, Sarangi 2023, San Roman 2024
- Cooperation: Atkisson etal. 2020, Cheng, Huang, Xing 2021
- Games/Peers: Chen, Zenou, Zhou 2018, Walsh 2019, Zenou, Zhou 2023, Jackson, Zenou, Zhou 2024
- Diffusion: Yagan, Gligor 2012, Bianconi, Radicchi 2016, Kobayashi, Onaga 2023, Larson Rodriguez 2023

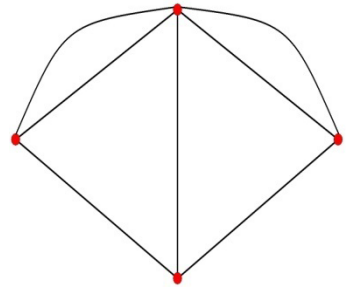
Background Motivation



- BBCDJK 2024: Introduction of microfinance changes social networks

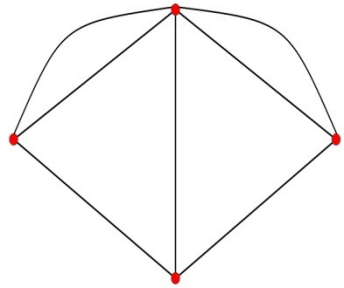
- Not only borrowing/lending networks, but also advice networks disappear

Background - Microfinance



- Karnataka India 75 villages:
  - 43 villages people were offered microfinance loans
  - 32 controls (no loans offered)

Background - Microfinance



- Karnataka India 75 villages:
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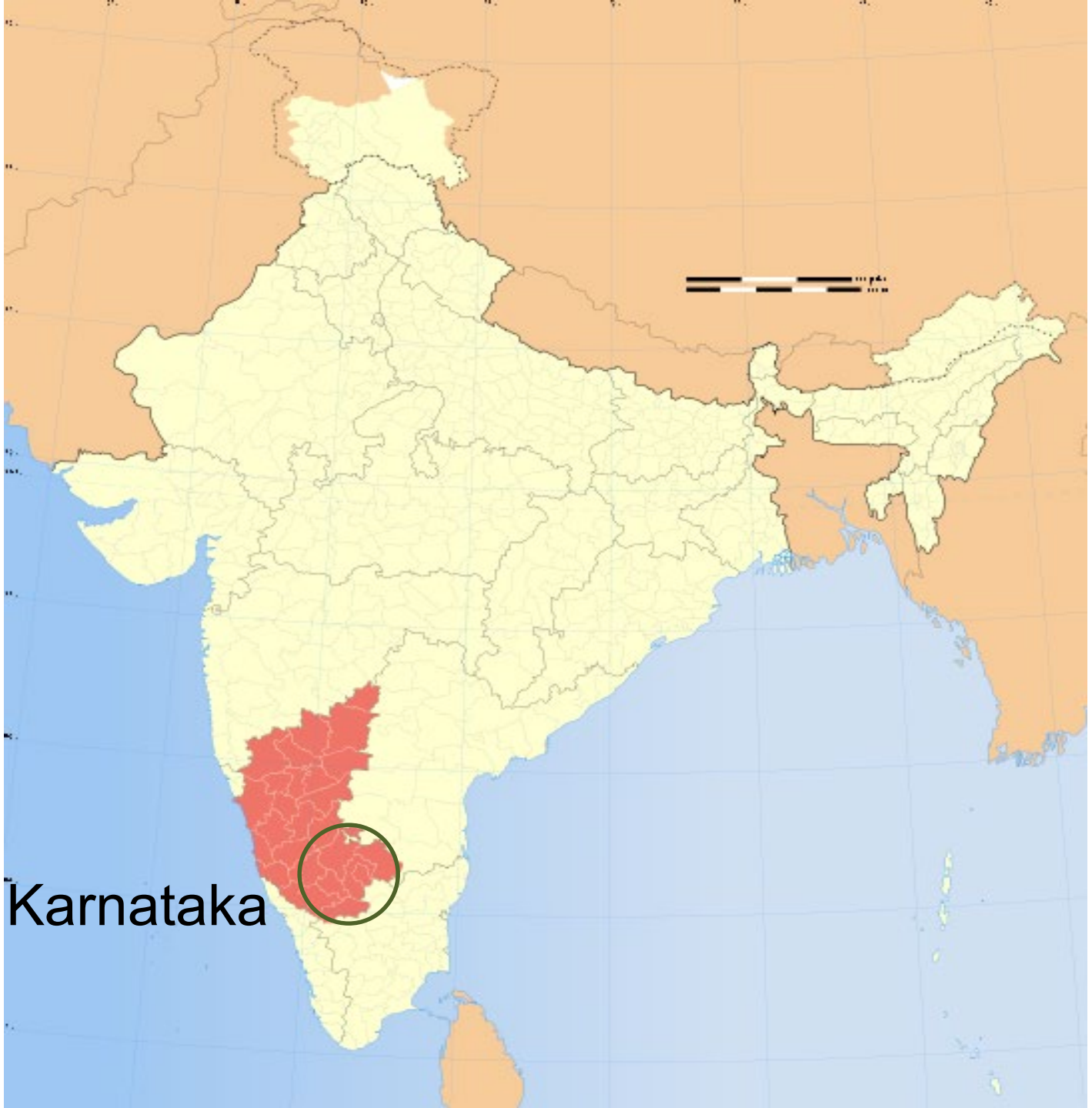
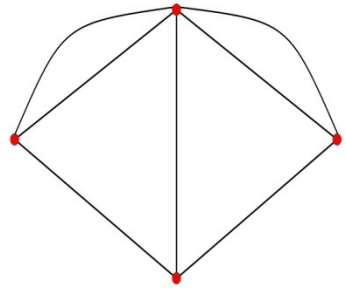
Banerjee, Chandrasekhar, Duflo, Jackson (Science 2013, Restud 2019)

Banerjee, Breza, Chandrasekhar, Duflo, Jackson, Kinnan (Restud, 2024)

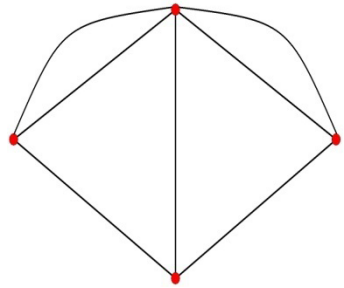

Karnataka

Timeline



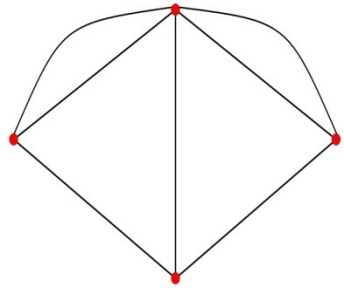
- 2006 We surveyed 75 villages that the bank intended to enter
- 2007-2010 Bank entered 43 villages offered loans, not other 32
- 2011-2012 We resurveyed all villages

Kenneth Arrow 1999



``This leads to an important and long-standing question: does the market (or, for that matter, the large, efficient, bureaucratic state) destroy social links that have positive implications for efficiency?''

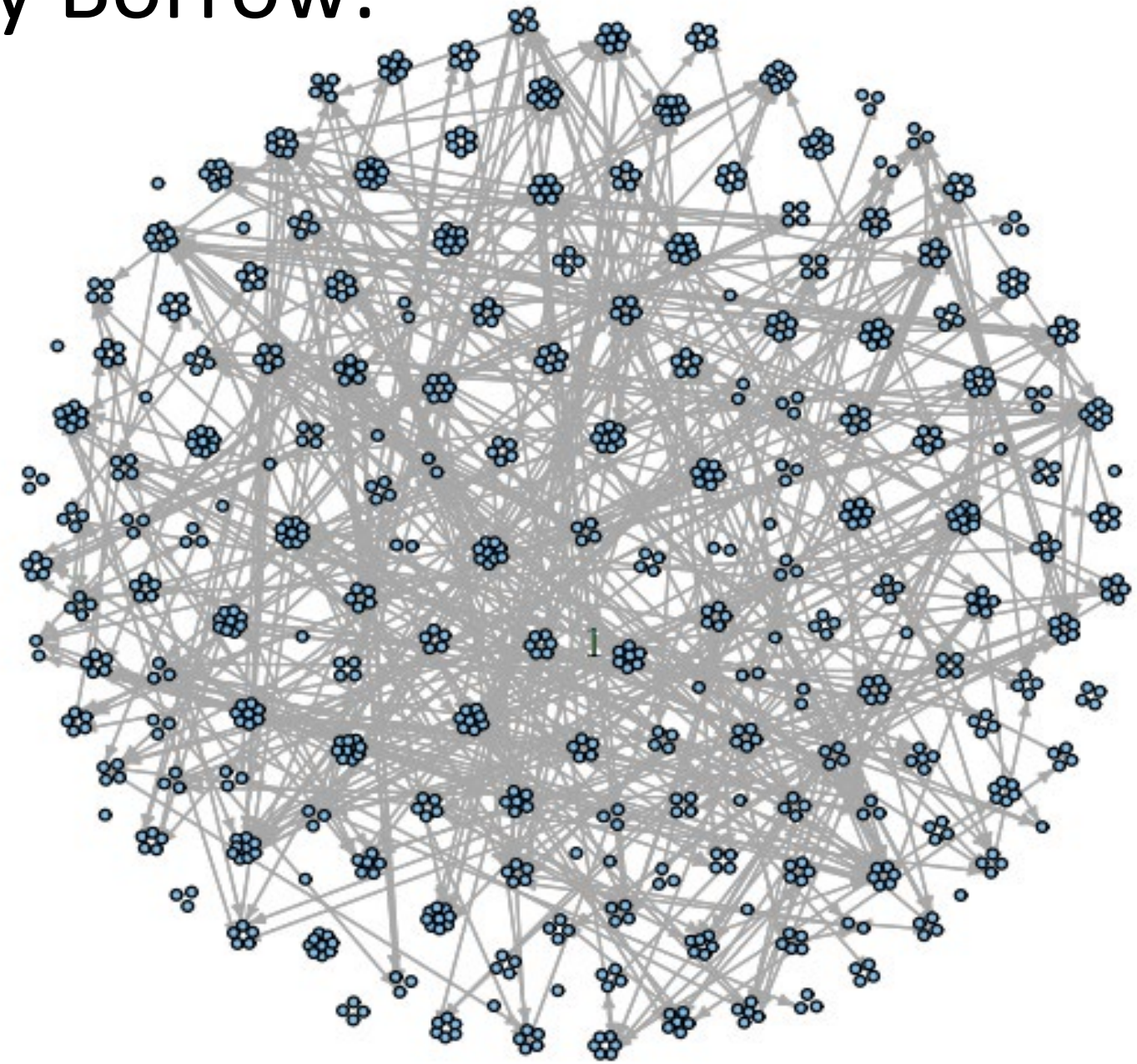
Kenneth Arrow 1999



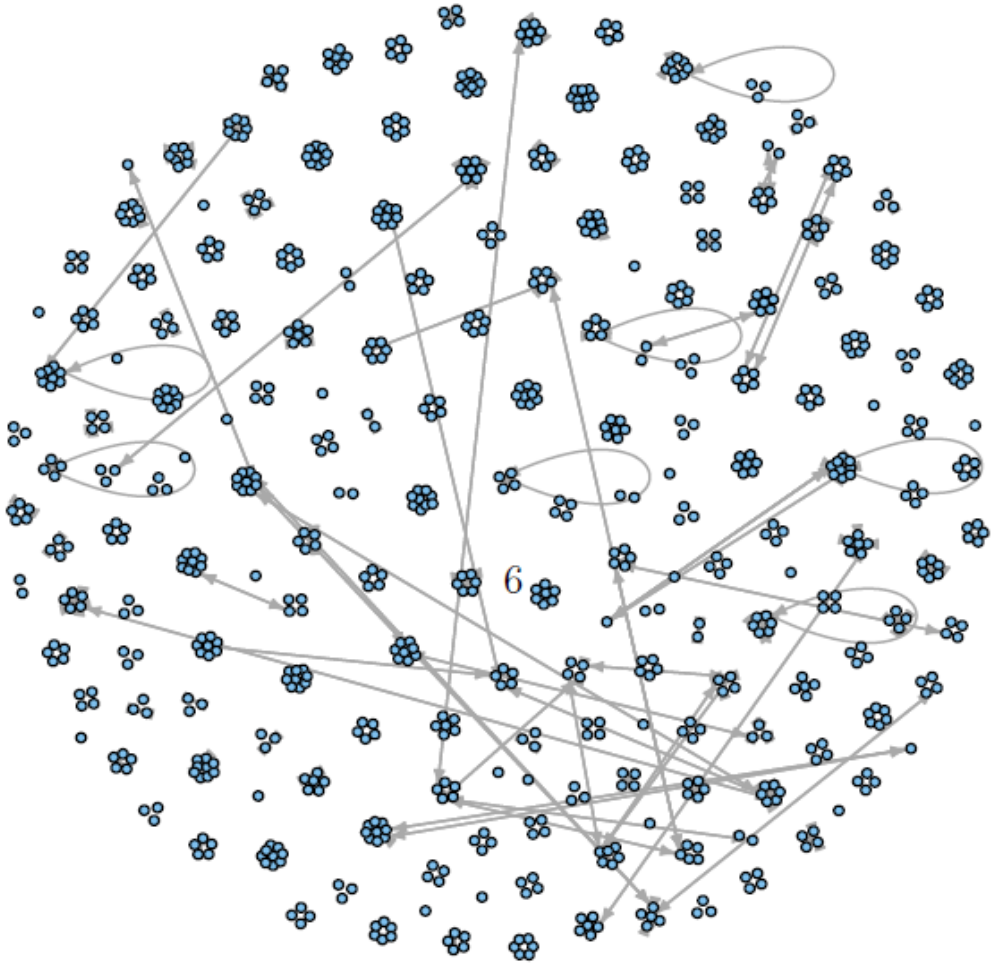
``This leads to an important and long-standing question: does the market (or, for that matter, the large, efficient, bureaucratic state) destroy social links that have positive implications for efficiency?''

Here:
Does availability of formal loans change informal networks?
Which networks?

