# Online-Appendix: <br> Entropy Balancing for Causal Effects: <br> A Multivariate Reweighting Method to Produce Balanced Samples in Observational Studies 

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#### Abstract

This online appendix presents various additional results referenced in the main paper. In particular, it contains two additional simulations studies and two additional empirical applications.


## I. Monte Carlo Simulations

In addition to the simulation experiment presented in the main paper we conducted two smaller simulation experiments which we describe in the sections below. We also show additional results for the third simulation study that is presented in the main text.

## A. Monte Carlo Experiment I: EPBR Data

The first experiment follows the design presented in Diamond and Sekhon (2006) for a setting that meets the EPBR conditions. We use 50 treated observations and 100 control observations, with three baseline covariates $X$ that are multivariate normal with zero covariances. For the treated observations the means of the $X$ variables are .20 and for the control observations the means are equal to 0 . We generate outcomes with a linear mapping $Y=X \beta+\epsilon$ where $\epsilon \sim N(0, .5)$ and $\beta=(1,1,1)^{\prime}$. The true treatment effect is zero for all units. We consider three variants of this design:

- Design A: the variances of the covariates in $X$ are unity in both groups; the easiest, but presumably most unrealistic case.
- Design B: the variances of the covariates in $X$ are .5 in the control group and and 1.5 in the treatment group. This provides a somewhat more difficult but realistic case; in many empirical cases variances may differ between the two groups.
- Design C: the variances of the covariates in $X$ are equal to unity, but we include all three squared terms in the preprocessing adjustment. Since these squared terms are omitted from the outcome equation (ie. the mapping from $X$ to $Y$ ), this scenario mirrors a case where a researcher adjusts for three irrelevant covariates (as in Brookhart, Schneeweiss, Rothman, Glynn, Avorn and Sturmer (2006)). We include this case because adjusting for squared terms is often recommended in practice.

For the propensity score methods we use the correctly estimated propensity score (from a logistic regression that is linear in $X$ ), instead of the true score because the former is
known to be more efficient. We run 1,000 simulations and report the mean estimate which indicates the bias (multiplied by 100) and the root mean squared error (MSE).

## A.1. Results for Design A: Equal Variances

The results for design A (equal variances), design B (unequal variances), and design C (irrelevant covariates) are shown in the upper, middle, and lower panel of Table I respectively. We find that across all three designs the entropy balancing adjustment is unbiased and highly efficient; it outperforms all other propensity score methods and multivariate matching methods in terms of MSE. In particular for design A, the entropy balancing adjustment has an MSE that is more than four times lower than that of the conventional propensity score weighting estimator where a probit regression is used to estimate the score. This is expected because the entropy balancing adjustment fully incorporates the information about the known sample moments. The MSE of entropy balancing is 13 times lower compared to propensity score matching, about 8 times lower than Mahalanobis matching and about 11 times lower than the joint propensity score Mahalanobis distance matching. Consistent with Diamond and Sekhon (2006), Genetic matching dominates the other matching techniques in terms of MSE, but its MSE is still more than 3 times larger than that of entropy balancing. The fact that the matching adjustment (except Genetic matching) are generally less efficient is consistent with the results from Abadie and Imbens (2006). We also find that the multivariate matching methods are all biased. This is consistent with Abadie and Imbens (2006) who show that the bias of matching is of order $O\left(N^{-1 / k}\right)$ where $k$ is the number of continuous covariates. The patterns are very similar for design B with unequal variances, except that the the differences in MSE are now amplified due to the higher variances in the treatment group. The results for design C (lower panel) shows that the inclusion of the irrelevant variables does not adversely affect entropy balancing, but the other methods now exhibit lower MSE compared to design A.

In summary, the first monte carlo experiment shows that in this setting where the conditions necessary for EPBR hold, entropy balancing outperforms the other preprocessing techniques.

## B. Monte Carlo Experiment II: Non-EPBR Data

## B.1. Design

The second experiment follows the second experiment in Diamond and Sekhon (2006). The covariates are taken from the Dehejia and Wahba (1999) experimental sample of the LaLonde (1986) data (see Diamond and Sekhon (2006) for details). The covariates are not ellipsoidally distributed and thus the EPBR conditions do not hold. Assuming a constant treatment effect of $\$ 1,000$ the fictional earnings are a non-linear function of only two covariates:

$$
Y=1000 D+.1 \exp [.7 \log (\mathrm{re} 74+.01)]+.7 \log (\mathrm{re} 75+.01)])+\epsilon
$$

where $\epsilon \sim N(0, .5)$, re74 and re75 are real earnings in 1974 and 1975 and $D$ is the treatment indicator. The true propensity score is:

$$
\pi_{i}=\operatorname{logit}{ }^{-1}\left[1+.5 \cdot \mu+.01 \log \left(\mathrm{age}^{2}\right)-.3 \log \left(\mathrm{educ}^{2}\right)-.01 \log (\mathrm{re} 74+.01)^{2}+.01 \log (\mathrm{re} 75+.01)^{2}\right]
$$

where the linear predictor $\mu$ is obtained from regressing the actual treatment indicator on age ${ }^{2}$, educ ${ }^{2}$, black, hispanic, married, nodegree, re74 ${ }^{2}$, re75 ${ }^{2}$, u74, and u75 in the Dehejia Wahba sample. So the true propensity score is a combination of this logistic regression plus the extra variables specified in the equation above.

In the Monte Carlo replications we use the following incorrect functional form to estimate the propensity score:

$$
\begin{aligned}
\hat{\mu} & =\alpha_{0}+\alpha_{1} \text { age }+\alpha_{2} \text { educ }+\alpha_{3} \text { black }+\alpha_{4} \text { hispanic }+ \\
& \alpha_{5} \text { married }+\alpha_{6} \text { nodegree }+\alpha_{7} \text { re } 74+\alpha_{8} \text { re } 75+\alpha_{9} \text { u } 74+\alpha_{10} \text { u } 75
\end{aligned}
$$

We run 1,000 simulations and report the mean estimate which indicates the bias (multiplied by 100) and the root mean squared error (MSE). We also report the average computing time per simulation measured in seconds.

## B.2. Results

The results are displayed in Table II. Entropy balancing achieves the second lowest bias and the lowest MSE across all adjustments. It is also much faster compared to Genetic

Matching, which achieves the second lowest MSE in this experiment. The propensity score methods perform badly given the incorrect specification of the propensity score model. This indicates that entropy balancing retains good finite sample properties in this situation where the EPRB conditions do not hold.

## C. Additional Tables for Monte Carlo Experiment III

Tables III and IV display the results for the third simulation discussed in the main paper for the samples sizes $N=600$ and $N=1,500$ respectively. Overall the results are very similar to the case of $N=300$ except that the propensity score methods (with a correctly estimated score) improve as the sample size grows. Entropy balancing achieves the lowest MSE across all simulations.

