Simultaneous Ascending Auctions

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Econ 285, Market Design

February 2009

FCC Spectrum Auctions

- Auctions to allocate radio spectrum
  - Pioneered by FCC in 1994, and followed by UK, Germany, Netherlands, Belgium, Mexico, India, etc.
  - Generally perceived as quite successful, raising hundreds of billions of dollars in revenue.
- Structure of typical auction
  - FCC specifies a set of licenses to be sold, with each license conveying the right to use a portion of the spectrum in a certain geographic area.
  - Licenses allocated using SAA format proposed by Milgrom-Wilson-McAfee; some changes over time.

SAA Rules

- Auction consists of multiple rounds. In each round
  - FCC sets minimum acceptable bid for each object, some increment above current high bid.
  - Each bidder can submit bids on any number of items, subject to “activity rules”.
  - If multiple bids on a license, FCC selects one randomly to become standing high bid.
- Auction ends when no new bids are submitted.
- Information revealed:
  - FCC has typically revealed all bids after each round.
  - In recent auctions, FCC has anonymized the bids.

Activity Rules

- FCC uses Milgrom-Wilson "activity rule"
  - Each bidder j starts with some eligibility $e_j(1)$ determined by initial deposit, measured in "bid units".
  - "Activity" in a round consists of new bids and standing high bids from the prior round: must have $A_j(n) \leq e_j(n)$.
  - A bidder's "eligibility" evolves as $e_j(n+1) = \min(e_j(n), \alpha A_j(n))$, where α is close to but possibly larger than 1.
- Activity rule keeps the auction moving, but we will see later that it also has strategic consequences.

Why multiple rounds?

- Relative to sealed bidding, information revelation…
  - Allows bidders to identify target licenses “on the fly”
  - Mitigates inefficiency due to the winner’s curse
  - Helps bidders to assess “roaming” opportunities.
- The SAA design has some other virtues..
  - It’s transparent, and easy to check up on the gov’t.
  - Activity rule prevents super-slow bidding.
- Skeptics might argue…
  - Design is vulnerable to demand reduction/collusion.
  - Design does not facilitate new entry or “package” bidders.

Roadmap for Lecture

- Non-strategic theory
  - SAA is conceptually similar to matching theory algorithm (eg Kelso-Crawford).
  - With substitutes demand, “straightforward” bidding leads to approx. competitive equilibrium.
- Strategic bidding
  - Demand reduction, collusion may be a problem.
  - Bidders that want to buy some minimal set of licenses face complex “exposure” problems.

Non-strategic theory

- $\{1,\ldots,L\}$ is a set of *indivisible* licenses with typical subset S.
- Bidders' payoffs are the value of licenses acquired minus the amount paid $v_j(S)-m_j$. (assume free disposal).
- Demand "correspondence"

$$D_j(p) = \arg\max_S v_j(S) - p(S)$$

- "Personalized price" p^{jn}_k for bidder j on item k at round n is the lowest price at which j might conceivably acquire k
  - the high bid if j is the *standing high bidder* on k
  - the high bid plus one increment otherwise

Definitions

- Bidder *j demands* set *S* at price vector *p*, if $S \subseteq D_j(p)$.
- Licenses are *substitutes* (standard definition) if:

$$\left(k \in D_j(p), p' \geq p, p'_k = p_k\right) \Rightarrow k \in D_j(p')$$

- Examples
  - A bidder who wants just one license.
  - A bidder who wants spectrum in several areas, but has declining marginal value for bandwidth in each area.
- Bidder *j bids straightforwardly* if in each round she bids on a preferred set of licenses given her current standing high bids and next price increments for other licenses.

Substitutes and "no regret"

- ***Theorem:*** *Assume that licenses are substitutes for j. If j bids straightforwardly at every round n,* $S_j^n \subseteq D_j(p^{jn})$.
  - That is, at every round *j* demands its preferred licenses at its personalized prices.

- This means *j* never "gets stuck" with a standing high bid on a license it no longer wants as other prices rise. There is *no regret*.
  - This property depends crucially on substitutes.

The Exposure Problem

- Suppose a bidder has value
  - 10 for either A or B alone
  - 30 for A and B together

- The “exposure problem”
  - If both license prices reach 12, straightforward bidding means bidding on both A and B.
  - If bidding on A stops and B’s price subsequently climbs to 20, the bidder will regret purchasing A.

Market clearing prices?

- Bidder 1 values: 17 for A, 22 for B, 34.5 for both.
- Bidder 2 values: 20 for A, 20 for B, 37.5 for both.

Round	A's price: p_A	B's price: p_B	A's High Bidder	B's High Bidder
25	11	16	1	1
26	12	17	2	2
27	13	17	1	2
28	14	17	2	2
29	14	18	2	1

Describing Outcomes

- Auction won't necessarily find *exact* market clearing prices, but it gets close…

- In particular, the auction outcome with straightforward bidding will be an <u>exact</u> competitive equilibrium for a <u>nearby</u> set of values.
- The nearby values are constructed as follows:
  - Identify the goods that bidder *j* wins at the auction.
  - Define *j*'s modified values for any set of goods *T* to be the original value minus one bid increment for each good in *T* that *j* does <u>not</u> win.

