Filling Positions without Transfers: Screening on Outside Options

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Motivation

Consider a designer allocating

- vertically-differentiated positions
- to agents with private outside options
- without transfers.

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Question: How can the designer fill the greatest number of positions?

Motivation

Example: volunteer assignment in the U.S. Peace Corps

- Assigns volunteers to development projects in 60+ countries. ▸ Assignment
- Volunteers have outside options: other jobs or graduate studies. ▸ Options
- Some locations are hard to fill; some are over-demanded. ▸ Positions
- Stipends are fixed. ▸ Stipends

Motivation

- **Job scheduling.** Each agent has a job and a deadline, and prefers to be scheduled earlier (Crès and Moulin (2001)).

- **Section assignment.** Assigning students to sections to maximize enrollment.

- Assignment of teachers, police officers, civil servants ...

Literature Review

- **Two-sided matching:** Gale and Shapley (1962), Akbarpour et al. (2022), Cowgill (2024).

- **Assignments without transfers:** Hylland and Zeckhauser (1979), Bogomolnaia and Moulin (2001), Budish et al. (2013), Miralles (2012), Ortoleva et al. (2021), Ashlagi and Shi (2016).
  Quantity objectives: Crès and Moulin (2001), Krysta et al. (2014), Bogomolnaia and Moulin (2015)

- **Ex-post participation decisions:** Compte and Jehiel (2007), Compte and Jehiel (2009), Haberman and Jagadeesan (2025).

Model

Model

- Continuum of agents and positions; finite set of types.
- Mass 1 of positions of increasing qualities $x \in X = \{x_0, x_1, \ldots, x_{N-1}\}$, PMF g.
- Mass D of agents with outside options $\theta \in \Theta = X$, with PMF f and CDF F.
- Agents have unit demand and increasing utility $u : X \to \mathbb{R}_+$.

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- Agents have unit demand and increasing utility $u : X \to \mathbb{R}_+$.
- This talk: common cardinal preferences, qualities normalized so $u(x) = x$.
- Designer assigns agents to positions without using transfers.
- *Ex-post participation decisions*: type θ accepts assignment x only if $x \geq \theta$.

Model: Objective

- Let $\boldsymbol{s}$ be a vector, where s_k is the mass of agents who accept position x_k.
- Designer maximizes $V(\boldsymbol{s})$, where $V : \mathbb{R}^N \to \mathbb{R}$ and upper semicontinuous
- Example: $V_{\text{fill}}(\boldsymbol{s}) := \sum_{k=0}^{N-1} \boldsymbol{s}_k$

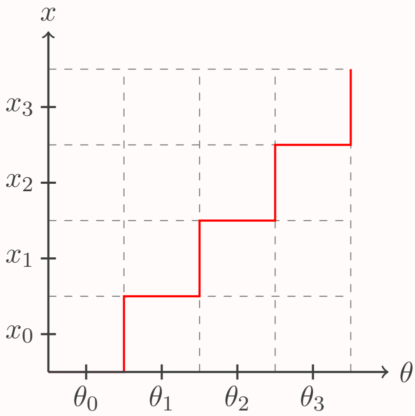
By the revelation principle, restrict attention to direct mechanisms:

$$a : \Theta \to \Delta(X \cup \{\varnothing\}).$$

Before writing the IC and feasibility constraints, some examples.

Example

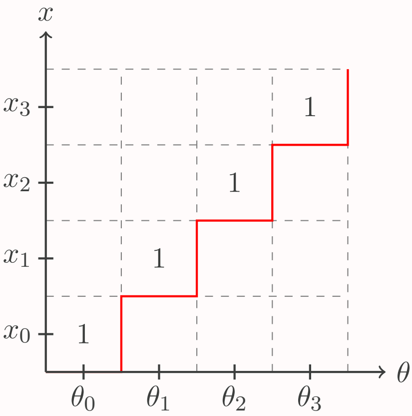
Suppose $D = 1$; $X = \{0, 1, 2, 3\}$; uniformly distributed x and θ; objective V_{fill}.



Example

Suppose $D = 1$; $X = \{0, 1, 2, 3\}$; uniformly distributed x and θ; objective V_{fill}.