## II. Additional Results for LaLonde Application

Table V presents additional statistics for the covariate balance in the LaLonde application described in the main text. We can see that both the means and the variances are much more similar after entropy balancing when comparing the treatment and control group in the preprocessed data (the last few columns present the balance results form propensity score weighting as a benchmark). Figure 1 shows the QQ-plots that compare the distributions for all four continuous variables: pretreatment earnings in 1975 and 1974, age, and education. The black dots represent empirical quantile estimates for the raw data. The gray dots represent quantile estimates for the reweighted data. The distributions of all four variables are much more similar after the entropy balancing adjustment.

## III. Additional Results for News Media Persuasion Application

Table VI shows additional balance statistics for the data used in the Ladd and Lenz (2010) study on the effect of news media persuasion in the 1997 British general election. As discussed in the main text, entropy balancing exactly adjusts all the means and almost all of the variances in this data.

## IV. Additional Application: The Fox News Effect

In this section we provide another application of entropy balancing by reanalyzing data from DellaVigna and Kaplan (2007) who study the effect of media bias on voting. The authors exploit the fact that between 1996 and 2000 the conservative Fox News Channel was introduced in the cable programming of about 20 percent of U.S. towns. Using voting data that is aggregated at the town level, the study compares the gain in Republican two-party vote share from the 1996 to 2000 Presidential election between towns that broadcasted Fox News by 2000 in their cable programming and towns that did not. The data covers 9,256 towns overall, 1,807 of which had Fox News availability by 2000 (the treated towns). Using various regression specifications, the authors find that the introduction of Fox News increased Republican vote share gains by 0.4 to 0.7 percentage points.

The authors control for a range of confounders that capture town characteristics measured in the 2000 census including the population size, median income, unemployment rate, and other socio-demographic characteristics for race, gender, urban, education, and married. They include additional controls that measure the trend in each of these variables from the 1990 to 2000 census. The authors also control for the number of cable channels and the number of potential cable subscribers in 2000 . There are 26 covariates overall $[$ T

Estimating the effect of Fox News from this data is difficult for various reasons. First, the effect is fairly small according to the original study. Second, all of the control variables are continuous and given the heterogeneity across towns their distributions are often heavily skewed as can be seeen in Figure 2 which visualizes the distributions using histograms. Third, since the introduction of Fox News into local cable markets was highly selective, the treatment and control towns strongly differ on many important characteristics. In particular, towns with Fox News availability in 2000 were much larger markets with more channels and potential cable subscribers; the standardized difference in means exceeds the

[^0]extreme level of one on both of these key variables. The areas with Fox News were also more urban, richer, and more highly educated. To correct for these imbalances we conduct entropy balancing on all covariates and their squared terms to equalize the means as well as the variances between the two groups ( 52 covariates overall). As in the other examples we also apply the other perprocessing methods and for this purpose estimate the propensity score with a logistic regression of the treatment indicator on the raw variables and squared terms (below we replicate the same analysis while omitting the squared terms).

Figure 3 displays the standardized means (left panel) and p-values for the difference in means tests (right panel) for the raw data and after the various adjustment methods. After entropy balancing the means between the two groups are equal for all 52 covariate combinations. Given that we also include squared terms for each of the raw covariates, the variances on these variables are also equal as can be seen in Table VII which shows additional balance statistics. According to these metrics the balance is higher than that produced by the other adjustment methods. The other methods often leave several of the most important covariates imbalanced (the differences in means remain large and significant) and in several cases the imbalance on key moments is actually increased over the unadjusted data which can be avoided in entropy balancing by including the relevant moments in the reweighting.

A comparison of the average Republican two-party vote share between the treated and control towns in the preprocessed data yields an insignificant effect estimate that is very close to zero. To investigate the model dependency we again examine the effect estimates across a wide range of possible specifications. We create a dataset that includes all raw covariates, their squared terms, and all pairwise interactions (377 covariate combinations overall). We then fit one million regressions of the outcome on the treatment variable and covariates that we randomly sample from the set of all possible subsets of the covariates ${ }^{2}$ We fit each regression in the raw and the preprocessed data (regressions are weighted by the entropy balancing weights). Figure 4 shows the densities of the estimates of the Fox News effect across the regression specifications. In the raw data the estimates vary rather widely

[^1]within $\pm 1.5$ percentage points of vote share, which may be expected given the limited overlap in the data. The model dependency is much reduced after the entropy balancing adjustment; the range of effect estimates is now narrowed down to $\pm .3$ percentage points ${ }^{3}$

Finally, to more closely mirror common practice in applied work, Figures 5 and 6 show a replication of the balance figures for the same analysis where we now omit all squared terms from the preprocessing; the propensity score is estimated with a logistic regression of the treatment indicator on all raw covariates. Such simple propensity score models are widely used in practice where researchers often do not include all squared terms. We can see that the balancing property of the propensity score is now much worse. In fact, when squared terms are omitted the balance on many variables is significantly worse after the propensity score weighting adjustment compared to the raw data (the results look slightly better for propensity score matching). This shows that ill-estimated propensity scores can fail to produce good balance. In this case weighting on the logistic propensity score increases the imbalance over the unadjusted data on many covariates. This may be expected given that the simple model with only the raw covariates does a poor job of capturing the assignment process and the procedure also assigns some very extreme weights since the logistic propensity score is close to zero for some units (Rosenbaum, 1987).

Taken together the application suggests that entropy balancing delivers a high degree of balance in this dataset (as measured by standard metrics). Higher balance reduces model dependence for the estimation of causal effects. It is important to recognize that this replication is intended to simply illustrate the use of entropy balancing in an interesting dataset, it does not invalidate the results of the original study which contains many additional tests and evidence that we do not consider here. We are grateful to the authors for making their data freely available.

[^2]
## V. Additional Application: The Financial Returns to Political Office

In this section we provide another application of entropy balancing by reanalyzing data from Eggers and Hainmueller (2010), who study the financial returns to serving in parliament using data on the estates of recently deceased British politicians. We focus on their sample of 223 conservative candidates that ran for the House of Commons during the 1950-1970 period (see the article for a detailed discussion of the data). The treatment variable is a binary indicator that is coded as one for the 104 candidates that ran successfully and served in parliament and zero for the 119 control candidates that lost and did not enter parliament. The outcome variable is logged wealth at death, which is measured using probate values that capture the value of the candidate's estate at the time of death (in real 2007 British Pounds). In order to account for the selection into political office the authors control for a variety of background covariates including the candidate's gender, year of birth, year of death, as well as educational, occupational, and aristocratic background. There are 18 covariates in total.

Columns 1-4 in Table VIII display the covariate balance in the unmatched data. As discussed by the authors there are important imbalances in this data. In particular, successful candidates are more likely than unsuccessful candidates to be male and to have aristocratic backgrounds and elite educations (Eton Schooling and Oxbridge Degrees). Successful candidates are also less likely to be in white-collar professions (engineering, accounting, or public relations), journalism, and teaching professions, and less likely to have business backgrounds. The standardized bias exceeds $|.1|$ for all but three of the covariates.

