Demand II

- Recap: last lecture we covered:
  - Income Expansion Paths and Engel curves
  - Inferior and Normal Goods
  - Necessities and Luxuries
  - "Marshallian" Demand Curves

Example: Calculating IEPs and Engel Curves

- *Find the IEP and Engel Curve for a consumer with*

  Cobb Douglass Utility Function : $U(x,y)=x^{\alpha}y^{1-\alpha}$

  Budget Constraint : $p_x x + p_y y = I$
- To find the solution:
  - Solve for the Marshallian demand curves. This will automatically give you the Engel Curve
  - Solve each demand curve for income
  - Set these equations equal to each other to derive the IEP.

Solved Example

- Set up the Lagrangian:

$$L = x^{\alpha}y^{1-\alpha} - \lambda\left(p_x x + p_y y - I\right)$$

- Calculate the first order conditions:

$$\frac{\partial L}{\partial \lambda} = p_x x + p_y y - I = 0$$

$$\frac{\partial L}{\partial x} = \alpha\left[\frac{y}{x}\right]^{1-\alpha} - \lambda p_x = 0 \qquad \frac{\partial L}{\partial y} = \alpha\left[\frac{x}{y}\right]^{\alpha} - \lambda p_y = 0$$

Solved Example (II)

- Find the Marshallian demand curves:

$$x\left(p_x, p_y, I\right) = \alpha\frac{I}{p_x} \qquad y\left(p_x, p_y, I\right) = (1-\alpha)\frac{I}{p_y}$$

- These demand curves are the same as the Engel curves, since they show how the optimal levels of *x* and *y* change with income.
- Note that for Cobb-Douglass utility, Engel curves are linear in income.

Solved Example (III)

- Solve each demand curve for income:

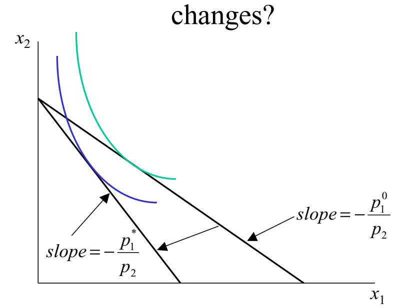
$$I = \frac{p_x x^*}{\alpha} \qquad I = \frac{p_y y^*}{1-\alpha}$$

- Setting these equations equal to each other gives the income expansion path:

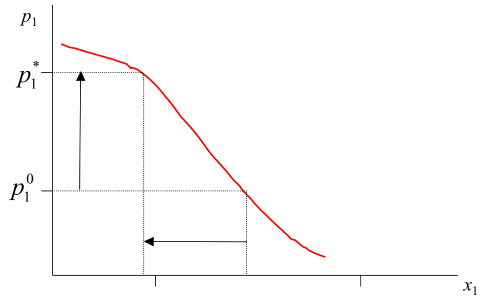
$$\frac{p_y y^*}{1-\alpha} = \frac{p_x x^*}{\alpha} \Rightarrow y^* = \frac{(1-\alpha)p_x x^*}{\alpha p_y}$$

- For Cobb-Douglass utility, the IEP is linear.

What happens to demand when price changes?



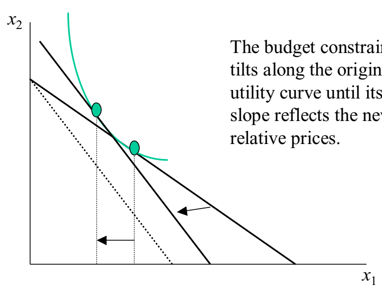
Marshallian Demand Curve



What Causes the Change in Demand?

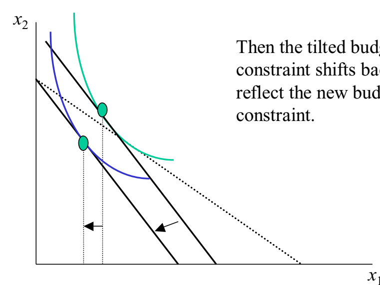
- 2 reasons why demand for x_1 changes
  - it is more expensive relative to x_2
  - consumer effectively has less income
- We label these 2 effects as:
  - the 'substitution effect'
    - ("Hicks" substitution effect)
  - the 'income effect'

The "Hicks" Substitution Effect



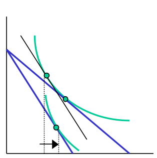
The budget constraint tilts along the original utility curve until its slope reflects the new relative prices.

