

Scanner linearity

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Abstract. We describe a linear scanner model that provides a useful characterization of the response of a scanner to diffusely reflecting surfaces. We show how the linear model can be used to estimate that portion of the scanner sensor responsivities that fall within the linear space spanned by the input signals. We also describe how the model can be extended to characterize a scanner's response to surfaces that fluoresce under the scanner illuminant.

1 Introduction

Computer-assisted color image editing systems include methods for scanning, displaying, and printing images. Image scanning is the first stage in the image editing process where image information can be lost or distorted. At each sample point in an image, the scanner converts a multidimensional spectral signal into a 3-D spectral signal—namely, the output of three spectral channels referred to as an *RGB* signal. This data compression is necessarily accompanied by information loss.

The *RGB* values returned by most scanners are *device dependent*; the values are uncalibrated, informing us only about the state of this one device. Were all scanner outputs reported in the same standardized units, calibration software could be greatly simplified.

There has been considerable interest in finding methods for converting scanner responses into a calibrated signal gen-

erally referred to as *device-independent* units. Because the human eye is the ultimate consumer of most image data, calibrated representations are generally based on units derived from the sensitivity of the human eye. The eye also compresses a multidimensional spectral signal into a 3-D signal *XYZ*. A scanner is calibrated and also achieves visually lossless data compression when we can convert the *RGB* values to the human response *XYZ*.

Calibration is made much simpler when scanners are designed with two simple properties. First, the scanner responses should be linear with respect to the input (*scanner linearity*), and second, the scanner *RGB* values to a sample should be within a linear transformation of the *XYZ* values (*colorimetric*).

If the scanner sensors are linear, then we can define meaningful spectral responsivities for each of the sensors. If the scanner sensors are within a linear transformation of the color image editing (CIE) functions, then coefficients l_{ij} exist such that

$$X = l_{11}R + l_{12}G + l_{13}B, \quad (1)$$

and similarly for *Y* and *Z*. The coefficients l_{ij} form a 3×3 matrix that maps the scanner responses *RGB* to the sample *XYZ* values.

A linear colorimetric system is easy to calibrate. Nonlinear or noncolorimetric systems can require complex calibration methods. For example, Hung¹ shows that global linear transformations fail badly if *RGB* data derived from the Sharp JX450 scanner are used. He describes a polynomial method for achieving calibrated data.

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An intermediate class of systems includes those that are linear but not colorimetric. Recently, there have been a number of studies of the properties of such systems and some analyses of how well these systems can perform at achieving calibrated output.²⁻⁴ The theoretical results on this topic are encouraging and suggest that a linear noncolorimetric scanner could produce satisfactory calibrated output by use of a single linear transformation.

Empirically, however, little attention has been paid to the basic question of scanner linearity. The studies we describe here were designed to evaluate scanner linearity. Our results include several examples in which scanner linearity provides a good model of the scanner response to diffusely reflecting surfaces. We outline how the model can be extended to characterize a scanner's response to surfaces that fluoresce under the scanner illuminant. We also illustrate how stray light can lead to failures of scanner linearity.

2 Preliminaries and Notation

We use matrix algebra notation. We represent functions of wavelength as vectors with $N_w = 31$ entries representing the function values over the range from 400 to 700 nm in 10-nm steps. The formulas we introduce here only apply to materials without fluorescence.

The entries of the surface reflectance vector \mathbf{s} are the values at the N_w sample wavelengths. We represent the illuminant by an $N_w \times N_w$ diagonal matrix \mathbf{E} whose entries contain the scanner light's spectral power distribution at the sample wavelengths. The scanner sensor responsivities at the sample wavelengths $\mathbf{X}_i(\lambda)$ are defined by the three columns of the $N_w \times 3$ matrix \mathbf{X} . We use a simple linear model to predict the scanner's response to a sample reflectance: $\mathbf{r} = \mathbf{X}\mathbf{E}\mathbf{s}$, where \mathbf{r} is a 3-D vector containing the RGB values.