To correct for these imbalances we conduct entropy balancing and specify moment conditions to equalize the means of all 18 covariates between the treatment and the reweighted control group. Columns 5-8 in Table VIII display the covariates means as well as the various balance metrics computed with the re-weighted control group. The mean differences are now reduced to zero on all covariates. Except for the year of birth and year of death measures, all variables are binary so by adjusting their means the variances are also adjusted. This constitutes a higher level of balance than previously achieved for
these metrics in this dataset. The difference in means between the treatment group and the reweighted control group yields an average treatment effect on the treated of .99 with a t-statistic of about 2.8, indicating that at serving in Parliament considerably increased wealth at death from conservative MPs. This estimate is close to the magnitude estimated by the authors (the original study used genetic matching).

As a comparison the last few columns present similar balance measures when propensity score weighting, based on a score that is estimated with a logistic regression of the treatment indicator on all covariates, is applied to the same data. While propensity score weighting leads to some balance improvements, important imbalances remain one some key variables such as Oxbridge Degrees, Barrister and Solicitor, and aristocratic background an even worse one some covariates like White Collar professions and year of death the imbalance actually increases over the unmatched data. These imbalances may be corrected by tinkering with the propensity score specification. However, with 18 covariates is is difficult and tedious to find a model that jointly balance all covariates. This shows the benefits of entropy balancing which provides balance by construction of the moment conditions. Figures 7 and 8 show the standardized bias and p-value for the difference in means tests for each covariate when we apply various other matching methods to the same data. Entropy balancing improves on these balance metrics over all other methods including Mahalanobis distance matching, genetic matching, and matching or weighting on the logistic propensity score. Among the other methods, genetic matching does best although some imbalances remain on aristocratic backgrounds and Oxbridge degrees.

## References

Abadie, A. and Imbens, G. (2006), 'Large Sample Properties of Matching Estimators for Average Treatment Effects', Econometrica 74(2), 235-267.

Brookhart, M., Schneeweiss, S., Rothman, K., Glynn, R., Avorn, J. and Sturmer, T. (2006), 'Variable Selection for Propensity Score Models', American Journal of Epidemiology 163(12), 1149-1156.

Diamond, A. J. and Sekhon, J. (2006), Genetic matching for causal effects: A general multivariate matching method for achieving balance in observational studies. Unpublished manuscript, Dept. of Political Science, UC Berkeley.

Rosenbaum, P. (1987), 'Model-based direct adjustment', Journal of the American Statistical Association 82(398), 387-394.
VI. Tables

| Table I: Results for First Monte Carlo Experiment |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Estimator: | RAW | PSM | MD | PSMD | GM | PSW | EB |
| Design A: equal variances |  |  |  |  |  |  |  |
| Bias | -60.19 | -2.34 | -9.66 | -9.83 | -11.36 | -0.58 | 0.02 |
| MSE | 45.51 | 4.87 | 2.31 | 2.47 | 3.25 | 1.21 | 0.23 |
| Bias / Bias EB | 2746.19 | 106.78 | 440.73 | 448.53 | 518.36 | 26.31 | 1.00 |
| MSE / MSE EB | 195.78 | 20.94 | 9.93 | 10.61 | 13.99 | 5.22 | 1.00 |
| Design B: unequal variances |  |  |  |  |  |  |  |
| Bias | -59.79 | -10.01 | -20.84 | -23.62 | $-13.37$ | -25.64 | -0.11 |
| MSE | 46.58 | 5.17 | 7.07 | 8.78 | 3.88 | 9.38 | 0.25 |
| Bias / Bias EB | 564.52 | 94.54 | 196.76 | 222.99 | 126.19 | 242.09 | 1.00 |
| MSE / MSE EB | 188.78 | 20.94 | 28.64 | 35.59 | 15.73 | 38.04 | 1.00 |
| Design C: equal variances and squared terms |  |  |  |  |  |  |  |
| Bias | -63.24 | -3.41 | -10.10 | -10.39 | -11.11 | -0.89 | 0.15 |
| MSE | 48.18 | 4.94 | 2.42 | 2.62 | 3.06 | 1.24 | 0.25 |
| Bias / Bias EB | 420.09 | 22.64 | 67.11 | 69.02 | 73.78 | 5.92 | 1.00 |
| MSE / MSE EB | 190.98 | 19.57 | 9.61 | 10.38 | 12.13 | 4.90 | 1.00 |
| Note: Three independent normal covariates $X$ drawn for 50 treated units with means .2 and for 100 control units with means 0 . Outcome mapping is linear $Y=X \beta+\epsilon$ with $\beta=(1,1,1)^{\prime}$ and $\epsilon \sim N(0, .5)$. The true treatment effect is zero for all units. Assumptions satisfy the conditions for EPBR. 1,000 simulations. Design A: Equal unit variances in both groups. Design B: Unequal variances: 1.5 in treatment and .5 in control group. Design C: Equal unit variances. Squared terms of $X$ are included for all estimators, but not the outcome. This mirrors the situation of adjusting for additional irrelevant covariates. Raw: Difference of means; PSM: Propensity score matching; MD: Mahalanobis Distance Matching, PSMD: MD matching on the PS and orthogonalized covariates; GM: Genetic Matching; PSW: weighting on the PS; EB: entropy balancing. Matching is 1:1 pair matching. The propensity score is estimated with a linear logit in $X$. |  |  |  |  |  |  |  |



Table III: Results for Monte Carlo Experiment (N=600)
Sample Design 1: Strong Separation and Normal Errors

| MSE | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Ratio CtoT 1 Y1 | 320 | 21 | 14 | 358 | 182 | 46 | 352 | 186 | 15 | 354 | 183 | 2 |
| Ratio CtoT 1 Y2 | 497 | 15 | 7 | 153 | 524 | 49 | 164 | 449 | 19 | 152 | 529 | 2 |
| Ratio CtoT 1 Y3 | 997 | 915 | 671 | 1326 | 1640 | 1764 | 1479 | 2026 | 742 | 1205 | 1526 | 172 |
| Ratio CtoT 3 Y1 | 323 | 18 | 12 | 363 | 185 | 39 | 353 | 190 | 10 | 356 | 185 | 2 |
| Ratio CtoT 3 Y2 | 499 | 13 | 5 | 153 | 525 | 41 | 155 | 459 | 12 | 156 | 532 | 2 |
| Ratio CtoT 3 Y3 | 1041 | 807 | 599 | 1186 | 1675 | 1601 | 1357 | 2035 | 592 | 1123 | 1547 | 168 |
| Ratio CtoT 5 Y1 | 324 | 17 | 14 | 368 | 186 | 33 | 352 | 190 | 8 | 360 | 182 | 2 |
| Ratio CtoT 5 Y2 | 499 | 12 | 7 | 155 | 525 | 34 | 143 | 467 | 9 | 158 | 526 | 2 |
| Ratio CtoT 5 Y3 | 1125 | 739 | 604 | 1133 | 1719 | 1431 | 1269 | 2022 | 627 | 1072 | 1589 | 186 |
| BIAS | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| Ratio CtoT 1 Y1 | 177 | 42 | 6 | 185 | 132 | 65 | 185 | 135 | 7 | 186 | 134 | 0 |
| Ratio CtoT 1 Y2 | 222 | 35 | 10 | 121 | 227 | 68 | 126 | 211 | 8 | 121 | 229 | 2 |
| Ratio CtoT 1 Y3 | 294 | 292 | 72 | 333 | 393 | 412 | 374 | 443 | 51 | 329 | 384 | 69 |
| Ratio CtoT 3 Y1 | 178 | 38 | 7 | 187 | 133 | 60 | 185 | 136 | 11 | 186 | 134 | 0 |
| Ratio CtoT 3 Y2 | 222 | 31 | 8 | 120 | 227 | 61 | 121 | 213 | 14 | 123 | 229 | 2 |
| Ratio CtoT 3 Y3 | 296 | 272 | 63 | 311 | 393 | 390 | 355 | 441 | 76 | 314 | 384 | 61 |
| Ratio CtoT 5 Y1 | 177 | 35 | 5 | 187 | 131 | 54 | 183 | 135 | 12 | 187 | 133 | -1 |
| Ratio CtoT 5 Y2 | 222 | 28 | 7 | 120 | 226 | 54 | 115 | 213 | 17 | 123 | 227 | 1 |
| Ratio CtoT 5 Y3 | 294 | 253 | 58 | 284 | 389 | 362 | 335 | 432 | 96 | 302 | 384 | 50 |


