

Shape, Orientation, and Apparent Rotational Motion

Joyce E. Farrell and Roger N. Shepard
Stanford University

Apparent rotational motion was investigated in polygonal shapes ranging in rotational symmetry from random to self-identical under 180° rotation. Observers adjusted the rate of alternation between two computer-displayed orientations of any given polygon to determine the point of breakdown of perceived rigid rotation between those two orientations. For asymmetric polygons, the minimum stimulus-onset asynchrony yielding apparent rigid rotation increased approximately linearly with orientational disparity, as anticipated on the basis of the extension of Korte's third law to rotational motion by Shepard and Judd. For nearly symmetric polygons, however, the critical time increased markedly as the disparities approached 180°, owing to the availability of a shorter, nonrigid rotation in the opposite direction. The results demonstrate the existence of competing mental tendencies to preserve the rigid structure of an object and to traverse a minimum transformational path.

There is more to seeing than meets the eye; we often have the perceptual experience of things that are not physically there. In the illusion of apparent motion, an object that is successively displayed for suitable durations in different locations or orientations is perceived as moving continuously through the intermediate positions in space. Under some circumstances, this automatic filling in or *impletion* of a connecting path can have a perceptual force comparable to that produced by real motion (e.g., DeSilva, 1929; Dimmick & Scahill, 1925; Morinaga, Noguchi, & Yokoi, 1966). Despite the absence of external guidance, this illusory filling in is also remarkably orderly. Hence, as the founders of Gestalt psychology were quick to realize (Koffka, 1935; Wertheimer, 1912/1961), parametric investigations of apparent motion should be particularly revealing of the inherent organizing principles of the brain.

Our own work on apparent rotational motion has led us to focus, particularly, on two general principles: (a) Although an ob-

ject could traverse innumerable paths between any two positions, only one motion is experienced on any one occasion, and that motion tends to be one that is in some sense as short or simple as possible. (b) If the second of two successive presentations of the same object does not follow the first too quickly, the minimization of path evidently is subject to the constraint that the rigid shape of the object be preserved over the impleted trajectory.

Traversal of Minimum Path

Following Wertheimer's (1912/1961) original demonstration of the importance of time relations in apparent motion, Korte (1915) established a monotonically increasing, approximately linear relationship between the optimum temporal interval between successive stimuli and their spatial separation, a relationship now generally known as "Korte's third law" (Boring, 1942; Kolers, 1972). More recently, Shepard and Judd (1976) extended this relationship between time and distance to the case of rotational apparent motion, for which distance is specified in terms of the angular disparity between the two successively presented views of an object. Shepard and Judd also obtained more precise, quantitative evidence for a specifically linear increase in critical time with spatial disparity and found that the critical

This work was supported by National Science Foundation Grant BNS 75-02806 to the second author. Portions of the work have previously been described in oral presentations by Shepard (Notes 1 and 2) and by Farrell (Note 3; Farrell & Shepard, Note 4).

Requests for reprints should be sent to Roger N. Shepard, Department of Psychology, Stanford University, Stanford, California 94305.

time for a given rotational difference was the same for rotations in the picture plane as for rotations in depth—just as had been found by Shepard and Metzler (1971) for the very different task of “mental rotation.” This last result, together with related earlier results by Ogasawara (1936), Corbin (1942), and Attneave and Block (1973), suggest that the implected path is constructed at a relatively deep, cognitive level of the perceptual system.

In the case of mental rotation, several kinds of evidence support the idea that the internal process passes through a series of representations corresponding to intermediate orientations in the external world (Cooper, 1976; Cooper & Shepard, 1973, 1978; Metzler & Shepard, 1974; Shepard, 1975). Likewise, in the case of apparent rotational motion, there are at least two kinds of evidence supportive of the notion that the implected trajectory is instantiated in a similarly concrete form. First, objects that differ by 180° in orientation can be experienced as rigidly rotating through either of two alternative paths (starting out in the clockwise or counterclockwise direction) and, despite the physical identity of the stimulating conditions, the two experiences are phenomenally distinct (Robins & Shepard, 1977; Shepard & Judd, 1976). Second, the precision of discriminative response concerning the relation of a spatiotemporally localized probe to such an apparently rotating object suggests that the probe is compared against an internal representation of the object as it would appear in successively more and more rotated orientations (Robins & Shepard, 1977).