- The first-best allocation
    - achieves $V_{\text{fill}} = 1$.
- Violates IC



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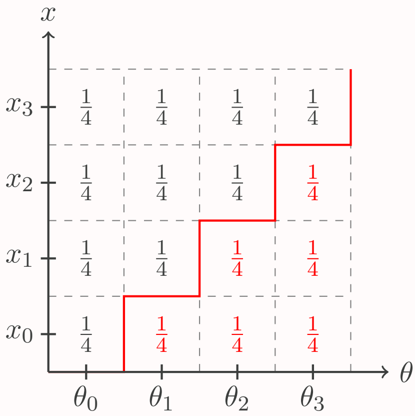
Common Lotteries

- Uniform common lottery
  - achieves $V_{\text{fill}} = 5/8$



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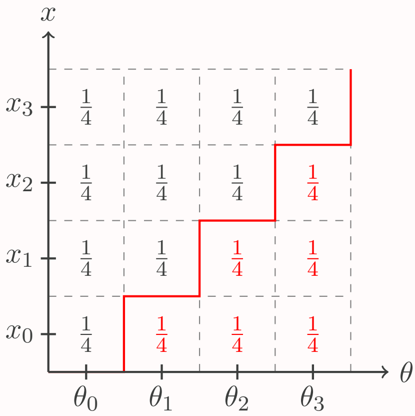
  - achieves $V_{\text{fill}} = 17/24$



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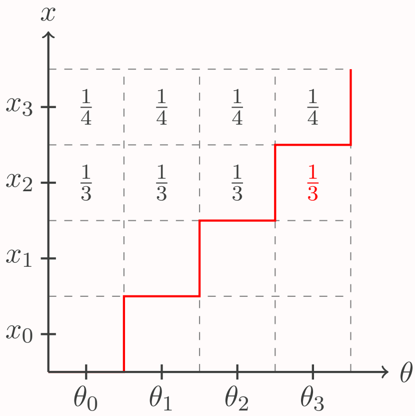

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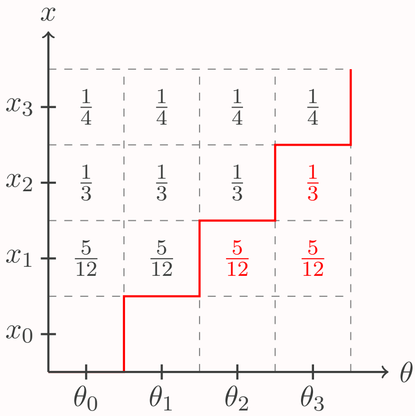

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Other Mechanisms

- Binary menu of lotteries
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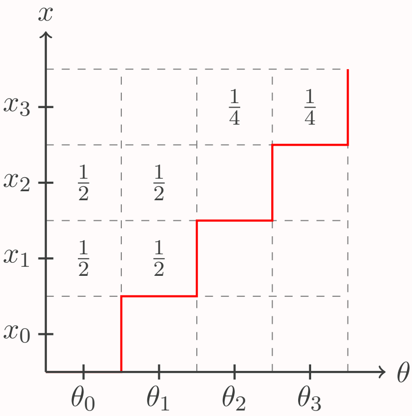

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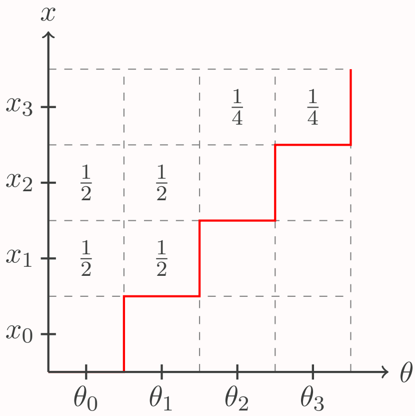
Other Mechanisms

- Binary menu of lotteries
  - achieves $V_{\text{fill}} = 5/8$
- CEEI (Hylland and Zeckhauser (1979))
  - achieves $V_{\text{fill}} = 11/16$



Preview of Results

What is the optimal mechanism?

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What is the optimal mechanism?

- Under a general condition on F, **common lotteries** are optimal
- Under this condition, straightforward to characterize the optimal mechanism.
- Otherwise, the optimal mech. may screen agents with a **menu of lotteries**.