It is convenient to group the illuminant and sensors together so that we have a single matrix, $\mathbf{T}_E = \mathbf{X}\mathbf{E}$ and

$$\mathbf{r} = \mathbf{T}_E \mathbf{s} \quad (2)$$

We call the matrix \mathbf{T}_E the *scanner transfer matrix*.

To solve for the scanner transfer matrix, we use scanner responses to, say, M known inputs. We create the matrix equation

$$\mathbf{R} = \mathbf{T}_E \mathbf{S} \quad (3)$$

where the $N_w \times M$ matrix \mathbf{S} contains the surface reflectance functions in its columns and the $3 \times M$ matrix \mathbf{R} contains the RGB values for each of the input samples.

3 Methods

We measured surface reflectance functions for a variety of samples, including 216 samples of offset lithographic prints, 216 samples of electrophotographic prints, 224 samples of Cibachrome prints, 24 samples in the Macbeth ColorChecker, 360 samples taken from the set of Munsell glossy surfaces, and 350 samples taken from the set of Munsell matte surfaces. The reflectance functions in these samples are not linearly independent, so that the spectral reflectance functions for each collection of surfaces \mathbf{S} can be described as weighted combinations of a subset of the samples. For example, six spectral basis functions can account for more than 99% of the variance in the spectral reflectance functions for the 24 different sur-

Table 1 Linearity errors expressed as the rms error based on $\|\mathbf{R} - \hat{\mathbf{T}}_E \mathbf{S}\|$.

Surfaces	Scanner	R	G	B
Macbeth	Sharp JX450	2.41	1.70	2.90
	HP Scanjet2C	2.72	1.98	2.06
Offset	Sharp JX450	2.48	1.84	6.62
	HP Scanjet2C	1.74	1.57	1.52
Cibachrome	Sharp JX450	6.45	5.93	6.04
	HP Scanjet2C	6.40	6.64	5.96

faces in the Macbeth ColorChecker, the 216 different spectral reflectance functions measured from Cibachrome prints, and the 216 different functions measured from offset lithographic prints.

We measured the mean scanner response by scanning the samples and averaging over 400 scanner values in the center of each test patch. In all of our measurement conditions the variance of the measurements was signal dependent. The standard deviation increased with the mean sensor value, ranging between 1 and 4 as the scanner responses ranged from 0 to 255.

To estimate the scanner transfer matrix, we minimized the rms error based on Eq. (3), $\|\mathbf{R} - \hat{\mathbf{T}}_E \mathbf{S}\|$. We use the matrix $\hat{\mathbf{T}}_E$, which minimizes this quantity, as our estimate of the scanner transfer function. We can only estimate the part of \mathbf{T}_E that falls in the column space spanned by the surface reflectance functions \mathbf{S} because only this part of the transfer matrix contributes to the error. (For a general analysis see Golub and van Loan⁵; for an analysis specifically related to color see, e.g., Marimont and Wandell.⁴) We estimated this part of \mathbf{T}_E by means of conventional methods. We factored the matrix \mathbf{S} with the singular value decomposition $\mathbf{U}\mathbf{D}\mathbf{V}'$, where $\mathbf{U}\mathbf{U}' = \mathbf{U}'\mathbf{U} = \mathbf{V}\mathbf{V}' = \mathbf{V}'\mathbf{V} = \text{identity}$ and \mathbf{D} is a diagonal matrix. We set the small entries of the diagonal matrix (i.e., the values beyond the eighth entry) to zero and formed a new matrix $\hat{\mathbf{D}}$. Finally, we use $\mathbf{V}(\hat{\mathbf{D}}^{-1})\mathbf{U}'$ as the pseudo-inverse of \mathbf{S} . The diagonal entries of $\hat{\mathbf{D}}^{-1}$ are set to the inverse of the nonzero elements of $\hat{\mathbf{D}}$ and to zero elsewhere.

