Learning and the complexity of monetary policy rules

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In recent years, there has been a renewal of interest in the use of rules for governing monetary policy. Theory tells us that there are advantages to precommitting to a policy rule. Given this interest, it is perhaps surprising that the rules under discussion are not rules that are optimal in the sense of having been computed from a optimal control problem. Instead, the rules that are widely discussed-notably the Taylor rule-are remarkable for their simplicity. One reason for the apparent preference for simple ad hoc rules might be the assumption of full information that is generally maintained for the computation of an optimal rule. This tends to make optimal control rules less robust to model specification errors than are simple ad hoc rules. In this paper, we drop the full information assumption and investigate the choice of policy rules when private agents must learn the rule that is used. To do this, we construct a small, forward-looking model of neo-Keynesian variety and estimate it with US data. We then conduct stochastic simulations on this model with agents following a strategy of least-squares learning. We find that in terms of steady-state performance, rules do not have to be very complex before they can mimic fairly well the performance of the globally optimal rule. We also find that the costs of learning a new rule can be substantial-but not so substantial that an optimising policymaker would need to take into account the transition costs of learning in choosing the complexity of its rule. Finally, we find that the costs of learning depend in an interesting way on what the incumbent rule is. In particular, a liberal policymaker may actually benefit from agents prior belief that a conservative rule is in place.

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1.0 Introduction

In recent years, there has been a renewed interest in the governance of monetary policy through the use of rules. This has come in part because of academic contributions including those of Hall and Mankiw (1994), McCallum (1987), Taylor (1993, 1994), and Henderson and McKibbin (1993). It has also arisen because of adoption in a number of countries of explicit inflation targets. New Zealand (1990), Canada (1991), the United Kingdom (1992), Sweden (1993) and Finland (1993) have all announced such regimes.

The academic papers noted above all focus on simple ad hoc rules. Typically, very simple specifications are written down and parameterised either with regard to the historical experience [Taylor (1993)], or through simulation experiments [Henderson and McKibbin (1993), McCallum (1987)]. Both the simplicity of these rules, and the evaluation criteria used to judge them stand in stark contrast to the earlier literature on optimal control. Optimal control theory wrings all the information possible out of the economic model, the stochastic shocks borne by the economy, and policymakers' preferences.

Optimal control theory has been criticised on three related grounds. First, the optimization is conditional on a large set of parameters, some of which are measured imperfectly and the knowledge of which is not shared by all agents. Some features of the model are known to change over time, often in imprecise ways. The most notable example of this is policymakers' preferences which can change either 'exogenously' through the appointment process, or endogenously through the accumulation of experience.[1] Second, optimal control rules are invariably complex. The arguments to an optimal rule include all the state variables of the model. In working models used by central banks, state variables can number in the hundreds. The sheer complexity of such rules makes them difficult to follow, difficult to communicate to the public, and difficult to monitor. Third, in forward-looking models, it can be difficult to commit to a rule of any sort. Time inconsistency problems often arise. Complex rules are arguably more difficult to commit to, if for no other reason than the benefits of commitment cannot be reaped if agents cannot discern commitment to a complex rule from discretion.

Simple rules are claimed to avoid most of these problems by enhancing accountability, and hence the returns to precommitment, and by avoiding rules that are optimal only in idiosyncratic circumstances. At the same time, simple rules still allow feedback from state variables over time, thereby avoiding the straightjacket of 'open-loop' rules, such as Friedman's k-percent money growth rule. The costs of this simplicity include the foregone improvement in performance that a richer policy can add.