Preservation of Rigid Shape

Kolers and Pomerantz (1971) and Shepard and Judd (1976) noted that when the spatial disparity was not too great in relation to a given temporal separation, the apparent motion was perceived as a rigid motion of the object as a whole. When the spatial disparity became sufficiently large in relation to the temporal separation, however, the perceived motion became nonrigid. Typically, local parts of the object in one display then appeared to deform into similarly shaped

parts that were nearby in the other display. Apparently, there is an intimate connection between the perception of the identity of an object and the perception of its position in space—a connection that has been insightfully discussed by Attneave (1974) as the “what–where connection.” (See, also, Rock, 1973; Shepard, 1981; and the differing points of view on the relative roles of shape and position in apparent motion taken by Orlandy, 1940, and by Kolers, 1972).

We undertook the present study to clarify the relationships among shape, orientation, and apparent motion when preservation of shape is pitted against minimization of transformational path. We chose rotation as the spatial transformation for two reasons: First, we and our associates had already developed techniques and considerable calibrational data for apparent rotation (e.g., Shepard & Judd, 1976). Second, the connection between the shape of an object and its spatial transformations is more intimate for rotational than for other rigid transformations (Shepard, Note 2). Except for the circle in two dimensions and sphere in three dimensions (shapes that are unique in being identical to themselves under all rotations), objects have the property that they resemble themselves to different degrees for different angular disparities. For example, although every object is identical to itself under 360° rotation, a rectangle is also identical to itself under 180° rotation and, depending on its length-to-width ratio, it attains some augmented degree of similarity to itself under 90° rotation as well. Indeed, in the spirit of autocorrelational approaches to the representation of form (e.g., Uttal, 1975), shape might be entirely characterized in terms of its self-similarity under all possible rigid transformations in space (Shepard, 1981; Shepard, Note 2). Moreover, in the case of compact objects, as opposed to spatially extended patterns (such as textures, tilings, latticeworks, etc.), rotational transformations will be the most important.

The aspect of shape that is of most central concern here, then, is approximation to symmetry under rotation. For simplicity in this first parametric study of the connection between shape and apparent motion, we systematically varied the degree of approxi-

mation of otherwise random polygons to a single type of rotational symmetry; namely, self-identity under 180° rotation within the picture plane. Correspondingly, we also restricted the rotational disparities between the two alternately displayed views of such a polygon to disparities of angular orientation within that same picture plane. For each combination of rotational symmetry and angular disparity, we determined the maximum rate of alternation for which observers experienced rigid apparent rotation. We sought in this way to develop a more quantitative characterization of the trade-off in apparent motion between extent of transformation and preservation of shape that had been qualitatively noted by Kolers and Pommerantz (1971) and by Shepard and Judd (1976), and, thus, to take a small step toward a more complete understanding of the mental mechanisms underlying perceptual impletion.

Method

Subjects

Eight Stanford students and members of staff participated individually as observers in three sessions consisting of 60 recorded trials each. Although the trials were self-paced, an average session lasted about 1 hr.

Stimuli

We first constructed three asymmetric polygons by picking one point on each of 18 equally spaced radii of polar graph paper and drawing straight line segments between the chosen points on adjacent radii. Two of the points, each chosen to lie 8 units from the origin in opposite directions, determined the vertical axis of the shape. We then determined the distance from the origin of each of the 16 other points by randomly selecting a number between 1 and 17 (excluding 8) without replacement.

To create polygons with 180° rotational symmetry, we cut each of the asymmetric polygons in half along its vertical axis. We then attached each half to a duplicated version of itself that had been rotated 180° . Thus we obtained three rotationally symmetric polygons from the left halves of the asymmetric polygons (called the left-side versions) and three rotationally symmetric polygons from the right halves of the asymmetric polygons (called the right-side versions).

Finally, we constructed polygons of intermediate degrees of symmetry as follows: Placing each symmetric polygon over the asymmetric shapes from which it was derived, so that corresponding halves coincided, we interpolated polygons of intermediate degrees of symmetry by connecting points along radii that fell between

a point on the asymmetric polygon and a corresponding point of the symmetric polygon. In the generation of a particular polygon, these interpolated points were all chosen to fall either one fourth, one half, or three fourths the distance from a point on the asymmetric polygon to the corresponding point on the symmetric polygon.

Our final set thus consisted of 27 distinct polygons: the 3 randomly generated original polygons with 0% experimentally imposed rotational symmetry and 3 left-side and 3 right-side versions at each of the following four levels of experimentally imposed 180° symmetry: 25%, 50%, 75%, and 100%. However, the qualification "experimentally imposed" should be kept in mind. Even a randomly generated polygon, unlike a perfect circle, must possess some appreciable (though haphazard) increments in self-similarity at certain angular disparities, and it is in fact functional dependence of these self-similarities on angle that characterizes the unique shape of such a polygon. However, these haphazard increments in self-similarity, unlike the experimentally imposed increments, would not be expected to be common to the different polygons within an ensemble (of three or six) at a given level of experimentally imposed 180° symmetry. Representative instances of the polygons are illustrated in Figure 1.

