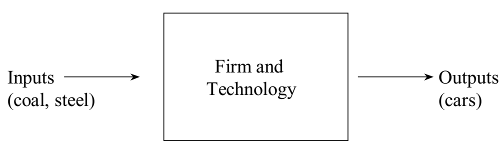
Theory of the Consumer

- What we have done so far:
- Derived "downward" sloping demand curves from underlying principles
  - budgets
  - preferences
  - optimizing behavior

Theory of the Firm

- We will derive "upward sloping" supply curves from underlying principles
  - Technology
  - Firm objectives (profit maximization / cost minimization)
  - Firm optimization

Technology



Technology

We represent technology by a production function:

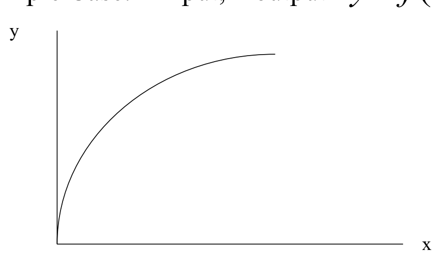
$$y = f(K, L)$$

For example: Cobb-Douglass production function:

$$y = AK^{\alpha}L^{\beta}$$

Production Functions

Simple Case: 1 input, 1 output $y = f(x)$



Properties of Production Functions

1. Slope of the production function is equal to the marginal product of the input
   - The marginal physical product of the input (MP_x) is defined as the incremental increase in output associated with an increase in the input.

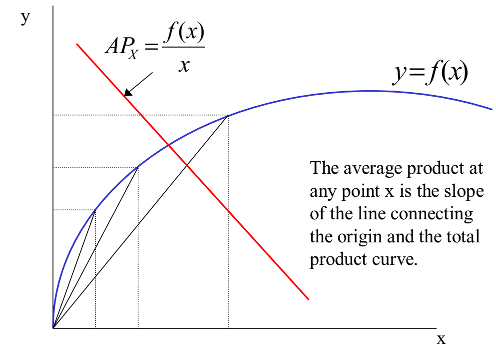
$$\frac{\partial y}{\partial x} = MP_x$$

2. Marginal physical products of inputs are positive

$$MP_X > 0$$

3. The average physical product of an input is the ratio of output to the amount of input used

$$AP_X = \frac{f(x)}{x}$$



4. If $MP_X > AP_X$, then AP_X is increasing
   - $MP_x > AP_x$ means that the last unit of input will produce more than the average level of output up to that point.
   - Thus this last unit of input will increase the average product produced.
5. If $MP_X < AP_X$, then AP_X is decreasing
   - The last unit of input will produce less than the average level of output up to that point, driving down the average

Proof that AP_x is maximized when $AP_x = MP_x$

- Given: $AP_x = f(x)/x$
- The necessary first order condition to maximize AP_x is:

$$\frac{dAP_x}{dx} = 0$$

Proof (continued)

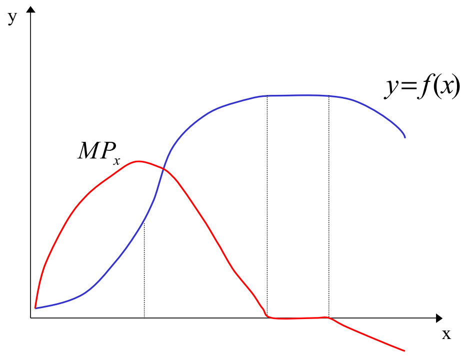
$$\frac{dAP_x}{dx} = \frac{f'(x)}{x} - \frac{f(x)}{x^2} = 0 \Rightarrow$$

$$f'(x) = \frac{f(x)}{x} \quad \forall x \neq 0 \Rightarrow$$

$MP_x = AP_x$ when AP_x is at a maximum.

Law of Diminishing Returns

- Eventually, each input has decreasing marginal product
- ‘S-shaped’ production functions
  - increasing marginal product at low levels of x
  - decreasing marginal product at high levels of x



More than One Input

- Typically, descriptions of technology will require the transformation of several inputs into one (or more outputs).
- Multiple output production technologies are beyond the scope of this class.
- We will look at production technologies with two inputs: e.g. capital and labor.

Marginal Physical Product with Two Inputs $Y = F(K, L)$

- MP is defined in exactly the same way as with one input, except now you need partial derivatives:

$$MP_K = \frac{\partial F}{\partial K}; \quad MP_L = \frac{\partial F}{\partial L}$$

Cobb-Douglass Example

$$Y = AK^{\alpha}L^{\beta}$$

$$MP_K = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}L^{\beta}; \quad AP_K = \frac{Y}{K} = AK^{\alpha-1}L^{\beta}$$

$$MP_L = \frac{\partial Y}{\partial L} = \beta AK^{\alpha}L^{\beta-1}; \quad AP_L = \frac{Y}{L} = AK^{\alpha}L^{\beta-1}$$

Returns to Scale

- When there is only 1 input, marginal returns to the input exactly equal returns to scale!
- Increasing returns to scale: if you 'double' all inputs, output more than 'doubles'
- Decreasing returns to scale: if you 'double' all inputs, output less than 'doubles'
- Constant returns to scale: if you 'double' inputs, then output 'doubles'

Homogeneity and Returns to Scale

- Recall the definition of k^{th} degree homogeneity.
  - A function is homogenous of degree k if

    $$f(tx_1, tx_2, \ldots, tx_n) = t^k f(x_1, x_2, \ldots, x_n) \ \forall t > 0$$

- A production function which is 1^{st} degree homogenous exhibits constant returns to scale.

