How many labels do you have?
Some perspectives on gold standard labels

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The "standard" story in statistics & ML

Data →

$$\{(X_i, Y_i)\}_{i=1}^{N}$$

$$y = X\beta + \varepsilon$$

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } x \in X \end{aligned}$$

$$g(x) \in \int_{\mathcal{S}} \partial F(x; s) dP(s)$$

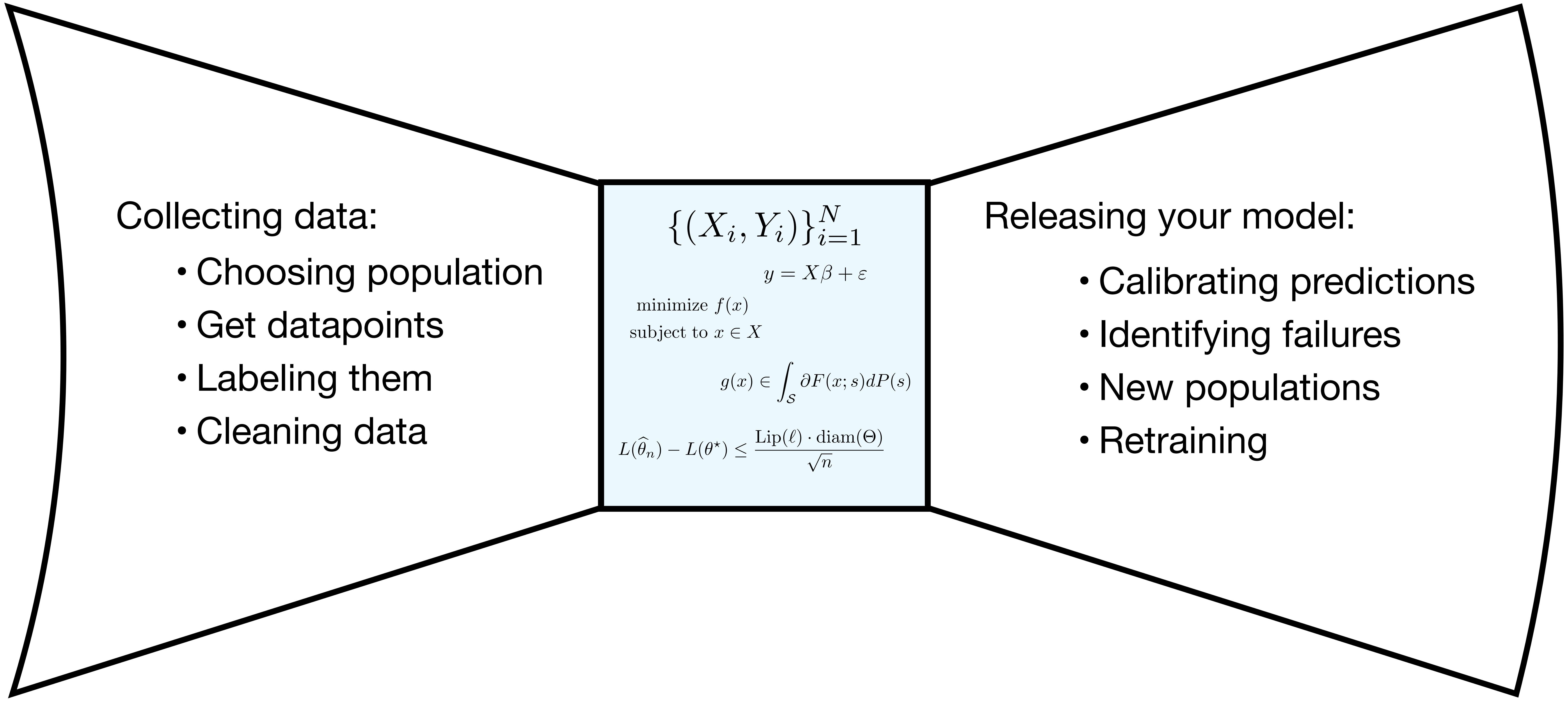
$$L(\widehat{\theta}_n) - L(\theta^\star) \leq \frac{\mathrm{Lip}(\ell) \cdot \mathrm{diam}(\Theta)}{\sqrt{n}}$$

Statistics and machine learning

→ Great model

The big picture

- Excited about the full pipeline of statistical machine learning



Motivation

Dave Donoho, “50 Years of Data Science”

> It is no exaggeration to say that the combination of a Predictive Modeling culture together with Common Task Framework is the ‘secret sauce’ of machine learning

Common Task Framework:

1. A publicly available training dataset
2. A set of enrolled competitors whose common task is to infer a class prediction rule from the training data
3. A scoring referee to which competitors submit their prediction rule(s)