Direct Mechanisms

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- Total probability that θ_i accepts an offer:

$$P(\theta_i) = \sum_{k=i}^{N-1} a(x_k;\theta_i).$$

- Mass of agents accepting position x_k:

$$s_k(a) = D\sum_{i=0}^{k} a(x_k;\theta_i)f(\theta_i).$$

Designer's Problem

$$
\begin{aligned}
&\max_{a:\Theta\to\Delta(X\cup\{\varnothing\})} V(\boldsymbol{s}(a)), && \\
\text{s.t.}\quad & U(\theta_i,\theta_i) \geq U(\theta_j,\theta_i), \qquad \forall i,j && (\mathrm{IC}_{i,j}) \\
& s_k(a) \leq g(x_k), \qquad \forall k && (\text{Position-Feasibility}) \\
& P(\theta_i) \leq 1, \qquad \forall i && (\text{Agent-Feasibility})
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$$

- a is **feasible** if it satisfies all the constraints above.
- a is **optimal** if it is feasible and solves the designer's problem.

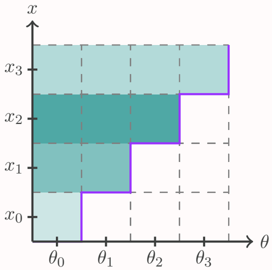
Common Lotteries

▸ Common Lotteries Equivalent Representation

Definition (Common Lottery)

Direct mech. a is a *common lottery* if there exists $\tilde{a} \in \Delta(X \cup \{\varnothing\})$ with

$$a(x_k; \theta_i) = \begin{cases} \tilde{a}(x_k), & \theta_i \leq x_k \\ 0, & \theta_i > x_k. \end{cases}$$



Results

Result I: When Common Lotteries Are Enough

Theorem 1 (Sufficiency of Common Lotteries)

Suppose $1/F$ is convex on Θ. Then any feasible direct mechanism a can be replaced by a feasible common lottery $\tilde{a}$ with $\boldsymbol{s}(\tilde{a}) = \boldsymbol{s}(a)$, which therefore attains the same objective value V.

- Screening agents with a menu of lotteries does not benefit the designer.

Result I: When Common Lotteries Are Enough

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- Continuous analogue: increasing virtual cost (procurement regularity).

▸ When Is 1/F Convex?

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▸ When Is 1/F Convex?

- Sufficient for $1/F$ convex: f log-concave, f/F decreasing, f decreasing

- Satisfied by uniform, exponential, Weibull, Fréchet, Pareto, normal, lognormal distributions ...

Result II: When Screening May Be Necessary

Theorem 2 (Partial Converse)

Suppose $1/F$ is not convex on Θ and g has full support. If V is strictly increasing, there exists D such that no common lottery is optimal.

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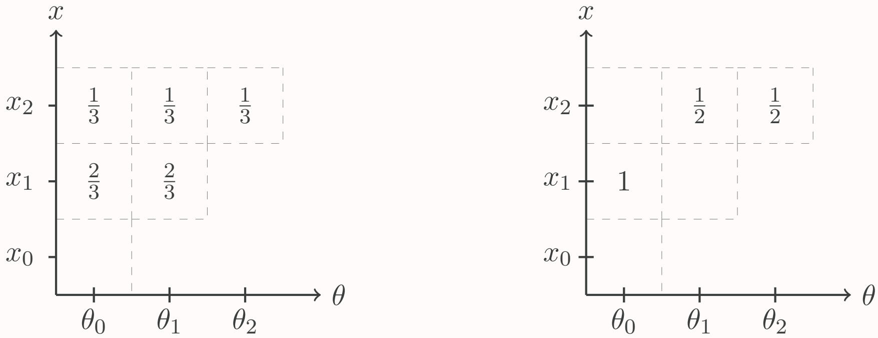
Theorem 2 (Partial Converse)

Suppose $1/F$ is not convex on Θ and g has full support. If V is strictly increasing, there exists D such that no common lottery is optimal.

- Optimal mechanism may require screening agents with a menu of lotteries.

Example: A Binary Menu Can Be Optimal

$D = 1$, $X = \{0, 1, 2\}$, objective V_{fill}. Positions distributed uniformly.
Type distribution: $f(\theta_0) = \frac{1}{3}, f(\theta_1) = \frac{1}{12}, f(\theta_2) = \frac{7}{12}$.