| MSE | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ratio CtoT 1 Y1 | 144 | 8 | 5 | 126 | 82 | 18 | 126 | 88 | 3 | 121 | 78 | 1 |
| Ratio CtoT 1 Y2 | 223 | 4 | 3 | 53 | 222 | 13 | 44 | 191 | 3 | 50 | 221 | 1 |
| Ratio CtoT 1 Y3 | 514 | 440 | 273 | 458 | 823 | 859 | 722 | 1143 | 219 | 372 | 767 | 95 |
| Ratio CtoT 3 Y1 | 145 | 8 | 5 | 126 | 85 | 16 | 124 | 89 | 2 | 123 | 79 | 1 |
| Ratio CtoT 3 Y2 | 224 | 4 | 3 | 51 | 227 | 12 | 42 | 199 | 3 | 50 | 224 | 1 |
| Ratio CtoT 3 Y3 | 557 | 387 | 259 | 431 | 861 | 775 | 659 | 1149 | 200 | 354 | 783 | 103 |
| Ratio CtoT 5 Y1 | 146 | 8 | 8 | 131 | 86 | 14 | 123 | 92 | 2 | 124 | 80 | 1 |
| Ratio CtoT 5 Y2 | 225 | 4 | 4 | 54 | 230 | 10 | 38 | 204 | 3 | 51 | 225 | 2 |
| Ratio CtoT 5 Y3 | 629 | 362 | 387 | 458 | 939 | 708 | 630 | 1138 | 215 | 330 | 786 | 130 |
| BIAS | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| Ratio CtoT 1 Y1 | 118 | 25 | 1 | 107 | 87 | 40 | 109 | 91 | 1 | 107 | 87 | 0 |
| Ratio CtoT 1 Y2 | 148 | 15 | 2 | 68 | 147 | 33 | 63 | 136 | 1 | 68 | 147 | 1 |
| Ratio CtoT 1 Y3 | 195 | 200 | 15 | 175 | 269 | 284 | 257 | 329 | 6 | 173 | 267 | 18 |
| Ratio CtoT 3 Y1 | 117 | 23 | 2 | 107 | 87 | 37 | 107 | 91 | 3 | 108 | 87 | 0 |
| Ratio CtoT 3 Y2 | 148 | 13 | 2 | 67 | 147 | 31 | 60 | 138 | 4 | 68 | 147 | 1 |
| Ratio CtoT 3 Y3 | 199 | 184 | 18 | 161 | 271 | 266 | 241 | 326 | 22 | 165 | 267 | 14 |
| Ratio CtoT 5 Y1 | 117 | 20 | -1 | 107 | 86 | 32 | 104 | 91 | 2 | 107 | 86 | -1 |
| Ratio CtoT 5 Y2 | 147 | 10 | 1 | 67 | 147 | 26 | 54 | 138 | 4 | 67 | 147 | -0 |
| Ratio CtoT 5 Y3 | 198 | 171 | 6 | 146 | 270 | 246 | 226 | 316 | 22 | 154 | 261 | 3 |

Sample Design 3: Medium Separation and Leptokurtic Errors

| MSE | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ratio CtoT 1 Y1 | 218 | 16 | 12 | 239 | 138 | 32 | 233 | 150 | 10 | 234 | 139 | 1 |
| Ratio CtoT 1 Y2 | 343 | 10 | 6 | 103 | 398 | 31 | 108 | 346 | 15 | 115 | 409 | 2 |
| Ratio CtoT 1 Y3 | 1049 | 794 | 583 | 843 | 1346 | 1512 | 1219 | 1721 | 506 | 862 | 1210 | 234 |
| Ratio CtoT 3 Y1 | 216 | 14 | 11 | 237 | 138 | 27 | 227 | 148 | 8 | 234 | 139 | 1 |
| Ratio CtoT 3 Y2 | 341 | 9 | 5 | 104 | 396 | 27 | 99 | 348 | 13 | 116 | 405 | 2 |
| Ratio CtoT 3 Y3 | 1069 | 680 | 577 | 761 | 1399 | 1352 | 1044 | 1644 | 512 | 789 | 1218 | 222 |
| Ratio CtoT 5 Y1 | 224 | 14 | 13 | 249 | 144 | 25 | 232 | 152 | 7 | 246 | 141 | 2 |
| Ratio CtoT 5 Y2 | 343 | 9 | 6 | 108 | 402 | 23 | 92 | 358 | 11 | 118 | 407 | 2 |
| Ratio CtoT 5 Y3 | 1172 | 655 | 820 | 723 | 1462 | 1244 | 1006 | 1670 | 524 | 780 | 1233 | 238 |
| BIAS | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| Ratio CtoT 1 Y1 | 146 | 36 | 7 | 150 | 115 | 54 | 150 | 121 | 25 | 151 | 116 | 0 |
| Ratio CtoT 1 Y2 | 184 | 29 | 11 | 98 | 198 | 53 | 101 | 184 | 35 | 105 | 201 | 4 |
| Ratio CtoT 1 Y3 | 301 | 273 | 74 | 247 | 351 | 381 | 339 | 407 | 177 | 274 | 339 | 121 |
| Ratio CtoT 3 Y1 | 145 | 32 | 6 | 149 | 114 | 50 | 147 | 119 | 24 | 151 | 116 | 0 |
| Ratio CtoT 3 Y2 | 184 | 25 | 10 | 98 | 197 | 49 | 96 | 185 | 33 | 106 | 200 | 3 |
| Ratio CtoT 3 Y3 | 302 | 250 | 63 | 217 | 357 | 358 | 309 | 395 | 181 | 263 | 339 | 117 |
| Ratio CtoT 5 Y1 | 146 | 31 | 5 | 151 | 115 | 45 | 147 | 119 | 20 | 154 | 116 | -0 |
| Ratio CtoT 5 Y2 | 183 | 23 | 10 | 98 | 197 | 43 | 90 | 186 | 28 | 105 | 199 | 3 |
| Ratio CtoT 5 Y3 | 297 | 237 | 42 | 195 | 351 | 335 | 292 | 388 | 167 | 245 | 333 | 103 |