ImageNet

The (probably) currently preferred image classification benchmark

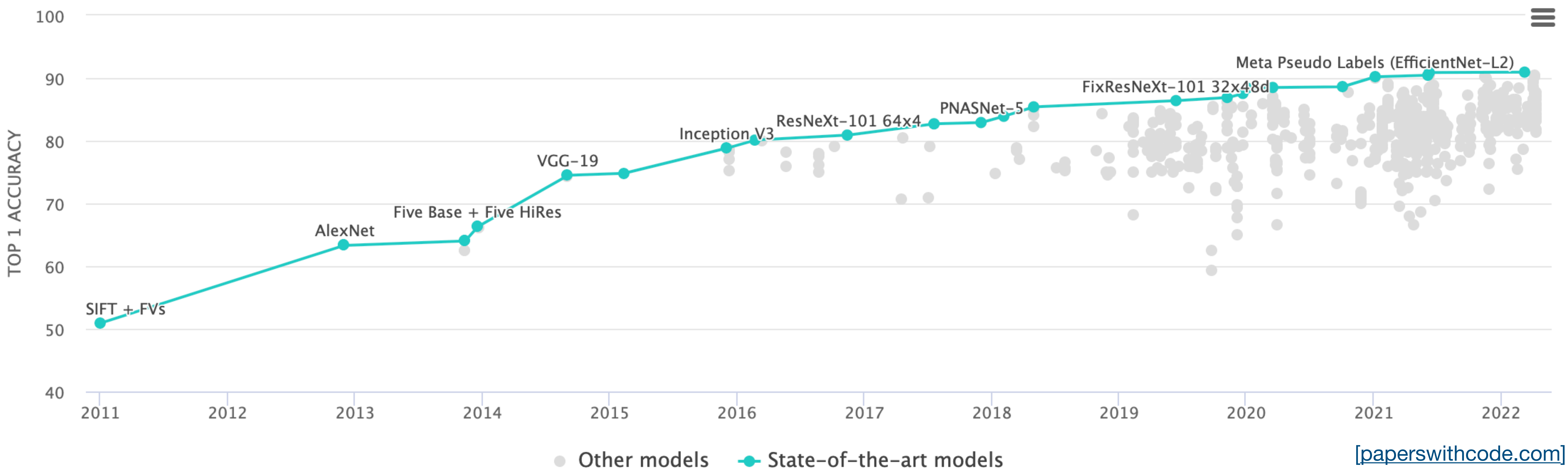
Input data X

Goal: assign label Y to this image (In this case, Y = Golden Retriever)

Dataset description: For each of 1000 image categories (e.g. cherry, bow and arrow, golden retriever, dachshund) there are 1000 representative images

[Deng et al., “ImageNet: A large-scale hierarchical image database” CVPR 2009]

ImageNet Progress



[paperswithcode.com]

Little exaggeration to say deep learning descends from ImageNet

Supervised Learning

The construction of ImageNet isn't really what we teach

- Usual machine learning story

Input data X

Noisy Label Y

"Dog"

"Golden Retriever Puppy"

"Cat"

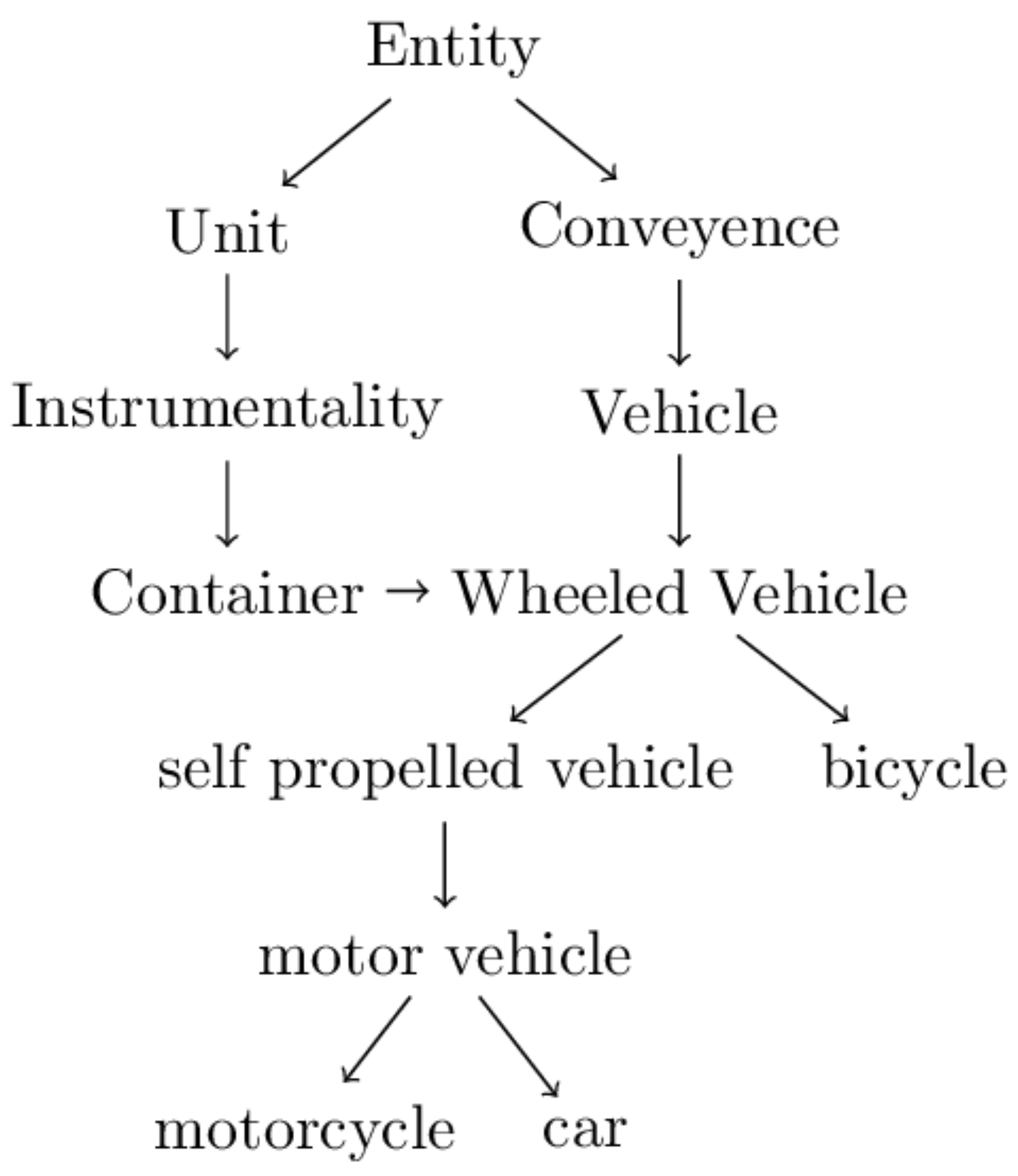
Machine learning pipeline:

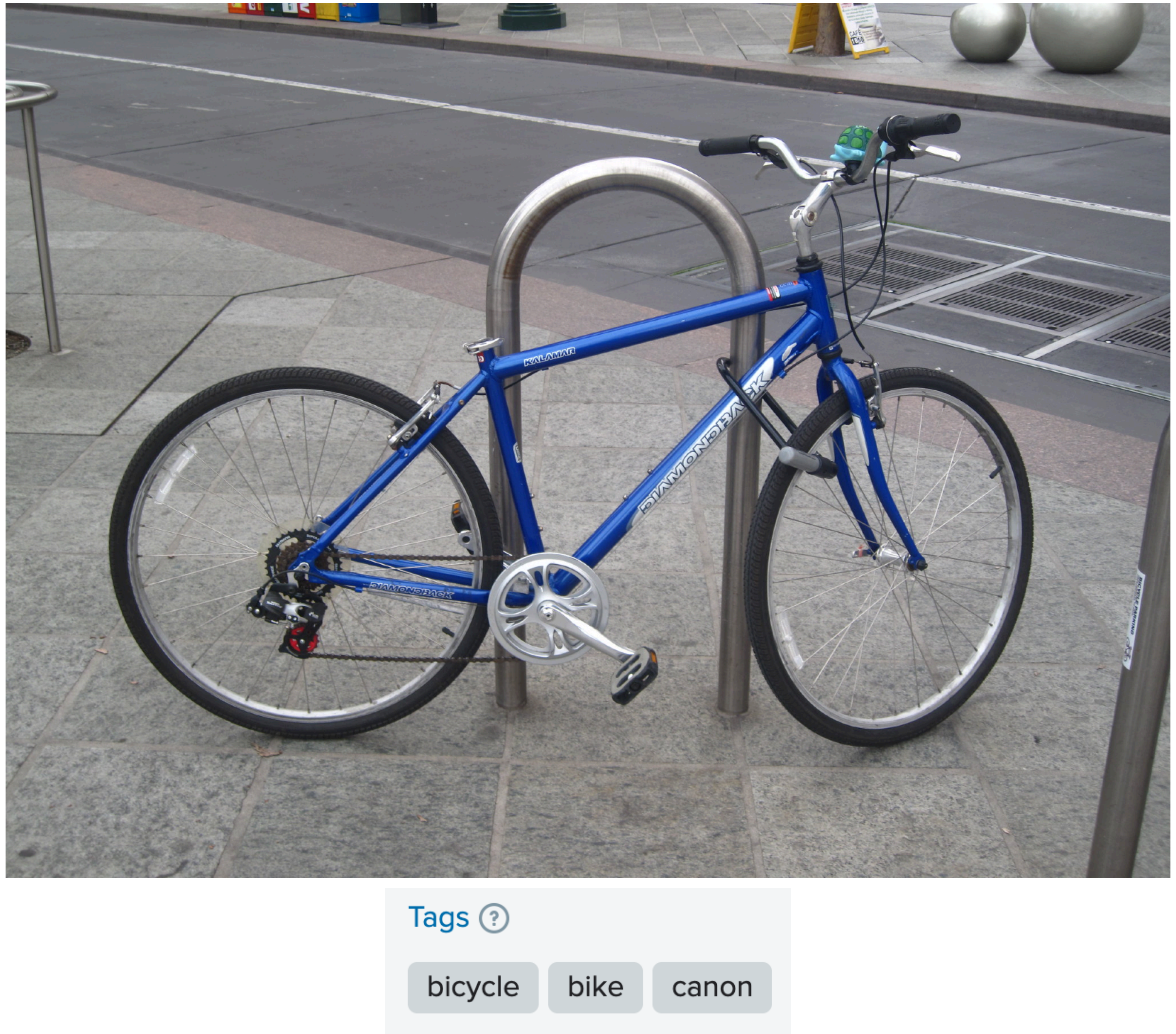
Feed in a bunch of pairs $\{(X_i, Y_i)\}_{i=1}^N$ ——(magical fitting…)——> Output a model $\widehat{Y} = f(X)$

ImageNet construction

flickr

WordNet hierarchy





[Deng et al., "ImageNet: A large-scale hierarchical image database" CVPR 2009]

ImageNet construction

Select all the images that contain a bicycle

[Deng et al., “ImageNet: A large-scale hierarchical image database” CVPR 2009]

ImageNet construction

Select all the images that contain a bicycle

Bicycle object class:

Include X as an example of Y = bicycle if selection frequency $\geq 70\%$

Repeat for each of 1000 classes

What this is:

Ingenious, clever, surprisingly effective

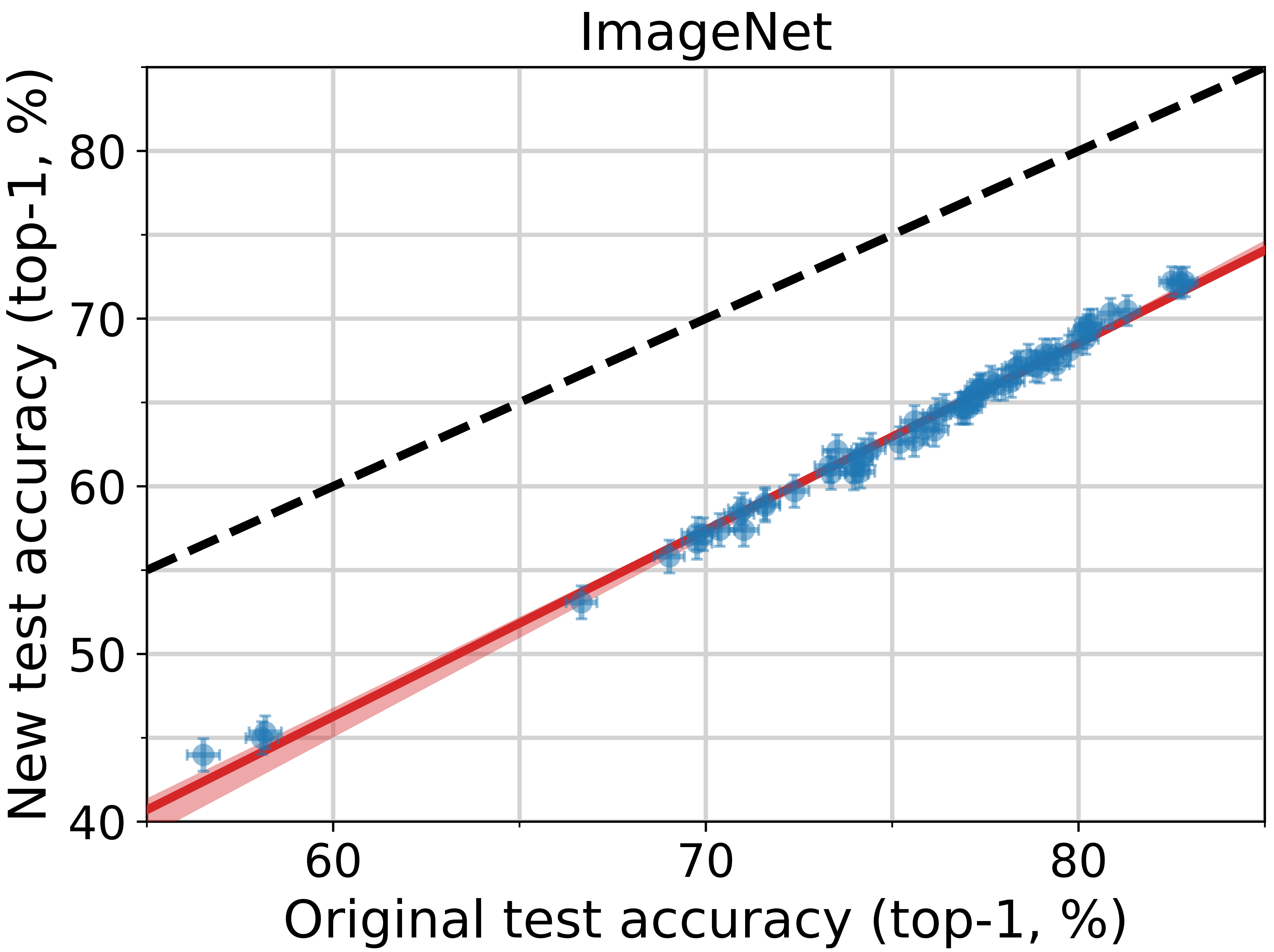
What this is not:

Noisy labels Y given X

[Deng et al., "ImageNet: A large-scale hierarchical image database" CVPR 2009]

How much does data construction matter?

Even when we're careful, things get weird



Also, methods are quite overconfident in predictions (e.g., predict classes with 90+% certainty)

[Recht, Roelofs, Schmidt, Shankar, "Does ImageNet generalize to ImageNet?", ICML 2019]

Remainder of this talk

Collecting data:

- Choosing population
- Get datapoints
- Labeling them
- Cleaning data

A very stylized model of part of this process

$$\{(X_i, Y_i)\}_{i=1}^N$$

$$y = X\beta + \varepsilon$$

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in X \end{array}$$

$$g(x) \in \int_{\mathcal{S}} \partial F(x; s) dP(s)$$

$$L(\widehat{\theta}_n) - L(\theta^\star) \le \frac{\text{Lip}(\ell) \cdot \text{diam}(\Theta)}{\sqrt{n}}$$

Releasing your model:

- Calibrating predictions
- Identifying failures
- New populations
- Retraining

Steps:

1. Propose a model
2. Analyze the model
3. It makes some predictions: test them!

The model

- Binary classification with *m* labelers

$$Y \in \{-1, 1\}, \quad X \in \mathbb{R}^d$$

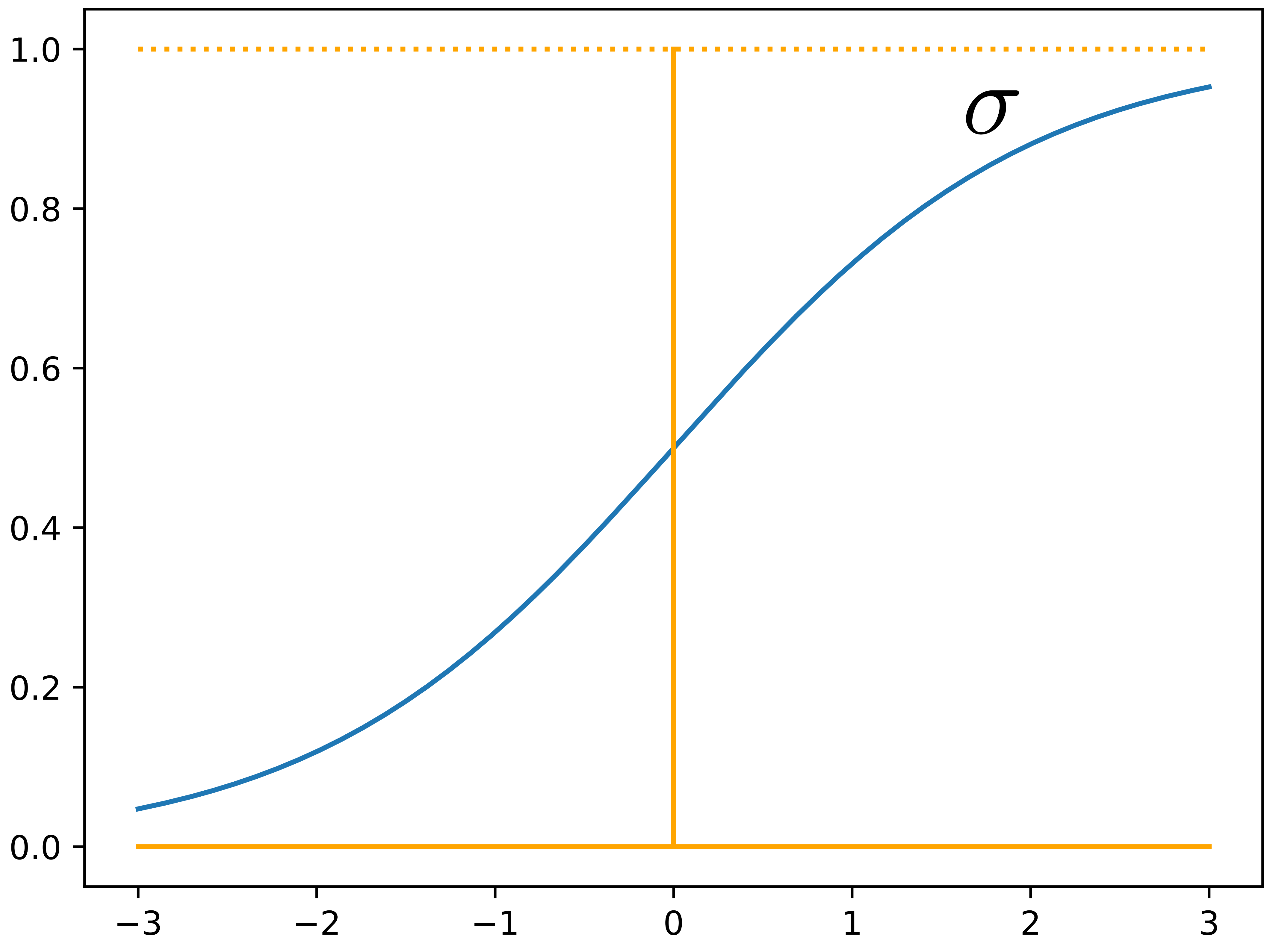
- Symmetric link function

$$\mathbb{P}(Y = y \mid X = x) = \sigma(y x^\top \theta^\star)$$

- Data in tuples (*n* total tuples)

$$(X, Y_1, \ldots, Y_m), \qquad Y_j \mid X \stackrel{\text{iid}}{\sim} \mathbb{P}(\cdot \mid X)$$

- Covariate vectors $X_i \stackrel{\text{iid}}{\sim} \mathsf{N}(0, I_d)$



The model (continued)

- Margin-based loss ℓ satisfying

$$\ell'(t) = -\sigma(-t)$$

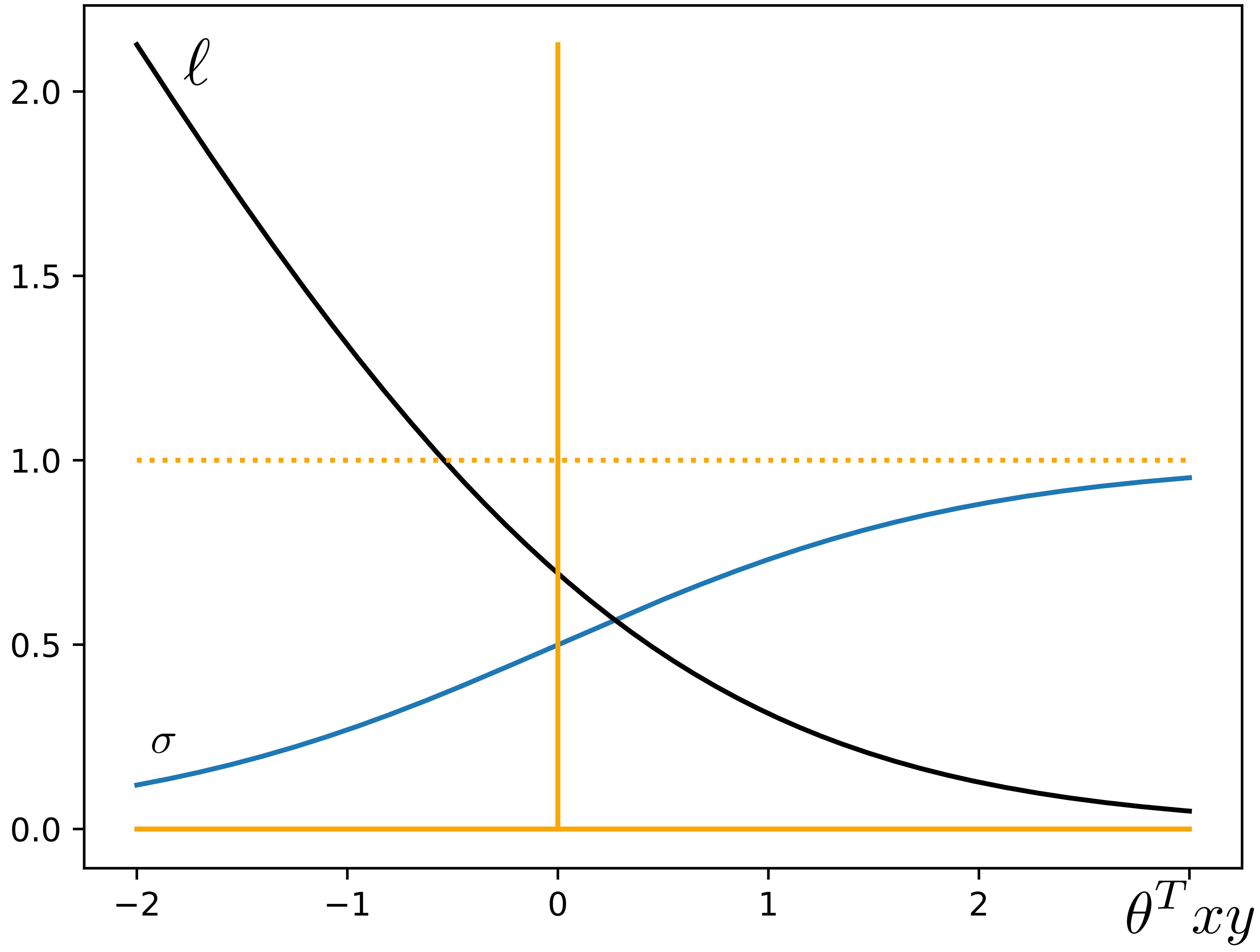
- Loss of parameter θ on (x, y)