We presented each observer with 15 of these distinct polygons: 3, based on the original three random polygons, at each of the five levels of symmetry from 0% to 100%. For half of the observers, the partially symmetric polygons (between 25% and 100% symmetric) were left-side versions, and for the other half they were right-side versions. The 3 asymmetric polygons (0% symmetric) were the same for all observers.

We displayed only one polygon at a time, always centered on the face of a computer-driven cathode ray tube (Tektronix Model 604 with P31 phosphor) situated in a dimly illuminated room. At the standard viewing dis-

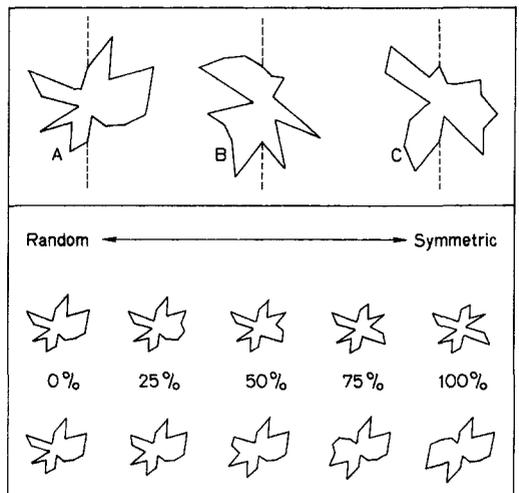


Figure 1. The three originally generated random polygons (A, B, C) and the five levels of 180° rotational symmetry illustrated for the left-side and right-side versions derived from Polygon A.

tance, each polygon subtended a visual angle of approximately 4.8° . We presented the polygon, chosen for a given trial, in continuous alternation between two orientations: the arbitrary upright orientation defined by the so-called "vertical axis" used in the generation of the polygon and an orientation that departed from that "upright" orientation by a fixed angle, throughout that trial, of 30° , 60° , 90° , 120° , 150° , or 180° , clockwise or counterclockwise. On the basis of earlier indications that onset-onset time or *stimulus-onset asynchrony* (SOA) is the most critical temporal determinant of apparent motion (Kahneman, 1967; Korte, 1915; Neuhaus, 1930; Shepard & Judd, 1976, and our own unpublished explorations), we arranged that regardless of the durations of the alternately presented fields, each stationary view immediately replaced the other, without any interstimulus interval.

Procedure

At the beginning of a trial, the duration of each of the two alternately presented views, that is, the SOA, was always initialized at 1 sec. We asked the observers to "try to imagine the shape rigidly rotating from one orientation into the other, as if the shape were actually physically rotating in the picture plane." In most cases they saw the polygon as thus rigidly rotating either at or soon after the alternation began on each trial. They then tapped a key on one side of the response panel, and in this way decreased the duration of each field by 10%. They continued to tap this key and, hence, to speed up the rate of alternation between the two views until they reached a rate at which the apparently rigid rotation gave way (often rather abruptly) to a qualitatively different, apparently nonrigid motion. They then depressed a key at the center of the response panel whereupon the computer recorded the SOA at which the appearance of rigid rotation had broken down, referred to as SOA 1. Observers then began tapping a key on the other side of the response panel, which increased the SOA each time by the same 10% steps. When rigid motion first reappeared, they once again depressed the center key and, thus, recorded the SOA for the first reemergence of apparently rigid motion, referred to as SOA 2. This terminated the trial for that polygon at that angular disparity.

Observers were told that if, on a particular trial, they could not succeed in seeing the polygon as rigidly rotating at the initial 1 field/sec rate or no matter how much they slowed the rate of alternation, they should indicate this as well as the nature of the perceived nonrigid motion on a sheet with numbered spaces for comments on individual trials. They then proceeded to the next trial.

Beyond a series of practice trials that followed the initial instructions, and a few warm-up trials at the start of each ensuing session, each observer proceeded in this way through a randomly arranged sequence of 180 recorded trials, 1 for each polygon in one of the two sets of 15 at each of the six clockwise and six counterclockwise angular disparities (including the 180° disparities, which were identical for clockwise and counterclockwise rotations). The assignment of the speeding-up and slowing-down functions to the left and right response keys,

like the assignment of left-side and right-side versions of the polygons, was counterbalanced over observers.