4 Results

Table 1 lists the rms error we obtain by minimizing the error in Eq. (3). For the offset and Macbeth samples, the linear regression predictions provide a useful summary of the observations. For example, for the offset samples measured with the Scanjet2C, differences between the observed and predicted values were less than 1.9 units in 80% of the measurements. Linear predictions at this precision have some practical value, but any rigorous statistical test will reject the linearity hypothesis. The linear predictions for the Cibachrome samples, however, are too large even to be of practical value. Only 40% of the linear predictions fall within ± 2.4 units of the observed value. Twenty percent of the observations deviated from the observation by more than 6.0 units.

There are at least two reasons why the linear model may fail to predict scanner responses. First, stray light within the scanner can cause systematic deviations from linearity. Second, the fluorescence present in many paper and ink samples

invalidates the model assumptions. All our samples were fluorescent to some extent, but the Cibachrome samples were both glossy and fluorescent.

In the next section, we describe some further analyses of the linear data sets. Then we discuss some measurements of stray light and comment on the challenges posed by surface fluorescence.

4.1 Scanner Transfer Matrix Estimation

Minimizing the error in Eq. (3) yields an estimate of the scanner transfer matrix \mathbf{T}_E . To the extent that the linear model offers a useful summary of the data, the estimated scanner transfer matrix should be close to the scanner sensor responsivities. Both of these scanners perform an electronic matrixing of the raw sensor responses prior to output,^{6,7} so the scanner transfer function is related to the responsivities by a linear transformation. In this section, we describe our estimates of the system transfer function and compare our estimate with the scanner sensor responsivities.*

The samples define one important limit on how well we can estimate the scanner transfer matrix. If the input samples each reflect a different narrowband of light, we could derive the spectral responsivities of the device completely. Real samples, however, are far from such an ideal set of surfaces. As has been noted elsewhere, the reflectance functions of the Macbeth ColorChecker and other typical ink sets are described well by low-dimensional linear models.^{4,8-12}

Most of the variance in the Macbeth surfaces is described by a linear model consisting of a few smooth functions. We plot the first six basis functions of the linear model that describe the Macbeth ColorChecker surfaces in Fig. 1(a). The amplitude of these functions represents their significance. Plainly, beyond these functions there is very little variance left to explain in the data. The presence of sensor noise makes it impossible to measure the small contributions of components beyond the first few. In the calculations that follow, we report estimates assuming that only the first six terms contribute to the scanner responses. The main features of our data remain unchanged if we use from five to eight terms.

The limited input set implies that we cannot estimate the sensors perfectly; our estimates are constrained to be a weighted sum of the six functions shown in Fig. 1(a). Figure 1(b) shows how closely we can estimate Sharp sensor functions (rms error sense) using these six curves as the basis functions. Because the surface contains no material with a steep transition in the short-wavelength region, our estimates cannot capture the narrow tuning of the blue sensor. These curves show the best one can do in estimating the sensors using the Macbeth samples.

Next, we derived an estimate of the scanner transfer matrix from our measurements according to the linear model in Eq. (3). In Fig. 1(c), we plot the estimated transfer matrix curves.

Finally, to compensate for the internal electronic matrixing, we found the linear transformation that maps the scanner transfer matrix into the scanner sensor curves [Fig. 1(d)]. Our empirically derived estimates of the sensor sensitivity are

about as close to the underlying sensor sensitivities as the best estimate we can obtain for these samples [Fig. 1(c)].

5 Departures from Linearity

For some samples, Eq. (3) served as a useful model. For other sample sets, however, the linear model failed. Because we know that the CCD sensor encoding is linear, failures of linearity are likely to arise from the imaging process and errors implicit in the formulation of our simple linear model. In this section, we discuss two sources of error that we have observed in our data set. First, we discuss the failure of the linear model to take into account surface fluorescence. We then describe how to extend the linear model to characterize the scanner response to surfaces that fluoresce as well as reflect. Second, we describe failures in scanner linearity that result from stray light in the imaging process.