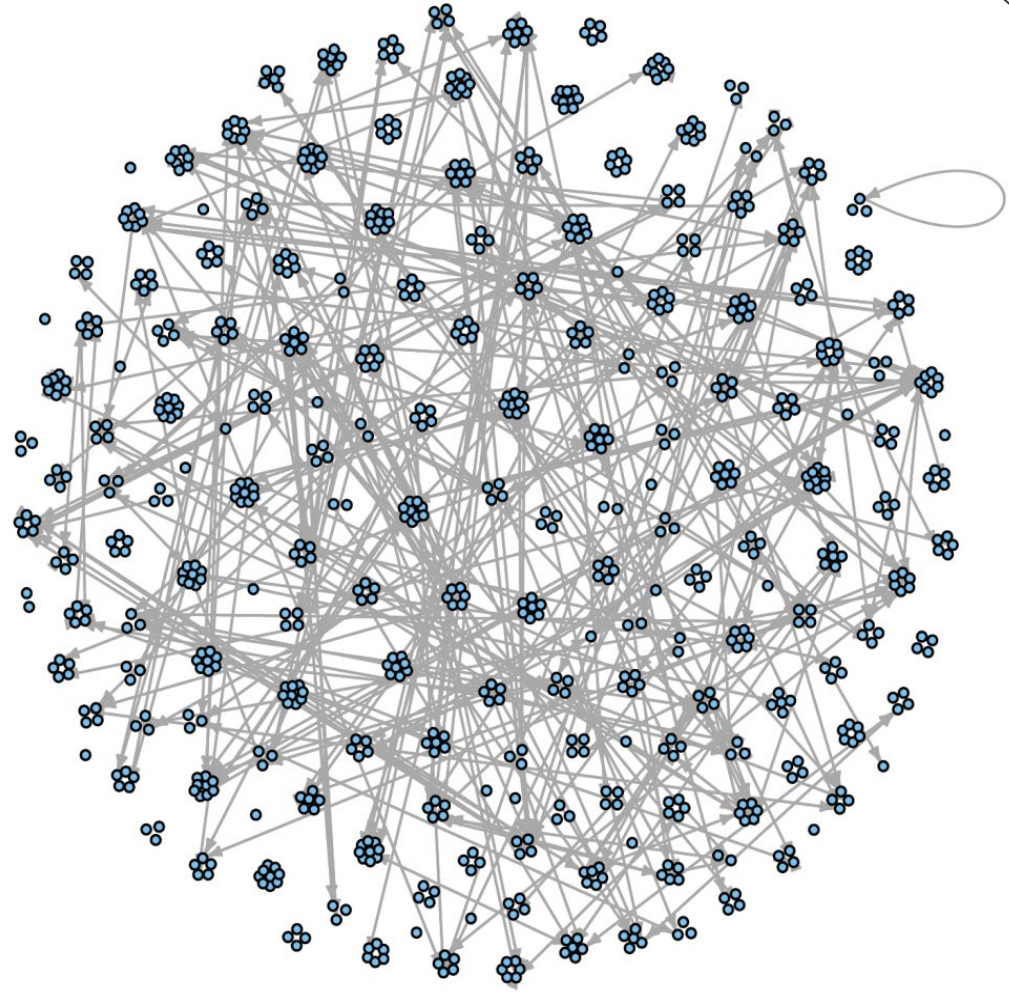
Money Borrow:

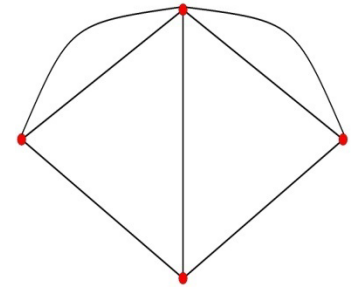


Temple

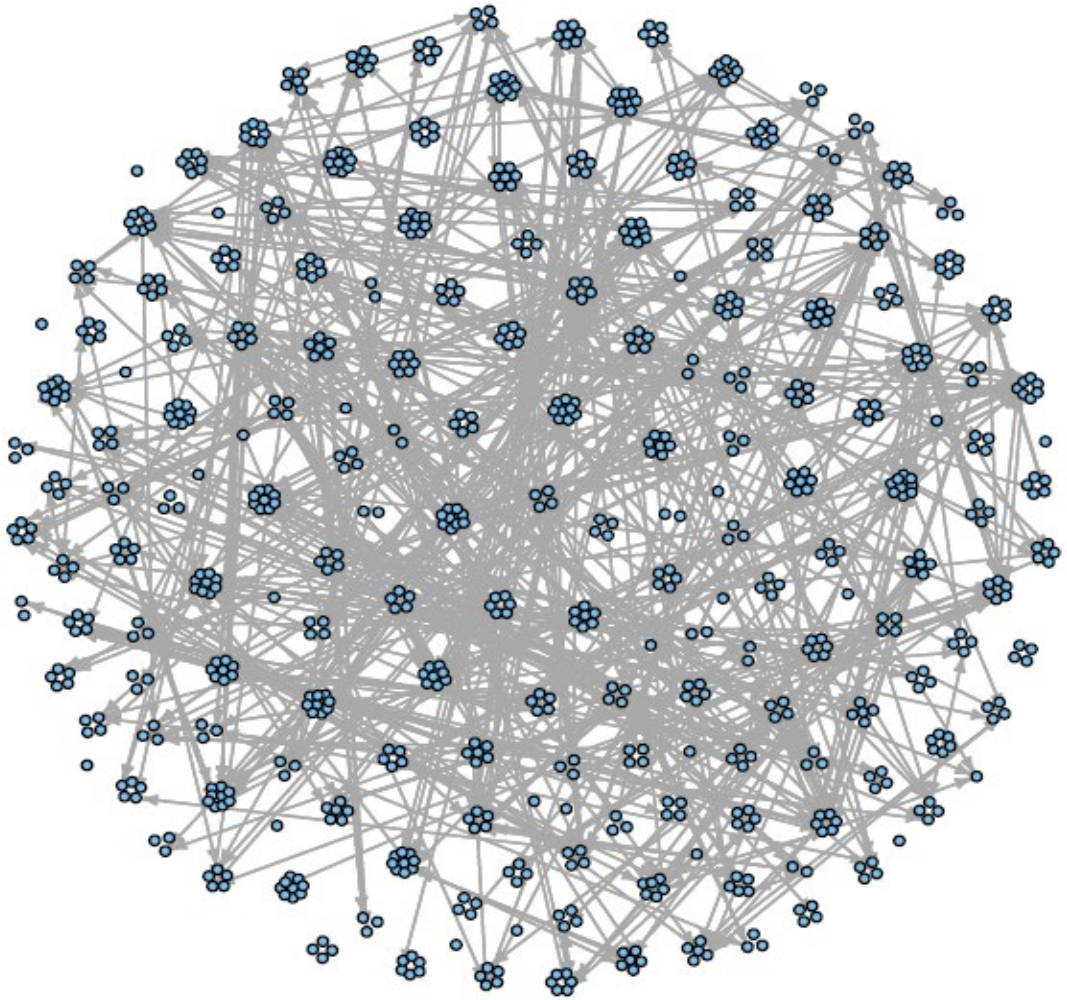


Advice

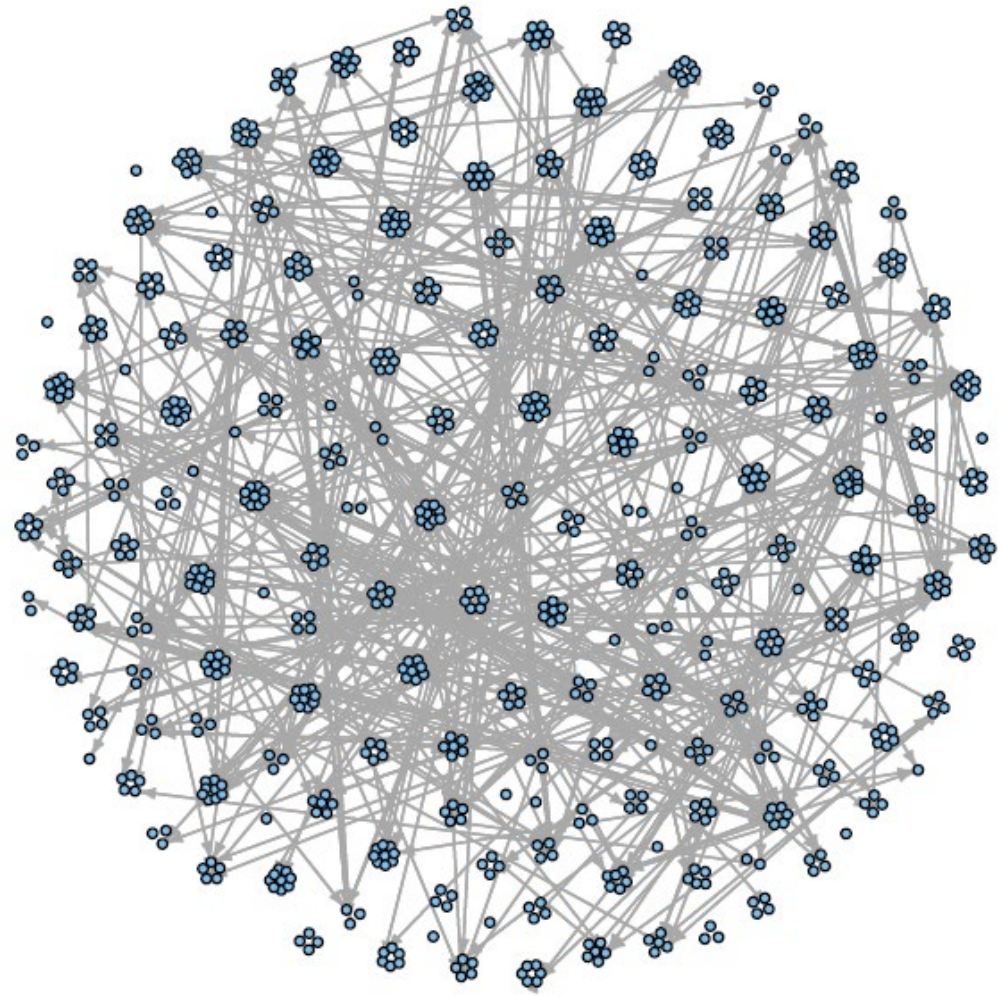


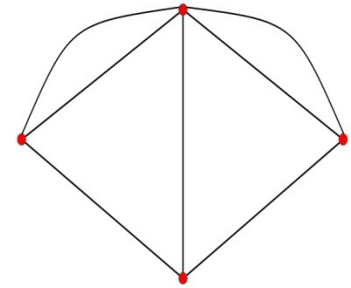


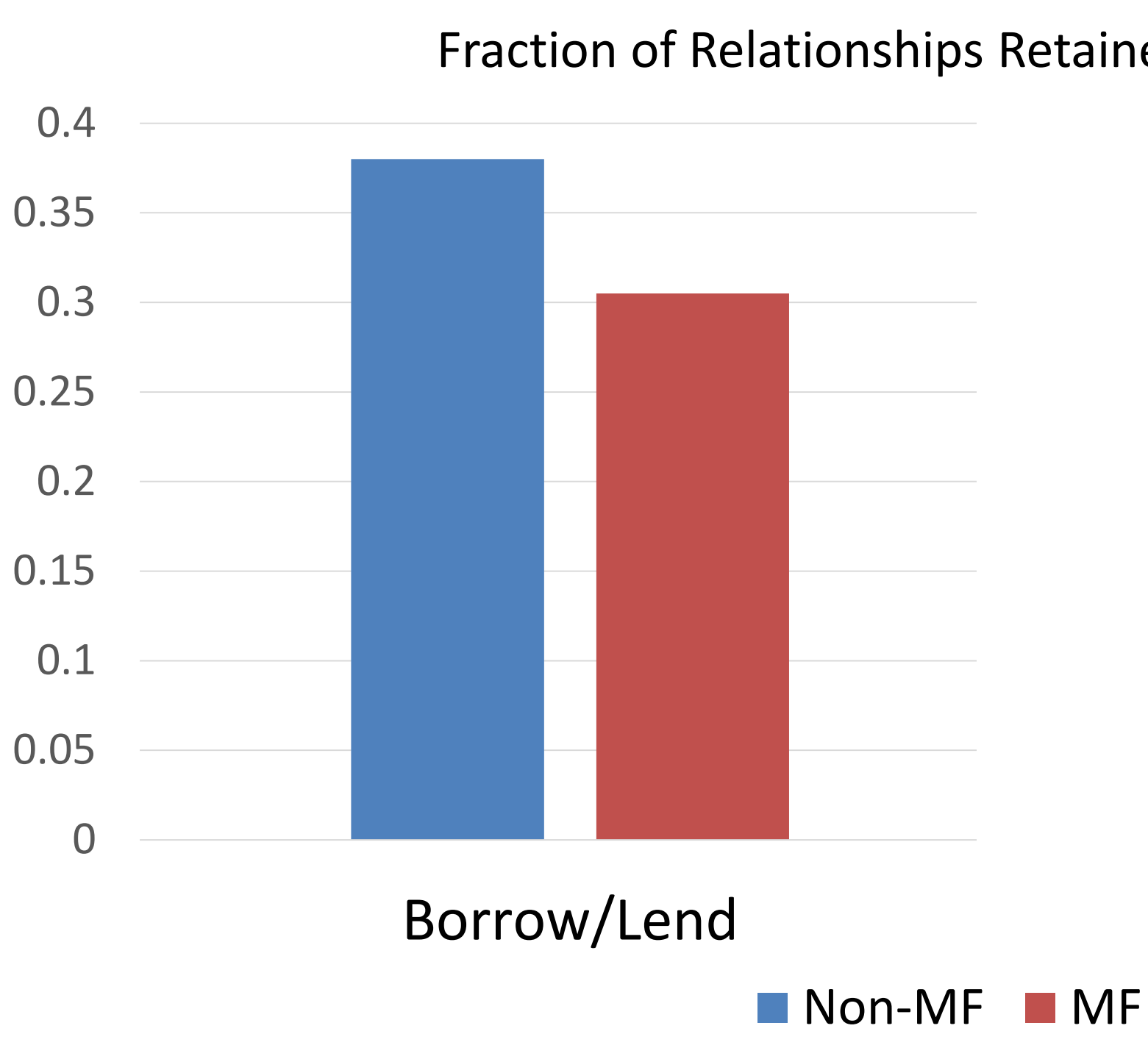
Kero-Come

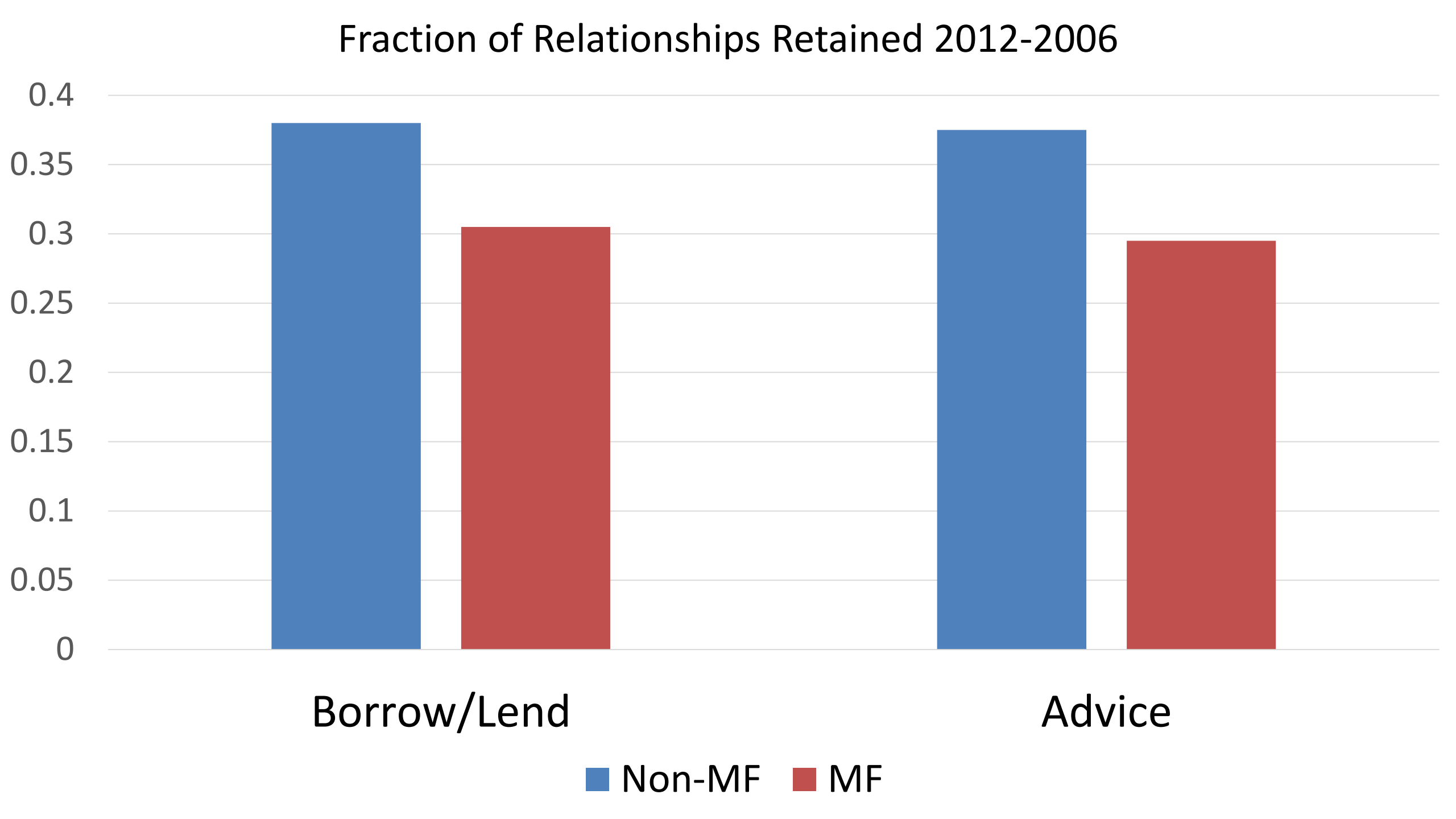


Medic






Fraction of Relationships Retained 2012-2006
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Borrow/Lend
Non-MF
MF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0.4
0.35
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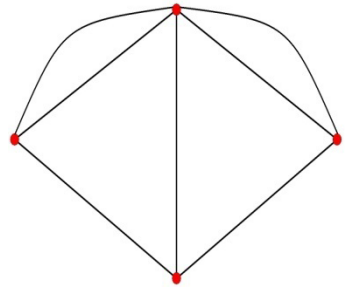
Advice
Non-MF
MF

Network Changes: Theory

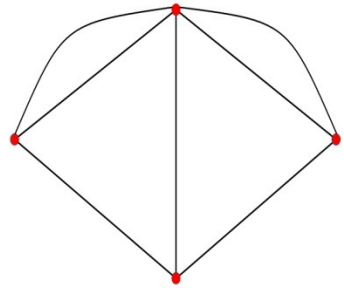


- BBCDJK 2024: Model
  - Takes time/effort to socialize to form/maintain relations
  - Loan participants decrease socializing
  - Then so do non-participants…

- Socializing changes *all* relationships, not just borrow/lending

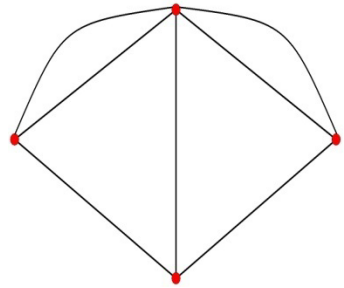
- Multiplexing! What are its consequences?

Outline



- Brief empirical look at multiplexing
- Do multiple layers affect diffusion (in an RCT)?
- Theory on how multiplexing impacts diffusion (simple, complex)
- Demographics and multiplexing

Outline



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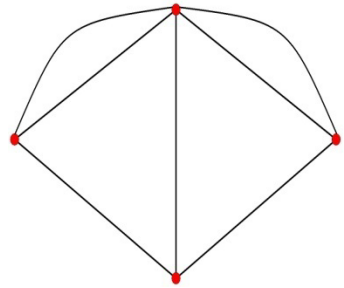
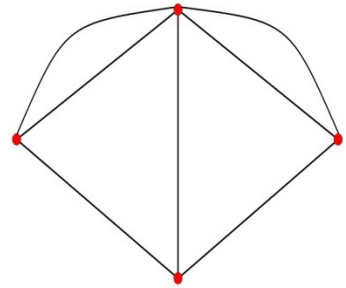
Two Data Sets



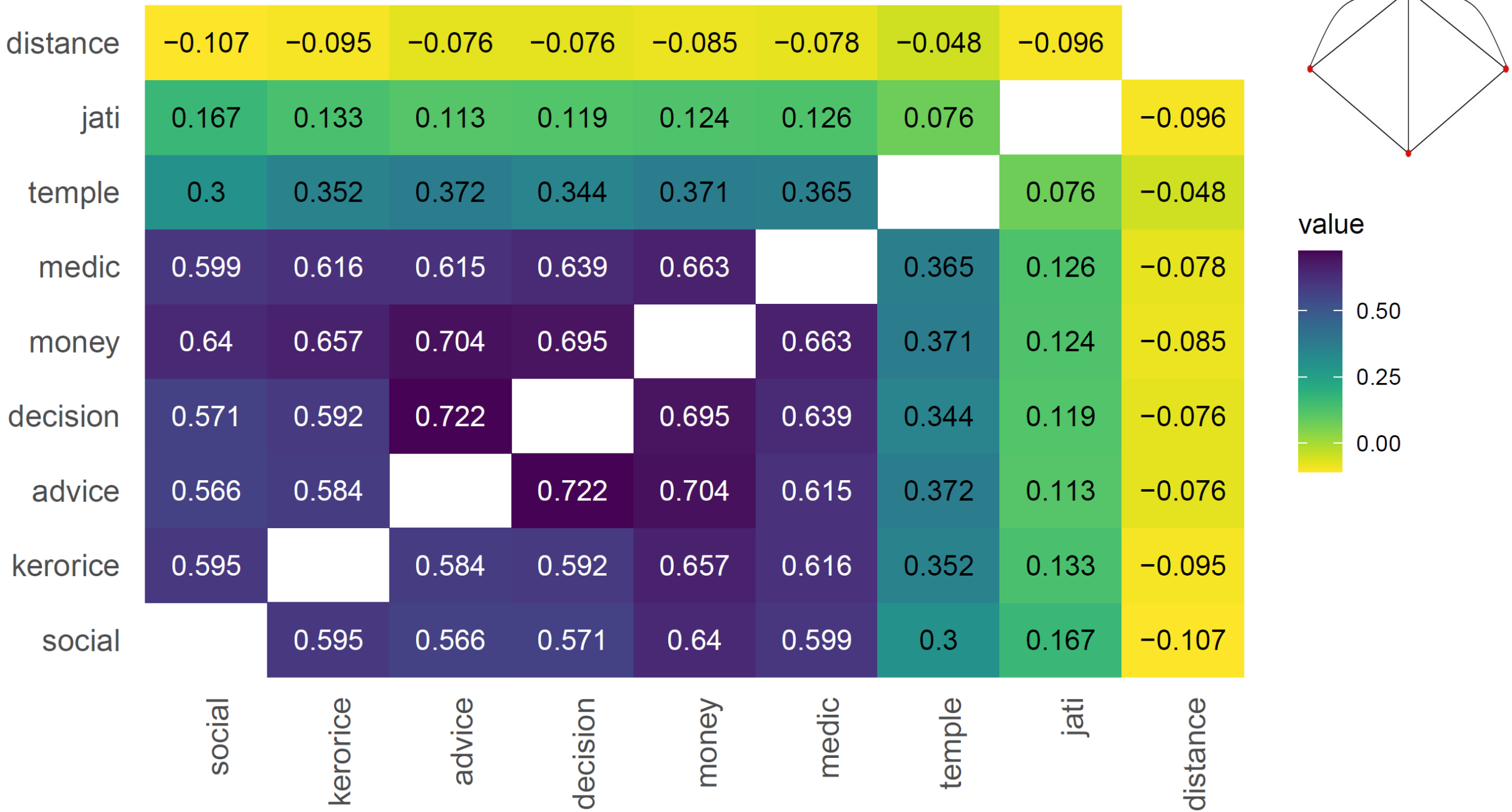
- 75 Microfinance villages from BCDJ 2013,19, BBCDJK 2024
  - 9 layers of networks

- 68 different villages (also Karnataka) from RCT of diffusion BCDJ 2019
  - 5 layers of networks

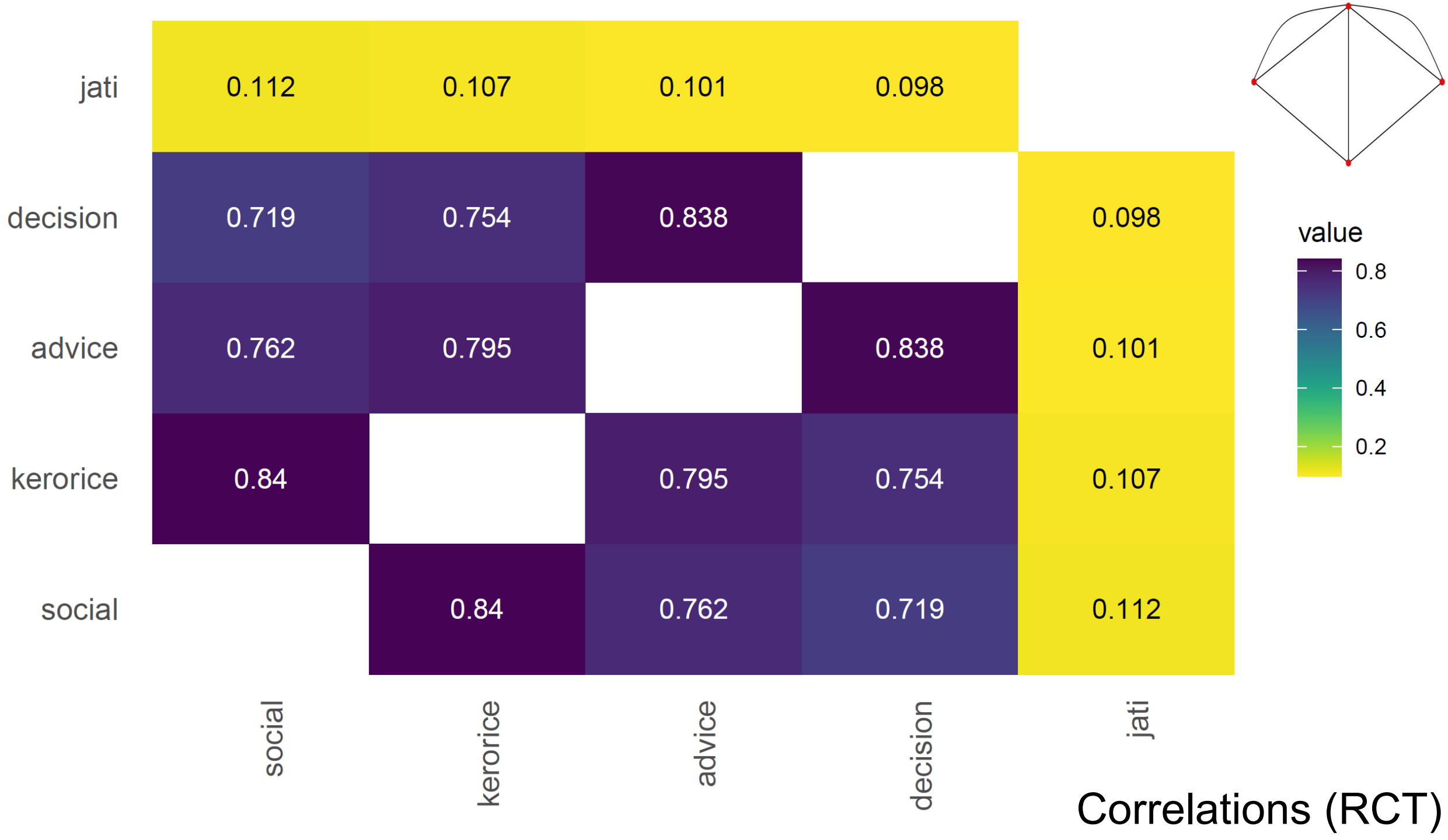
Networks:



- MF villages 9 total:
  - Kero-Rice
  - Money
  - Socialize: relatives, non, visit
  - Medical help
  - Temple
  - Advice
  - Decision Help
  - Jati
  - Geography
- RCT villages 5 total:
  - Kero-Rice/Money
  - Socialize
  - Advice
  - Decision help
  - Jati