Results

Mean Dependence of Critical Time on Angular Disparity

Despite an appreciable hysteresis (to be considered later), plots of the SOAs for breakdown (SOA 1) and reemergence (SOA 2) of rigid apparent motion revealed that these two measures were approximately linearly related to each other and were related in similar ways to the principal independent variables, namely, angular disparity and degree of rotational symmetry. For the purposes of an overall examination of these relations, therefore, we averaged corresponding descending and ascending measures and will refer to each such mean SOA at the transition between rigid and nonrigid apparent motion as the *critical SOA*.

Figure 2 shows the dependence of this mean critical SOA on angular disparity, plotted as a separate curve for each of the five levels of 180° rotational symmetry of the polygons. Each point in this figure is averaged over the corresponding shapes derived from the three original polygons, over equivalent clockwise and counterclockwise disparities, and over all eight observers. However, since observers were not always able to see rigid rotation for the more symmetric polygons at the larger angular disparities, some points (particularly for 75% symmetry at 150° and, especially, at 180°) are based on fewer measurements. We believe the patterns of these curves to be quite interpretable.

For the more asymmetric polygons (say 0% and 25% symmetric), critical SOA increased monotonically and, indeed, approximately linearly with angular disparity. This general trend, if linearly extrapolated back to 0° , goes from about 150 msec at the point of no angular disparity to about 550 msec at the maximum 180° disparity. Such a dependence of critical time on difference in orientation is broadly consonant with the linear increase reported by Shepard and Judd (1976) for apparent rotation of asymmetric three-dimensional objects; namely, "from about 110 msec at 0° to 300 msec at 180° ." Overall, the critical SOAs estimated here do average some 145 msec longer owing to a greater elevation and, especially, a greater slope of the new linear trend. However, this experiment differs from Shepard and Judd's with respect to sample of observers, type of stimuli, and procedural and computational methods for estimating critical SOAs.

Effect of Approximation to 180° Rotational Symmetry

The new information provided by the present experiment concerns the consequences of experimentally imposing rotational symmetry. For the polygons with complete 180° rotational symmetry, the critical SOA increased to 90° much as it did for asymmetric polygons. Beyond 90° , however, it decreased

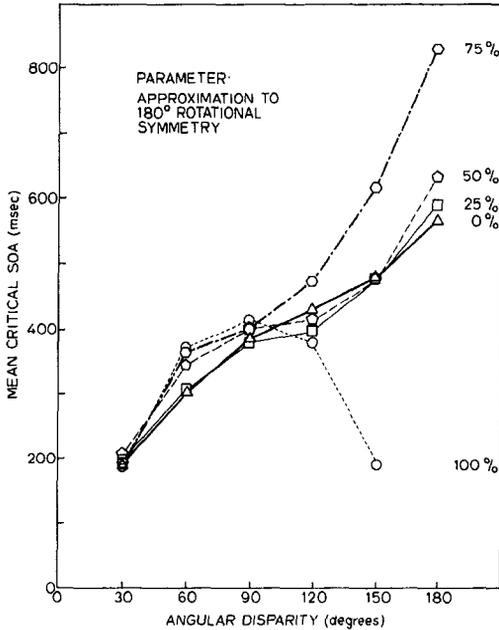


Figure 2. Mean stimulus-onset asynchrony (SOA) for breakdown of rigid apparent rotation plotted as a function of angular disparity for each of the five levels of rotational symmetry of the polygons.

with a slope of equivalent magnitude to an SOA at 150° that is nearly identical to that at 30°. This is entirely to be expected. For these symmetric polygons, a disparity that exceeds 90° by some amount is exactly equivalent to a disparity in the opposite direction that falls short of 90° by that same amount. Hence, the observers experienced a rigid rotation, but through the smaller, supplementary angle in the opposite direction. No point is plotted at 180° for these completely symmetric objects, since the supplementary angle then becomes 0°, and no motion of any kind can be seen.

Of particular interest are the polygons that approximate but do not attain complete rotational symmetry. It is for these polygons that we expect a conflict to arise, beyond disparities of 90°, between a strictly rigid rotation through the larger, obtuse angle and a necessarily nonrigid rotation through the oppositely directed supplementary acute angle. This anticipated conflict in fact manifests itself in our data in two ways: First, as we already implied, the fraction of trials on which observers were able to see rigid ro-

tation markedly decreased as the 75% symmetric polygons reached angular disparities of 150° and, especially, 180°. Second, even on those occasions when they were able to see rigid rotation through these large angles, Figure 2 shows that it was only by going to substantially slower rates of alternation than were required for the asymmetric polygons. The observers claimed that it was as if a strong "pull" toward the similarly shaped and rotationally closer alternative impeded their establishment of a connecting path to the identically shaped but rotationally more remote alternative.