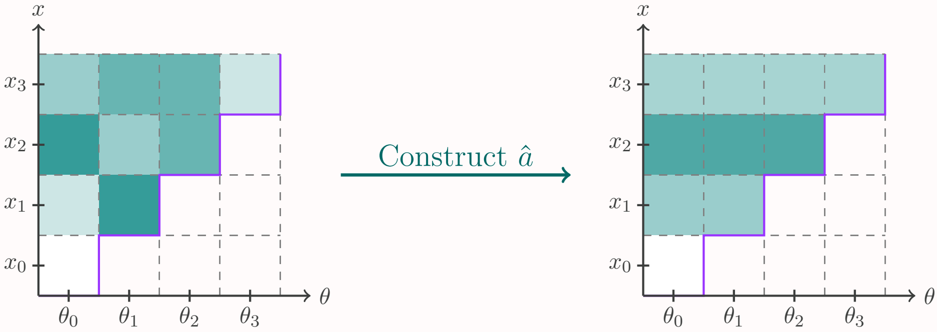
(a) Common Lottery Mechanism

(b) Binary-Menu Screening Mechanism

Common lottery fills $\frac{11}{18}$ of positions; binary menu fills $\frac{2}{3} > \frac{11}{18}$ of positions.

Sufficiency of Common Lotteries: Proof Sketch ▸ Alternative Proof

- Start with a feasible mechanism a.
- Construct common allocation $\hat{a}(x_k;\theta_i) := \mathbf{1}_{\theta_i \leq x_k} \cdot \frac{s_k(a)}{D \cdot F(x_k)}$; show it is feasible.



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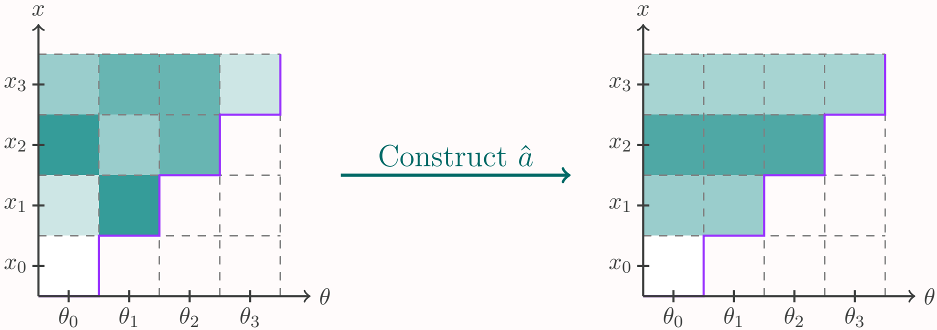
  - By construction, position-feasible ✓
  - By construction, preserves objective ✓
  - By construction, incentive-compatible ✓
  - Agent-feasibility **?** Only need to check $\hat{P}(\theta_0)$.



Agent-feasibility at the Lowest Type ▸ Alternative Proof

- In the initial feasible mechanism, $\mathrm{IC}_{i,j} := U(\theta_i, \theta_i) - U(\theta_j, \theta_i) \geq 0$. Decompose assignment probability of θ_0:

$$P(\theta_0) = \underbrace{\hat{P}(\theta_0)}_{\substack{\text{common-allocation}\\ \text{term}}} + \underbrace{\left(\sum_{i=0}^{N-1} \lambda_{i,i+1} \cdot \mathrm{IC}_{i,i+1} + \sum_{i=1}^{N-1} \sum_{j=0}^{i-1} \lambda_{i,j} \cdot \mathrm{IC}_{i,j} \right)}_{\text{screening wedge}}.$$

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  - local upward and *global downward* ICs are non-redundant; ▸ Local IC insufficient
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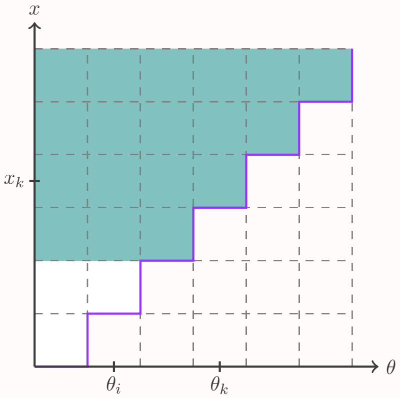
- Thus, $\hat{P}(\theta_0) \leq P(\theta_0) \leq 1$ ✓

Partial Converse: Proof Sketch ‣ Detail

Suppose $1/F$ is not convex at x_k.

