Welfare Properties of Market Outcomes

- Last time, we covered equilibrium in one market—partial equilibrium.
- We found that under perfect competition, the equilibrium price and quantity maximized the sum of producer and consumer surplus.
- In an exchange economy, we found that trade at the equilibrium market price led automatically to Pareto optimal outcomes (on the contract curve).
- The next two lectures will ask whether these nice welfare properties hold in general equilibrium under perfect competition.

The Complexity of General Equilibrium

- In partial equilibrium analysis, there is a clean and sharp theoretical distinction between producers and consumers.
- In general equilibrium, one person may participate simultaneously in many different markets.
  - Sometimes as a consumer
  - Sometimes as a producer
- There are billions of people in the economy.
- Outcomes in one market affect outcomes in countless other markets.
- What is needed is some mechanism to concisely convey to everyone information about everyone else's needs and about the difficulty in fulfilling them.

The Role of Market Prices

- The role of prices is to convey signals to every participant about relative scarcity in all markets simultaneously.
- We will see that under competitive conditions, market prices succinctly convey all necessary information about an unimaginably complicated reality so that all markets are in equilibrium simultaneously.

Competitive Equilibrium

- Consumers, taking prices as given, choose consumption goods and supply inputs (capital, labor) to firms to maximize utility.
- Producers, taking prices as given, buy inputs from consumers and make consumption goods to maximize profits.
- Consumers own firms and collect any profits based on how many shares they own.
- In competitive equilibrium, all input and output markets clear and firms make zero profits.

Contingent Commodities and Futures

- Contingent commodities are goods that are delivered depending upon the state of the world.
  - Fire insurance pays if there is a fire.
  - Bets against the Lakers must be paid when they win.
- Futures are commodities that are delivered at some future time for a price paid today.
  - Cattle futures
  - Organ futures
- Economists have constructed proofs that show that general equilibrium exists even when contingent and futures markets are allowed.

Plan for the Rest of the Lecture

- Rather than showing the most general forms of the proof of general equilibrium (which requires advanced math), I will consider the existence of general equilibrium in two models:
  - Walras' exchange economy with many markets but no production.
  - An economy with two production goods and two factors of production.

Walras' Exchange Economy

- n goods ($x_1 \ldots x_n$) in fixed supply, $S_1 \ldots S_n$.
- Each good has an associated price, $P_1 \ldots P_n$.
- There are K people and each take prices as given.
- "Income" is the value of each person's holdings at market prices
  - Person k's income is $I_k = \sum_{i=1}^{n} P_i S_{ik}$
  - Person k's budget constraint is: $\sum_{i=1}^{n} P_i x_{ik} = I_k$

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Market Demand in Walras' Economy

- Each person picks his optimal consumption bundle to maximize utility
- This leads to n individual demand functions for each individual which are a function of all market prices.
  - $D_{1k}(P_1 \ldots P_n)$, $D_{2k}(P_1 \ldots P_n)$,… $D_{nk}(P_1 \ldots P_n)$
- The sum of individual demand functions yields n market demand functions:
  - $D_1(P_1 \ldots P_n)$, $D_2(P_1 \ldots P_n)$,… $D_n(P_1 \ldots P_n)$

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Equilibrium in Walras' Economy

- Equilibrium is a set of prices $P^* = \left(P_1^*, P_2^*, \ldots P_n^*\right)$ such that all of the markets clear: $D_i\left(P^*\right) = S_i \;\; \forall\, i$
- n excess demand equations:

$$ED_i\left(P^*\right) = D_i\left(P^*\right) - S_i = 0 \;\; \forall\, i$$

- There are n equations with n unknowns
  - This means there will automatically be a solution, right?
  - No! Since the n equations characterizing the equilibrium are non-linear, there is no guarantee that there will be any solutions.

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Walras' Law

- The total value of demand must equal the total value of supply in the economy.
  - This is true even when non-equilibrium prices hold.
- Walras' Law follows directly from summing the individual budget constraints.
  - Each person's budget constraint is: $\sum_{i=1}^{n} P_i D_{ik}(P) = \sum_{i=1}^{n} P_i S_{ik}$
  - Summing all the budget constraints yields:

$$\text{Total value of demand} = \sum_{i=1}^{n} P_i D_i(P) = \sum_{i=1}^{n} P_i S_i = \text{Total value of supply}$$

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Zero Degree Homogeneity of Demand

- Now there are n + 1 equations in P, and only n unknowns.
  - If this were a linear system, this would mean there are infinitely many solutions.
- This happens because demand is homogenous of degree zero.
  - Suppose we have found the equilibrium.
  - Doubling all prices would also double income
  - We have seen that doubling prices and income at the same time does not change demand.

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Only Relative Prices Are Identified

- The system of demand equations identifies only n-1 relative prices, not all n absolute prices.
- This means we can pick any n - 1 of the equilibrium excess demand equations, which, in principle should be able to identify all n - 1 relative prices.
  - We're back to where we were--no guarantee of a solution, since the system is non-linear.

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Mathematical Digression-- Brouwer's Fixed Point Theorem

- Any continuous mapping F(X) of a closed, bounded, convex set into itself has at least one fixed point (X^*) such that F(X^*)=X^*
  - Mapping: A rule associating points in a set with another set of points.
  - Closed: The set contains its edge.
  - Convex: If the set contains two points, it also contains all points on the line connecting the two points.
  - Bounded: The set's dimensions are finite.

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Example for a Univariate Mapping

- Intuition: Consider a continuous function $f(x)$ with domain and range [0,1].
  - A continuous function is a continuous mapping.
  - The set X = [0,1] is closed, bounded, and convex.
- A fixed point of $f(x)$ is a point x^* such that $f(x^*)=x^*$.
- On a graph of $f(x)$, it's fixed points are on the 45° line.