Substitutes: Competitive Equilibrium

- ***Theorem:*** Suppose the licenses are substitutes and that all bidders bid straightforwardly. Let (p*,S*) be the final standing high bids and license assignment and suppose the minimum bid increment vector is q. Then the final allocation is "nearly efficient" and (p*,S*) is a competitive equilibrium for a nearby economy with individual valuations defined by: $\hat{v}_j(T) = v_j(T) - q \cdot 1_{T \backslash S_j^*}$

Example, continued…

Round	A's price: p_A	B's price: p_B	A's High Bidder	B's High Bidder
29	14	18	2	1

- Bidder 1 values: 17 for A, 22 for B, 34.5 for both.
- Bidder 2 values: 20 for A, 20 for B, 37.5 for both.
- Nearby values
  - Bidder A: (16, 22, 33.5) and Bidder B (20, 19, 36.5)
  - Final prices (14,18) and allocation clear the market using the nearby values.

Summary of non-strategic SAA theory

- Suppose bidders view licenses are substitutes and bid straightforwardly in the SAA.

  - **Arbitrage:** The final prices for identical items will differ by at most one bid increment.
  - **Efficiency:** If the bid increments are small, the final license allocation will be efficient.
  - **Competitive Equilibrium** The final prices will “close” to competitive equilibrium prices.

Good market design

- UK sale of 3G spectrum (1999)
  - 5 national licenses, 2 larger than the others.
  - 4 incumbent (2G) operators, plus entrants.
  - Each bidder could win at most one license.

- What happened in the auction
  - Straightforward bidding a natural strategy.
  - Outcome widely perceived as efficient.
  - British government raised 22 billion pounds.

Bad market design

- Netherlands 3G auction in 1999.
  - 5 nationwide licenses, pretty similar.
  - 5 incumbent (2G) operators.
  - Prior to auction, major outside telecom firms (Deutsche Telekom, DoKoMo, Hutchinson Whampoa) all reach partnership agreements with an incumbent.
  - Only one additional entrant, startup called Versatel.

- What happened in the auction
  - On day 1, Telfort (owned by BT) sends Versatel a letter saying that it “can’t win” and should drop out immediately!
  - Versatel shortly drops out: total revenue of 3bn euros – at UK prices, auction would have raised 10bn euros.

Strategic Demand Reduction

- German GSM auction (2000)
  - 10 nationwide licenses, almost identical.
  - Starting price of zero.
  - Bid increments of DM 10m.
  - Bidders: Mannesman, T-Mobile (large) and small guys.

- What happened in the auction
  - Round 1: Mannesman bids 36.6m for each of 5 bands, and reduces eligibility.
  - Round 2: T-Mobile (Deutsche Telekom) bids 40m for the other five bands, reduces eligibilty.
  - No bids in round 3!

Complexity and strategy

- US sale of AWS spectrum (2006)
  - 90 MHz of nationwide spectrum, 1122 licenses
  - Regional licenses (10,10,20 MHz), 6 to cover US
  - Smaller licenses (10,20,20 MHz), ≥176 to cover
  - Total of 168 bidders, including major incumbents, small firms and two potential national entrants.

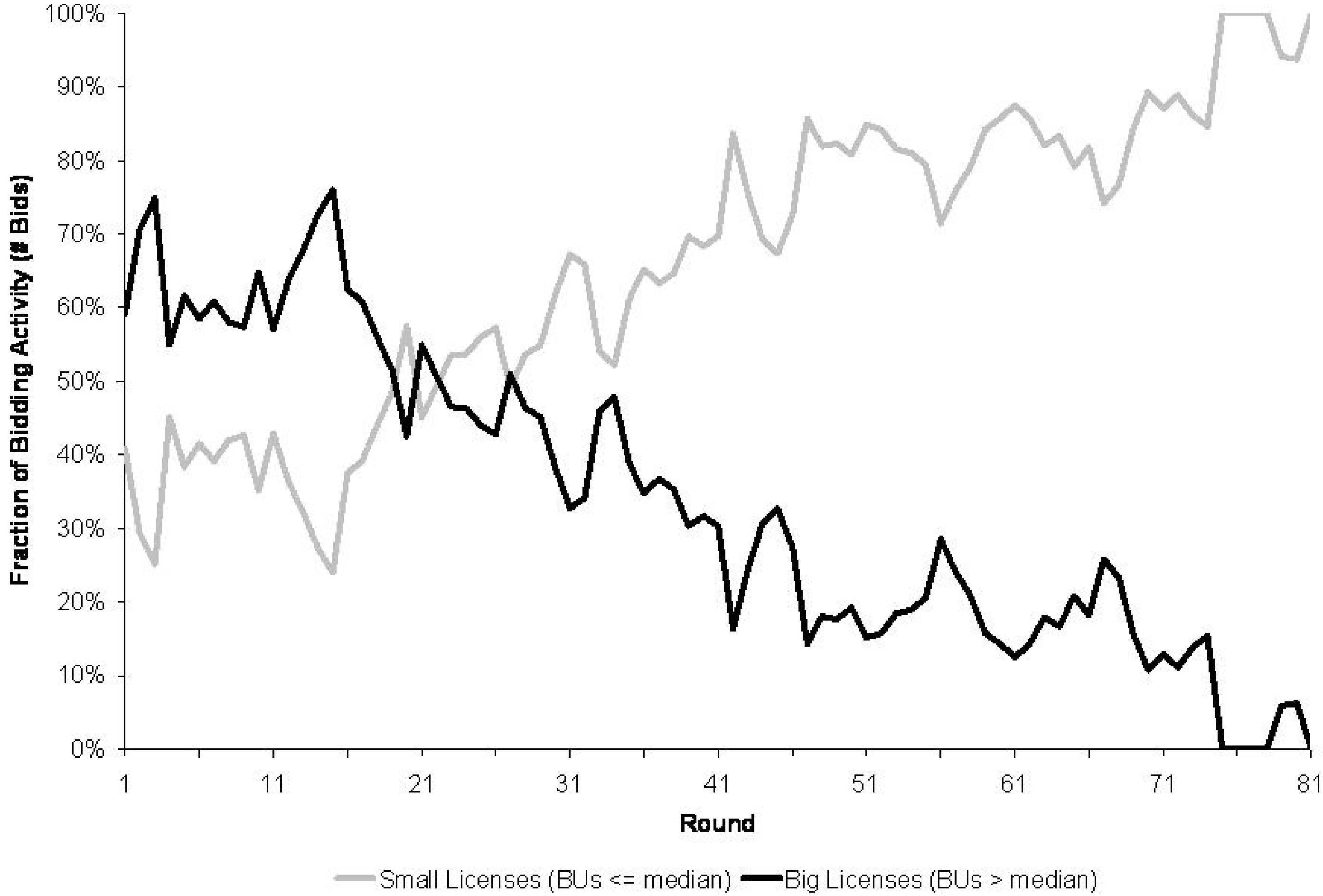
- Entrants face a difficult problem
  - Theory doesn't provide much guidance on how to bid in a way that avoids the exposure problem…
  - This has been a standard concern in spectrum auctions.