The "Hicks" Income Effect

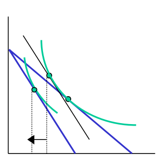


Then the tilted budget constraint shifts back to reflect the new budget constraint.

- The substitution effect <u>must</u> be negative
- The income effect can be positive or negative. Why?
  - some goods are inferior



x_1 inferior

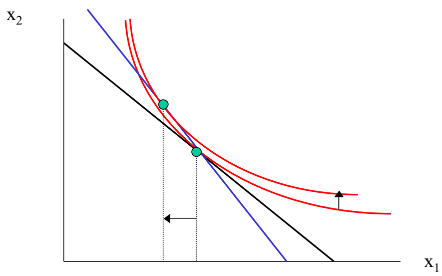


x_1 normal

"Hicks" vs. "Slutsky"

- The "Hicks" substitution effect holds utility constant
  - rotate along the indifference curve
- The "Slutsky" substitution effect holds purchasing power constant
  - rotate around the original consumption bundle
- It's easier to derive the Slutsky equation and the size of the income effect from the latter

The “Slutsky” Substitution Effect



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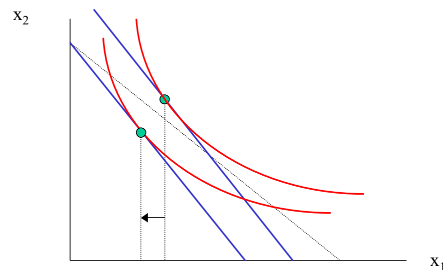
The “Slutsky” Income Effect



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Change in Total Demand

$$\Delta X_1 = X_1^* - X_1^0$$

$$= (X_1^* - \tilde{X}_1) + (\tilde{X}_1 - X_1^0)$$

(+ or -) Income Effect (-) Substitution Effect

$$\Delta X_1 = \Delta X_1^I + \Delta X_1^S$$

The “Slutsky” Equation

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How large is the substitution effect?

- The substitution effect represents a movement along an indifference curve.
  - The size of the substitution effect depends upon how much of a change is needed to get to the point where the MRS is equal to the slope of the new budget constraint.
- This distance depends upon the curvature of the indifference curve.
  - If the indifference curve is flat, the substitution effect will be large.
  - If the indifference curve is “very convex,” the substitution effect will be small.

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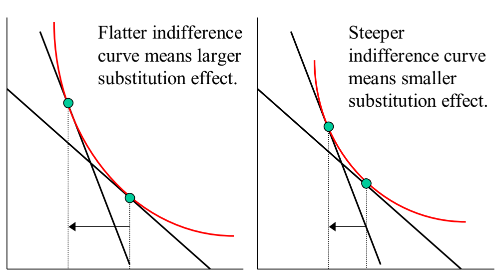
How large is the substitution effect?



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How large is the income effect?

- Intuition: The income effect will be larger, the more x_1 originally purchased.
- How to see this: Think about how much extra income is needed to get back the original bundle of goods when one price increases.

$$new \quad price = p_1^*$$

$$old \quad price = p_1^0$$

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How Large is the Income Effect?

$$p_1^0 x_1^0 + p_2^0 x_2^0 = I_1 \quad \text{Original}$$
$$-\left(p_1^* x_1^0 + p_2^0 x_2^0\right) = -I_2 \quad \text{New}$$
$$\Delta p_1 x_1^0 \qquad -\Delta I \quad \text{Income Needed}$$

This quantity is larger if x_1^0 is larger. Thus:

$$-\Delta I = \Delta p_1 x_1^0$$

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Compensated and Uncompensated Changes in Demand

- "Uncompensated" Change is the total change resulting from a price change.
  - Marshallian Demand
  - What we observe
- We separate the uncompensated change into 2 effects
  - substitution effect
  - income effect
- "Compensated" demand is the change holding utility constant, i.e., the substitution effect.

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Slutsky's Equation

- An algebraic decomposition of the total change in demand into income and substitution effects
- What do the relative size and sign of the two effects imply for the change in demand?