A similar consideration seems to explain other, more subtle regularities of the plotted curves. Quite generally, the critical SOA is elevated above the expected linear trend to the extent that there is a relatively short alternative path of transformation. Given this hypothesis, the 0% curve for the asymmetric polygons is most linear because there are no strongly competing alternatives. But, even for this curve, the slight upswing at 180° may reflect the existence of an alternative rigid rotation in the opposite direction around the 360° circle and, also, an alternative, nonrigid "flip" in depth to be discussed later.

The more nearly the polygons approach 180° rotational symmetry, at the other extreme, the more nearly would rotations close to 90° in one direction compete with rotations through similar, supplementary angles in the opposite direction. Such a competition between opposite paths might thus explain the elevation of the middle points of the curve for the 100% symmetric polygons (at 60°, 90°, and 120°) above the inverted V-shaped curve that would coincide (to the left of 90°) with the linear curve for the asymmetric polygons. It might also explain the S-shaped deviations from linearity of the curves for intermediate degrees of symmetry: To the extent that the polygons become more symmetric, their critical SOA curves should more closely approximate the middle section of the inverted-U curve for completely symmetric polygons. This hypothesized role of competing paths appears to be illustrated especially clearly at 60°. Although the curves for all five levels of similarity are virtually superimposed at 30° dis-

parity, by 60° they have fanned out so that their critical SOAs increase in exactly the predicted order: 0%, 25%, 50%, 75%, and 100% symmetry.

Questions of Linear Trend and Hysteresis

Our interpretation of these systematic departures from an underlying linear trend is at least as tentative as our conclusion that the underlying trend is linear. For large disparities, close to 180°, the assumed linear trend would be perturbed by the increase in SOA that we conjecture to arise from competition between alternative transformations, whether in the picture plane or in depth (see later discussion of Figure 3). And at small angular disparities, there are at least two reasons to suspect departures from a linear trend: (a) The hysteretic difference (SOA 2 - SOA 1), though generally greater than 100 msec for angular departures of 60° or more, dropped to about 70 msec for angular departures of 30° (as well as for the supplementary departures of 150° for the 100% symmetric polygons). (b) Previous work (e.g., by Anstis, 1978; Braddick, 1974; as well as preliminary explorations that we have begun) suggests that different criteria for rigid motion, perhaps depending on more peripheral sensory mechanisms, may come into play for angular disparities much smaller than 30°. Certainly, our observers (like those of DeSilva, 1929, p. 289) reported that the illusion of motion over the longer paths, when achieved, required greater mental effort and was less perceptually compelling than motions experienced over shorter paths. If, as we have assumed, there nevertheless is an underlying linear trend (except, possibly, for very small angular disparities), our current estimate of its slope could fall anywhere between 2.4 msec per degree, as calculated for the whole 30° to 180° range, and 2.0 msec per degree, as calculated after exclusion of the problematic extreme points at 30° and 180°.

Reliability of the Pattern of Critical SOAs in Individual Observers

Although the critical SOAs for individuals were noticeably more variable than the

means for the group plotted in Figure 2, they all exhibited the basic trends noted in those means. First, the mean critical SOA, averaged over the five levels of symmetry, not only increased from 30° to 90° but increased monotonically in that range for all eight observers. Second, for all but the completely symmetric polygons, the SOA consistently continued this increase beyond 90° for all eight observers. Third, for completely symmetric polygons, the SOA beyond 90° reversed direction and decreased for all eight observers. And fourth, the upswing above the other curves for the 75% symmetric polygons beyond 90° is consistent. Although the largest angular departures for which rigid apparent motion of the 75% shapes was attainable was 180° for three observers, 150° for three, and 120° for two, in seven out of eight cases the last point of the curve was higher than the corresponding points of all four curves for the other levels of symmetry, and for the eighth observer (for whom rigid motion could be maintained only to 120°) the last point fell above all but one of the other four curves.

Dependence of Frequency and Type of Breakdown on Angular Disparity and Degree of Symmetry

Except for the case of the 100% symmetric polygons, for which observers almost always saw rigid rotation, the proportion of trials on which observers experienced rigid motion consistently decreased as angular displacement and amount of symmetry increased. After applying the arc-sine transformation to these proportions, we subjected them to an analysis of variance using observers as a repeated measure. The effects of angular disparity, degree of symmetry, and the interaction of these two factors were all significant ($p < .001$). The interaction between angular disparity and degree of symmetry ($F = 5.86$) describes a trade-off between the preservation of the identity of shape and the tendency to choose the shortest of alternative paths connecting its two successive positions.