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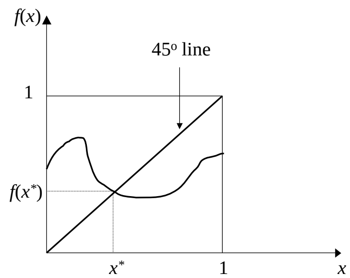
Example: Brouwer's Fixed Point Theorem



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Back to Walras' Economy

- The next step to finding an equilibrium is to "normalize" the n prices so that they add to one.
  - For the new price set, divide each price by the sum of all the prices. $P_j' = \frac{P_j}{\sum_{i=1}^{n} P_i} \ \forall j$
  - Redefining prices in this way will not change demand because of zero degree homogeneity.

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Applying Brouwer's FP Theorem

- After normalizing prices, the price set is closed, convex, and bounded.
- Use the excess demand functions to define a mapping on the normalized price set.

$$F^i(P) = P_i + ED_i(P) \ \forall i$$

- To be rigorous, we need to worry about what happens to demand when some price is zero.
  - Demand could be less than supply for a good with zero price.
  - We will ignore this case here.

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An Equilibrium Exists

- By Brouwer's FP Theorem, a fixed point must exist.

$$F^i(P^*) = P_i^* + ED_i(P^*) = P^* \ \forall i$$

- At the fixed point P*, all n of the excess demand functions equal zero.

$$ED_i(P^*) = D_i(P^*) - S_i = 0 \ \forall i$$

- All markets clear, so a competitive equilibrium must exist.

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Two-Good Two-Input Economy

- Now, an economy that includes production.
- Two types of firms, each producing one of two goods X and Y, which are sold to consumers.
- The firms buy inputs K and L from consumers to produce the outputs using production technologies X=F(K, L) and Y=G(K,L).
- Consumers own the firms--they collect any profits made by the firms.

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Consumer and Producer Goals

- Consumers (i = 1…n) maximize utility subject to their budget constraint.
  - Income from labor and capital: $r L_i + w K_i$
  - Income from profits: $\alpha_i \pi_X + \beta_i \pi_Y$
- Producers maximize profits π_X and π_Y.
- Profit shares sum to one: $\sum_{i=1}^{n} \alpha_i = 1 \quad \sum_{i=1}^{n} \beta_i = 1$
- Labor and capital constraints:

$$\sum_{i=1}^{n} L_i = \overline{L} \qquad \sum_{i=1}^{n} K_i = \overline{K}$$

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Demonstrating Equilibrium

- The formal approach to demonstrating the existence of competitive equilibrium is similar to the approach in the Walras exchange economy.
  - Two factor demand equations; two factor prices w & r.
  - Two product demand equations; two good prices P_X and P_Y.
  - Profits equal zero in competitive equilibrium.
  - Adding all participants budgets show Walras' Law, so only relative prices can be identified.
- Instead, equilibrium will be demonstrated graphically.

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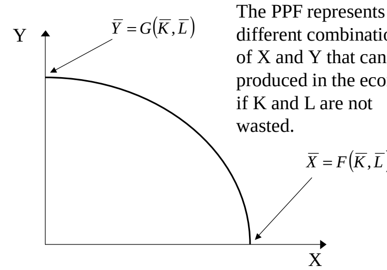
Production Possibility Frontier



The PPF represents the different combinations of X and Y that can be produced in the economy if K and L are not wasted.

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Rate of Product Transformation

- The rate of product transformation is defined as (-1 times) the slope along the production possibility frontier.

$$RPT(X \text{ for } Y) = -\frac{dY}{dX} = \frac{MC_X}{MC_Y}$$

- *RPT* represents how much *X* can be traded for *Y* while keeping inputs *K* and *L* productively employed.
- *RPT* is equal to the ratio of marginal costs of production.
  - At every point on the PPF, K and L are efficiently employed
  - By definition, this means that costs are minimized.
  - Since inputs are in fixed supply, minimum costs of producing X and Y will be constant.

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Why is the PPF Concave?

- Diminishing returns.
  - Increasing output of X raises its marginal cost.
  - Increasing output of Y raises its marginal cost.
- Specialized inputs
  - Some inputs may be better suited to the production of one good, rather than another
  - Increasing the production of one good eventually requires using inputs that are poorly suited for that good's production.
- Differing factor intensity
  - X and Y may require K and L in different proportions.
  - Then, even under constant returns to scale and non-specialized inputs, making more of X or Y may require the use of relatively more of the less intensively required factor, raising marginal costs.

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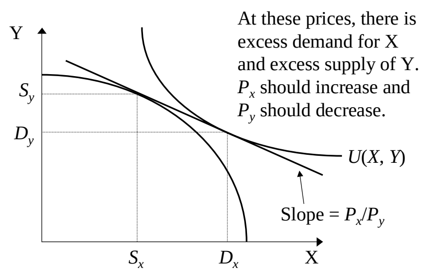
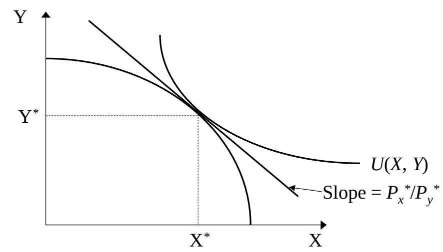
PPF, Consumer Utility and Prices



Equilibrium in Two Good Economy



Properties of Equilibrium

$$\frac{P_x^*}{P_y^*} = RTS = \frac{MC_x}{MC_y} = \frac{MU_x}{MU_y} = MRS$$

- At P^*_x and P^*_y, both product markets clear.
- Profits and utility are maximized.
- Labor and capital are efficiently used since final consumption is on the PPF.
- Profits equal zero. (Need to discuss input markets to show this).