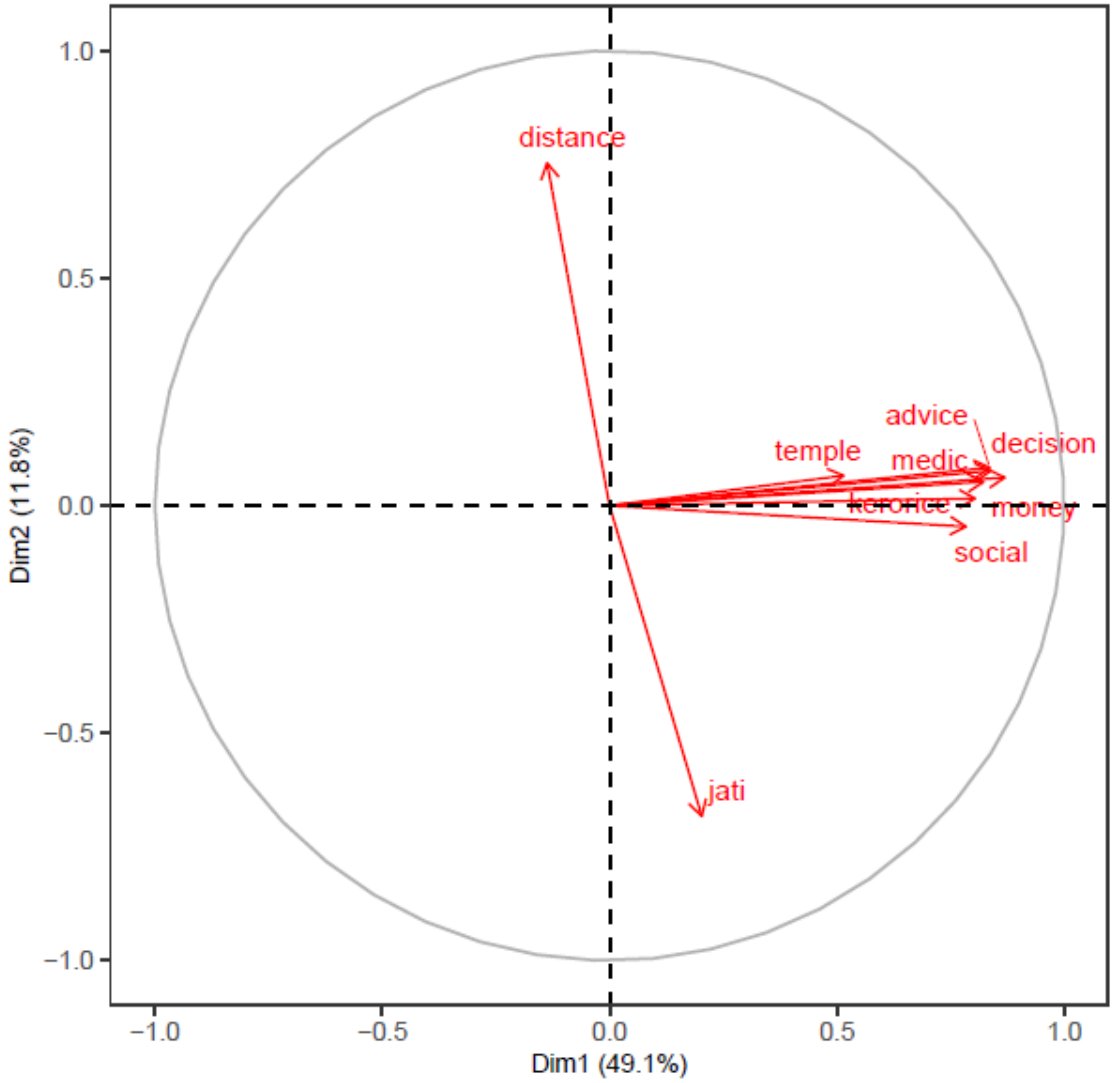


Correlations (MF)

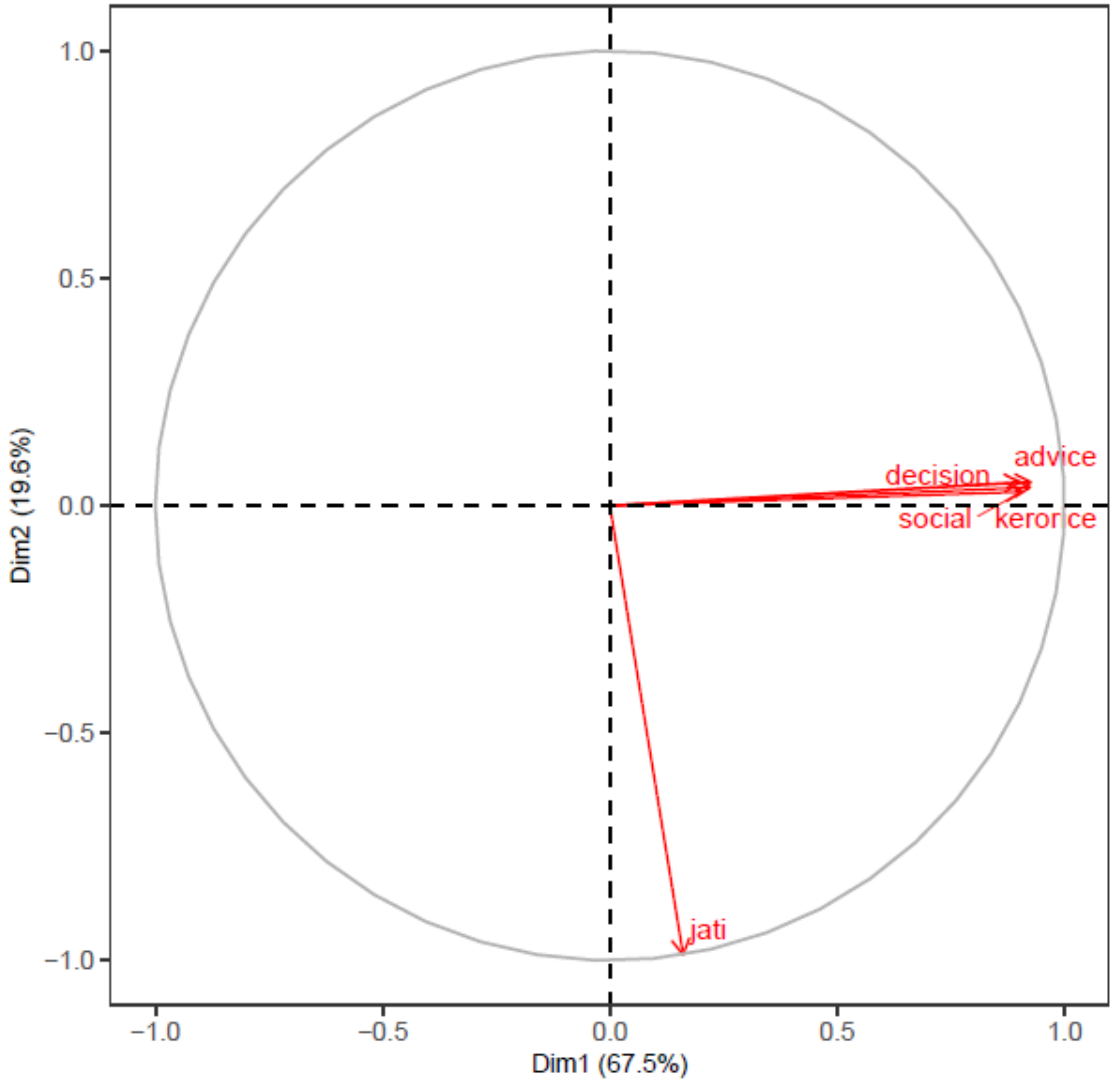


Correlations (RCT)

Principal Component Analysis

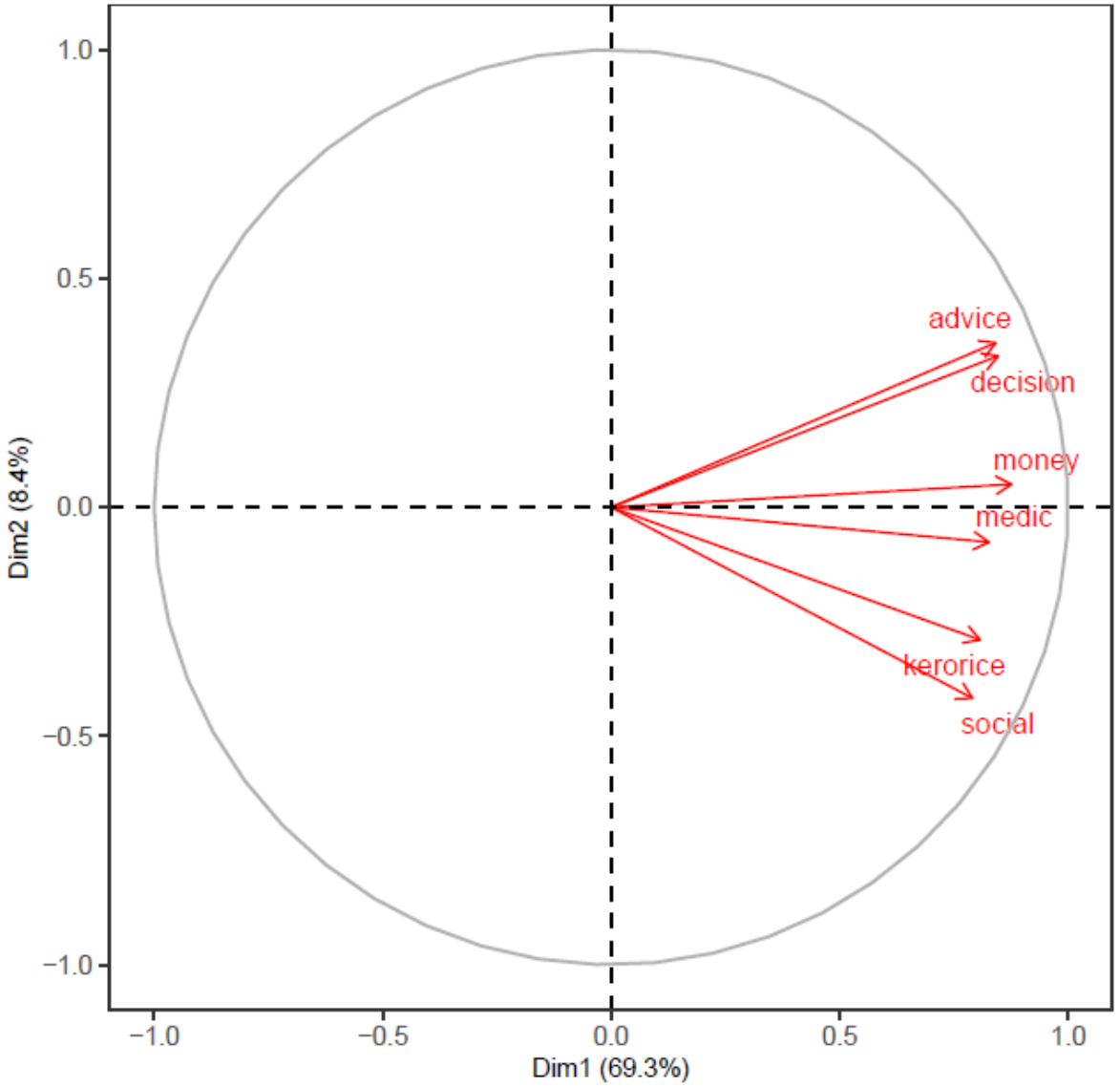


MF villages

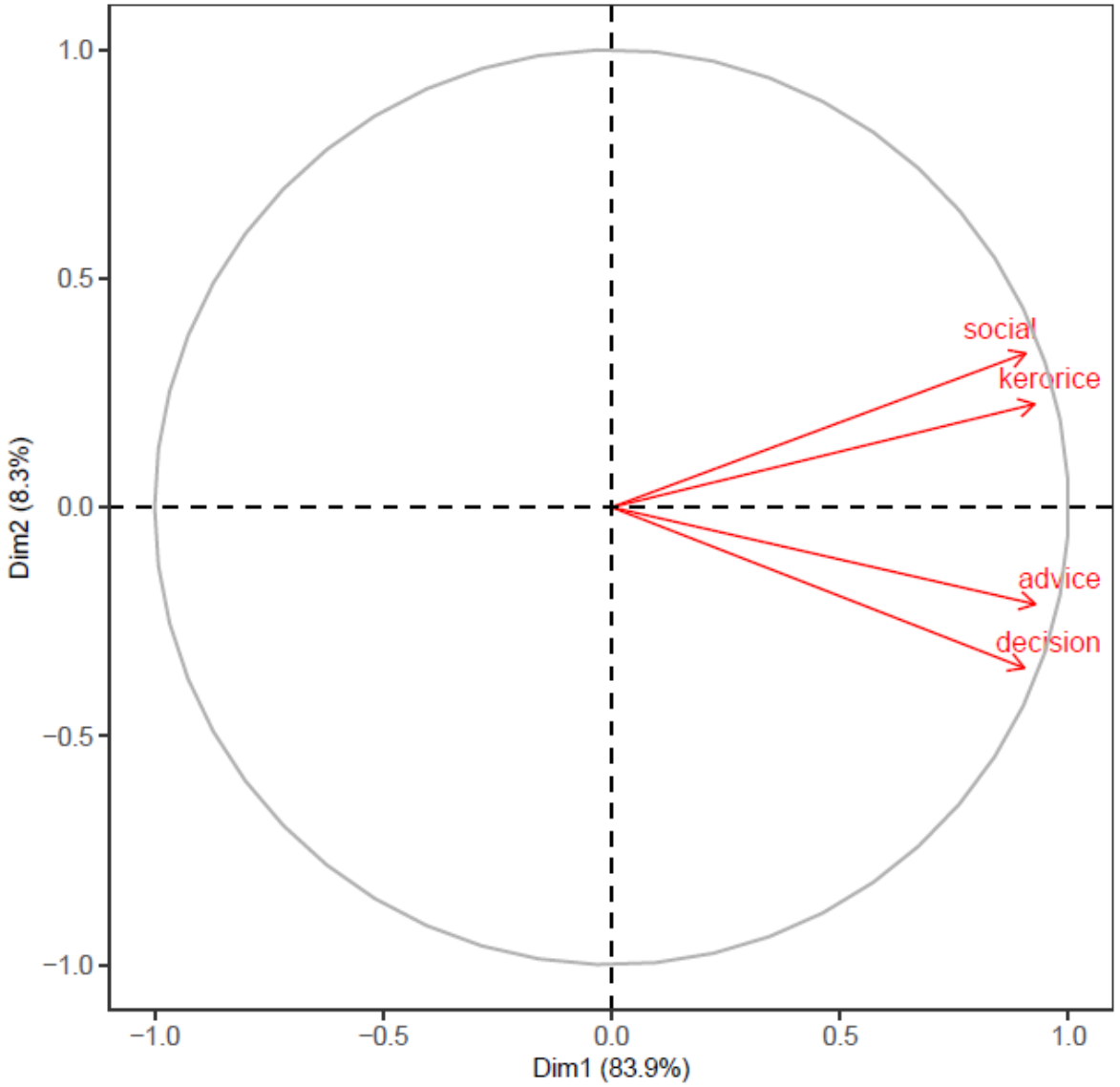


RCT villages

Principal Component Analysis (excluding jati and geography)

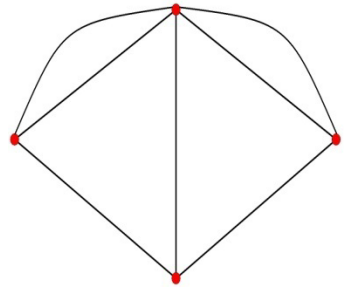


MF villages



RCT villages

Outline



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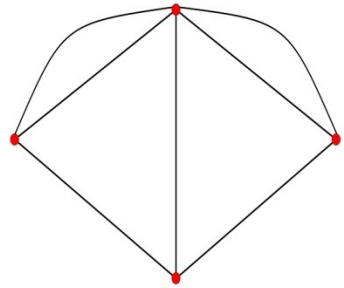
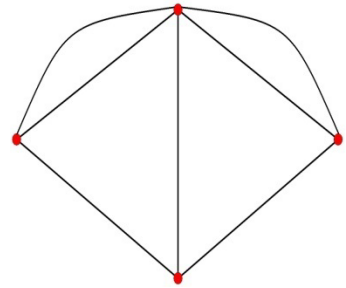
RCT on Diffusion



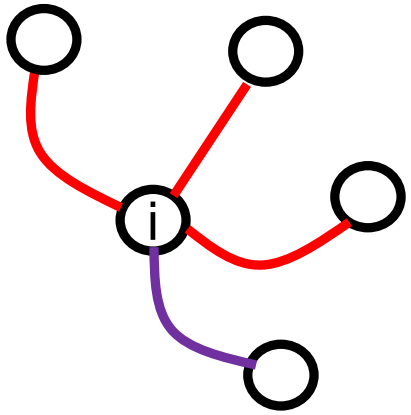
- Spread information about a chance to win a cell phone
- Randomly tell 3 to 5 people in each village and ask them spread information
- Measure diffusion of information by how many people participate in the cell-phone giveaway

Diffusion

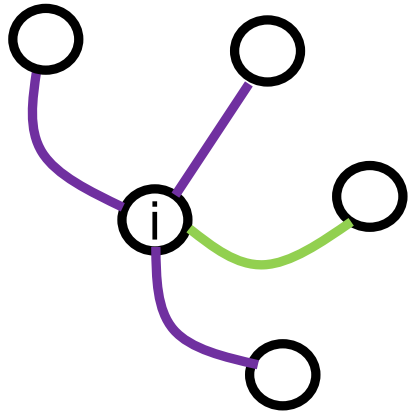


- Which network layer(s) best predict the diffusion?
  - Is it more important to have ``seeds'' central in advice or in kero/rice or in jati…?