Timing problems & opportunities

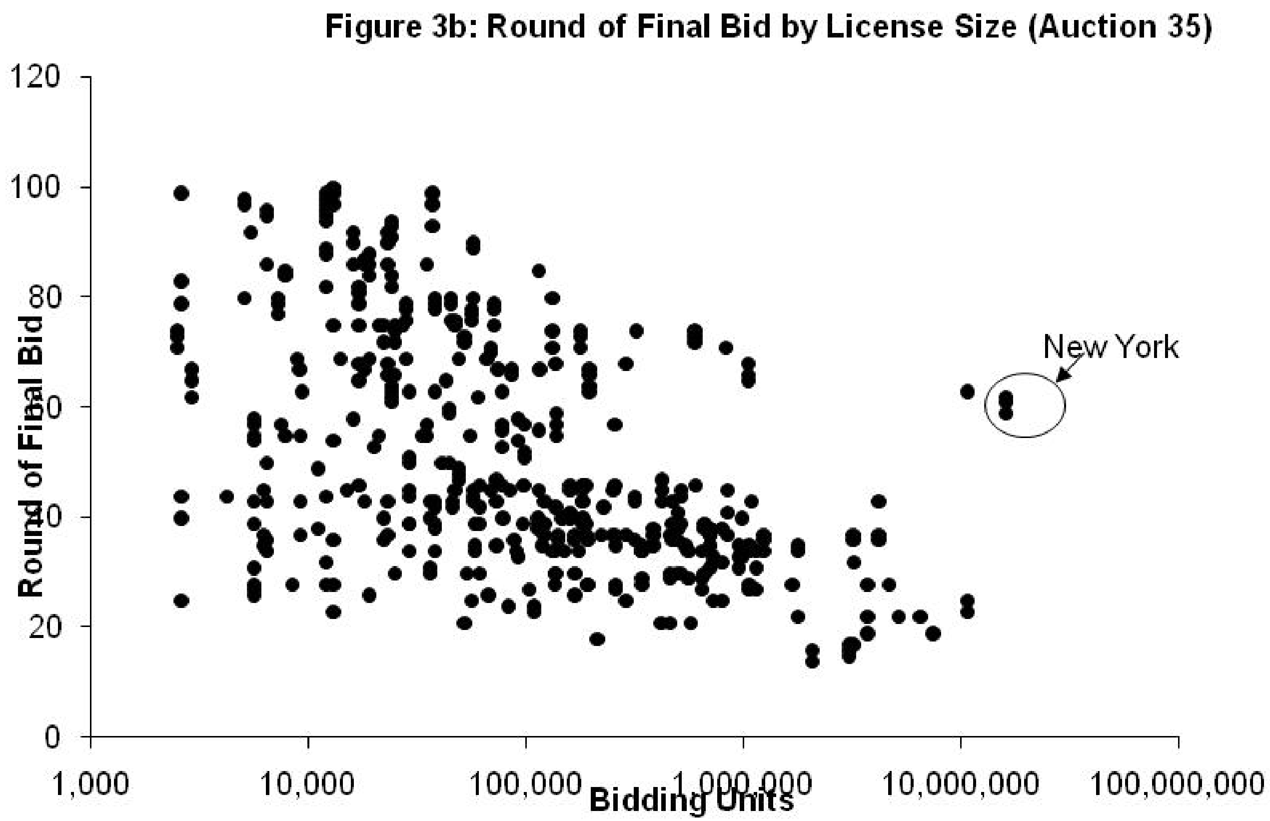
- Activity rules force bidders to make early commitments
  - Creates exposure problem for entrants (package bidders)
  - Creates difficulties for bidder with budget constraint
  - Not so easy to arbitrage different size licenses: easy to substitute from a big license to smaller licenses, but not so easy to get back!

- *Empirical proposition:* bidding tends to start on large licenses, and these licenses tend to clear first.