Note: Results show MSE and Bias across 1,000 simulations. Six covariates with a mixture of continuous, binary, and categorical
variables. Experimental factors are: 3 sample designs (sample design 1: strong separation and normal errors; sample design 2: weaker separation and normal errors; sample design 3: medium separation and leptokurtic errors), 3 outcome designs (Y1 linear: $Y 1=$ $X_{1}+X_{2}+X_{3}-X_{4}+X_{5}+X_{6}+\eta ;$ Y2 somewhat non-linear Y2 $=X_{1}+X_{2}+0.2 X_{3} X_{4}-\sqrt{X_{5}}+\eta$; Y3 highly non-linear: $Y 3=\left(X_{1}+X_{2}+X_{5}\right)^{2}+\eta$ ), and 3 controls-to-treated ratios (Ratio CtoT 1, 3, and 5). Estimators are Raw: Difference of means; MD: Mahalanobis distance matching, GM: Genetic matching; PSM: Propensity score matching; PSMD: MD matching on the PS and orthogonalized covariates; PSW: weighting on the PS; EB: entropy balancing. All matching is $1: 1$ pair matching. We use three specifications (labeled with a 1,2 , or 3 postfix) for all propensity score based methods (PSM, PSW, PSMD). The first propensity score model is correct for sample designs 1 and 2 , and slightly misspecified for sample design 3 . Propensity score models 2 and 3 are increasing in misspecification (as measured by the linear correlation between the true and the estimated score). 1000 simulations for each scenario; the true treatment effect is zero.

Table IV: Results for Monte Carlo Experiment ( $\mathrm{N}=1,500$ )
Sample Design 1: Strong Separation and Normal Errors

| MSE | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Ratio CtoT 1 Y1 | 318 | 11 | 6 | 343 | 180 | 28 | 327 | 180 | 8 | 342 | 181 | 1 |
| Ratio CtoT 1 Y2 | 496 | 8 | 2 | 147 | 526 | 30 | 134 | 467 | 10 | 146 | 530 | 1 |
| Ratio CtoT 1 Y3 | 952 | 562 | 301 | 1097 | 1540 | 1262 | 1078 | 1863 | 488 | 1060 | 1498 | 95 |
| Ratio CtoT 3 Y1 | 317 | 9 | 5 | 343 | 181 | 23 | 321 | 180 | 4 | 341 | 180 | 1 |
| Ratio CtoT 3 Y2 | 495 | 6 | 2 | 143 | 522 | 24 | 123 | 468 | 6 | 149 | 524 | 1 |
| Ratio CtoT 3 Y3 | 936 | 475 | 258 | 970 | 1547 | 1099 | 960 | 1833 | 280 | 977 | 1511 | 75 |
| Ratio CtoT 5 Y1 | 318 | 9 | 5 | 343 | 180 | 20 | 318 | 181 | 4 | 348 | 181 | 1 |
| Ratio CtoT 5 Y2 | 495 | 6 | 2 | 143 | 522 | 20 | 115 | 473 | 5 | 155 | 523 | 1 |
| Ratio CtoT 5 Y3 | 988 | 420 | 263 | 837 | 1603 | 972 | 875 | 1833 | 263 | 910 | 1548 |  |
| BIAS | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| Ratio CtoT 1 Y1 | 178 | 31 | 3 | 184 | 133 | 52 | 180 | 133 | 3 | 184 | 134 | 0 |
| Ratio CtoT 1 Y2 | 222 | 26 | 6 | 120 | 229 | 54 | 115 | 216 | 5 | 120 | 230 | 2 |
| Ratio CtoT 1 Y3 | 300 | 232 | 44 | 318 | 388 | 352 | 323 | 429 | 30 | 316 | 384 | 54 |
| Ratio CtoT 3 Y1 | 177 | 28 | 3 | 184 | 133 | 47 | 178 | 133 | 9 | 184 | 134 | -0 |
| Ratio CtoT 3 Y2 | 222 | 22 | 4 | 119 | 228 | 48 | 109 | 216 | 12 | 121 | 228 | 1 |
| Ratio CtoT 3 Y3 | 296 | 213 | 32 | 297 | 388 | 327 | 304 | 424 | 61 | 304 | 385 | 41 |
| Ratio CtoT 5 Y1 | 177 | 26 | 2 | 183 | 133 | 42 | 176 | 133 | 12 | 185 | 134 | 0 |
| Ratio CtoT 5 Y2 | 222 | 20 | 4 | 118 | 227 | 43 | 105 | 216 | 17 | 123 | 228 | 1 |
| Ratio CtoT 5 Y3 | 297 | 197 | 37 | 266 | 391 | 305 | 287 | 421 | 91 | 291 | 387 | 39 |


| MSE | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ratio CtoT 1 Y1 | 140 | 4 | 2 | 114 | 77 | 10 | 109 | 81 | 1 | 114 | 77 | 0 |
| Ratio CtoT 1 Y2 | 221 | 2 | 1 | 47 | 219 | 7 | 32 | 197 | 2 | 47 | 220 | 0 |
| Ratio CtoT 1 Y3 | 444 | 238 | 95 | 329 | 774 | 557 | 465 | 972 | 103 | 298 | 734 | 41 |
| Ratio CtoT 3 Y1 | 140 | 4 | 2 | 115 | 78 | 9 | 104 | 82 | 1 | 115 | 77 | 0 |
| Ratio CtoT 3 Y2 | 220 | 1 | 1 | 47 | 221 | 6 | 28 | 201 | 1 | 47 | 219 | 0 |
| Ratio CtoT 3 Y3 | 448 | 206 | 104 | 289 | 754 | 495 | 409 | 954 | 80 | 277 | 731 | 42 |
| Ratio CtoT 5 Y1 | 142 | 4 | 3 | 118 | 80 | 8 | 105 | 86 | 1 | 118 | 79 | 1 |
| Ratio CtoT 5 Y2 | 221 | 2 | 1 | 48 | 222 | 5 | 28 | 205 | 1 | 49 | 220 | 1 |
| Ratio CtoT 5 Y3 | 488 | 186 | 135 | 278 | 782 | 437 | 381 | 952 | 90 | 262 | 738 | 55 |
| BIAS | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| Ratio CtoT 1 Y1 | 117 | 18 | 1 | 105 | 87 | 31 | 103 | 89 | 0 | 106 | 87 | 0 |
| Ratio CtoT 1 Y2 | 148 | 10 | 1 | 67 | 147 | 25 | 55 | 139 | 1 | 67 | 148 | 0 |
| Ratio CtoT 1 Y3 | 196 | 150 | 11 | 163 | 271 | 232 | 210 | 308 | 1 | 164 | 267 | 10 |
| Ratio CtoT 3 Y1 | 117 | 16 | -0 | 105 | 87 | 28 | 100 | 89 | 2 | 106 | 87 | -0 |
| Ratio CtoT 3 Y2 | 148 | 9 | 0 | 67 | 148 | 23 | 51 | 140 | 3 | 68 | 147 | 0 |
| Ratio CtoT 3 Y3 | 196 | 139 | 6 | 152 | 267 | 218 | 195 | 304 | 18 | 158 | 266 | 8 |
| Ratio CtoT 5 Y1 | 117 | 15 | -0 | 106 | 87 | 25 | 100 | 91 | 3 | 107 | 87 | -0 |
| Ratio CtoT 5 Y2 | 148 | 7 | 0 | 67 | 147 | 20 | 49 | 141 | 4 | 68 | 147 | -0 |
| Ratio CtoT 5 Y3 | 196 | 127 | 0 | 136 | 266 | 200 | 184 | 299 | 26 | 150 | 264 | 2 |