- Start with the best common lottery yielding $\boldsymbol{s}$.



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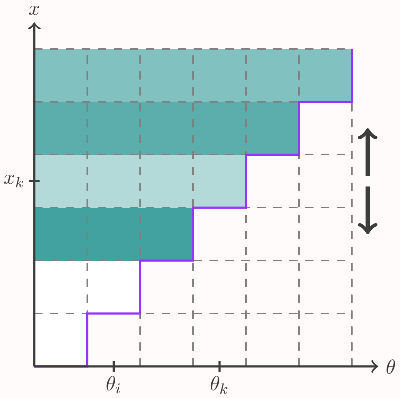

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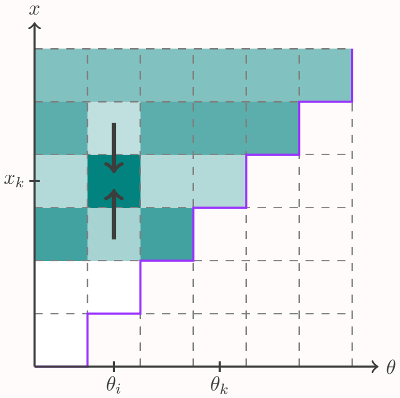
- **Step 2**: Mean-preserving contraction for low type θ_i.
- Maintains IC and allocates $\boldsymbol{s}$ with slack.



Optimal Common Lotteries

▸ Example: Optimum for V_{fill}

- Theorem 1 reduces the designer's problem to a **consumer choice problem**.
- To fill mass s_k in position x_k: offer position x_k to $\frac{s_k}{F(x_k)}$ of agents.

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- Designer has a "**budget**" of agents with mass D.

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Corollary (Consumer Problem)

If $1/F$ is convex on Θ, then the designer's problem solves

$$\max_{s_k \leq g(x_k)} V(\boldsymbol{s}) \qquad \text{s.t.} \quad \sum_{k=0}^{N-1} \frac{1}{F(x_k)} s_k \leq D.$$

Conclusion

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- A designer assigning agents to positions may have *quantity objectives*.
  How should she optimally assign agents?

- Under a general convexity condition, we show **common lotteries** are optimal.

- Common lotteries are straightforward to implement and optimize.

- Otherwise, designer may optimally screen agents with a **menu of lotteries**.

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Thank You!

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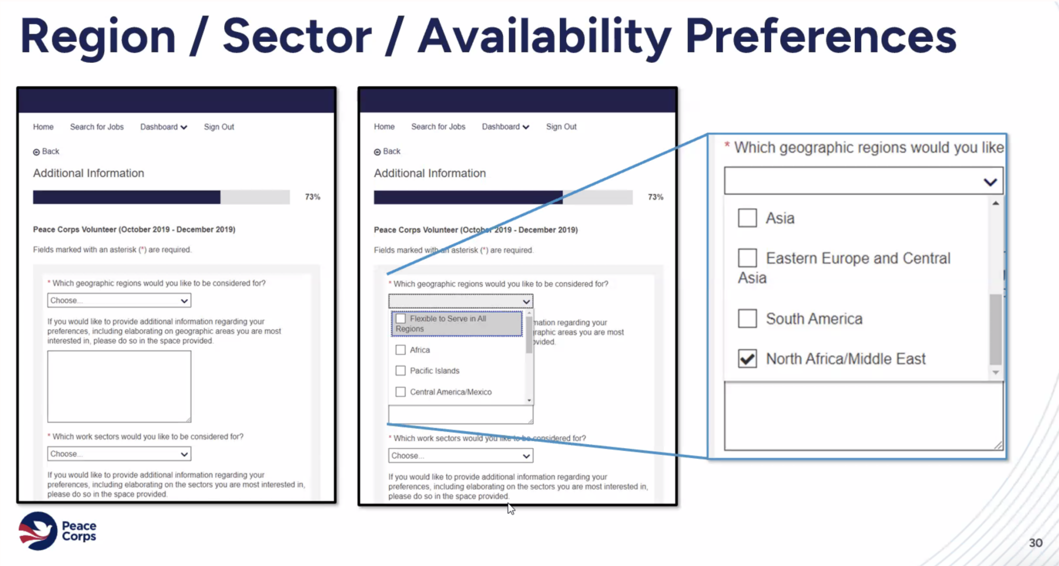
Back: Motivation