On the basis of the observers' reports we divided the trials on which they failed to see rigid motion into two categories: (a) nonrigid motion that appeared to occur largely within

the picture plane (usually a deformation together with some rotation in that plane), and (b) nonrigid motion in depth (usually a deformation together with a reflection or "flip" in depth). Figure 3 shows that reports of both types of nonrigid motion increased with angular displacements beyond 90°. But, whereas shapes with a high degree of experimentally imposed rotational symmetry were more often seen to deform in the plane, the shapes intended to be asymmetric were more often seen to "flip" in depth. These latter, reflectional deformations appear to have capitalized on some chance approximations to bilateral symmetries in the original random polygons. For example, since Polygon C (at the top right in Figure 1) happened to be approximately symmetrical about an axis inclined about 30° clockwise from vertical, when it was alternated with a copy of itself rotated 180° in the picture plane, it tended to be seen as nonrigidly flipping in depth about its orthogonal, long axis. Similar phenomena occurred for the other two random polygons owing to the presence of axes of weak bilateral symmetry. In Polygon A, for example, there are two orthogonal such axes inclined at about 45°.

Discussion

Connections Between the Perception of Objects and Their Transformations

The results of our parametric manipulation of angular disparity and rotational symmetry in apparent motion have confirmed the existence of an intimate relation between the perception of shape, on the one hand, and the perception of orientation and rotational movement, on the other. When an object possesses an approximation to some sort of symmetry, ambiguities arise at certain transformational disparities. An object alternately presented in two different positions may then be seen to move through one or another of the shortest paths that preserve rigid structure, especially if the SOA is sufficiently long, or through a much shorter path that does not preserve rigid structure, particularly if the SOA is short. Moreover, the critical SOA for the transition from rigid to nonrigid motion appears (a) to increase

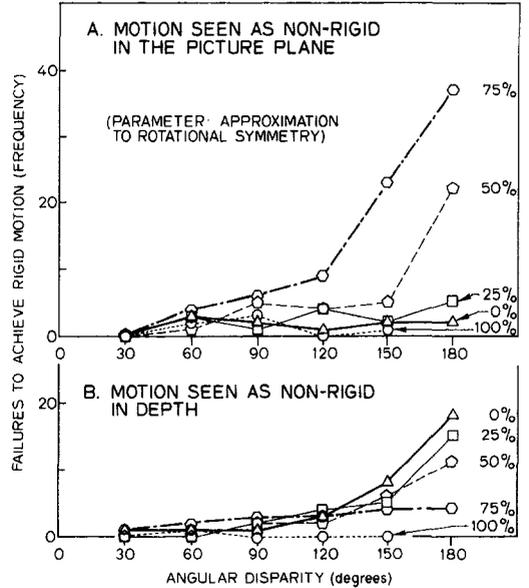


Figure 3. Number of trials on which rigid apparent rotation was not achieved, plotted separately for instances in which the nonrigid motion appeared to be in the picture plane (A) or in depth (B), as a function of angular disparity. (Again, each curve is for a different level of rotational symmetry).

approximately linearly with the extent of the transformation if there are no alternative paths with a competitive trade-off between directness and rigidity and (b) to be elevated above this linear trend if there are. Evidently, the time available between the microgenesis of structural information about each successively presented view, which is determined by the SOA, is critical for the perceived rigidity of the intervening motion and, hence, of the object itself. This points again to the intimate connection between the perception of a shape and the perception of its spatial transformations (cf. Shepard, 1981; Shepard, Note 2).

Our results appear to implicate the operation of internalized rules of object conservation and least action. Having glimpsed two successive visual patterns, the brain attempts to interpret these as manifestations of one enduring object. To the extent that the two patterns differ, the brain strives to represent the intervening existence of the object concretely in the form of a spatial transformation (a) that preserves as much as possible of the rigid structure of the ex-

ternal object and (b) that is the most direct or shortest such transformation. When such a transformation is found, its concrete instantiation satisfies a principle of least action subject to conservation of structural identity. Sometimes, however, a rigid transformation cannot be found because the external object has in fact undergone a nonrigid deformation or because insufficient time was available for the brain to implete the rigid transformation. In this case the brain necessarily accepts a weaker criterion of structural identity and, accordingly, instantiates the most direct nonrigid transformation compatible with that weaker criterion. Correspondingly, the object itself, although still perceived as one enduring object, appears as nonrigid, flexible, rubbery, hinged or jointed.