Sample Design 3: Medium Separation and Leptokurtic Errors

| MSE | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ratio CtoT 1 Y1 | 213 | 8 | 5 | 226 | 133 | 19 | 216 | 143 | 7 | 226 | 137 | 1 |
| Ratio CtoT 1 Y2 | 339 | 5 | 3 | 98 | 390 | 19 | 87 | 354 | 13 | 112 | 403 | 1 |
| Ratio CtoT 1 Y3 | 933 | 491 | 319 | 625 | 1264 | 1075 | 845 | 1468 | 379 | 745 | 1146 | 160 |
| Ratio CtoT 3 Y1 | 217 | 7 | 5 | 230 | 134 | 17 | 212 | 144 | 6 | 231 | 137 | 0 |
| Ratio CtoT 3 Y2 | 338 | 5 | 2 | 97 | 391 | 16 | 79 | 360 | 12 | 114 | 401 | 1 |
| Ratio CtoT 3 Y3 | 967 | 426 | 291 | 557 | 1266 | 955 | 719 | 1450 | 386 | 700 | 1163 | 154 |
| Ratio CtoT 5 Y1 | 216 | 7 | 6 | 229 | 135 | 15 | 208 | 144 | 5 | 235 | 136 | 1 |
| Ratio CtoT 5 Y2 | 338 | 4 | 2 | 96 | 393 | 13 | 72 | 365 | 9 | 113 | 399 | 1 |
| Ratio CtoT 5 Y3 | 1004 | 387 | 407 | 463 | 1307 | 850 | 640 | 1470 | 397 | 621 | 1189 | 166 |
| BIAS | RAW | MD | PSM1 | PSM2 | PSM3 | PSMD1 | PSMD2 | PSMD3 | PSW1 | PSW2 | PSW3 | EB |
| Ratio CtoT 1 Y1 | 145 | 27 | 4 | 149 | 114 | 43 | 146 | 119 | 24 | 149 | 116 | -0 |
| Ratio CtoT 1 Y2 | 184 | 21 | 7 | 98 | 197 | 42 | 92 | 188 | 34 | 105 | 200 | 3 |
| Ratio CtoT 1 Y3 | 296 | 218 | 44 | 228 | 350 | 324 | 286 | 380 | 174 | 265 | 335 | 111 |
| Ratio CtoT 3 Y1 | 146 | 25 | 3 | 150 | 115 | 40 | 144 | 119 | 23 | 151 | 117 | -0 |
| Ratio CtoT 3 Y2 | 183 | 19 | 7 | 97 | 197 | 39 | 87 | 189 | 33 | 106 | 200 | 3 |
| Ratio CtoT 3 Y3 | 301 | 202 | 34 | 210 | 349 | 305 | 262 | 376 | 178 | 257 | 337 | 109 |
| Ratio CtoT 5 Y1 | 146 | 24 | 4 | 149 | 114 | 36 | 142 | 118 | 20 | 152 | 116 | -0 |
| Ratio CtoT 5 Y2 | 183 | 18 | 6 | 96 | 197 | 35 | 82 | 190 | 29 | 105 | 198 | 3 |
| Ratio CtoT 5 Y3 | 298 | 190 | 29 | 176 | 351 | 284 | 242 | 375 | 176 | 239 | 338 | 107 |

variables. Experimental factors are: 3 sample designs (sample design 1: strong separation and normal errors; sample design 2: weaker separation and normal errors; sample design 3: medium separation and leptokurtic errors), 3 outcome designs (Y1 linear: $Y 1=$ $X_{1}+X_{2}+X_{3}-X_{4}+X_{5}+X_{6}+\eta ;$ Y2 somewhat non-linear Y2 $=X_{1}+X_{2}+0.2 X_{3} X_{4}-\sqrt{X_{5}}+\eta$; Y3 highly non-linear: $Y 3=\left(X_{1}+X_{2}+X_{5}\right)^{2}+\eta$ ), and 3 controls-to-treated ratios (Ratio CtoT 1, 3, and 5). Estimators are Raw: Difference of means; MD: Mahalanobis distance matching, GM: Genetic matching; PSM: Propensity score matching; PSMD: MD matching on the PS and orthogonalized covariates; PSW: weighting on the PS; EB: entropy balancing. All matching is $1: 1$ pair matching. We use three specifications (labeled with a 1, 2, or 3 postfix) for all propensity score based methods (PSM, PSW, PSMD). The first propensity score model is correct for sample designs 1 and 2 , and slightly misspecified for sample design 3 . Propensity score models 2 and 3 are increasing in misspecification (as measured by the linear correlation between the true and the estimated score). 1000 simulations for each scenario; the true treatment effect is zero.
Table V: Covariate Balance in Lalonde Data