Auction 35: bidding activity



Auction 35: time of last bids



The exposure problem

- New entry may require a package of licenses
  - Because markets clear at different times, could easily end up with some very expensive spectrum but not enough for viable entry.
- Fundamentally a problem of uncertainty.

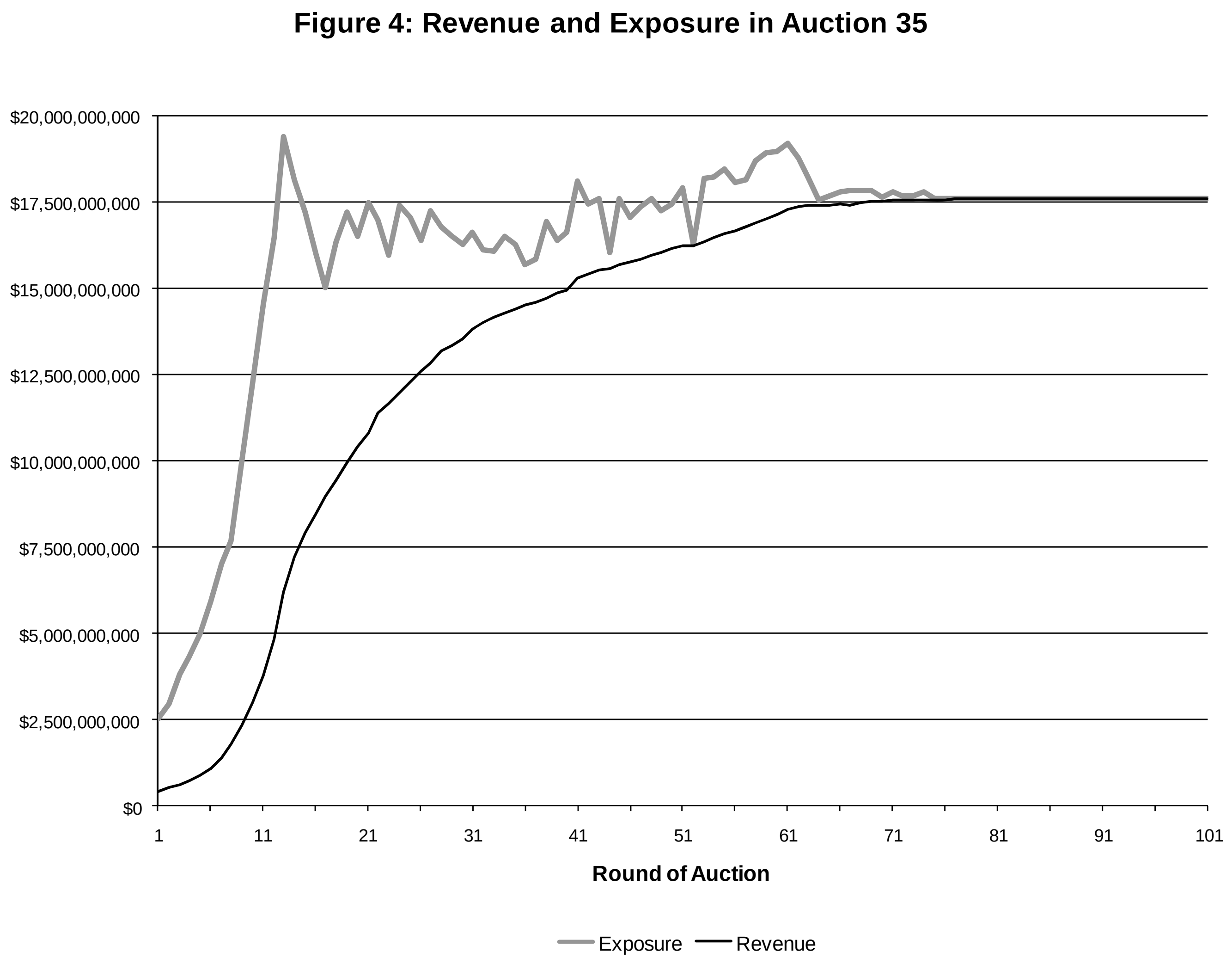
Prices vary enormously!

No.	Description	When?	$ Billion	$/MHz-Pop
5	PCS C Block	1996	13.4	1.77
10	C Block Re-auction	1996	0.7	1.50
11	PCS DEF Blocks	1997	2.7	0.36
22	PCS	1999	0.5	0.20
34	800 MHz	2000	0.3	0.18
35	PCS C&F	2001	17.6	4.37
58	Broadband PCS	2005	2.3	1.05