This paper examines the friction between simplicity and optimality in the design of monetary policy rules. With complete information, rational expectations, and full optimisation, the correct answer to the question of the best rule is trite: optimal control is optimal. However, rational expectations can be expected to prevail only in the steady state, since only then will agents have sufficient

1 Levin et al. (1999) examine the performance of rules in three models as a check on robustness of candidate rules.

knowledge to formulate a rational expectation. This means that changes in policy must consider not only the relative merits of the old and prospective new policies, but also the costs along the transition path to the new rule brought about by the induced break from rational expectations. With this in mind, we allow two elements of realism into the exercise that can alter the trite result. First, we consider optimal rules subject to a restriction on the number of parameters that can enter the policy rule-a simplicity restriction. We examine the marginal cost of this restriction. Second we restrict the information available to private agents, requiring them to learn the policy rule that is in force. In relaxing the purest form of the rational expectations assumptions, we follow the literature on learning in macroeconomics associated with Taylor (1975) and Cripps (1991) and advanced by Sargent (1993). We are then in a position to ask the question: if the Fed were to precommit to a rule in the presence of a skeptical public, what form should the rule take? If the Fed knew the true structure of the economy, would the rule that is optimal under full information still be optimal when private agents have to learn the rule? Or would something simpler, and arguably easier to learn, be better in practice? Or would learning a new rule be so costly that an optimising monetary authority would choose to accept the incumbent rule no matter what it might be?

To examine these questions, we estimate a small forward-looking macro model with Keynesian features and model the process by which agents learn the features of the policy rule in use. The model is a form of a contracting model, in the spirit of Taylor (1980) and Calvo (1983), and is similar to that of Fuhrer and Moore (1995). With this model, we conduct stochastic simulations of a change in the policy rule, with agents learning the structural parameters of the linear rule using recursive least squares, and discounted recursive least squares. Doing these sorts of experiments in an economy that is forward-looking obliges us to exploit modern efficient algorithms for computing state-space representations of the forward-looking model in real time.

Consistent with our institutional affiliation, the model is estimated using US data, and the experiments, quite naturally, relate most closely with the US experience. It is our belief, however, that the paper’s conclusions apply much more broadly than the present exercise, and include the case of a small open economy. This belief is supported by some extensive sensitivity analysis, only some of which is reported in the paper itself. Indeed, one might argue that the message of this paper is particularly germane to fledgling regimes such as that of New Zealand where a lengthy record of policy commitment has not yet been built up.

The rest of this paper proceeds as follows. In section 2 we discuss the simple, macroeconomic model. The third section outlines our methodological approach. Section 4 provides our results. The fifth and final section offers some concluding remarks.

2.0 The model

We seek a model that is simple, estimated, and realistic from the point of view of a monetary authority. Towards this objective, we construct a simple New Keynesian model along the lines of Fuhrer and Moore (1995b). The key to this model, as in any Keynesian model, is the price equation or Phillips curve. Our formulation is very much in the same style as the real wage contracting model of Fuhrer and Moore (1995a). By making use of the Fuhrer-Moore formulation, we ‘slip the derivative' in the price equation, thereby ruling out the possibility of costless disinflation.[2] However, instead of the fixed-term contract specification of Fuhrer-Moore, we adopt the stochastic contract duration formulation of Calvo. In doing this, we significantly reduce the state space of the model, thereby accelerating the numerical exercises that follow.

Equations (1) and (2) together comprise a forward-looking Phillips curve, with π and c measuring aggregate and core inflation respectively, and y is the output gap, a measure of excess demand. Equation (1) gives inflation as a weighted average of inherited inflation, π_{t-1} and expected core inflation, $E_{t-1}c_t$. Following Calvo (1978), the expiration of contracts is given by an exponential distribution with hazard rate, δ. Assuming that terminations of contracts are independent of one another, the proportion of contracts negotiated s periods ago that are still in force today is $(1-\delta)\,\delta^{t-s}$.

