Motivation for Welfare Analysis

- In the last class, we found that the Consumer Price Index (CPI) overstates a "true" cost-of-living. without defining a "true" cost-of-living index.
- Suppose,

Initial situation: $\vec{p}_0, I_0 \Rightarrow \vec{x}_0$

New situation: $\vec{p}_1, I_1 \Rightarrow \vec{x}_1$

Is the consumer better off?

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- Is the consumer better off?
  - To answer this question, we need to make use of our utility framework.
  - Given both old and new prices and income, we can calculate the consumer's demand for goods.
  - Then we plug these back into the consumer's utility function (deriving the indirect utility function) and compare.
  - But utility is an ordinal measure, we want a cardinal measure so that we can know how much better (or worse) off the consumer is.
  - We want a "monetary" value of welfare.

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Three measures of the change in welfare

- Compensating Variation (CV)
- Equivalent Variation (EV)
- Change in Consumer Surplus (ΔCS)

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Compensating Variation in Income (CV)

- Given a price change from p^0 to p^* what is the minimum income needed to get to the original level of utility, U^0, at the new prices p^*?
- "How much must I compensate you to make you as well off as you were before the price change?"

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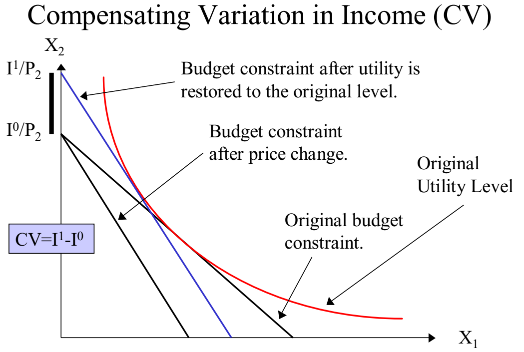
Compensating Variation in Income (CV)



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Compensating Variation and Expenditure Minimization

- In the graph, CV = $I^1 - I^0$
- I_1 is the minimum expenditure needed to reach utility U_0 at prices p^*:
  - $E(U^0, p^*)$
- I_0 is the minimum expenditure needed to reach utility U_1 at prices p^0
  - $E(U^0, p^0)$

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Expenditure Minimization

- The expenditure minimization problem is the dual to the utility maximization problem:

$$\min_{x_1, x_2} p_1 x_1 + p_2 x_2$$
$$s.t. U_0 = U(x_1, x_2)$$

- The Lagrangian for this problem is:

$$L = p_1 x_1 + p_2 x_2 - \mu\left(U(x_1, x_2) - U_0\right)$$

Solution to Expenditure Minimization

- The solution to the expenditure minimization problem are the Hicksian ("compensated") demand functions:

$$x_1 = D_1^{Hicksian}(U, p_1, p_2) \qquad x_2 = D_2^{Hicksian}(U, p_1, p_2)$$

- Plugging these back into $p_1 x_1 + p_2 x_2$ gives the minimum expenditure function:
  - $E(U^0, p_1, p_2)$

Relation Between Minimum Expenditure Function and Hicksian Demand

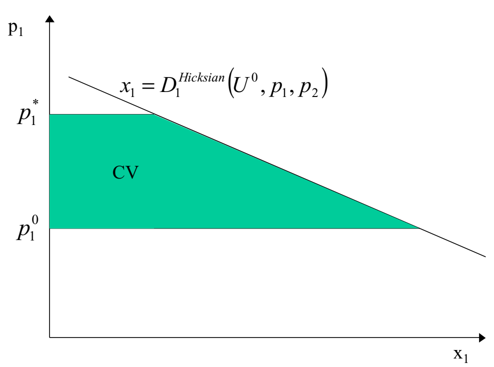
- You can use the Envelope Theorem to prove that the Hicksian demand functions are partial derivatives of the minimum expenditure function, $E(U, p_1, p_2)$

$$x_1 = D_1^{Hicksian}(U, p_1, p_2) = \frac{\partial E(U, p_1, p_2)}{\partial p_1}$$
$$x_2 = D_2^{Hicksian}(U, p_1, p_2) = \frac{\partial E(U, p_1, p_2)}{\partial p_2}$$

Compensating Variation and Hicksian Demand

- CV is the area to the left of the Hicksian Demand Curve.
  - Why? Recall that $CV = E(U^0, p^*) - E(U^0, p^0)$ and suppose only p_1 changes.

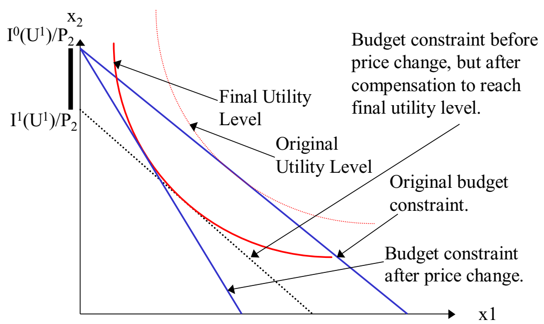
$$\int_{p_1^0}^{p_1^*} D_1^{Hicksian}(U^0, p_1, p_2)\,dp_1 = \int_{p_1^0}^{p_1^*} \frac{\partial E(U^0, p_1, p_2)}{\partial p_1} dp_1 =$$
$$= E(U^0, p_1^*, p_2) - E(U^0, p_1^0, p_2) = CV$$



Equivalent Variation in Income (EV)

- EV is the maximum amount the consumer would be willing to pay to avoid a price change.
- Given a price change from p^0 to p^*, how much extra/less income is required to reach final utility, U^1 at the original prices p^0?