Peace Corps Matching

Back: Motivation

Example of a Fill Report Tool

REGION	POST	POSITION TITLE	AA	SECTOR	VR
IAP	Belize	Youth and Sports Development Coordinator	162	YD	22
IAP	Belize	Urban Youth Development Volunteer	164	YD	4
IAP	Eastern Caribbean	English Literacy Resource Teacher (Primary Schools)	170	EDU	10
IAP	Eastern Caribbean	English Literacy Resource Teacher (Primary Schools)	171	EDU	36
IAP	Ecuador	Health and Well-being Promoter	154	HE	4
IAP	Ecuador	Health and Well-being Promoter	155	HE	11
IAP	Ecuador	Youth and Families Development Promoter	162	YD	11
IAP	Ecuador	Youth and Families Development Promoter	164	YD	4
AF	Eswatini	Health Extension Volunteer	155	HE	10
AF	Eswatini	Urban Youth Development Volunteer	164	YD	10
EMA	Georgia	English Language Co-Teacher and Youth Educator	171	EDU	30
IAP	Guyana	Science and Environment Educator	104	ENV	15
IAP	Guyana	Adolescent Health Promoter	155	HE	15
IAP	Guyana	Primary Literacy Promoter	171	EDU	14
EMA	Kyrgyz Republic	English Education Teaching	171	EDU	34
EMA	Mongolia	English Education/Community Development	171	EDU	50
IAP	Panama	Youth Leadership Development Facilitator	162	YD	38
AF	Togo	Community Health Educator	155	HE	9
AF	Togo	Secondary Education English Teacher	171	EDU	8
IAP	Vanuatu	Community Health Facilitator	155	HE	15
IAP	Vanuatu	English Literacy Facilitator	171	EDU	15

This example report shows how a Placement Officer may filter to find a good match for the previous candidate on the previous slide. The color-coding shows which are programmatic priorities. Red being the highest priority. This could be due to low applications, specialized requirements, etc.

Peace Corps

Peace Corps: Outside Options

"In the early days when Peace Corps began, it really was the only opportunity for people who wanted to work or live overseas. ... **Our applicants have lots of choices now**."
- Carrie Hessler-Radelet, PC Director ("Peace Corps Makes Big Changes", NPR)

"Part of the reason for the application system overhaul is so people can receive invitations to places they want to serve for a full 27 months. It's not talked about a lot, but **many people end up turning down their invitations or taking advantage of a grad school/job offer if their invitation isn't as exciting as they'd desired**. Peace Corps realizes that it is also competing to recruit and keep the best candidates as much as applicants are competing for Peace Corps."
- Reddit, Returned Peace Corps Volunteer

Back: Motivation

Peace Corps: Heterogeneous Positions

> Former Recruiter's perspective here: We'd roll our eyes hard if someone mentioned Costa Rica, Thailand, Morocco, Philippines or Fiji. Application ratios were upwards of 20-30:1 open spot. Anywhere where Americans frequently travel was always very high on that list and never in need of applications. I think it's just easier mentally to apply to a place where there's some knowledge of. Applying for South Africa seems more familiar than applying for... Zambia, for example.
>
> There were several counties they'd beg us to talk about (usually countries with no name recognition, think Guinea, Tuvalu, Kyrgyz Republic, etc.) Their ratio of applicants could be as low as 1:1 or 2:1.
>
> Of course this varies but in general and I left in 2020 but I'm sure it's still pretty relevant.

Reddit, Former Peace Corps Recruiter

▸ Back: Motivation

Peace Corps Stipend

The Peace Corps Manual states, “Volunteers live modestly by the standards of the people they serve, yet not in a manner that would endanger their health or safety.” While Volunteers do get paid, the stipend will be very modest – one might say small – by American standards. This is an important part of the Peace Corps experience, and helps to facilitate cultural integration.

Peace Corps Blog: How Much are Peace Corps Volunteers Paid?