|  | Raw Data |  |  |  |  | Entropy Balancing |  |  |  | PS Weighting |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Means |  | $\begin{gathered} \text { Std. } \\ \text { Diffs } \end{gathered}$ | Var <br> Ratio | $\begin{gathered} \mathrm{T} \\ \text { pval } \end{gathered}$ | $\begin{array}{r} \text { Mean } \\ \text { Controls } \end{array}$ | $\begin{aligned} & \text { Std. } \\ & \text { Diffs } \end{aligned}$ | $\begin{array}{r} \text { Var } \\ \text { Ratio } \end{array}$ | $\begin{gathered} \text { T pval } \\ \text { pval } \end{gathered}$ | $\begin{array}{r} \text { Mean } \\ \text { Controls } \end{array}$ | Std. Var T |  |  |
|  | Treated | Controls |  |  |  |  |  |  |  |  | Diffs | Var Ratio | pval |
| Age | 25.82 | 33.23 | -0.95 | 0.42 | 0.00 | 25.82 | 0 | 1.00 | 1 | 27.76 | -0.37 | 0.92 | 0.01 |
| Schooling | 10.35 | 12.03 | -0.83 | 0.49 | 0.00 | 10.35 | 0 | 1.00 | 1 | 10.22 | 0.09 | 1.18 | 0.51 |
| Black | 0.84 | 0.07 | 3.96 | 1.95 | 0.00 | 0.84 | 0 | 1.00 | 1 | 0.88 | -0.13 | 1.21 | 0.36 |
| Hispanic | 0.06 | 0.07 | -0.07 | 0.84 | 0.47 | 0.06 | 0 | 1.00 | 1 | 0.04 | 0.10 | 1.35 | 0.47 |
| Married | 0.19 | 0.71 | -1.62 | 0.75 | 0.00 | 0.19 | 0 | 1.00 | 1 | 0.16 | 0.12 | 1.16 | 0.39 |
| HS Dropout | 0.71 | 0.30 | 1.27 | 1.00 | 0.00 | 0.71 | 0 | 1.00 | 1 | 0.74 | -0.10 | 1.07 | 0.49 |
| Earnings 1974 | 2095.57 | 14024.14 | -1.75 | 0.26 | 0.00 | 2095.57 | 0 | 1.15 | 1 | 1684.68 | 0.13 | 1.36 | 0.37 |
| Earnings 1975 | 1532.06 | 13642.53 | -1.84 | 0.12 | 0.00 | 1532.06 | 0 | 0.90 | 1 | 1307.01 | 0.10 | 1.07 | 0.48 |
| Unemployed 1974 | 0.71 | 0.12 | 2.50 | 1.97 | 0.00 | 0.71 | 0 | 1.00 | 1 | 0.76 | -0.17 | 1.13 | 0.24 |
| Unemployed 1975 | 0.60 | 0.11 | 2.18 | 2.48 | 0.00 | 0.60 | 0 | 1.00 | 1 | 0.66 | -0.18 | 1.07 | 0.22 |
| Age*Age | 717.39 | 1225.91 | -0.92 | 0.30 | 0.00 | 717.39 | 0 | 1.13 | 1 | 826.13 | -0.36 | 1.03 | 0.01 |
| Schooling*Age | 266.98 | 395.54 | -1.17 | 0.35 | 0.00 | 266.98 | 0 | 1.02 | 1 | 282.20 | -0.24 | 1.09 | 0.09 |
| Schooling*Schooling | 111.06 | 152.90 | -0.88 | 0.34 | 0.00 | 111.06 | 0 | 1.01 | 1 | 107.82 | 0.12 | 1.16 | 0.39 |
| Black*Age | 21.91 | 2.40 | 2.97 | 1.65 | 0.00 | 21.91 | 0 | 1.00 | 1 | 24.65 | -0.33 | 1.00 | 0.02 |
| Black*Schooling | 8.70 | 0.81 | 3.58 | 1.99 | 0.00 | 8.70 | 0 | 1.03 | 1 | 8.90 | -0.07 | 1.25 | 0.62 |
| Hispanic*Age | 1.36 | 2.38 | -0.16 | 0.37 | 0.01 | 1.36 | 0 | 0.97 | 1 | 0.96 | 0.11 | 1.36 | 0.45 |
| Hispanic*Schooling | 0.58 | 0.73 | -0.07 | 0.70 | 0.40 | 0.58 | 0 | 0.94 | 1 | 0.42 | 0.10 | 1.28 | 0.48 |
| Married*Age | 5.56 | 25.85 | -1.55 | 0.42 | 0.00 | 5.56 | 0 | 1.04 | 1 | 4.71 | 0.10 | 1.15 | 0.46 |
| Married*Schooling | 1.96 | 8.56 | -1.55 | 0.49 | 0.00 | 1.96 | 0 | 0.98 | 1 | 1.58 | 0.14 | 1.20 | 0.34 |
| Married*Black | 0.16 | 0.05 | 0.74 | 3.05 | 0.00 | 0.16 | 0 | 1.00 | 1 | 0.13 | 0.11 | 1.17 | 0.44 |
| Married* Hispanic | 0.02 | 0.05 | -0.23 | 0.32 | 0.00 | 0.02 | 0 | 1.00 | 1 | 0.01 | 0.03 | 1.21 | 0.81 |
| HS Dropout*Age | 17.97 | 10.09 | 0.65 | 0.59 | 0.00 | 17.97 | 0 | 0.98 | 1 | 20.42 | -0.26 | 0.89 | 0.07 |
| HS Dropout*Black | 0.61 | 0.03 | 4.25 | 7.74 | 0.00 | 0.61 | 0 | 1.00 | 1 | 0.66 | -0.16 | 1.07 | 0.26 |
| HS Dropout*Hispanic | 0.05 | 0.04 | 0.07 | 1.25 | 0.53 | 0.05 | 0 | 1.00 | 1 | 0.03 | 0.10 | 1.40 | 0.47 |
| HS Dropout*Married | 0.14 | 0.20 | -0.20 | 0.77 | 0.03 | 0.14 | 0 | 1.00 | 1 | 0.11 | 0.11 | 1.19 | 0.44 |
| Earnings 1974*Age | 54074.04 | 509069.25 | -1.57 | 0.10 | 0.00 | 54074.04 | 0 | 1.14 | 1 | 43947.39 | 0.12 | 1.33 | 0.41 |
| Earnings 1974*Schooling | 22898.73 | 171241.56 | -1.62 | 0.20 | 0.00 | 22898.73 | 0 | 1.23 | 1 | 18132.78 | 0.13 | 1.52 | 0.37 |
| Earnings 1974*Black | 1817.20 | 840.59 | 0.36 | 1.54 | 0.01 | 1817.20 | 0 | 1.21 | 1 | 1451.45 | 0.12 | 1.46 | 0.40 |
| Earnings 1974*Hispanic | 151.40 | 893.68 | -0.26 | 0.09 | 0.00 | 151.40 | 0 | 0.79 | 1 | 126.10 | 0.03 | 0.94 | 0.83 |
| Earnings 1974*Married | 760.63 | 11809.15 | -1.48 | 0.12 | 0.00 | 760.63 | 0 | 1.37 | 1 | 608.57 | 0.07 | 1.73 | 0.64 |
| Earnings 1974*HS Dropout | 1094.15 | 3432.61 | -0.45 | 0.21 | 0.00 | 1094.15 | 0 | 1.06 | 1 | 896.94 | 0.09 | 1.26 | 0.54 |
| Earnings 1975*Age | 41167.28 | 489047.95 | -1.61 | 0.06 | 0.00 | 41167.28 | 0 | 1.00 | 1 | 35832.82 | 0.08 | 1.14 | 0.58 |
| Earnings 1975*Schooling | 15880.57 | 167310.76 | -1.69 | 0.07 | 0.00 | 15880.57 | 0 | 0.93 | 1 | 13332.83 | 0.11 | 1.14 | 0.44 |
| Earnings 1975*Black | 1257.04 | 804.32 | 0.17 | 0.69 | 0.05 | 1257.04 | 0 | 0.97 | 1 | 1082.19 | 0.08 | 1.14 | 0.56 |
| Earnings 1975*Hispanic | 153.73 | 884.98 | -0.27 | 0.07 | 0.00 | 153.73 | 0 | 0.65 | 1 | 122.50 | 0.04 | 0.88 | 0.76 |
| Earnings 1975*Married | 654.34 | 11366.04 | -1.47 | 0.08 | 0.00 | 654.34 | 0 | 1.14 | 1 | 549.80 | 0.06 | 1.42 | 0.69 |
| Earnings 1975*HS Dropout | 1134.96 | 3290.78 | -0.44 | 0.18 | 0.00 | 1134.96 | 0 | 0.89 | 1 | 983.57 | 0.07 | 1.06 | 0.60 |
| Unemployed 1974*Age | 18.78 | 3.60 | 1.97 | 1.63 | 0.00 | 18.78 | 0 | 0.99 | 1 | 21.92 | -0.32 | 0.95 | 0.02 |
| Unemployed 1974*Schooling | 7.26 | 1.42 | 2.04 | 1.58 | 0.00 | 7.26 | 0 | 1.01 | 1 | 7.71 | -0.13 | 1.17 | 0.35 |
| Unemployed 1974*Black | 0.60 | 0.01 | 6.44 | 23.63 | 0.00 | 0.60 | 0 | 1.00 | 1 | 0.68 | -0.22 | 1.10 | 0.11 |
| Unemployed 1974*Hispanic | 0.03 | 0.01 | 0.36 | 3.74 | 0.07 | 0.03 | 0 | 1.00 | 1 | 0.02 | 0.10 | 1.55 | 0.47 |
| Unemployed 1974*Married | 0.11 | 0.06 | 0.33 | 1.83 | 0.02 | 0.11 | 0 | 1.00 | 1 | 0.09 | 0.10 | 1.20 | 0.49 |
| Unemployed 1974*HS Dropout | 0.52 | 0.05 | 2.93 | 5.28 | 0.00 | 0.52 | 0 | 1.00 | 1 | 0.59 | -0.17 | 1.03 | 0.22 |
| Unemployed 1974*Earnings 1975 | 307.44 | 175.27 | 0.14 | 0.69 | 0.12 | 307.44 | 0 | 0.50 | 1 | 310.15 | 0.00 | 0.51 | 0.98 |
| Unemployed 1975*Age | 15.98 | 3.57 | 1.58 | 1.73 | 0.00 | 15.98 | 0 | 0.99 | 1 | 19.22 | -0.31 | 0.91 | 0.03 |
| Unemployed 1975*Schooling | 6.15 | 1.33 | 1.72 | 1.84 | 0.00 | 6.15 | 0 | 1.01 | 1 | 6.71 | -0.15 | 1.11 | 0.28 |
| Unemployed 1975*Black | 0.52 | 0.01 | 5.51 | 22.19 | 0.00 | 0.52 | 0 | 1.00 | 1 | 0.60 | -0.22 | 1.04 | 0.11 |
| Unemployed 1975*Hispanic | 0.03 | 0.01 | 0.32 | 3.61 | 0.10 | 0.03 | 0 | 1.00 | 1 | 0.02 | 0.10 | 1.59 | 0.49 |
| Unemployed 1975*Married | 0.09 | 0.06 | 0.12 | 1.31 | 0.30 | 0.09 | 0 | 1.00 | 1 | 0.07 | 0.07 | 1.17 | 0.61 |
| Unemployed 1975*HS Dropout | 0.43 | 0.04 | 2.84 | 7.08 | 0.00 | 0.43 | 0 | 1.00 | 1 | 0.50 | -0.19 | 0.98 | 0.19 |
| Unemployed 1975*Earnings 1974 | 43.85 | 203.65 | -0.14 | 0.08 | 0.00 | 43.85 | 0 | 0.67 | 1 | 35.68 | 0.02 | 0.77 | 0.87 |
| Unemployed 1975* Unemployed 1974 | 0.59 | 0.07 | 2.69 | 3.56 | 0.00 | 0.59 | 0 | 1.00 | 1 | 0.65 | -0.18 | 1.07 | 0.20 |