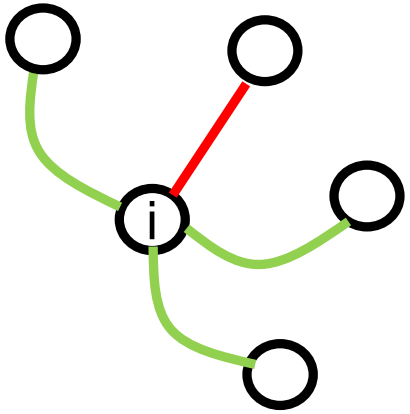
central in
advice network

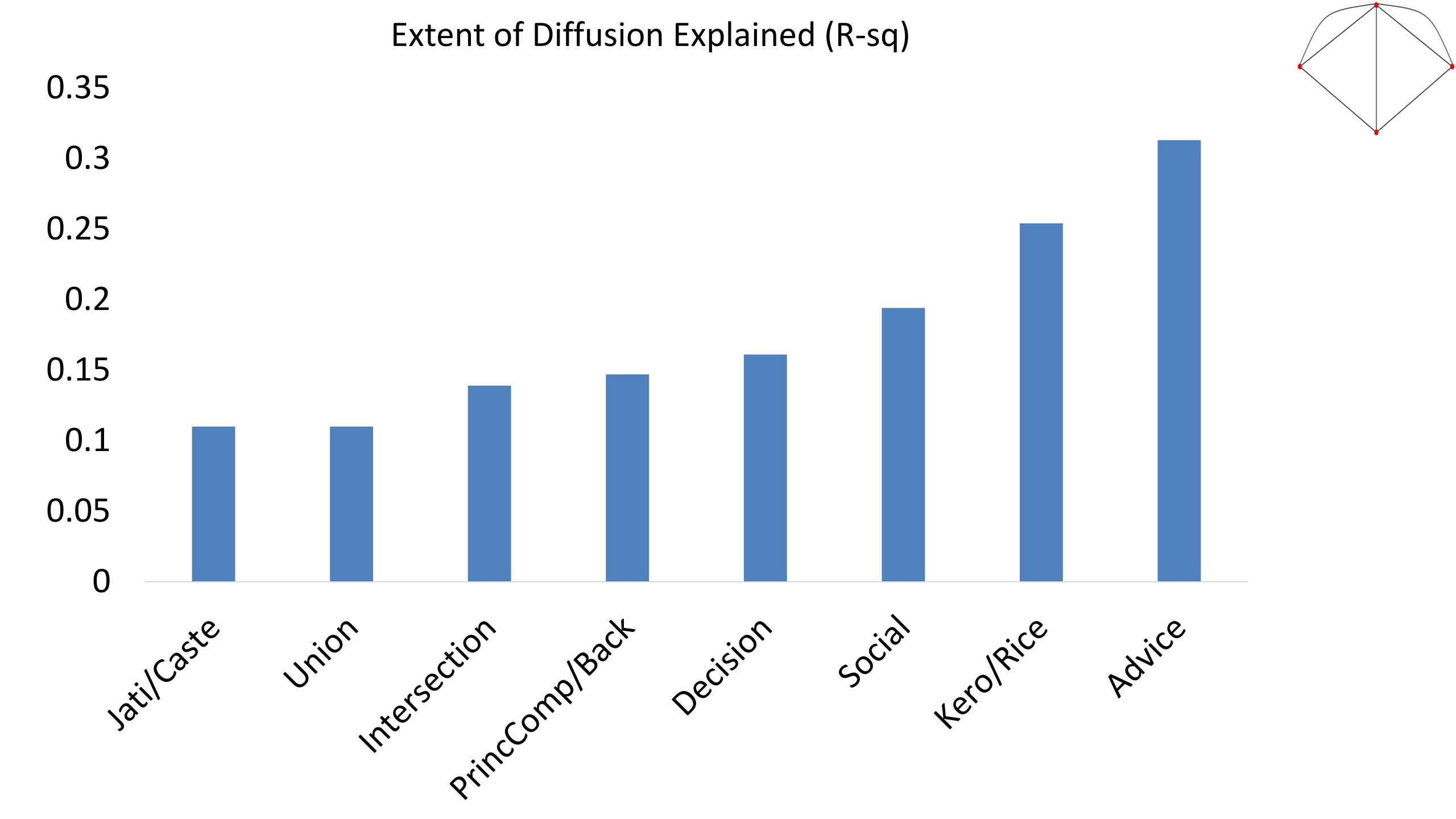


central in
kero/rice network

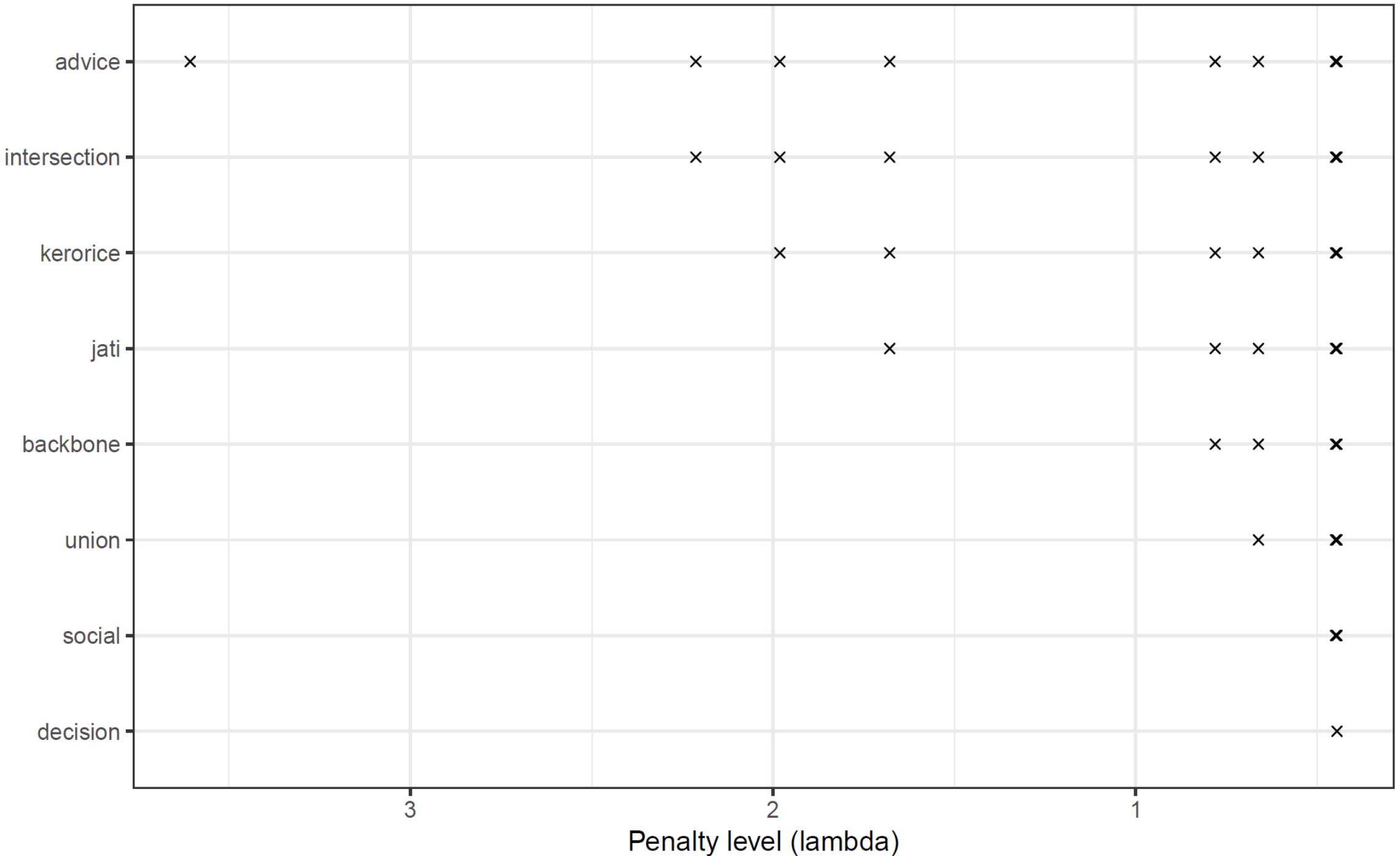


central in
jati network

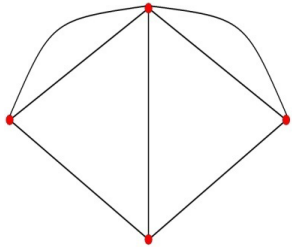



Extent of Diffusion Explained (R-sq)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Jati/Caste
Union
Intersection
PrincComp/Back
Decision
Social
Kero/Rice
Advice

Lasso


advice
intersection
kerorice
jati
backbone
union
social
decision
3
2
1
Penalty level (lambda)

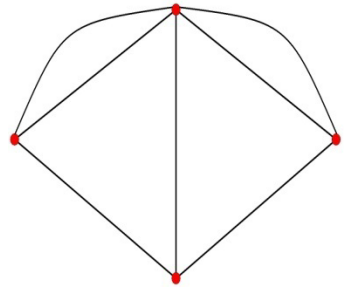
Cumulative Predictive Power:



layer	df	R.sq.	F-stat	p-val
advice	1	0.233	20.057	0.000
intersection	2	0.276	3.888	0.053
kerorice	3	0.281	2.134	0.127
jati	4	0.325	2.844	0.045
backbone	5	0.336	2.415	0.058
union	6	0.340	1.971	0.096
social	7	0.342	1.657	0.147
decision	8	0.344	1.419	0.215

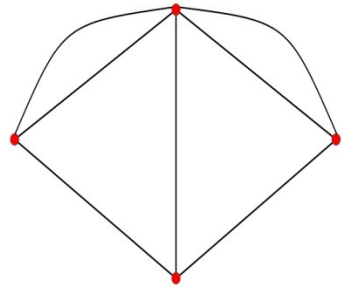
More than just advice matters:
combinations of layers matter in predicting diffusion

Which Networks?



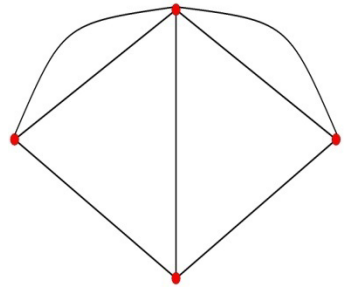
- Advice, kero/rice, social, are most predictive of diffusion
- Jati is not directly predictive of diffusion, but does significantly add when combined with advice and others…
- *Multiple layers matter, and matter differently*

Outline



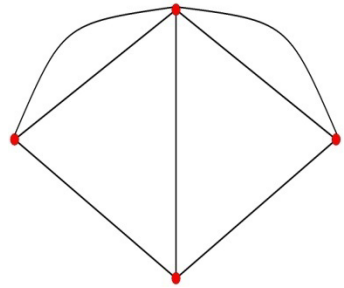
- Brief empirical look at multiplexing
- Do multiple layers affect diffusion (in an RCT)?
- Theory on how multiplexing impacts diffusion (simple, complex)
- Demographics and multiplexing

Theory: Diffusion



- Agents either infected or susceptible
- Transition back and forth in discrete periods
- If infected recover randomly at some rate δ each period
- If susceptible can be infected by contact with infected neighbor

Theory: Diffusion

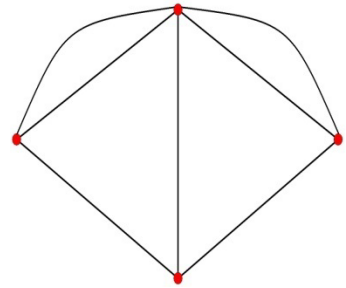


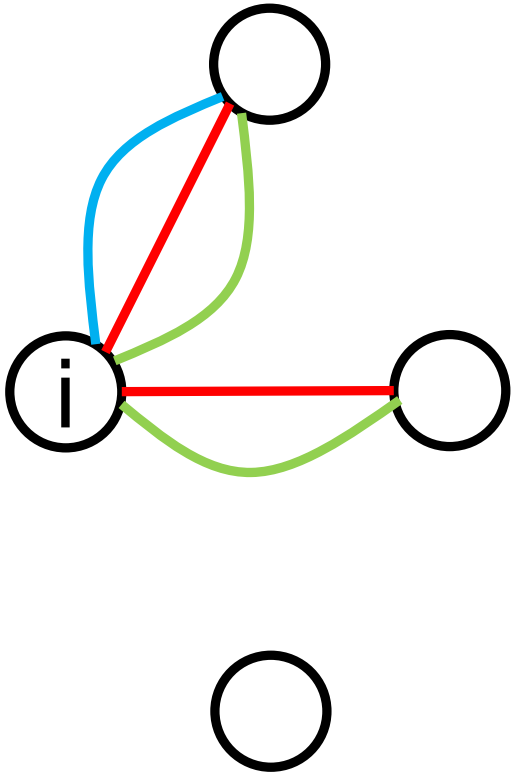
- Probability q^l that infected agent passes infection on layer l