Role of bidder budgets

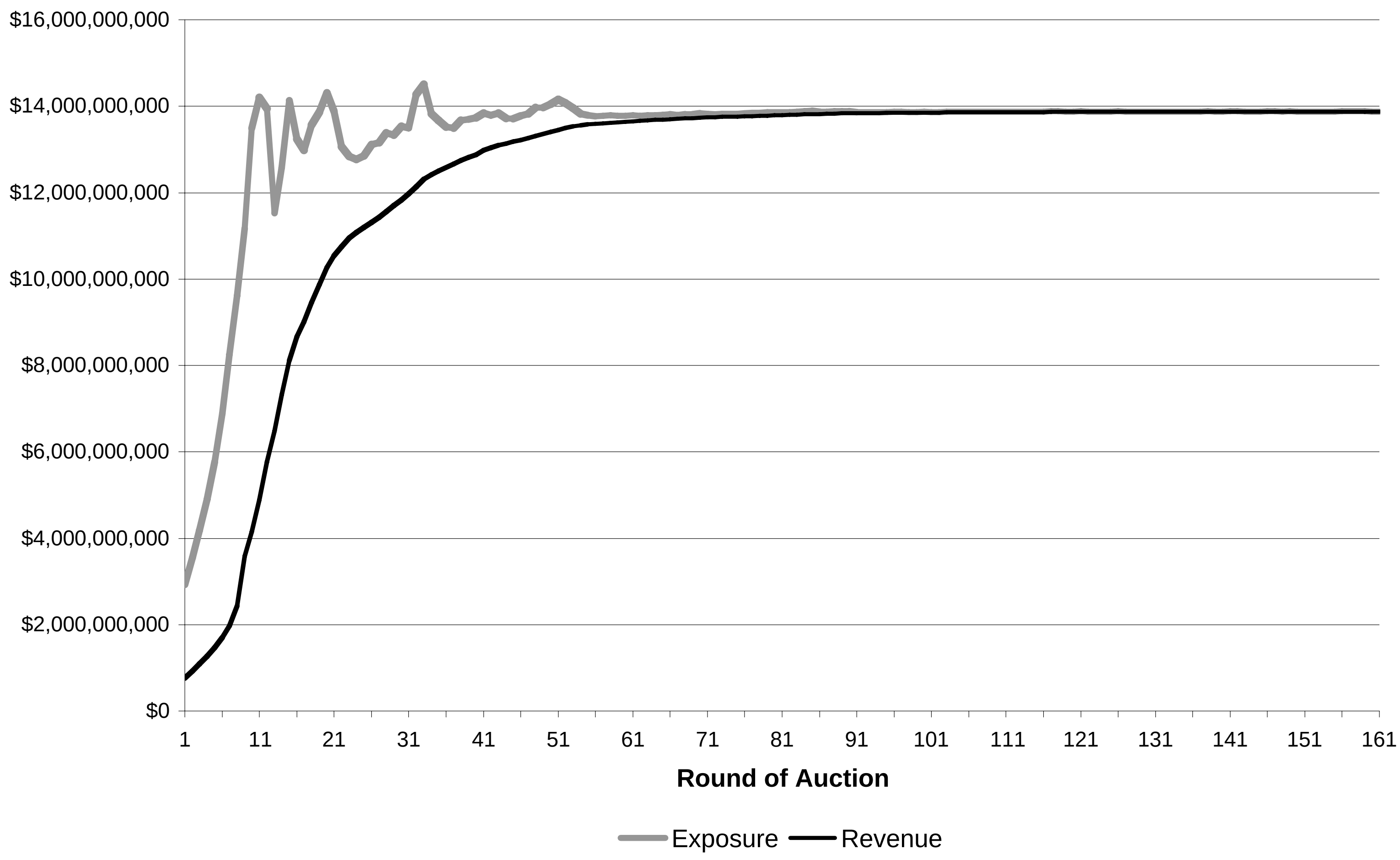
- Many bidders appear to be limited in their bidding by *budgets*, rather than *values*.
  - This neglected pattern is significant for both bidder strategy and auction design.
- *Empirical proposition*: at an aggregate level, budgets appear to play key role in determining prices.