$$\pi_t = \delta\,\pi_{t-1} + (1-\delta)\,E_{t-1}c_t \tag{1}$$

$$c_t = (1-\delta)\,E_{t-1}[\pi_t + \gamma\,y_t] + \delta\,E_{t-1}c_{t+1} + u_t^{\pi} \tag{2}$$

$$y_t = \phi_1\,y_{t-1} + \phi_2\,y_{t-2} + \phi_3\,rrl_{t-1} + u_t^{y} \tag{3}$$

$$rrl_t = \left(\frac{1}{1+D}\right) E_{t-1} \sum_{i=0}^{\infty} \left(\frac{D}{1+D}\right)^{i} (rs_{t+i} - \pi_{t+i}) \tag{4}$$

$$rs_t = rr^* + E_{t-1}\pi_t + \beta_{rs}\left[rs_{t-1} - \pi_{t-1}\right] + \beta_{\pi}\left[\pi_{t-1} - \pi^*\right] + \beta_y\,y_{t-1} + u_t^{rs} \tag{5}$$

In equation (2), core inflation is seen to be a weighted average of future core inflation and a mark-up of excess demand over inherited inflation.[3] Equations (1) and (2) differ from the standard Calvo model in only in that the dependent variables are rates of change rather than levels. Equation (3) is a very simple aggregate demand equation with output being a function of two lags of output as well as the lagged *ex ante* long-term real interest rate. Equation (4) follows Fuhrer and Moore (1995b) in using a constant approximation to Macaulay's (1938) duration formula to define the *ex ante* long-term real interest rate as a geometrically declining weighted average of current and future short-term real interest rates. Finally, equation (5) is a generic interest rate reaction function,

3 Our timing and notation convention for expectations is that $E_{t-1}(\cdot) = E(\cdot|\Omega_{t-1})$ where the information set, $\Omega_{t-1} \in \{\pi_{t-i}, y_{t-i}, rs_{t-i}, rrl_{t-i}\}\ \forall\, i \geq 1$. In words this means that the monetary authority chooses rs_t, and private agents form an expectation of c_t, prior to the revelation of stochastic shocks dated at time t.

written here simply to complete the model. The monetary authority is assumed to manipulate the nominal federal funds rate, rs_t and implicitly deviations of the real rate from its equilibrium level, rr-rr^*, with the aim of moving inflation to its target level, π^*, reducing excess demand to zero, and penalising movements in the instrument itself. Each of the state variables in the rule carries a weight of β_i where $i = \{\pi, y, rs\}$ These weights are related to, but should not be confused with, the weights of the monetary authority's loss function, about which we shall have more to say below.

The model is stylised, but it does capture what we would take to be the fundamental aspects of models that are useful for the analysis of monetary policy. We have already mentioned the stickiness of inflation in this model. Other integral features of the model include that policy acts on demand and prices with a lag. This rules out monetary policy that can instantaneously offset shocks as they occur. The model also assumes that disturbances to aggregate demand have persistent effects, as are the effects of demand itself on inflation. These features imply that in order to be effective, monetary policy must look ahead, setting the federal funds rate today to achieve objectives in the future. However, the stochastic nature of the economy implies that these plans will not be achieved on a period-by-period basis. Rather, the contingent plan set out by the authority in any one period have to be updated as new information is revealed regarding the shocks that have been borne by the economy.

We estimated the key equations of the model on US data from 1972Q1 to 1996Q4.[4] Since the precise empirical estimates of the model are not fundamental to the issues examined here, we will keep our discussion of them concise. A couple of important points should be mentioned however. We measure goods-price inflation, π, with quarterly change in the chain-weight GDP price index, a producer's price. However, we proxy $E_{t-1}c_{t+1}$ with the median of the Michigan survey of expected future inflation. The survey has some good features as a proxy. First, it is an unbiased predictor of future inflation. At the same time, it is not efficient: other variables do help in predicting movements in future inflation. Second, it measures consumer price inflation expectations, precisely the rates that would theoretically go into wage bargaining decisions, and thereby into unit labour costs. GDP price inflation can then be thought of as a pseudo-mark-up over these expected future costs. The disadvantage is that the survey is for inflation over the next twelve months, which does not match the quarterly frequency of our model. However, most of the predictive power of the survey to predict inflation over the next twelve months comes from its ability to predict inflation in the very short term rather than later on, so this problem is not too serious.[5]

Equation (2) can be substituted into equation (1) to yield a restricted Phillips curve. The estimates of this equation along with two others are presented in table 1 below. Unemployment gaps defined as the deviation of the demographically adjusted unemployment rate less the NAIRU performed better in estimation than did output gaps, and so the former appears in equation (A) of the table. We then supplemented the empirical model with a simple Okun's Law relationship, equation (C),

4 Recursive least squares estimates indicated significant parameter instability prior to the early 1970s.

5 It does, however, induce some spurious serial correlation in the residuals which we handle with the Newey-West (1987) variance-covariance matrix.

and then substituted in order to arrive at the appropriate estimates for equations (1) through (5).