- Agent becomes infected when getting at least τ `messages'
  - $\tau = 1$ simple contagion
  - $\tau > 1$ complex contagion

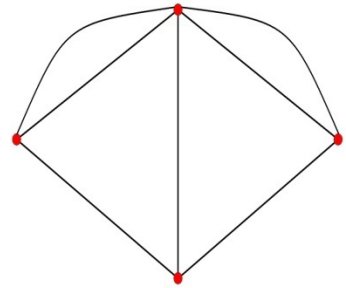
- Passing along layers independent (not too negatively correlated)

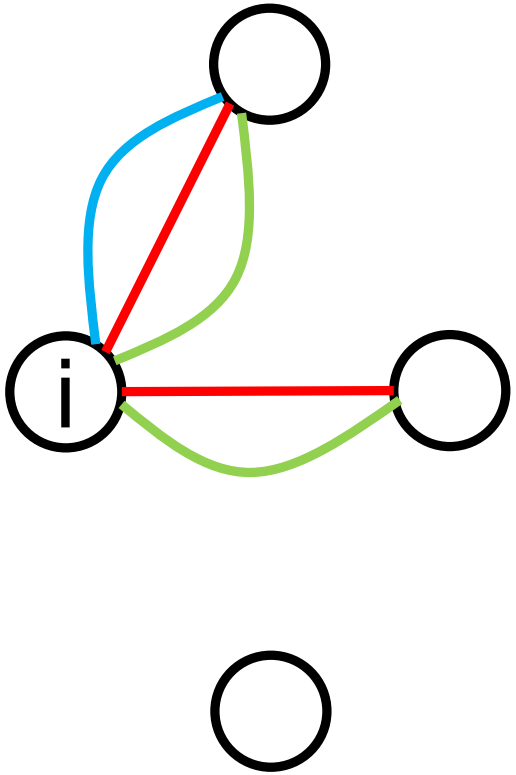
Less Multiplexed

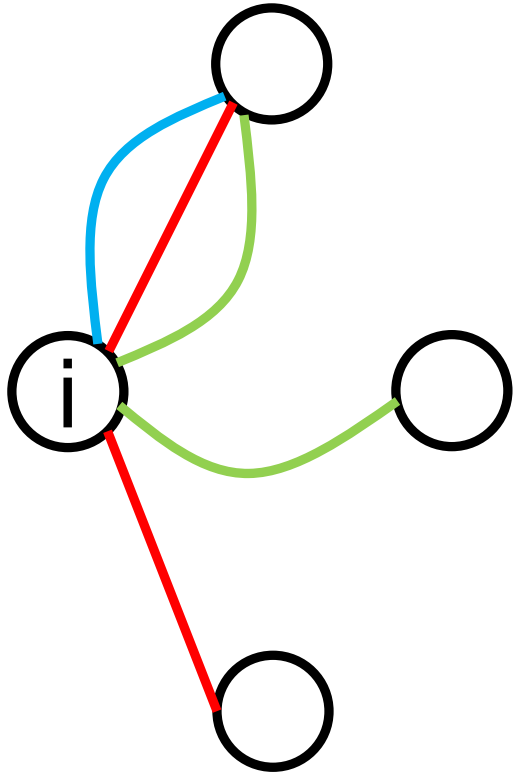




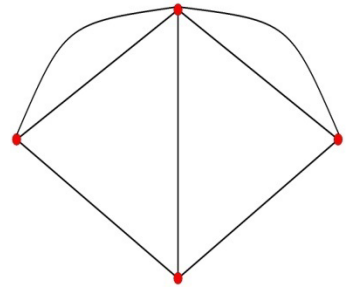
Less Multiplexed

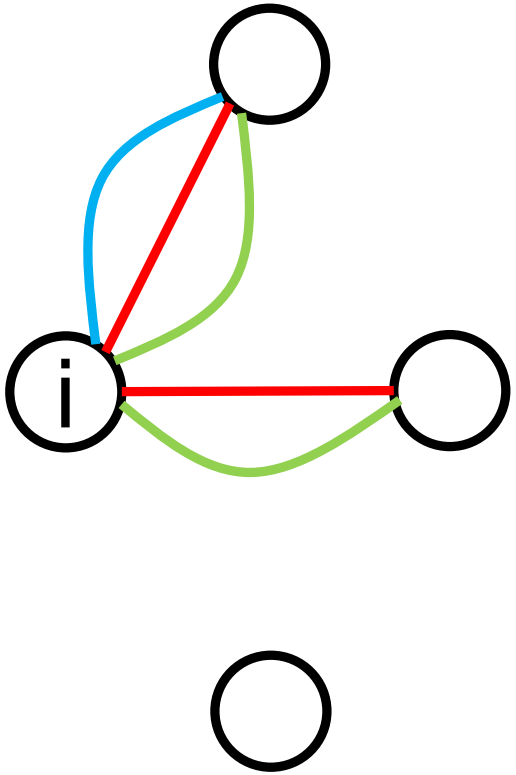


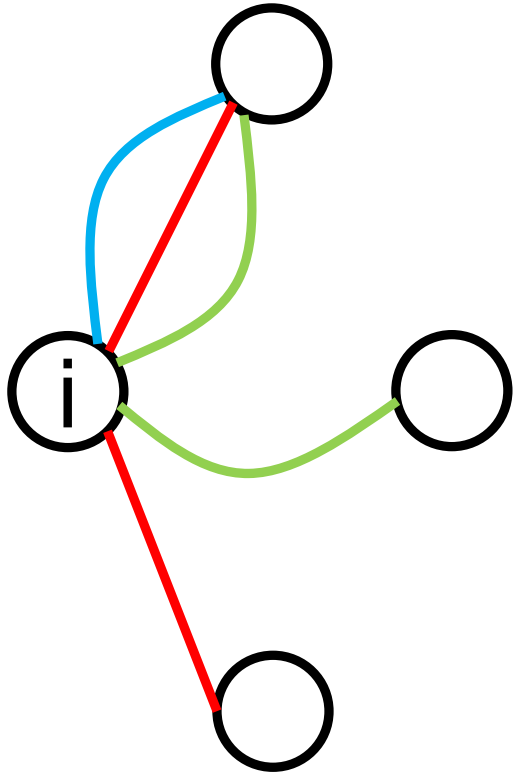


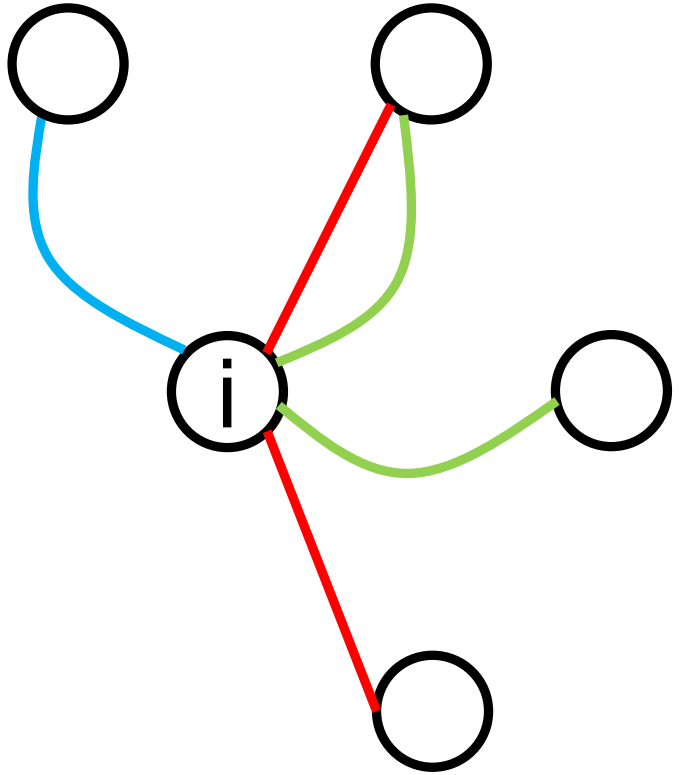


Less Multiplexed

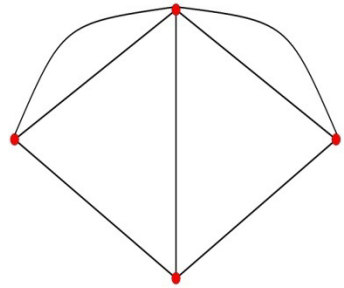








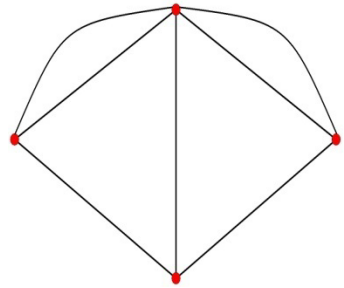
Proposition: Multiplexing Hurts Diffusion under *Simple Diffusion*



Consider two agents *i* and *j*, with *i* is less multiplexed.

If *i*'s and *j*'s neighbors are each infected with probability p then *i* is more likely to be infected.

Proposition: Multiplexing Hurts Diffusion under *Simple Diffusion*

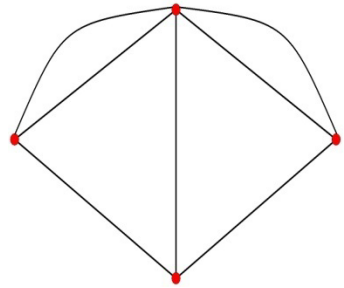


Consider agents *i* and *j*, with *i more* multiplexed than *j*.

If *i*'s and *j*'s neighbors are each infected with probability p, then *i* is *less* likely to be infected.

Less multiplexing – more diffusion/contagion

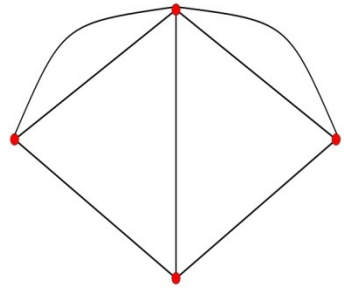
Proposition: Multiplexing Hurts Diffusion under *Simple Contagion*



In an SIS (or SIR model), the mean-field steady-state infection rate is **decreasing** in the multiplexing of the network.

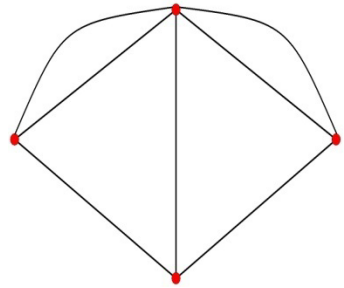
Less multiplexing - more diffusion/contagion

Intuition/Proof Logic:



- p chance neighbor is infected
- Infection on one multiplexed relation: $q^A\, p + q^B\, p - q^A\, q^B\, p$
- Infection on two un-multiplexed relations: $q^A\, p + q^B\, p - q^A\, q^B\, p^2$

Intuition/Proof Logic:



- p chance neighbor is infected
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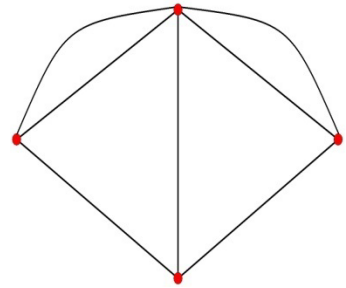
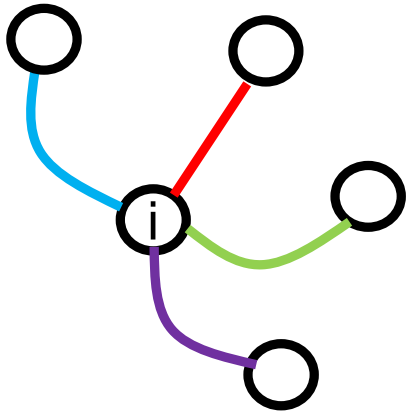
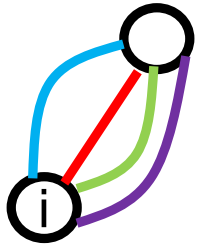
Back to the Data



Theory: more multiplexed networks less simple diffusion/contagion



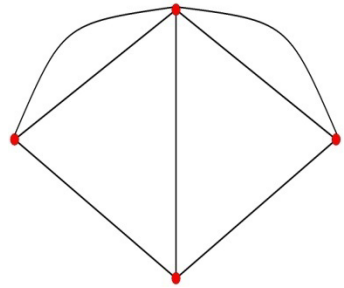


more diffusion

less diffusion

Do we see less diffusion in more multiplexed villages?