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Derivation of Slutsky's Equation

$$\Delta x_1 = \Delta x_1^S + \Delta x_1^I$$
$$\Delta x_1^I = \frac{\Delta x_1^I}{\Delta I} \Delta I$$
$$= \frac{\Delta x_1}{\Delta I} \bullet (-\Delta p_1 x_1^0)$$
$$\Delta x_1 = \Delta x_1^S - \frac{\Delta x_1^I}{\Delta I} \Delta p_1 x_1^0$$

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Slutsky's Equation (in terms of rates of change)

$$\Delta x_1 = \Delta x_1^S - \frac{\Delta x_1^I}{\Delta I} \Delta p_1 x_1^0$$
$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x^S}{\Delta p_1} - \frac{\Delta x_1^I}{\Delta I} x_1^0$$

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Interpretation

- The rate of change in demand as price changes (holding income fixed) is equal to
  - the rate of change in demand as prices change, adjusting income; and
  - the rate of change in demand as income changes holding prices fixed

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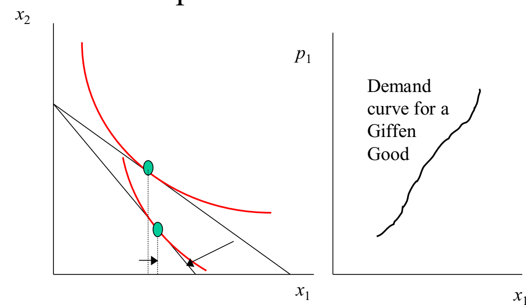
Law of Demand and Giffen Goods

- The change in demand can be positive or negative since the income effect can be positive or negative.
- Case I: 'Law' of Demand
  - Occurs if: $\frac{dx_1}{dp_1} < 0$
    - x_1 is normal, or
    - x_1 is inferior and substitution effect > income effect
- Case II: 'Giffen Good'
  - Occurs if: $\frac{dx_1}{dp} > 0$
    - $x1$ is inferior, and
    - income effect > substitution effect

Example of a Giffen Good



(Own) Price Elasticity of Demand

$$\eta_1 = \frac{dx_1}{dp_1}\frac{p_1}{x_1}$$

- Fact: the price elasticity of demand tells us how the total expenditure on a good changes with price
- Let T = total expenditure on x_1 = $x_1 p_1$
- How total expenditure changes with price = $\frac{dT}{dp_1}$

$$\frac{dT}{dp_1} = p_1 \frac{dX_1}{dp_1} + x_1$$

$$= x_1 \frac{p_1}{x_1}\frac{dx_1}{dp_1} + x_1$$

$$= x_1 \left[\frac{p_1}{x_1}\frac{dx_1}{dp_1} + 1\right]$$

$$= x_1 [\eta_1 + 1]$$

Price Elasticity of Demand

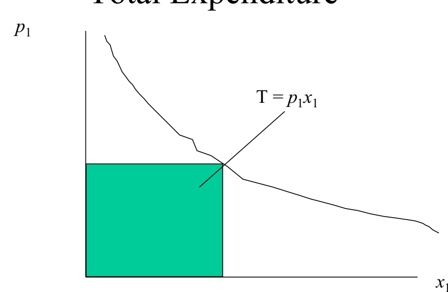
$$\frac{dT}{dp_1} = x_1[\eta_1 + 1]$$

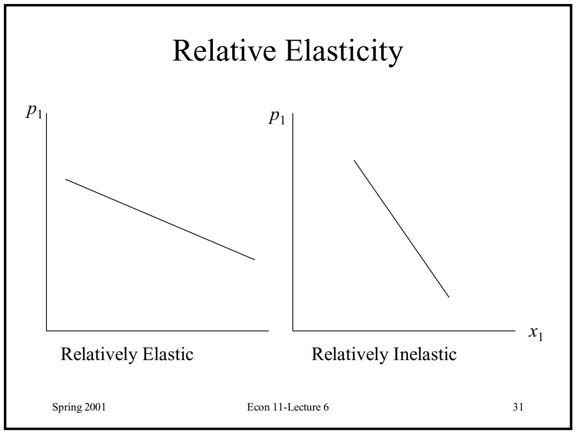
If $\infty < \eta_1 < -1 \Rightarrow \frac{dT}{dp_1} < 0$ Good 1 is relatively elastic

If $-1 < \eta_1 < 0 \Rightarrow \frac{dT}{dp_1} > 0$ Good is relatively inelastic

If $\eta_1 > 0 \Rightarrow$ Good 1 is a Giffen Good

Total Expenditure




Relative Elasticity
p_1
p_1
x_1
Relatively Elastic
Relatively Inelastic
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