5.1 Fluorescence

Many common inks and papers fluoresce. A surface fluoresces when light absorbed in one waveband, usually a short-wavelength band, generates emissions in longer wavelength bands. This process is often linear in the sense that the emissions from the sum of illuminants is equal to the sum of the emissions from each illuminant separately. (We refer the reader to p. 235 et seq. of Ref. 13 for a lucid discussion of the properties and calibration procedures of fluorescent samples.)

We can describe the light reflected from a fluorescent sample as the weighted sum of two terms. The first term is the diffuse reflection component. The second term is the fluorescent component. Both terms depend on the illuminant. Thus, both the diffusely reflected light and the fluorescence depend on the spectral power distribution of the scanner light. Whereas diffusely reflecting surfaces can be represented by a vector or a diagonal matrix, surfaces that fluoresce as well as reflect must be represented by a full matrix that includes measurements of the fluorescence.

As we noted earlier, the failures of linearity we observed for fluorescent surfaces most likely arise from the formulation of our simple linear model. Equation (2) is predicted on the assumption that we know the signal incident on the scanner sensors. But the equation makes no provision for fluorescence, assuming a diffuse illuminant-surface interaction. When surfaces fluoresce, the linear equations contain an incorrect term for the scanner input. This appears as an error when we solve the linear equations.

It is possible to extend our linear model of the scanner responses, but this requires that we *measure* the light incident on the scanner detectors. In other words, we must measure the color signal produced by the input surface and the scanner illumination. We were not able to make these measurements at the time of this publication. But we briefly describe the analysis one would perform to determine whether a scanner responds linearly to fluorescent surfaces.

Let \mathbf{C} be a $N_w \times M$ matrix containing the color signals corresponding to N surfaces and \mathbf{X} be a $N_w \times 3$ matrix whose columns contain the detector responsivities at the sample wavelengths. We can solve for \mathbf{X} , using the scanner responses to M inputs,

$$\mathbf{R} = \mathbf{X}'\mathbf{C} \quad (4)$$

*The sensor responsivities have been measured independently for the Sharp JX450 and HP Scanjet2C. We thank Pooh Chie Hung for estimates of the Sharp sensors and Doug Gennetten for the HP data.

