Firm Objectives

• Cost minimization: Given a fixed output level (without any story about how this output is determined) firms choose the minimum cost combination of inputs.
• Profit maximization: Firms choose that level of output that yields the highest level of profits.

Profit

• Profit = Revenue – Cost
• Last class, we analyzed costs in detail.
  – The cost minimization problem produces a function C(Q), which represents minimum costs given output Q.
• Revenue is output multiplied by the price at which that output sells—R(Q) = PQ.

Profit Maximization—Choosing Output

\[ \max \Pi = R(Q) - C(Q) \]

First order condition: Interpretation: To maximize profits, set marginal revenue (\(dR/dQ\)) equal to marginal cost (\(dC/dQ\)).

Supply: How much will firms produce?

• If a firm produces at all, it will produce an amount such that MR = MC.
  – If the extra revenue generated from producing 1 extra unit of output (MR) exceeds the additional cost of producing that unit, then the firm can increase its profit by expanding output by 1 unit.

Second Order Condition

\[ \frac{d^2 \Pi}{dQ^2} = \frac{d^2 R}{dQ^2} - \frac{d^2 C}{dQ^2} < 0 \]

• The SOC is important because of “S” shaped cost curves.
• Also, if price falls below AVC, the firm (if it produced positive amounts of output) would earn a loss. Instead, it should go out of business.

Graphical Presentation
Nicholson Example Problem

- Would a lump-sum profits tax affect the profit maximizing quantity of output? How about a proportional tax on profits? How about a tax assessed on each unit of output?

Lump-Sum Tax

- Profit maximization under a lump-sum profits tax: \( T \) is the lump-sum tax
  
  \[ \Pi = PQ - C(Q) - T \]

- First order condition is exactly the same:
  
  \[ \frac{d\Pi}{dQ} = \frac{dR}{dQ} - \frac{dC}{dQ} = 0 \]

Lump-Sum Tax (continued)

- However, it may be optimal for firms to go out of business if the lump sum tax is high enough:

Proportional Tax on Profits

- Under a proportional tax on profits (say, \( t \)) the firm’s problem is:
  
  \[ \Pi = (PQ - C(Q))(1-t) \]

- The first order condition is:
  
  \[ \frac{d\Pi}{dQ} = \left( \frac{dR}{dQ} - \frac{dC}{dQ} \right)(1-t) = 0 \Rightarrow \frac{dR}{dQ} = \frac{dC}{dQ} \]

Proportional Tax (continued)

- The firm makes less profits, but the marginal and limit conditions are the same:

Tax on Output

- Under a tax (say, \( t \)) on output the firm’s problem is:
  
  \[ \Pi = PQ(1-t) - C(Q) \]

- The first order condition is:
  
  \[ \frac{d\Pi}{dQ} = \frac{dR}{dQ}(1-t) - \frac{dC}{dQ} = 0 \Rightarrow \frac{dR}{dQ}(1-t) = \frac{dC}{dQ} \]
Tax on Output (continued)

- This tax distorts the firm’s FOC—the optimal $Q$ differs from $Q^*$:

The firm is more likely to go out of business with this risky tax scheme.

Prices and Industry Structure

- The price that firms can charge will depend upon the structure of the industry.
  - In a competitive industry, firms cannot affect prices by cutting back on (or increasing) output.
  - In a monopolistic or oligopolistic industry, changes in output affect market price.
- In general, output prices that firm faces will be a function of the firm’s output:
  - $P = P(Q)$
  - This is exactly the (inverse) market demand function.

Competitive Supply

- In a competitive industry, firms take prices as fixed.
  - If firms charge prices higher than marginal costs, they lose all their business.
  - If firms charge prices lower than marginal costs, they make negative profits.
- For price taking firms, $dR/dQ = P$
- The first order condition is $P = MC = dC/dQ$
  - The firm will choose output at a point where price is equal to marginal cost.

Supply Curve

- An increase in marginal cost = a shift in the supply curve
  - For example, an increase in wages shifts supply back since it increases marginal costs.

Monopoly Supply

- Monopolists do not sit back and take prices.
- They manipulate prices by curtailing output below competitive levels.
  - For monopolists, prices are a function of quantity produced (the inverse market demand curve).
- However, like firms in a competitive industry, monopolists still maximize profits. The FOC is:
  $$\frac{dR}{dQ} = \frac{dP(Q)}{dQ} Q + P(Q) = \frac{dC}{dQ}$$
**Monopoly Supply (Graphical)**

- **P** — Price
- **Q** — Quantity
- **dR/dQ** — Marginal Revenue curve
- **P(Q)** — Demand curve
- **C’(Q)** — Marginal Cost curve
- Area in box = Total revenue

**Nicholson Example Problem #2**

- Universal Widget produces high-quality widgets at its plant in Gulch, Nevada, for sale throughout the world. The cost function for total widget production (q) is given by $TC = 0.25q^2$. Widgets are demanded only in Australia (where demand is $q = 100 - 2p$), and Lapland (where demand is $q = 100 - 4p$). If Universal Widget can control $q$ in each market, how many should it sell in each location in order to maximize profits? What price will be charged in each location?

**Example #2 Solved**

- Let $q_1$ be the amount sold in Australia at price $p_1$.
- Let $q_2$ be the amount sold in Lapland at price $p_2$.
- The firm’s total revenues from the two locations equal $p_1 q_1 + p_2 q_2$.
- The firm’s total costs of producing $q_1 + q_2$ are $0.25(q_1 + q_2)^2$.

**Example #2 Solution (continued)**

- The firm chooses $q_1$ and $q_2$ to maximize profits:
  
  $$\max_{q_1, q_2} \Pi = q_1 p_1 + q_2 p_2 - (q_1 + q_2)^2$$

- Inverting two demand functions yields:
  
  $$p_1 = -\frac{1}{2} q_1 + 50 \quad p_2 = -\frac{1}{4} q_2 + 25$$

**Example #2 Solution (III)**

- Plugging the inverse demand functions back into the profit function gives the firm’s maximization problem in terms of $q_1$ and $q_2$:
  
  $$\max_{q_1, q_2} \Pi = q_1 \left( -\frac{1}{2} q_1 + 50 \right) + q_2 \left( -\frac{1}{4} q_2 + 25 \right) - (q_1 + q_2)^2$$

**Example #2 Solution (IV)**

- The first order conditions for the firm’s problem and their solution are:
  
  $$-\frac{3}{2} q_1 - \frac{1}{2} q_2 + 50 = 0 \quad \Rightarrow \quad q_1 = 30 \Rightarrow p_1 = 35$$
  
  $$-\frac{1}{4} q_1 - q_2 + 25 = 0 \quad \Rightarrow \quad q_2 = 10 \Rightarrow p_2 = 22.5$$
Producer Surplus

- Producer surplus = Revenue - Variable Cost
- Profit = Revenue - Total Cost
- Producer surplus = Profit if no fixed costs or if in the long-run
- AKA ‘operating profit’

3 Ways to Measure PS

- PS = Revenue - Variable Cost
- PS = Area where Price > MC
- PS = Area to the left of the supply curve

Industry Supply

- The industry supply curve is the horizontal sum of the existing firms’ supply curves