Geometrical Aspects of Path Minimization

Our results are in good agreement with the quantitative predictions of a geometrical model by means of which one of us (Shepard, 1978, 1981; Shepard, Note 2) has attempted to represent shortest paths of rigid transformation as geodesic trajectories in a curved "constraint manifold" and of nonrigid deformations as even shorter paths through a higher dimensional "embedding space" (compare, particularly, the present Figure 2 with Figure 10.9 in Shepard, 1981). We hope to undertake a fuller consideration of that geometrical model and its relation to the results of the present and other experiments in a later report.

Here, we remark only that the notion of a shortest transformational path is not completely defined without specifying, further, the metric of the underlying space (or manifold) within which that path is constructed. That this issue is of empirical significance is most clearly illustrated by a study in which Foster (1975) investigated paths of apparent motion between two views of a simple object (a rectangle) differing by a translation as well as a rotation. The path of experienced motion generally was not the path in which the center of the object traversed a straight line while the object rotated about that (moving) center. Rather, the center of the object was generally seen to describe a curved path—in fact, a close approximation

to the unique circular path by which the one view could be mapped into the other by a single rigid rotation of the entire two-dimensional plane about some fixed center, without any additional, translational component. Even when a nonrotational, translational component is introduced by the experimenter, then, the perceptual system seems to favor an interpretation in terms of a single rigid rotation (but one in which the center of rotation may have to be at some distance from the object) over an interpretation in terms of a combined translation plus rotation. As Foster noted, we can still suppose that the path of experienced motion tends to be a minimum path, but the metric with respect to which it is minimum may be the metric of the group of rigid rotations "SO(3)" rather than the metric of the familiar Euclidean space of the visual objects themselves.

Perhaps the brain has internalized the abstract geometrical principle that any rigid motion of the plane into itself can be described as a rotation about some center—with pure translations corresponding to the limiting case in which the center of rotation moves off to infinity. The same principle presumably underlies the phenomenon reported by Glass (1969), in which two identical random dot patterns when superimposed with a slight random misalignment give rise, in general, to the appearance of a kind of moiré pattern of concentric circles. We conjecture that the brain has also internalized the analogous principle for three-dimensional space, namely, that any rigid transformation can be described as a rotation together with a translation along the axis of rotation—that is, a "screw displacement" (Coxeter, 1961; cf. Shepard, in press; Shepard, Note 2). In agreement with this expectation, in other experiments on apparent motion, we have observed that when the two alternately presented views differ by a size scaling (dilatation) as well as a rotation, observers experience a rigid object advancing and receding in a helical or screwlike motion (see Shepard, 1981; Bundesen, Larsen, & Farrell, Note 5).

We believe that the Gestalt psychologists were correct in regarding the illusory implectation that occurs in apparent motion as par-

ticularly revealing of fundamental organizing principles of the perceptual system. We suggest, however, that neither they nor most subsequent investigators of apparent motion have fully appreciated (a) the extent to which these organizing principles represent an internalization of some of the most abstract geometrical constraints governing transformations in three-dimensional Euclidean space and (b) the extent to which the perception of shape itself is inextricably related to, and perhaps even dependent on, the representation of its possible transformations in space.

Reference Notes

1. Shepard, R. N. *Trajectories of apparent transformations*. Paper presented at the annual meeting of the Psychonomic Society, Washington, D.C., November 10, 1977.
2. Shepard, R. N. Connections between the representation of shapes and of their spatial transformations. In R. Bajcsy & N. Badler (Eds.), *Representation of three-dimensional objects*. Proceedings of a workshop held at the University of Pennsylvania in May 1979.
3. Farrell, J. E. *What goes where in apparent motion of forms*. Paper presented at the annual meeting of the American Psychological Association, New York, September 1, 1979.
4. Farrell, J. E., & Shepard, R. N. *The effect of form on apparent motion*. Paper presented at the European conference on visual perception, Noodwijkerhout, The Netherlands, October 15–18, 1979.
5. Bundesen, C., Larsen, A., & Farrell, J. E. *Visual apparent movement: Transformations of size and orientation*. Manuscript submitted for publication, 1980.