Cobb-Douglass and Returns to Scale

$$Y = F(K, L) = AK^{\alpha}L^{\beta}$$

$$F(tK, tL) = A(tK)^{\alpha}(tL)^{\beta}$$
$$= t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}Y$$

So Cobb-Douglass production functions are constant returns to scale if $\alpha + \beta = 1$. They are increasing returns to scale if $\alpha + \beta > 1$

They are decreasing returns to scale if $\alpha + \beta < 1$

Spring 2001 Econ 11--Lecture 12 19

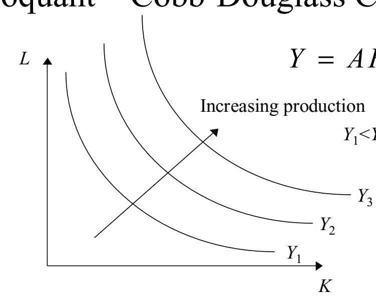
Isoquants

- These are defined as the set of all combinations of inputs that produce a given level of output.
- Notice the similarity between the definition of isquants and the definition of indifference curves—the set of all consumption combinations that produce a given level of utility.

Spring 2001 Econ 11--Lecture 12 20

Isoquant—Cobb-Douglass Case

$$Y = AK^{\alpha}L^{\beta}$$



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Rate of Technical Substitution

- The marginal rate of technical substitution is defined as the rate at which one input can be substituted for another along an an isoquant.
- It is defined to be (-1 times) the slope of the isoquant:

$$RTS_{L,K} = -\left.\frac{dK}{dL}\right|_{Y \text{ constant}}$$

Spring 2001 Econ 11--Lecture 12 22

Nicholson Example Problem

- *The production of barstools (q) is characterized by a production function of the form* $q = \sqrt{KL}$
  - *What is* AP_L *and* AP_K*?*

$$AP_L = \frac{q}{L} = \sqrt{\frac{K}{L}} \qquad AP_K = \frac{q}{K} = \sqrt{\frac{L}{K}}$$

  - *Graph the* AP_L *curve for K = 100.*

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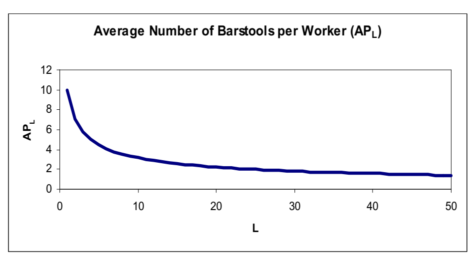
AP_L graph for K= 100



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Nicholson Example (continued)

- *For this particular function, show that $MP_L = 0.5\ AP_L$ and $MP_K = 0.5\ APK$. Using that information, add a graph of the MPL function to the graph in part (b) (at K= 100). What is unusual about this curve?*

$$MP_L = \frac{\partial q}{\partial L} = \frac{1}{2}\sqrt{\frac{K}{L}} = \frac{1}{2}AP_L \qquad MP_K = \frac{\partial q}{\partial K} = \frac{1}{2}\sqrt{\frac{L}{K}} = \frac{1}{2}AP_K$$

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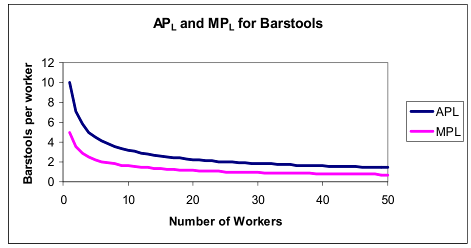
AP_L and MP_L graphs



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Nicholson Example (Continued)

- This graph is unusual because AP_L never intersects MP_L. Since $MP_L < AP_L$, every extra worker hired decreases average productivity!
- *Sketch the q = 10 isoquant for this production function.*

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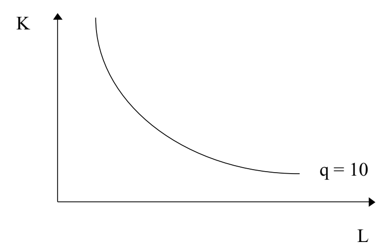
q = 10 Isoquant



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Nicholson Example (continued)

- *What is the rate of technical substitution (RTS) on the q=10 isoquant at the points:*
  - *K = L = 10*
  - *K = 25; L = 4*
  - *L = 25; K = 4*
- *Does this function exhibit a diminishing RTS?*

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RTS in the Example Problem

- Recall that RTS is (-1) times the slope of the isoquant.
- To derive this in terms of what we already know, take the production function and totally differentiate:

$$q = \sqrt{KL}$$

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RTS (continued)

$$dq = MP_K dK + MP_L dL = 0 \Rightarrow$$

$$\left.\frac{dK}{dL}\right|_{q\text{ constant}} = -\frac{MP_L}{MP_K} = -\frac{\sqrt{K/L}}{\sqrt{L/K}} = -\frac{K}{L} \Rightarrow$$

$$RTS_{L,K} = -\left.\frac{dK}{dL}\right|_{q\text{ constant}} = \frac{K}{L}$$

$RTS_{L,K}$ is Diminishing in L

L	K	RTS
4	25	6.25
10	10	1
25	4	0.16