Area Exam Syllabus

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1 Topic 1: Riemannian Geometry

- Fermi Coordinates - definition for internal coordinates, fermi coordinates off a higher codimension submanifold, metric in these coordinates, vanishing of christoffel symbols, gauss lemma, Laplacian in Fermi coordinate codim 1
- Geodesics - Existence via energy, Hopf-Rinow theorem, cut-locus, PDE for geodesics in coordinates, Jacobi equation for family of geodesics
- Curvature - Myers theorem, Bishop-Gromov inequality, Bochner formula, comparison theorem for distance functions ($\Delta\rho$)
- Submanifolds - Gauss-Codazzi equations, first and second variation of area, gauss-bonnet theorem
- Minimal Submanifolds - Definition, statement of regularity for $n \leq 8$, Examples in $\mathbb{R}^3$, non-existence of stable minimal submanifolds when ambient $\text{Ric}_g \geq 0$, Barta theorem on stable minimal

2 Topic 2: PDE

- Microlocal Analysis - Symbol calculus, Pseudodifferential operators, edge operators, theory of parametrices (in the large calculus), construction of parametrics for elliptic operators
- Calculus of Variations - existence of minimizers, Euler-Lagrange equation, Noether's theorem
- Functional Analysis - Lax-Milgram, Fredholm Alternative, existence of weak solutions to 2nd order elliptic equations
- Eigenvalues of the laplacian - Cheeger theorem for lower bounds, orthogonality of eigenfunctions on closed manifold, Faber-Kahn theorem on lower bound for first eigenvalue for $\Omega \subseteq \mathbb{R}^n$, variation of first eigenvalue with respect to the domain , co-area formula and usage in proof
- Schauder Estimates - statement of estimates, Simon's proof of estimates by scaling, applications (e.g. regularity of harmonic functions), Improved estimates for functions orthogonal to kernel

3 Topic 3: Miscellaneous

- The Allen-Cahn equation - 1-D solution, BV functions (definition), Modica-Mortola result for BV compactness/ Γ-convergence of nodal sets to minimal hypersurfaces, Stability operator, Stable solutions on $\mathbb{R}^2$, Modica Inequality, Monotonicity formula for $E(B_R(0), u)$, kernel classification of $L_* = \Delta_g - W''(g)$, Exponential decay of solutions away from nodal set, Solutions on S^n (nodal set is equator, two parallels)
- Wave equation - solution to linear wave equation, fundamental solution, Finite speed of propogation, monotonicity of energy functional, existence of solutions to linear wave equation, uniqueness of solutions to linear wave equation
- Ginzburg Landau (Statements only) - Comparison to Allen-Cahn, canonical 2D solutions, convergence of $\{u_\epsilon\} \to u^*$ an S^1 valued function, dependence of u^* on dirichlet data, harmonic functions in $C(\Omega \backslash \{p_i\}_{i=1}^n, S^1)$
- Poincare-Einstein metrics - graham normal form/evenness of PE metrics, boundary defining functions, Statement of correspondence between conformal infinity and einstein metric to the interior, Examples of PE spaces