$$\ell(\theta^T xy)$$

- E.g. logistic regression

$$\ell(t) = \log(1 + e^t)$$

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$



The two estimators

- Use all the labels

$$L_n(\theta) := \frac{1}{n}\frac{1}{m}\sum_{i=1}^{n}\sum_{j=1}^{m}\ell(Y_{ij}X_i^\top\theta)$$

(Log likelihood for multiple labels)

$$\widehat{\theta}_n = \operatorname*{argmin}_\theta L_n(\theta)$$

- Use majority vote

$$L_n^{\mathrm{mv}}(\theta) := \frac{1}{n}\sum_{i=1}^{n}\ell(\overline{Y}_i X_i^\top\theta)$$

where $\overline{Y}_i = \text{Majority}(Y_{i1},\ldots,Y_{im})$

$$\widehat{\theta}_n^{\mathrm{mv}} = \operatorname*{argmin}_\theta L_n^{\mathrm{mv}}(\theta)$$

Main quantities of interest:

- Calibration error $\left\|\widehat{\theta} - \theta^\star\right\|_2$
- Classification error $\left\|\widehat{u} - u^\star\right\|_2$ where $u = \theta/\left\|\theta\right\|_2$ is unit

Convergence of the MLE

$$L_n(\theta) := \frac{1}{n}\frac{1}{m}\sum_{i=1}^{n}\sum_{j=1}^{m}\ell(Y_{ij}X_i^\top\theta) \qquad\qquad L(\theta) = \mathbb{E}[\ell(YX^\top\theta)]$$

$$\widehat{\theta}_n = \operatorname*{argmin}_{\theta} L_n(\theta)$$

Theorem:

Under the well-specified model, we have asymptotic normality

$$\sqrt{n}\left(\widehat{\theta}_n - \theta^\star\right) \overset{d}{\to} \mathsf{N}\Big(0, \frac{1}{m}\nabla^2 L(\theta^\star)^{-1}\operatorname{Cov}(\dot{\ell}_{\theta^\star})\nabla^2 L(\theta^\star)^{-1}\Big)$$

and

$$\sqrt{n}\left(\widehat{u}_n - u^\star\right) \overset{d}{\to} \mathsf{N}\Big(0, \frac{1}{m\|\theta^\star\|_2^2}(I - u^\star u^{\star\top})\Big)$$

Convergence of majority vote

$$L_n^{\mathrm{mv}}(\theta) := \frac{1}{n}\sum_{i=1}^{n} \ell(\overline{Y}_i X_i^\top \theta) \quad \text{where } \overline{Y}_i = \mathsf{Majority}(Y_{i1}, \ldots, Y_{im})$$

Decompose $X = u^\star Z + (I - u^\star u^{\star\top})X = u^\star Z + W$

Theorem: Under the model, we have "overconfident" convergence

$$\widehat{\theta}_n^{\mathrm{mv}} \xrightarrow{p} t_m u^\star \qquad \text{where} \qquad t_m \asymp \sqrt{m}$$

and asymptotic normality

$$\sqrt{n}\,(\widehat{u}_n^{\mathrm{mv}} - u^\star) \xrightarrow{d} \mathsf{N}\Big(0, \frac{1}{t_m^2} H(t_m)^\dagger C(t_m) H(t_m)^\dagger\Big)$$

$$\stackrel{\mathrm{dist}}{=} \mathsf{N}\Big(0, \frac{c(1+o_m(1))}{\sqrt{m}}(I - u^\star u^{\star\top})\Big)$$

for matrices $H(t) = \frac{1}{4t}\mathbb{E}[WW^\top](1+o(1))$ and $C(t) = \frac{c}{t}\mathbb{E}[WW^\top](1+o(1))$

Robustness of majority vote

Theorem: With misspecified link, we have "overconfident" convergence

$$\widehat{\theta}_n^{\mathrm{mv}} \stackrel{p}{\to} t_m u^\star \qquad \text{where} \qquad t_m \asymp \sqrt{m}$$

and asymptotic normality (for fixed Σ)

$$\sqrt{n}\,(\widehat{u}_n^{\mathrm{mv}} - u^\star) \stackrel{d}{\to} \mathsf{N}\Big(0, \frac{1 + o_m(1)}{\sqrt{m}}\Sigma\Big)$$

Take home messages:

- Majority vote is (unfixably) uncalibrated and overconfident
- More robust (doesn't matter if the link is correct)
- Less efficient when the link *is* correct

Extensions: semiparametric estimates

- Corrected estimator: fit the model, refit the link, refit the model

$$\widehat{\theta}^{\mathrm{mv}} = \operatorname{argmin} L_n^{\mathrm{mv}}(\theta)$$

$$\widehat{\sigma} = \operatorname{argmin} \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} (2\sigma(\widehat{u}^\top X_i Y_{ij}) - 1 - Y_{ij})^2$$