“Exposure” forecasts prices



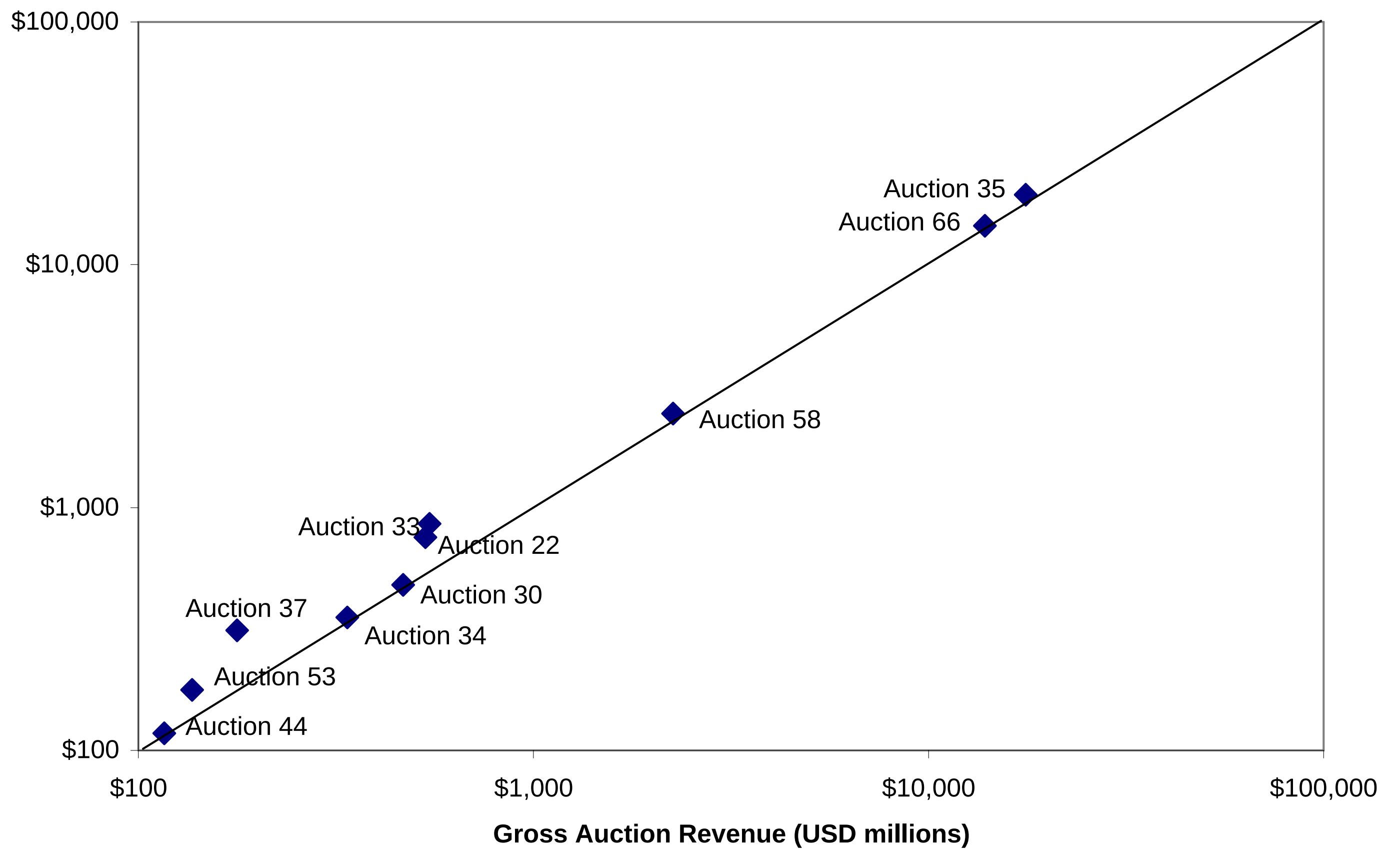
Forecasting in the AWS auction

Figure 5: Revenue and Exposure in Auction 66



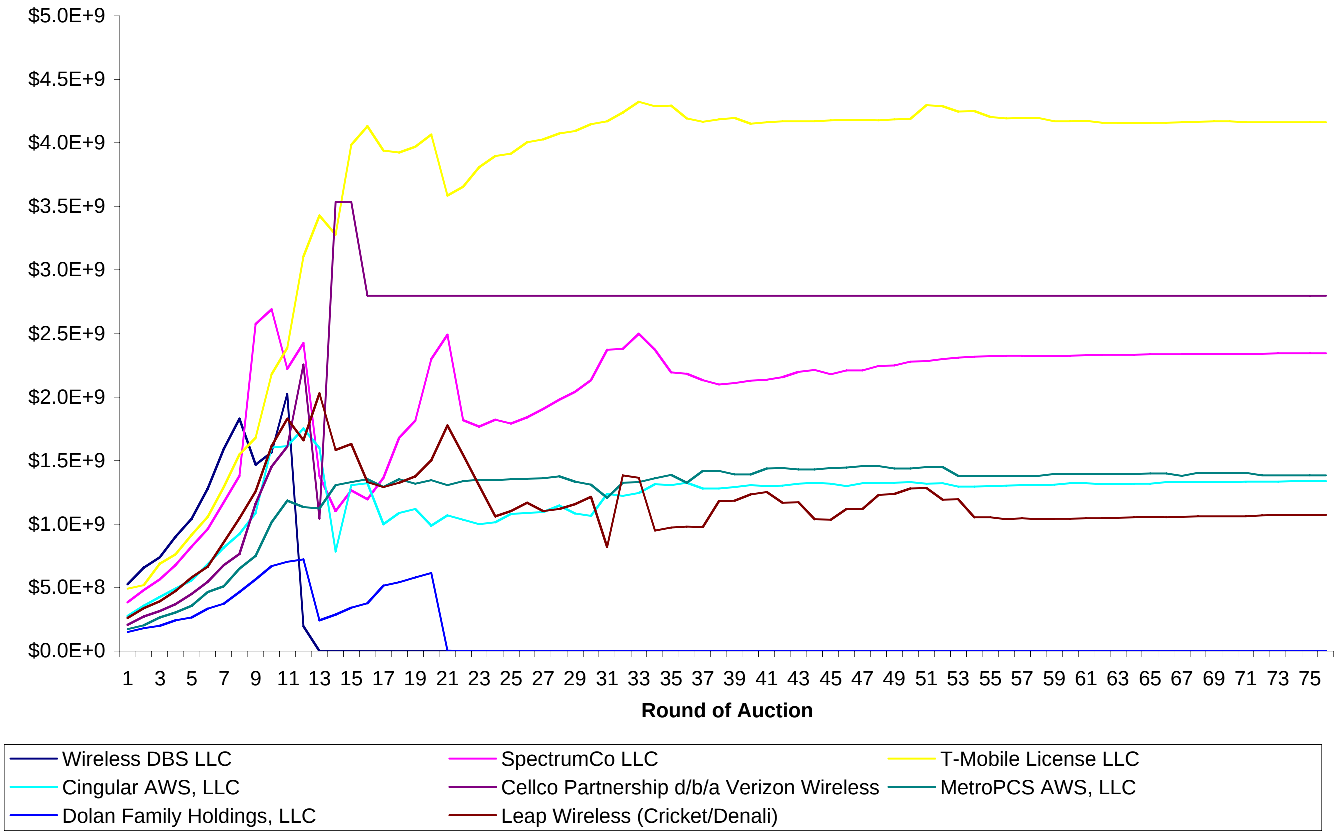
Peak/final exposure FCC sales

Figure 8: Budget Forecasting in Major FCC Auctions



Exposure by bidders in AWS

Figure 6b: Bidder Exposure in Auction 66



Use of budget forecasts

- To avoid the exposure problem:
  - Allows an entrant to identify if a desired aggregation is achievable at reasonable price.
- To acquire licenses cheaply:
  - Allows a bidder to anticipate price anomalies when individual licenses clear in sequence.