References

- Anstis, S. M. Apparent movement. In R. Held, H. W. Leibowitz, & H.-L. Teuber (Eds.), *Handbook of sensory physiology* (Vol. 8). Berlin, West Germany: Springer-Verlag, 1978.
- Attneave, F. Apparent movement and the what-where connection. *Psychologia*, 1974, 17, 108–120.
- Attneave, F., & Block, G. Apparent movement in three-dimensional space. *Perception & Psychophysics*, 1973, 13, 301–307.
- Boring, E. G. *Sensation and perception in the history of experimental psychology*. New York: Appleton-Century-Crofts, 1942.
- Braddick, O. A short-range process in apparent motion. *Vision Research*, 1974, 14, 519–528.
- Cooper, L. A. Demonstration of a mental analog of an external rotation. *Perception & Psychophysics*, 1976, 19, 296–302.
- Cooper, L. A., & Shepard, R. N. Chronometric studies of the rotation of mental images. In W. G. Chase (Ed.), *Visual information processing*. New York: Academic Press, 1973.
- Cooper, L. A., & Shepard, R. N. Transformations on representations of objects in space. In E. C. Carterette & M. P. Friedman (Eds.), *Handbook of perception*. Vol. 8: *Space and object perception*. New York: Academic Press, 1978.
- Corbin, H. H. The perception of grouping and apparent movement in visual depth. *Archives of Psychology*, 1942, No. 273, 1–50.
- Coxeter, H. S. M. *Introduction to geometry*. New York: Wiley, 1961.
- DeSilva, H. R. An experimental investigation of the determinants of apparent visual movement. *British Journal of Psychology*, 1929, 19, 268–305.
- Dimmick, F. L., & Scahill, H. G. Visual perception of movement. *American Journal of Psychology*, 1925, 36, 412–417.
- Foster, D. H. Visual apparent motion and some preferred paths in the rotation group SO(3). *Biological Cybernetics*, 1975, 18, 81–89.
- Glass, L. Moiré effect from random dots. *Nature*, 1969, 223, 578–580.
- Kahneman, D. An onset-onset law for one case of apparent motion and metacontrast. *Perception & Psychophysics*, 1967, 2, 577–584.
- Koffka, K. *Principles of gestalt psychology*. New York: Harcourt, Brace, 1935.
- Kolers, P. *Aspects of motion perception*. Oxford, England: Pergamon Press, 1972.
- Kolers, P. A., & Pomerantz, J. R. Figural change in apparent motion. *Journal of Experimental Psychology*, 1971, 87, 99–108.
- Korte, A. Kinematoskopische Untersuchungen. *Zeitschrift für Psychologie*, 1915, 72, 193–296.
- Metzler, J., & Shepard, R. N. Transformational studies of the internal representation of three-dimensional objects. In R. Solso (Ed.), *Theories in cognitive psychology: The Loyola Symposium*. Potomac, Md.: Erlbaum, 1974.
- Morinaga, S., Noguchi, K., & Yokoi, K. Direct comparison of real and apparent visual movement. *Perceptual and Motor Skills*, 1966, 22, 346.
- Neuhaus, W. Experimentelle Untersuchung der Scheinbewegung. *Archiv für gesamte Psychologie*, 1930, 75, 315–348.
- Ogasawara, J. Effect of apparent separation on apparent movement. *Japanese Journal of Psychology*, 1936, 11, 109–122.
- Orlansky, J. The effect of similarity and difference in form on apparent visual movement. *Archives of Psychology*, 1940, 246, 85.
- Robins, C., & Shepard, R. N. Spatio-temporal probing of apparent rotational movement. *Perception & Psychophysics*, 1977, 22, 12–18.
- Rock, I. *Orientation and form*. New York: Academic Press, 1973.
- Shepard, R. N. Form, formation, and transformation of internal representations. In R. Solso (Ed.), *Information processing and cognition: The Loyola Symposium*. Hillsdale, N.J.: Erlbaum, 1975.
- Shepard, R. N. The circumplex and related topological manifolds in the study of perception. In S. Shye (Ed.),

- Theory construction and data analysis in the behavioral sciences*. San Francisco: Jossey-Bass, 1978.
- Shepard, R. N. Psychophysical complementarity. In M. Kubovy & J. R. Pomerantz (Eds.), *Perceptual organization*. Hillsdale, N.J.: Erlbaum, 1981.
- Shepard, R. N. Structural representations of musical pitch. In D. Deutsch (Ed.), *Psychology of music*. New York: Academic Press, in press.
- Shepard, R. N., & Judd, S. A. Perceptual illusion of rotation of three-dimensional objects. *Science*, 1976, 191, 952-954.
- Shepard, R. N., & Metzler, J. Mental rotation of three-dimensional objects. *Science*, 1971, 171, 701-703.
- Uttal, W. R. *An autocorrelation theory of form detection*. Hillsdale, N.J.: Erlbaum, 1975.
- Wertheimer, M. Experimentelle Studien über das Sehen von Bewegung. *Zeitschrift für Psychologie*, 1912, 61, 161-265. (Translated in part in T. Shipley (Ed.), *Classics in psychology*. New York: Philosophical Library, 1961).

Received August 20, 1979 ■