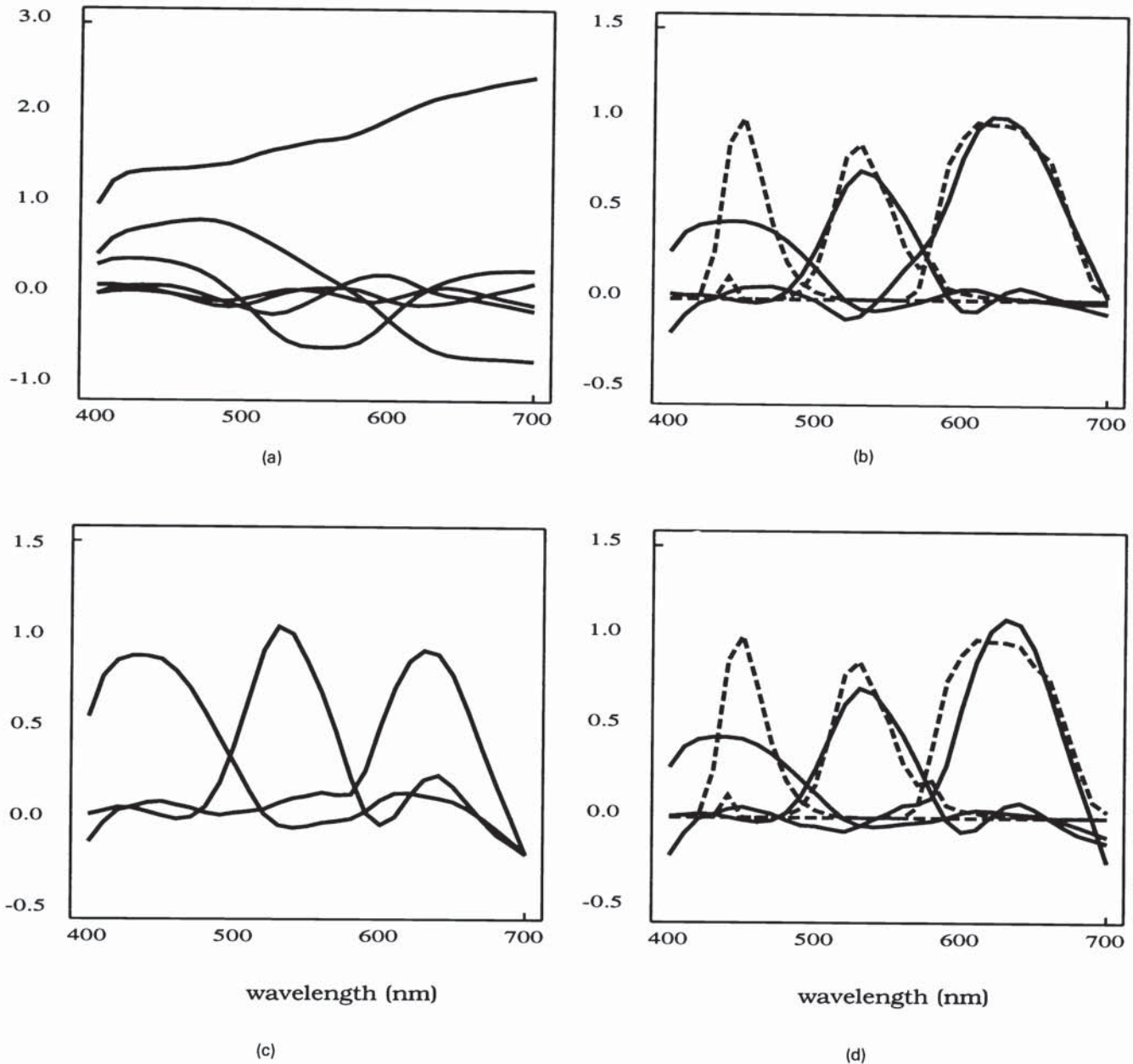


Fig. 1 (a) The first six basis functions of a linear model that approximates the Macbeth ColorChecker optimally in the least-squared error sense; (b) the measured Sharp sensors (dashed line) and the best estimate of these sensors (solid line) that can be obtained by the use of surfaces within the linear model defined in (a); (c) the scanner transfer matrix for the RGB signals; and (d) the measured Sharp sensors again and the sensors estimated by linearly transforming the scanner transfer matrix in (b) to approximate the Sharp sensors.

where the $N_w \times M$ matrix \mathbf{C} contains the color signals in its columns and the $3 \times M$ matrix \mathbf{R} contains the RGB values for each of the input samples. As shown in Sec. 4.1, we can only estimate the portion of the scanner sensor responsivities that falls in the space spanned by the columns of \mathbf{C} .

The method described earlier for estimating scanner detector responsivities can be applied to diffuse as well as fluorescent surfaces. In the case of fluorescence, however, the method requires that we measure the color signals \mathbf{C} produced by fluorescent surfaces under the scanner illuminant.

5.2 Stray Light

We have observed nonlinearities due to stray light created in the imaging process. To illustrate this effect, we scanned a gray target in two different contexts. When the target was scanned with a black surround, the mean scanner values were always less than when the target was scanned with a white surround.