Multiplexing Index



$$m_v := \frac{1}{n} \sum_i \frac{\sum_j \left(\sum_\ell g^\ell_{ij,v} / L \right)}{\sum_j 1\{\sum_\ell g^\ell_{ij,v} > 0\}}$$

	Participation
High Multiplexing x Seed Set Centrality	-.039**
	(.017)
Seed Set Centrality	.052***
	(.016)
High Multiplexing	-.023
	(.016)
Observations	68

	Participation
High Multiplexing x Seed Set Centrality	-.039**
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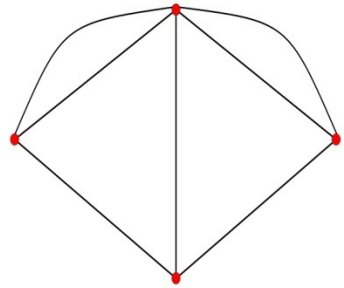
	(.016)
Observations	68

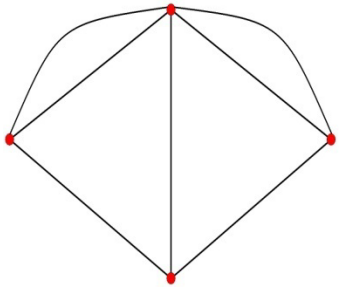
Complex Diffusion is more Complex

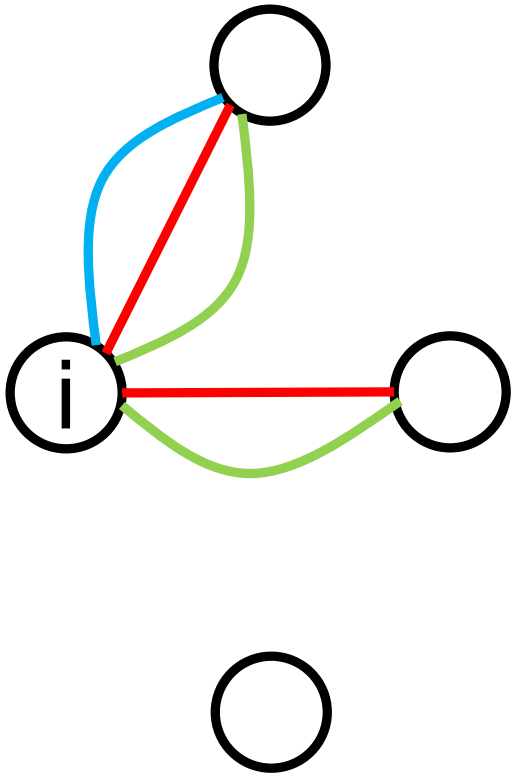


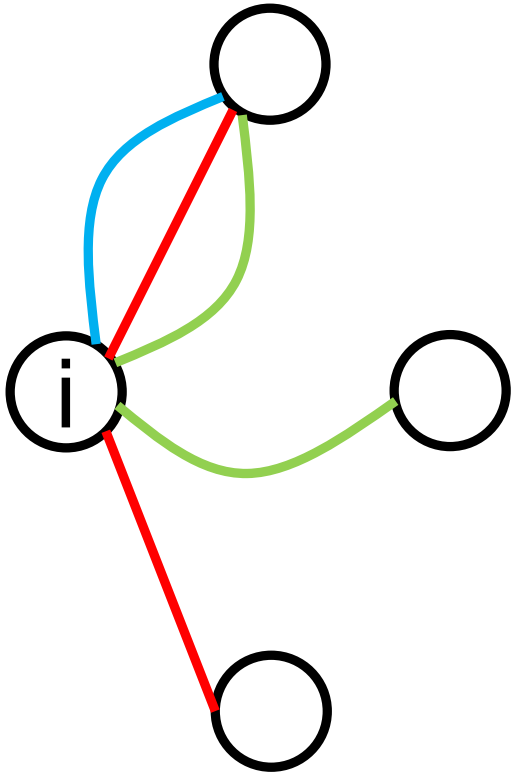
No longer can order based on multiplexing

Interaction with threshold

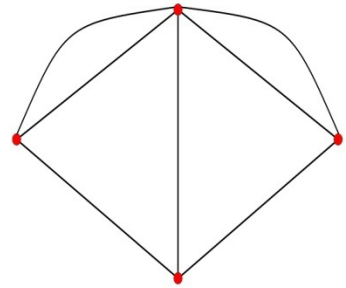
Complex Contagion

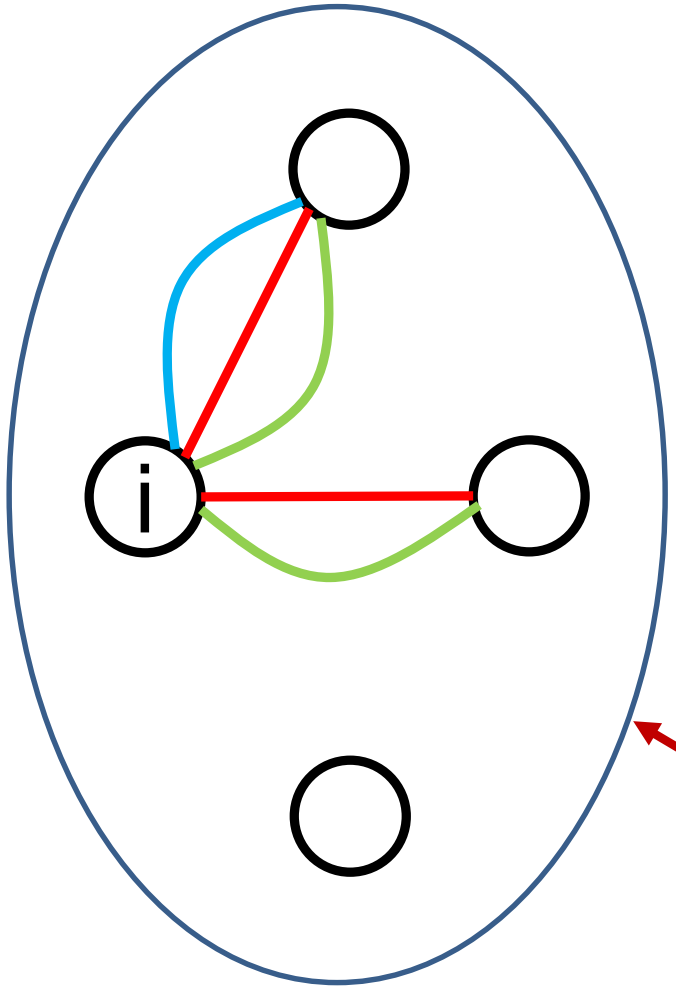






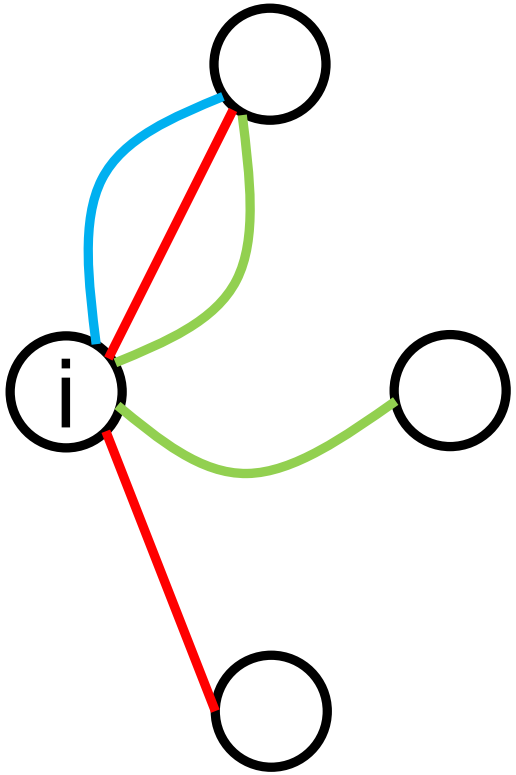
Complex Contagion





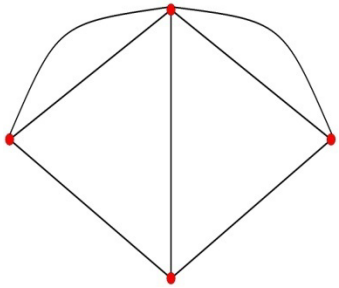
$\tau = 5$

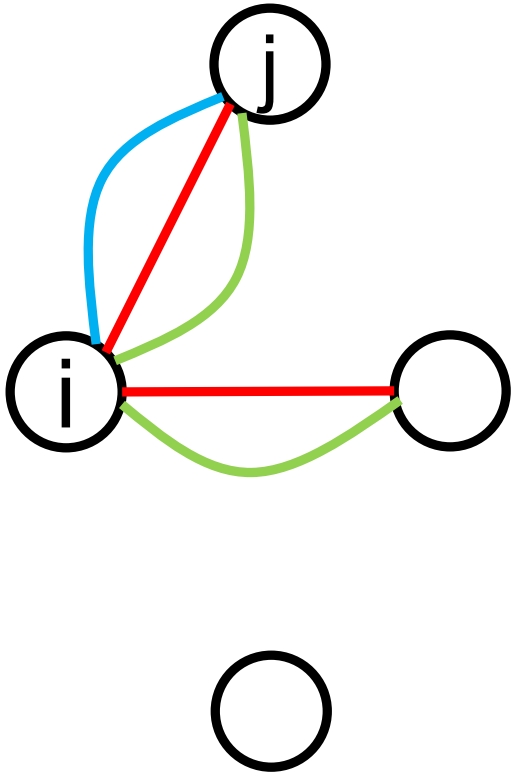
need all neighbors



Easier to infect

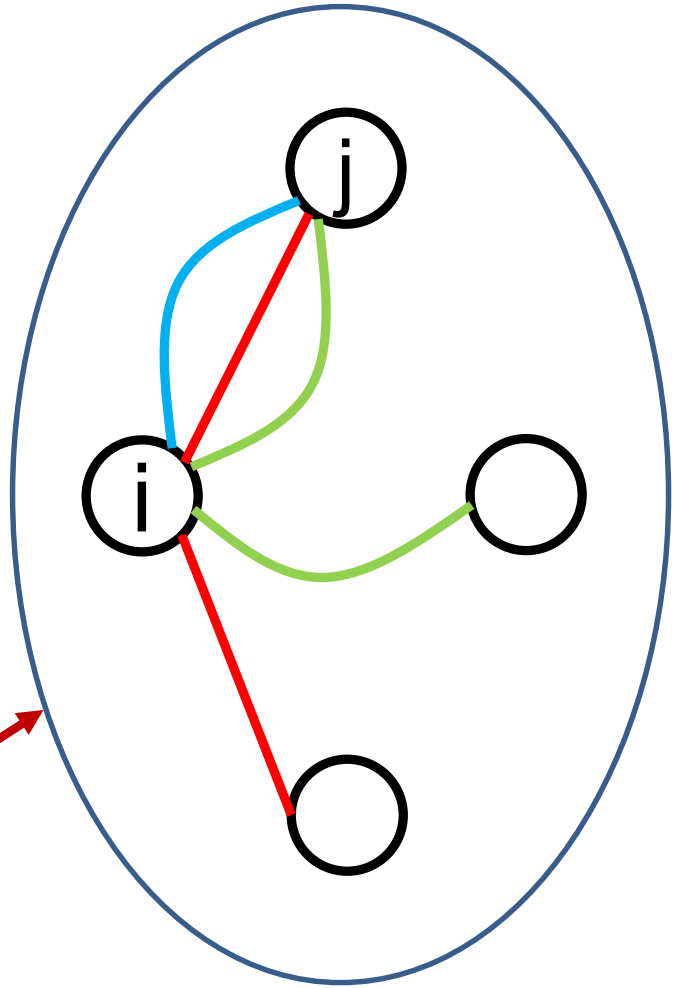
Complex Contagion





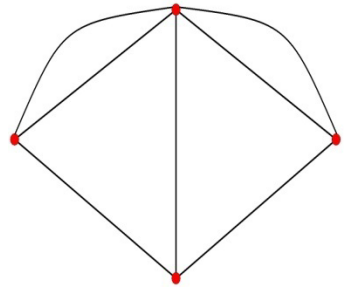
$\tau = 4$

need j and one other



Easier to infect

Complex Contagion

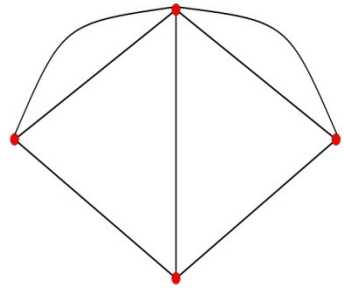


Competing forces

Less multiplexing – more chances to reach infected individuals

More multiplexing - more chances to get reinforced/multiple activations

Proposition: Complex Contagion

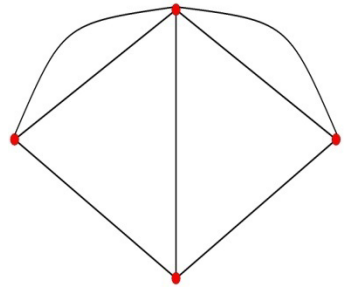


With **complex** contagion (threshold >1), total degree summed across layers is at least $\tau + 1$:

There exist $p' < p''$ (increasing in threshold) such that

- for $p < p'$ infection probability is **increasing** in multiplexing;
- for $p > p''$ infection probability is **decreasing** in multiplexing.

Complex Contagion



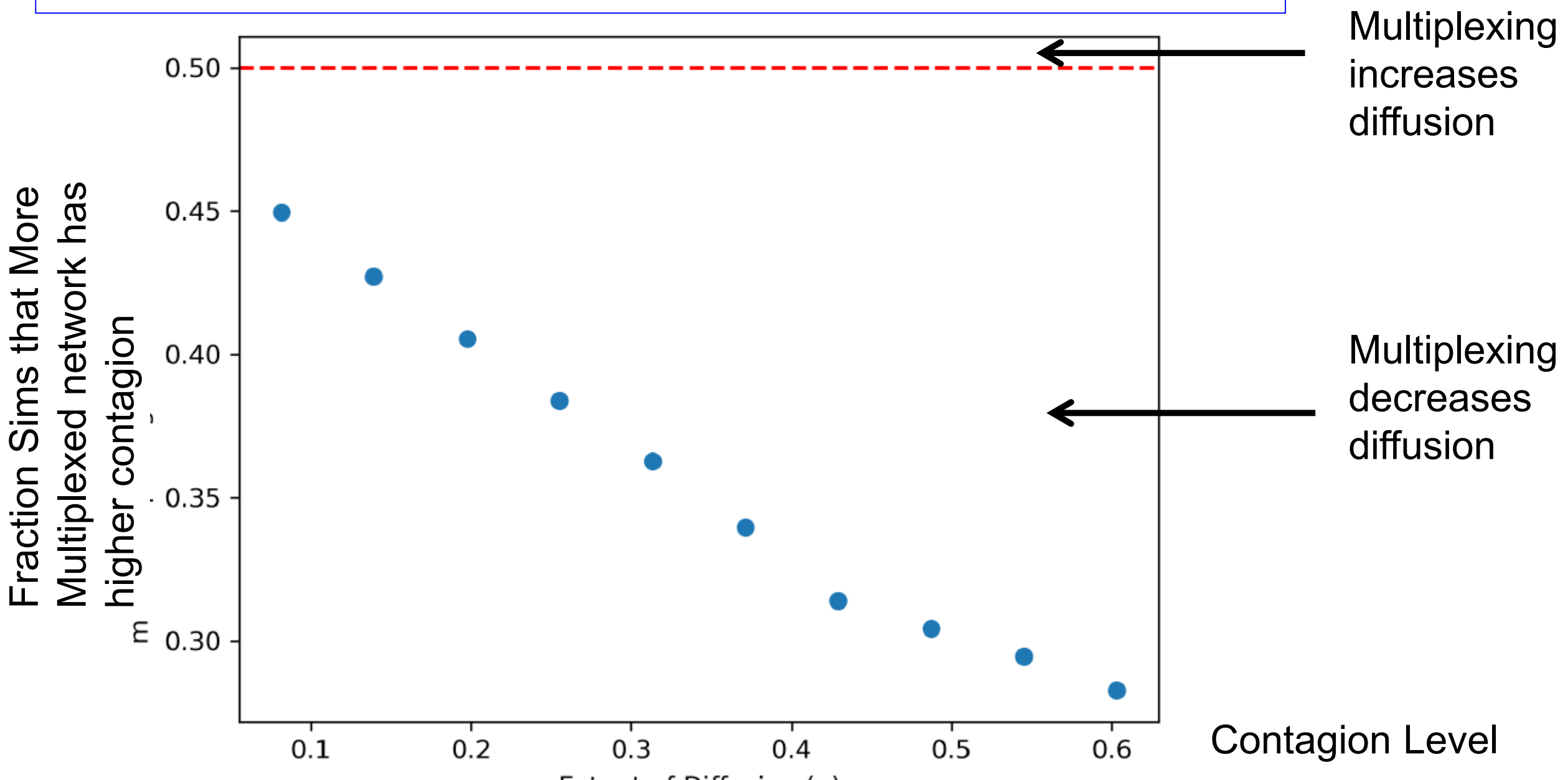
When the infection rate is low,

- Two neighbors being infected is unlikely.
- A multiplexed neighbor infected gives twice contact.