Equivalent Variation in Income (EV)



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Equivalent Variation in Income (CV)

- At old prices, "Equivalent Variation" is the amount of income necessary to get to the new level of utility.
- EV is also the area to the left of the Hicksian Demand Curve.
  - How? It's a different Hicksian Demand Curve! The one associated with the new level of utility.

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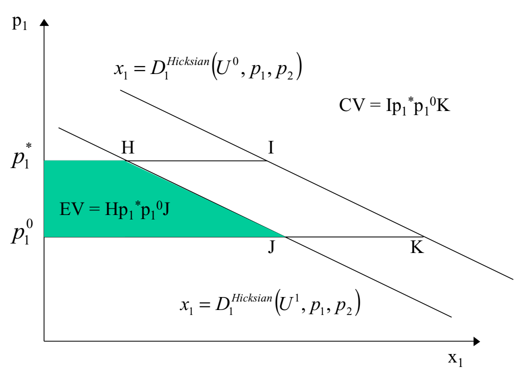

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How do CV and EV differ?

- Area under different Hicksian demand curves.
- When is one concept more useful than the other?
  - Los Angeles decides to build a new freeway which cuts through a neighborhood. How much would the city have to pay the residents of this neighborhood to keep them as well off as they were before? CV
  - What is the most the residents would pay not to have the freeway? EV

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Change in Consumer Surplus

- More common way to examine changes in consumer welfare.
  - Why? We don't observe Hicksian Demand curves.
- Consumer surplus (CS) is the area to the left of the ***Marshallian*** Demand Curve.
- Note: Sometimes CS is defined as the area under the ***Marshallian*** Demand Curve, but not in this class.
- While CV and EV are exact measures of the change in welfare, the change in CS is an approximate measure that is only valid for specialized preferences.

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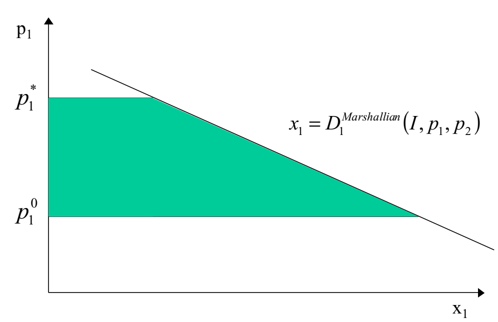
Change in Consumer Surplus



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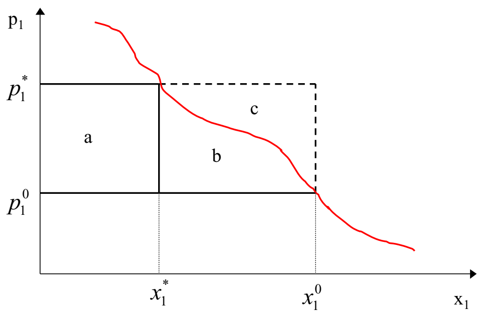
Linear Approximation of CV



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Linear Approximation of CV

CV =Area (a+b+c) – Area (c)

$$\approx (p_1^* - p_1^0)x_1^0 - \frac{1}{2}(p_1^* - p_1^0)(x_1^0 - x_1^*)$$

$$= (p_1^* - p_1^0)\left[x_1^0 - \frac{1}{2}(x_1^0 - x_1^*)\right]$$

$$= \frac{\Delta p_1}{2}\left[x_1^0 + x_1^*\right]$$

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Relationship between ΔCS, CV, EV

- The relationship between ΔCS, CV, and EV depends upon whether the good is normal or inferior.
- For a <u>normal</u> good, the Hicksian demand curve is steeper than the Marshallian demand curve.
  - Why? Income and substitution effects go in the same direction.
- For an <u>inferior</u> good, the Hicksian demand curve is flatter than the Marshallian demand curve.
  - Why? Income and substitution effects go in opposite directions.

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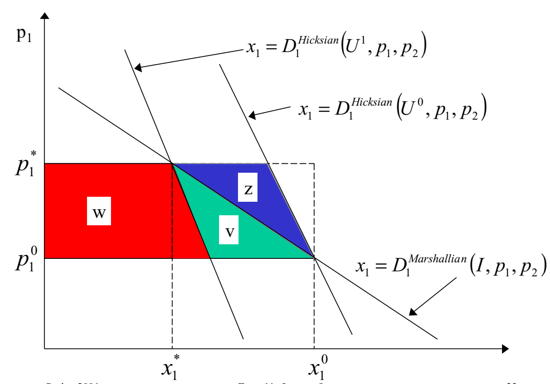

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Relationship between ΔCS, CV, EV

- CV = Area (w + v + z)
  - new prices, old utility
- ΔCS = Area (w+v)
  - utility not held fixed, income fixed
- EV = Area (w)
  - old prices, new utility
- For a price increase: CV > ΔCS > EV
- For a price decrease: CV < ΔCS < EV

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