Theorem: Under appropriate conditions,

$$\widehat{\theta} = \operatorname{argmin} \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} \ell_{\widehat{\sigma}}(\theta^\top X_i Y_{ij})$$

is efficient: $$\sqrt{n}\left(\widehat{\theta} - \theta^\star\right) \stackrel{d}{\to} \mathsf{N}\left(0, \frac{1}{m} I(\theta^\star)^{-1}\right)$$

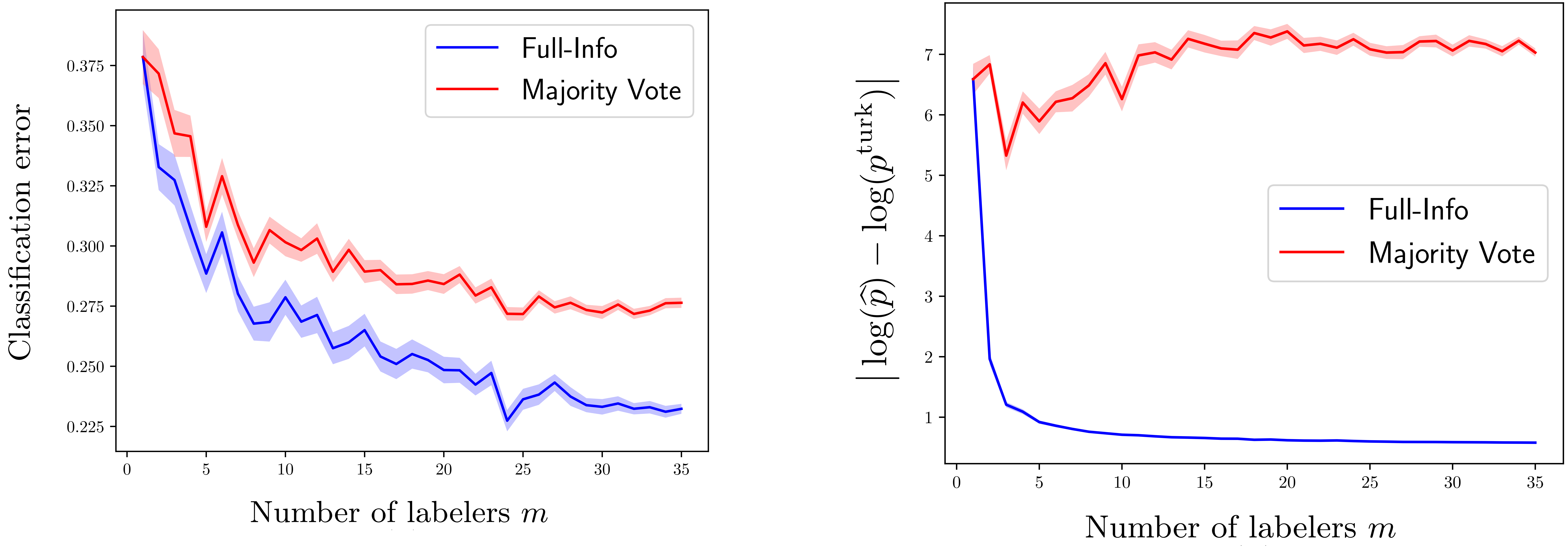
Experimental results

If our model is reasonable, it should make real predictions

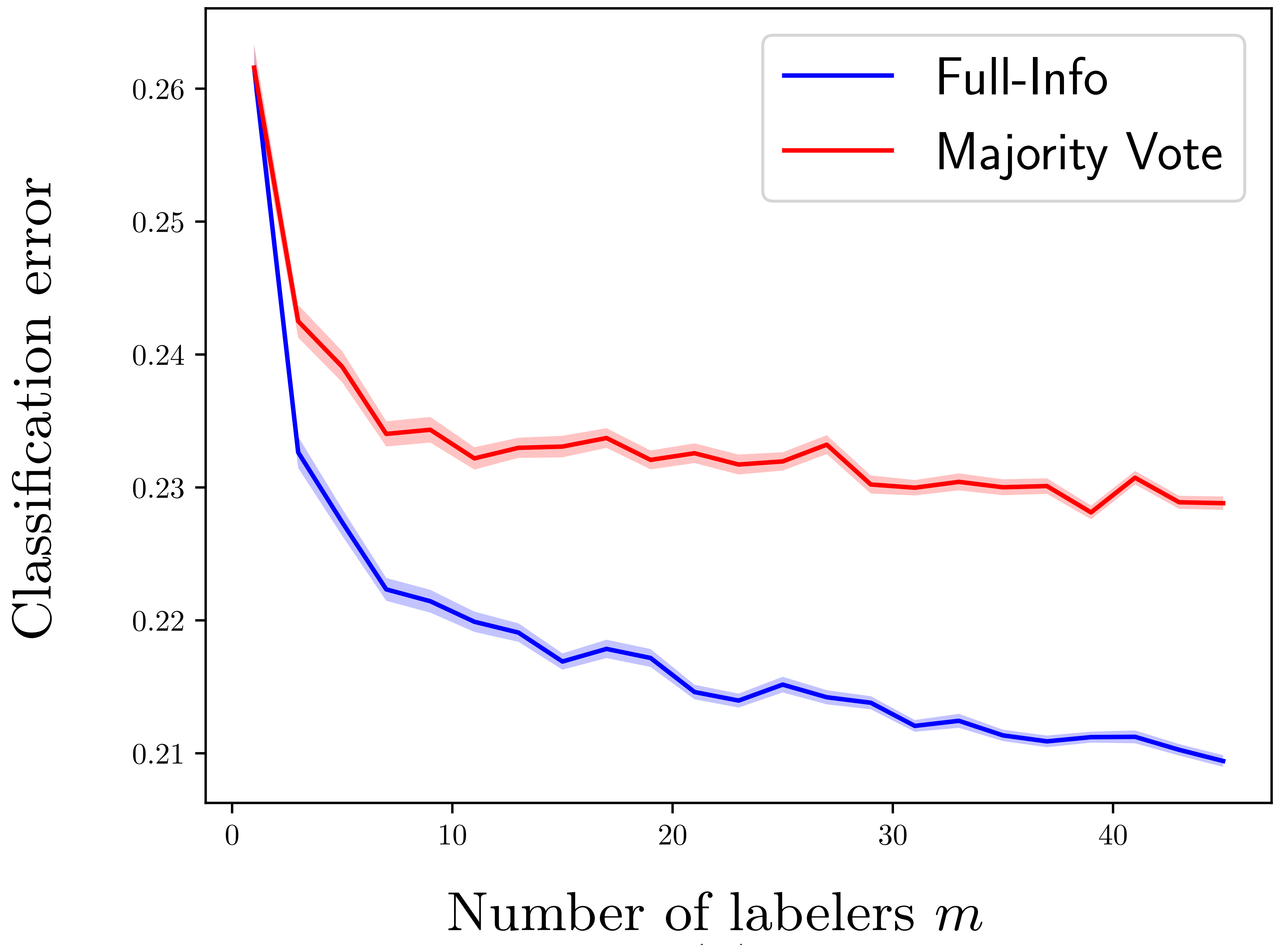
- BlueBirds: *Indigo Bunting* versus *Blue Grosbeak* [Welinder, Branson, Perona, Belongie 10]