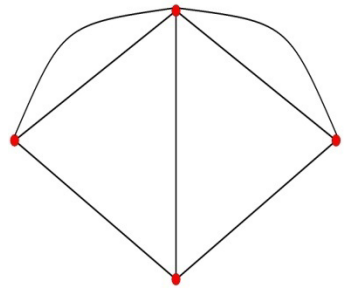
When the infection rate is higher,

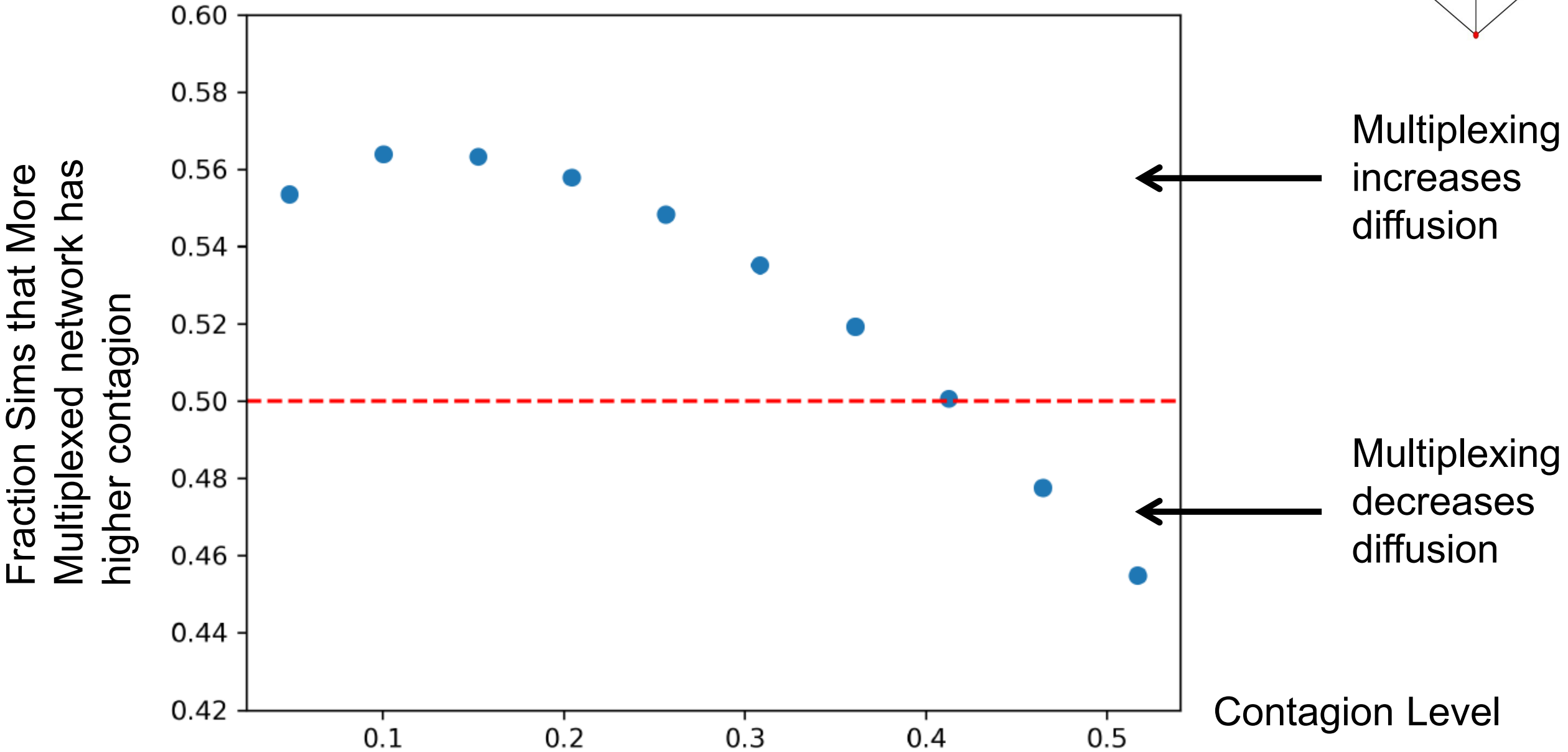
- More likely that only need one contact from these two links
- Non-multiplexed more likely to get at least one contact.

Simulations on Villages, Simple (τ =1)

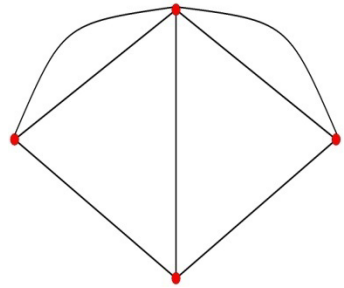


Simulations on Villages, Complex (τ =2)





Outline



- Brief empirical look at multiplexing
- Do multiple layers affect diffusion (in an RCT)?
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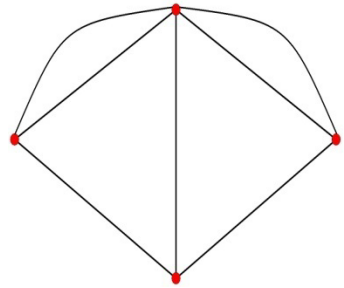
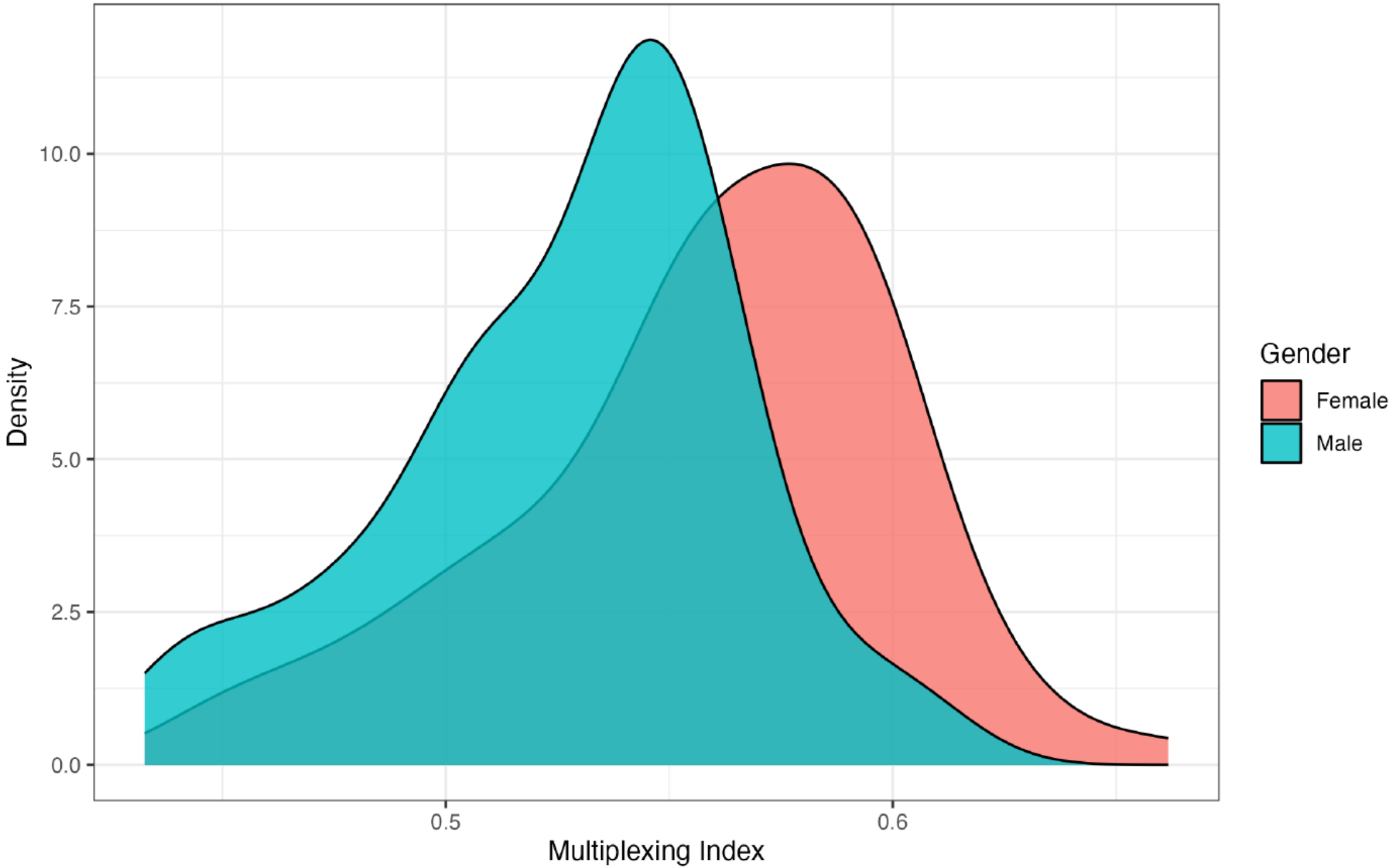
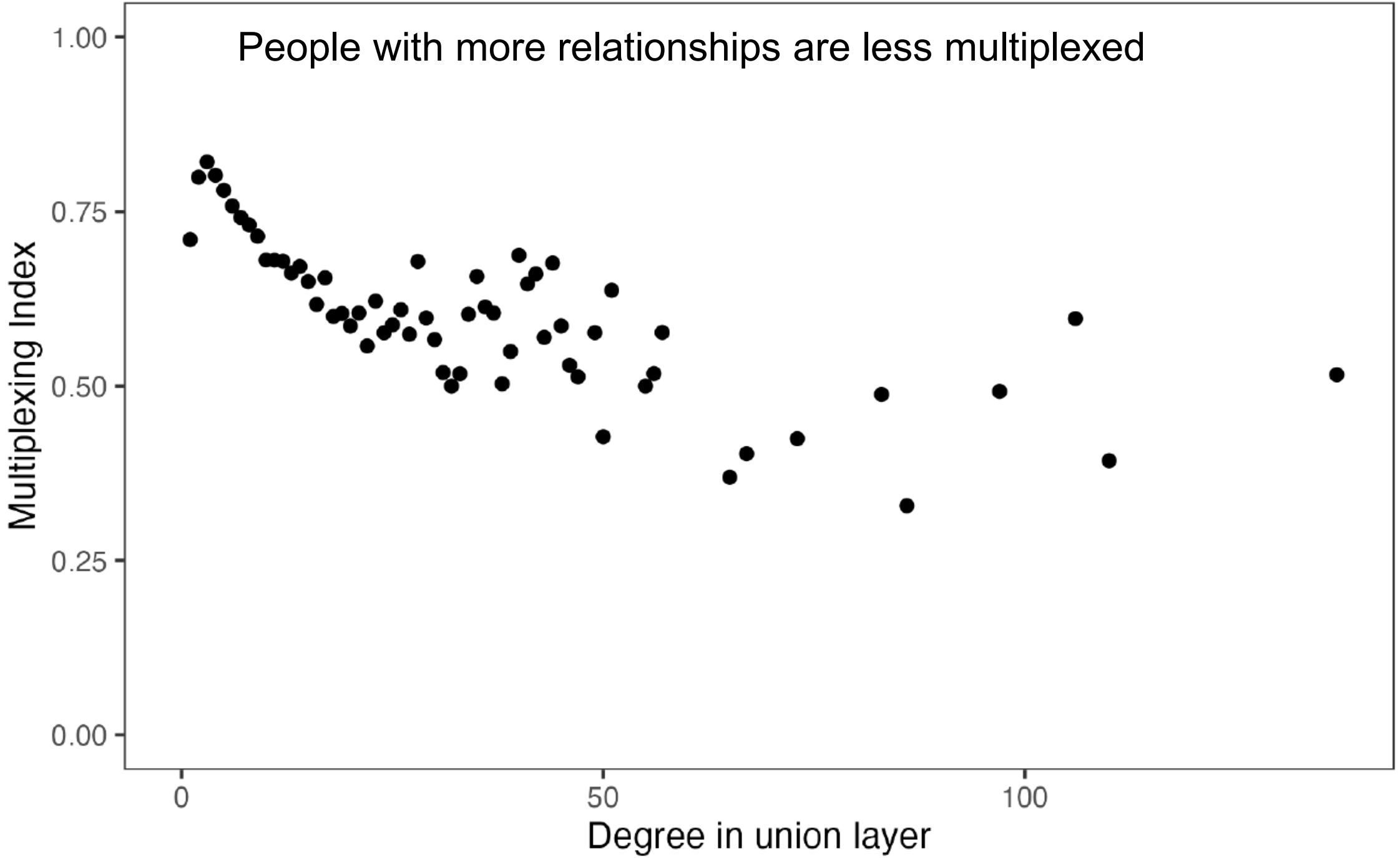
Who Multiplexes?

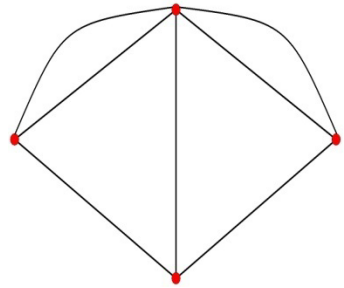


- Does multiplexing vary across individuals?
- Does multiplexing vary across locations?
- Which ones are more multiplexed?


10.0
7.5
5.0
2.5
0.0
Density
0.5
0.6
Multiplexing Index
Gender
Female
Male

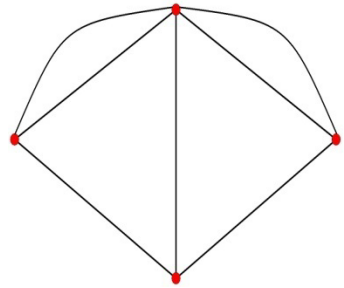

People with more relationships are less multiplexed
Multiplexing Index
1.00
0.75
0.50
0.25
0.00
0
50
100
Degree in union layer

Summary



- High multiplexing in data: high correlation between layers
- Some layers more predictive of diffusion, but multiple layers matter
- Multiplexing inhibits simple diffusion in a model and RCT, can aid complex
- Multiplexing higher among the less connected, females

Closing Thoughts



- Multiplexing matters in many contexts
  - Networks between nations (trade, migration, war)
  - Networks between companies (partnerships, lending, competition)
  - Networks among workers (communication, direction, collaboration)
  - Networks among students (friends, study partners, roommates)
  - ….

- Need for more theory/empirics of multiplexing and behaviors, network formation, methods