Table VI: Covariate Balance in News Media Persuasion Data

Table VII: Covariate Balance in Fox News Data


## VII. Figures

Figure 1: Covariate Balance: QQ plots of Continuous Covariates


Note: QQ plots of pretreatment earnings in 1975 and 1974 , age, and education. The black dots represent empirical QQ estimates for the raw data. The gray dots represent QQ estimates for the matched data. The superimposed 45 -degree line indicates identical distributions for the treatment and control group.

















Figure 3: Covariate Balance in Fox News Data


[^3]Figure 4: Model Dependency in Fox News Data


Note: Density of estimated treatment effects across one million randomly samples model specifications in the unadjusted data (dashed line) and the data preprocessed with entropy balancing (solid line).

Figure 5: Covariate Balance in Fox News Data - Using Only the Raw Covariates (Standardized Bias)


Note: Covariate-by-covariate standardized bias in the unadjusted data and after the various preprocessing methods. The standardized bias means, dots to the right (left) of zero indicate a higher mean among the treatment (control) group

Figure 6: Covariate Balance in Fox News Data - Using only the Raw Covariates (p-values)


Note: p-values for a covariate-by-covariate t-test for the differences in means in the unadjusted data and after the various preprocessing methods.

Figure 7: Covariates Balance in British MPs Data (Standardized Bias)


Note: Covariate-by-covariate standardized bias in the unadjusted data and after the various preprocessing methods. The standardized bias measures the difference in means between the treatment and control group (scaled by the standard deviation). Zero bias indicates identical means, dots to the right (left) of zero indicate a higher mean among the treatment (control) group.

Figure 8: Covariates Balance in British MPs Data (p-values)


Note: p-values for a covariate-by-covariate t-test for the differences in means after the unadjusted data and after the various preprocessing methods.


[^0]:    ${ }^{1}$ Notice that the authors also control for House district or county fixed effects in some of their analysis, but many districts and counties have no variation on the treatment variable and including these fixed effects if anything lowers the positive effect of Fox News on vote shares according to the authors' specifications. Also notice that the authors weight many of their regressions by population size which we ignore here (they do state that their results are robust to using no weights and a linear regression of the outcomes on all covariates confirms this).

[^1]:    ${ }^{2}$ Notice that there are $3.078282 \times 10^{113}$ possible subsets $\left(\sum_{i=1}^{377}\binom{377}{i}\right)$.

[^2]:    ${ }^{3}$ Notice that the variability in the preprocessed estimates is entirely driven by the fact that the interaction terms are not included in the reweighting adjustment but only in the outcome regressions. Since the entropy balancing includes all raw covariates and their squared terms, the regression estimates are identical across all subsets that do not involve interaction terms.

[^3]:    Note: Left panel shows plot of covariate-by-covariate standardized bias in the unadjusted data and after the various preprocessing methods. The standardized bias measures the difference in means between the treatment and control group (scaled by the standard deviation). Zero bias indicates identical means, dots to the right (left) of zero indicate a higher mean among the treatment (control) group. The right panel shows the p-value for a covariate-by-covariate t-test for the differences in means after the unadjusted data and after the various preprocessing methods.