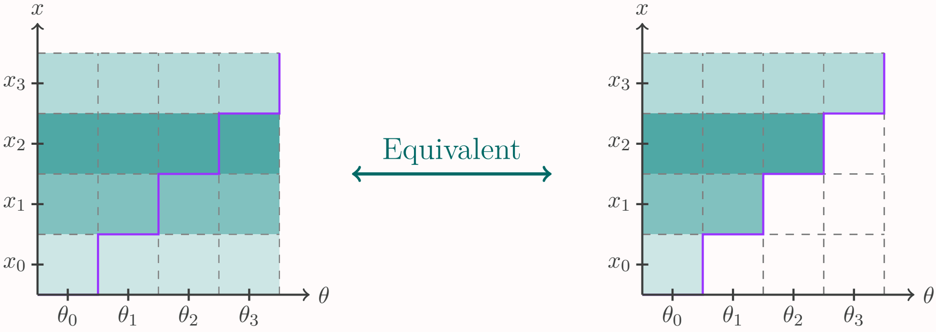
Back: Motivation

Common Lotteries Equivalent Representation

Back: Common Lotteries

- Any common lottery $a(\cdot;\cdot)$ has an outcome equivalent direct mechanism $\tilde{a}(\cdot;\cdot)$:

$$\tilde{a}(x_k;\theta_i) = \begin{cases} a(x_k;\theta_i), & \theta_i \leq x_k \\ 0, & \theta_i > x_k \end{cases}.$$



- We will use the second representation in the proofs.

Partial Converse: Proof

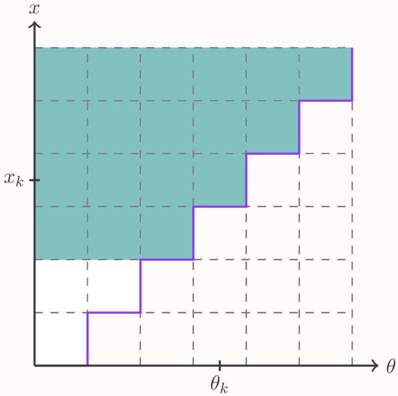
▸ Back: Partial Converse Proof Sketch

Suppose $1/F$ is not convex at x_k:

- Start with the best common lottery yielding $\boldsymbol{s}$.
- Must offer position k to mass $\frac{s_k}{F(\theta_k)}$ of agents.
- Total mass of agents offered:

$$D = \sum_{k=0}^{N-1} \frac{s_k}{F(\theta_k)}$$

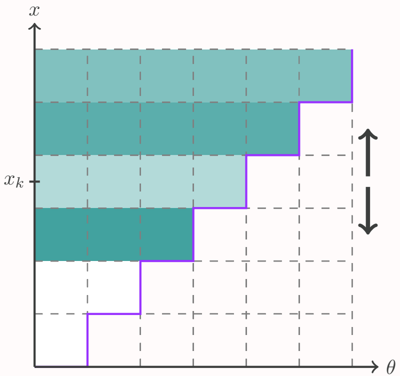
- Can we reduce the mass of agents required to obtain $\boldsymbol{s}$?



Partial Converse: Proof

▸ Back: Partial Converse Proof Sketch

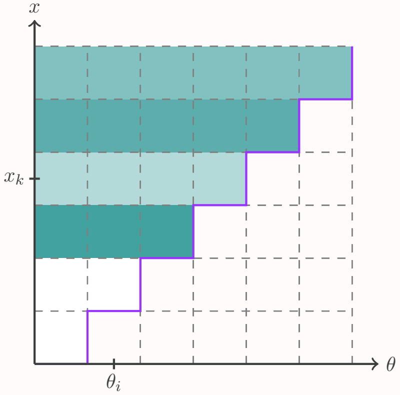
- Total mass of agents required: $\sum_{k=0}^{N-1} \frac{s_k}{F(\theta_k)}$
- **Step 1**: mean-preserving spread of $\boldsymbol{s}$ to obtain $\boldsymbol{s}'$.
- Because $1/F$ is locally concave, $\boldsymbol{s}'$ uses fewer applicants.



Partial Converse: Proof

▸ Back: Partial Converse Proof Sketch

- Reduces agents needed, but results in different allocation $\boldsymbol{s}'$. How to restore $\boldsymbol{s}$?
- **Step 2**: Mean-preserving contraction for low type θ_i.
- Critically: *maintains IC*. Withdrawal rights only makes riskier gambles more appealing.
- We now have allocated $\boldsymbol{s}$ with slack; can do strictly better than $V(\boldsymbol{s})$.