Figure 2 compares the mean scanner RGB values produced by the Sharp JX450 scanner for a 0.25-in.² matte gray target placed against a matte white background and for the same

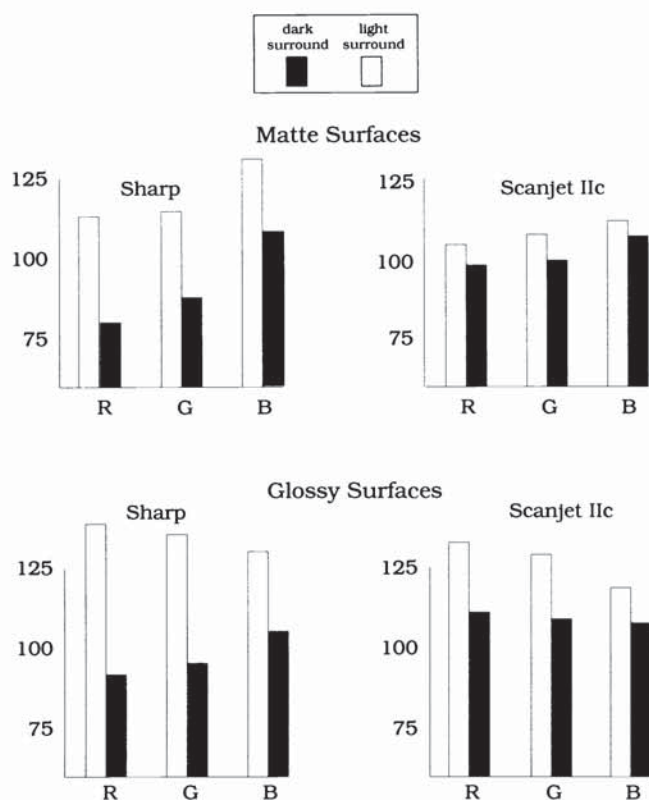


Fig. 2 Mean scanner values for gray targets placed against dark and light backgrounds. The top panels show the mean scanner values produced by the Sharp JX450 (left) and the HP Scanjet2C (right) scanners for matte gray targets placed against matte white and black backgrounds. The bottom panels show the mean scanner values for glossy surfaces.

target placed against a matte black background. For matte targets and backgrounds, the stray light reflected off the white background increases the *R*, *G*, and *B* values by nearly the same amount.

Figure 2 also shows the mean *RGB* values for a 0.25-in² glossy gray target placed against glossy white and glossy black backgrounds. For glossy targets and background, the stray light increases the *R* value more than the *G* value, and the *G* value more than the *B* value. Apparently, the stray light from glossy surfaces has a specular angle and differentially affects the sensors.

Figure 2 also shows the Hewlett-Packard Scanjet2C scanner values recorded for the matte gray and glossy gray targets placed against white and black backgrounds. For this scanner, there is a much smaller effect of stray light.

Stray light in the imaging process violates the linear model of scanner responses. Stray light implies that the scanner response depends on the surrounding surfaces, not just the surface reflectance of the sample. We believe that the stray light artifact did not violate the linearity in the data we report here because of the way in which we collected the sample data. The samples were large patches and our measurements were based on a small selection taken from the center of the target. Under these conditions the stray light that contributes to the measurements is the same as the light we measure. If the targets are small, or measured in varying contexts, this calibration artifact becomes significant.

6 Summary

Most present-day flatbed scanners are not colorimetric in the sense that the scanner *RGB* sensors are not within a linear transformation of the CIE colorimetric *XYZ* functions. The failures of transformations that map *RGB* into *XYZ* have usually been attributed to the fact that scanners are not colorimetric. Whereas, under certain circumstances, it is possible to produce colorimetric output from scanners that are linear but not colorimetric,^{4,14,15} it is not possible to produce colorimetric output from a scanner that does not respond linearly to the input materials.

In this paper, we describe a linear scanner model that provides a useful characterization of the response of a scanner to diffusely reflecting surfaces. The linear model can be used to estimate the portion of the scanner sensor responsivities that falls within the linear space of the input signals. We describe how to extend the model to characterize the scanner's response to surfaces that fluoresce under the scanner illuminant. We also describe conditions in which the linear model fails to predict scanner response.

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