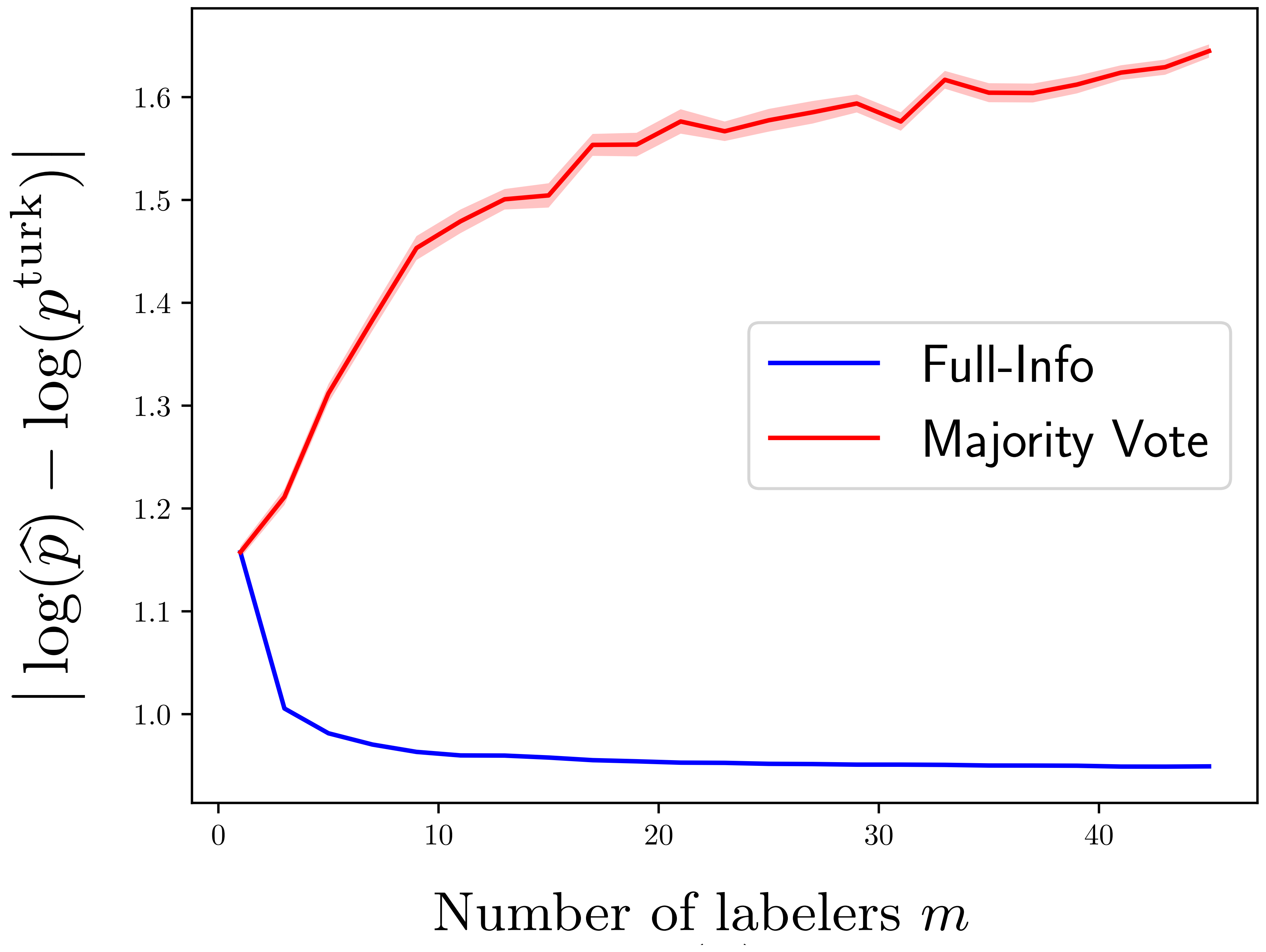
- CIFAR-10H: soft labels of CIFAR-10 test set [Peterson, Battleday, Griffiths, Russakovsky 19]

Experimental results: bluebirds



Experimental results: CIFAR-10H





Conclusions and next steps

- Interesting to think about dataset construction: a place for statistics to lay down some intellectual foundations
- Would obtaining data with (human) perceptual uncertainty help build better prediction methods?
- Currently limited datasets like those above: develop datasets to drive progress we want to see
- Fun to make (theoretical) predictions that can be wrong