Controlling auction prices

- Simultaneous ascending auction
  - Entrant wants two licenses: value $v_{12} > v_1+v_2$
  - Individual bidders: values $u_i \sim F_i$
- If the entrant can control the rate of price increases, how should it behave?

Optimal timing

Suppose prices are (p_1,p_2) and entrant wins 1

$$\pi_1(p_1,p_2) = v_1 - p_1 + Q_2(p_1,p_2)$$

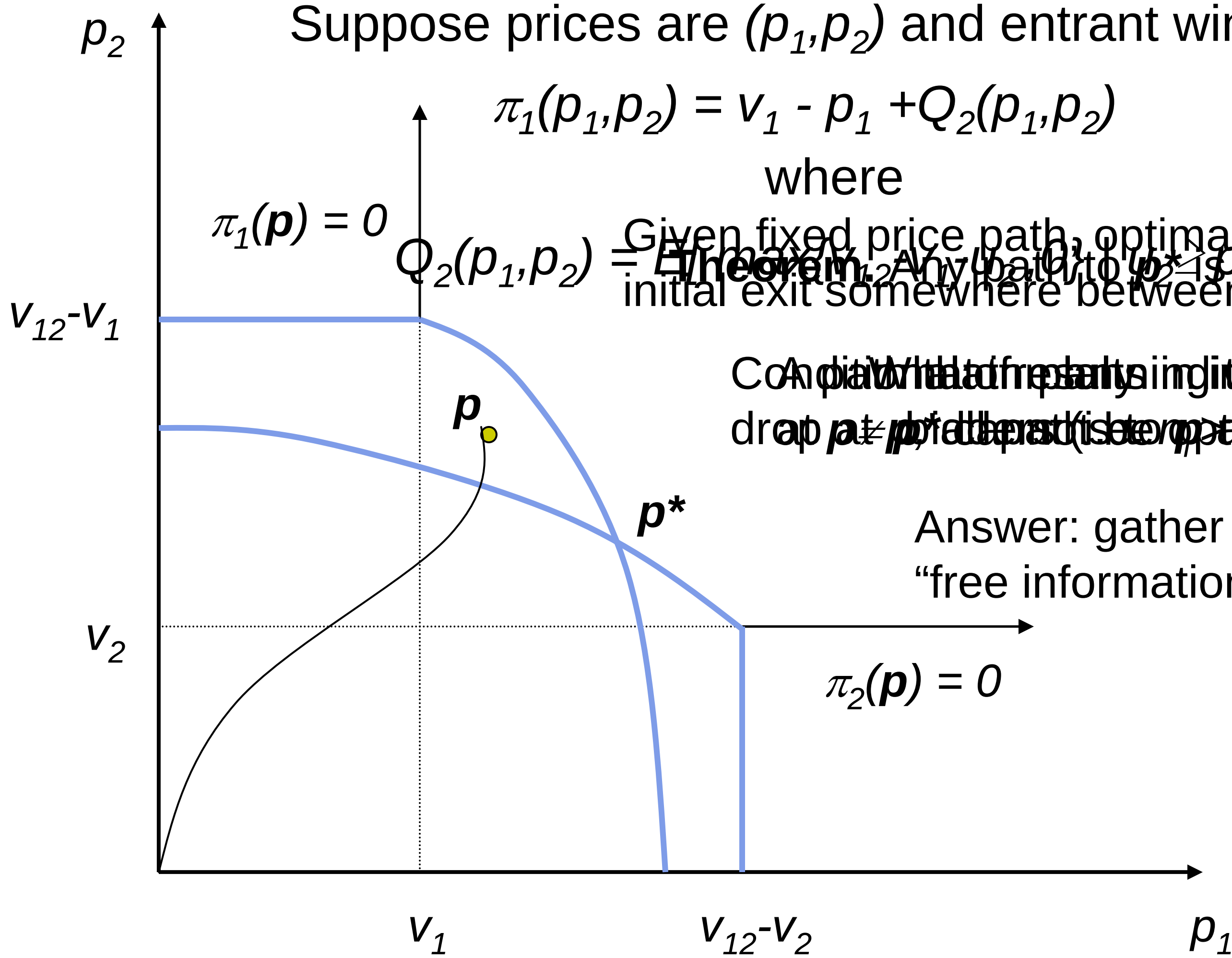
where

$Q_2(p_1,p_2) = E[\max(v_{12} - v_1 - p_2, 0) \mid \ldots]$

Theorem. Any path to $\boldsymbol{p}^*$ is optimal.

Given fixed price path, optimal to make initial exit somewhere between blue curves.

Answer: gather as much “free information” as possible.



Pacing and efficiency

- **Theorem.** Any change in pacing that benefits entrant also increases efficiency.
  - Entrant may win too many or too few licenses from efficiency perspective, but always pays social cost.

The AWS auction

- Recall basic structure of licenses:
  - “Large” regional licenses (40 MHz)
  - “Small” EA/CMA licenses (50 MHz)
- Competitive landscape: 168 bidders, major incumbents, and two potential national entrants
  - SpectrumCo: cable TV consortium
  - Wireless DBS: satellite TV consortium
- Prior to auction, appeared there would be room for at most one successful entrant, if any.