Partial Converse: Proof

Back: Partial Converse Proof Sketch

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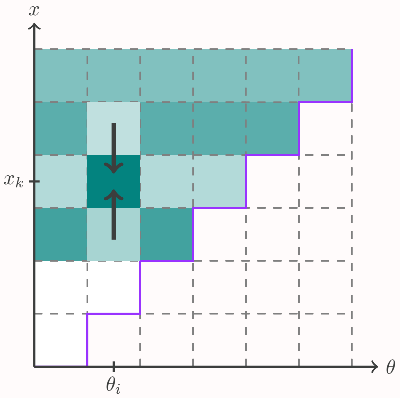


Optimal Common Lotteries for V_{fill}

▸ Back: Optimal Common Lotteries

- Marginal utility of filling each position is equal.
- Price of positions $1/F(x_k)$ decreases with x_k.
- Designer fills positions from highest to lowest until agent mass D is exhausted.

Optimal Mechanism for V_{fill}

Suppose $1/F$ is convex. Let $q_k \coloneqq \frac{1}{D}\frac{g(x_k)}{F(x_k)}$, and $Q_k \coloneqq \min\left\{1, \sum_{k'=k+1}^{N-1} q_{k'}\right\}$. An optimal direct mechanism a for designer objective V_{fill} satisfies

$$a(x_k;\theta_i) = \begin{cases} \min\{q_k, 1 - Q_k\} & \text{if } \theta_i \leq x_k \\ 0 & \text{if } \theta_i > x_k. \end{cases}$$

When Is $1/F$ Convex?

Back: Main Result

- Continuous analogue: procurement regularity, $c + \frac{F(c)}{f(c)}$ increasing.
- Sufficient for $1/F$ convex: f log-concave, f/F decreasing, f decreasing
- Satisfied by uniform, exponential, Weibull, Fréchet, Pareto, normal, lognormal distributions ...

Technical Results

Proposition (Monotonicity)

In a feasible mechanism, $P(\theta_i)$ is non-increasing.

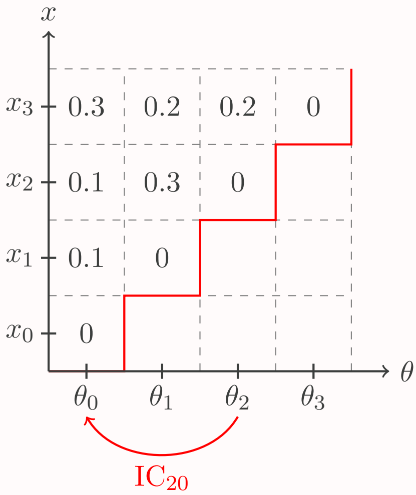
- Agents with lower-quality outside options must be allocated with a higher probability.

Technical Results

▸ Back: Main Proof Sketch

Do monotonicity and local IC's guarantee global IC's?

- Only upward global IC's are guaranteed.
- Suppose $D = 1$, $N = 4$, and uniform distributions for x and θ.
- All the local IC's hold.
- But IC_{20} fails.



Main Result: Duality Proof

Equalization feasible if a common lottery solves the following auxiliary problem for all feasible $\boldsymbol{s}$:

$$\min_{D \in \mathbb{R}_+,\ a:\Theta \to \Delta(X \cup \{\varnothing\})} D$$

subject to constraints:

$$\sum_{k=i}^{N-1} (x_k - \theta_i)\, a(x_k;\theta_i) \geq \sum_{k=i}^{N-1} (x_k - \theta_i)\, a(x_k;\theta_j), \qquad \forall i, j \qquad (\mathrm{IC}_{i,j})$$

$$D \sum_{i=0}^{k} a(x_k;\theta_i) f(\theta_i) = s_k, \qquad \forall k \qquad \text{(Position-Target)}$$

$$\sum_{k=0}^{N-1} a(x_k;\theta_i) \leq 1, \qquad \forall i \qquad \text{(Agent-Feasibility)}$$

- Shadow prices $\lambda_{IC_{i,j}}$ constructed by reversing the perturbation in our proof of the partial converse. Back: Main Proof Sketch

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