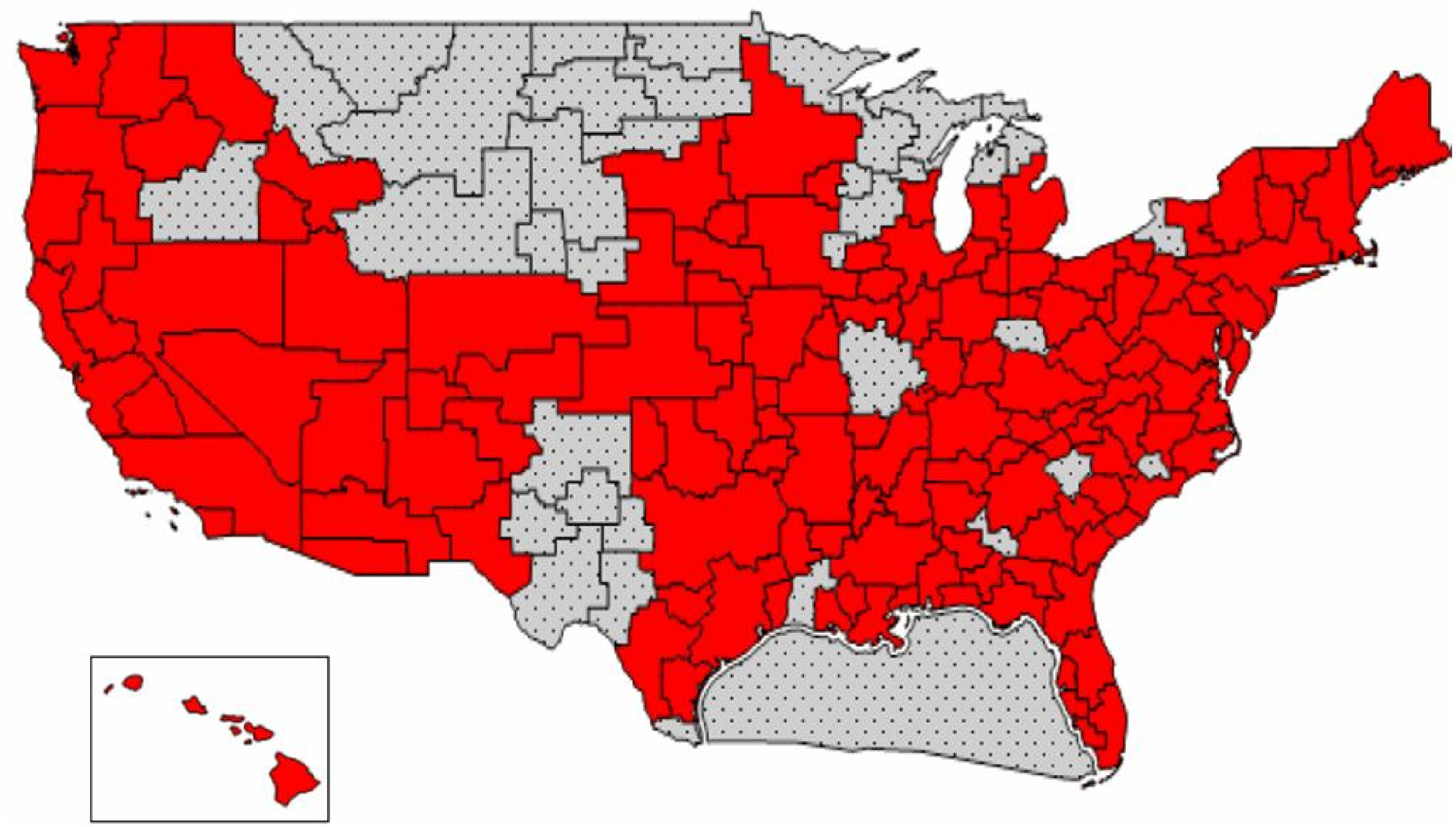
Controlling the pace

- Bidding started on large regional licenses.
  - But prices rose uniformly on coasts/interior, creating serious exposure problem…
- In response, SpectrumCo makes maximal ($750m) jump bid, *doubling* prices in Northeast and West.
  - What happens? Wireless DBS takes waivers, then exits
  - FCC eliminates jump bidding in subsequent auctions.

Budget forecasting

- As of round 13, the situation is
  - High bids on REAGs (40 MHz): $ 5.03 bn
  - High bids on EA/CMAs (50 MHz): $ 0.76 bn
  - Level where auction exposure had peaked: $ 14.2 bn
- SpectrumCo - alone among the major bidders - gives up REAGs and switches to smaller licenses.

SpectrumCo's licenses (20 MHz)



Failure of price arbitrage

Table 1: Prices Paid by the Five largest Buyers in Auction 66

Bidder	Total Winning Bids	Per MHz-Pop	SpectrumCo's Savings
SpectrumCo	$2,377,609,000	$0.45	--
Cingular	1,334,610,000	$0.55	$511 m
T-Mobile	4,182,312,000	$0.63	$943 m
Verizon	2,808,599,000	$0.73	$1,476 m
MetroPCS	1,391,410,000	$0.96	$2,699 m
Four incumbents	9,716,931,000	$0.68	$1,191 m