The equation of primary interest is our Phillips curve. As equation (A) in the table shows, we supplemented the basic formulation with a small number of exogenous supply shock terms, including oil prices, a variable to capture the effects of the Nixon wage-and-price controls, and a constant term. These are traditional and uncontroversial inclusions. Roberts (1996) has found oil prices to be important for explaining the inflation in estimation using Michigan survey data.

The key parameters are the 'contract duration' parameter $\hat{\delta}$, and the excess demand parameter, $\hat{\gamma}$ If this were a *level* contracts model, $\hat{\delta} = 0.41$ would be a disappointingly low number since it implies a very short average contract length. This might be taking this interpretation too far, however. An estimate of $\hat{\delta} = 0.41$ implies substantial persistence in inflation, much more so than any nominal

Table 1
Estimates of basic contract model (1972Q1-1996Q4)

Description	Label	Estimate	t-statistic	Summary statistics
$\pi = [1-(1-\delta)^2]^{-1}\left(\delta\pi_{t-1} + (1-\delta)^2\gamma u_{t-1} + (1-\delta)\delta\pi_{t,t+1}\right) + \Gamma Z$				(A)
Nixon price controls	$\hat{Z}_1$	-5.35	(2.20)	R^2
changes in oil prices	$\hat{Z}_2$	0.0019	(0.70)	SEE=1.02
unemployment	$\hat{\gamma}$	-0.23	(1.49)	B-G(1)=0.01
contract duration	$\hat{\delta}$	0.41	(4.65)	Constrained linear IV
	$y = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 rrf_{t-1}$			(B)
first lag of output	ϕ_1	1.21	(12.16)	R^2 =0.88 SEE=0.60
second lag of output	ϕ_2	-0.38	(4.02)	b-g(4)=0.05
real fed funds rate	ϕ_3	-0.24	(2.41)	OLS
	$u = \gamma_0 + \gamma_1 y_t \gamma_2 T + \gamma_3 (poil_t / p_t)$			(C)
output gap	$\hat{\gamma}_1$	-0.34	(9.01)	R^2 =0.80 SEE=0.60
time trend	$\hat{\gamma}_2$	0.0081	(1.86)	B-G(4)=0.00
relative price of oil	$\hat{\gamma}_3$	0.57	(3.07)	2SLS

wage contracting model could furnish. In fact, when equation (A) is solved, its reduced-form coefficient on lagged inflation is seen to be 0.846. This is substantial inflation stickiness by any measure.[6]

Data: $\Delta poil$ change in oil prices is a four-quarter moving average of the price of oil imported into the US; π is the quarterly change at annual rates of the chain-weight GDP price index; u is the demographically corrected unemployment rate, less the natural rate of unemployment from the FRB/US model database; c_{t+1} is proxied by the median of the Michigan survey of expected inflation, 12 months ahead; y is the output gap for the US from the FRB/US model database; rr is the real interest rate defined as the quarterly average of the federal funds rate less a four-quarter moving average of the chain-weight GDP price index; *poil/p* is the price of imported oil relative the GDP price index; and Nixon price controls equals unity in 1971Q4 and -0.6 in 1972Q1. All regressions also included an unreported constant term. Constants were never statistically significant. B-G(1) is the probability value of the Breusch-Godfrey test of first-order serial correlation.

Notes: Equation (A) is estimated with instruments: constant, time trend, lagged unemployment gap, four lags of the change in imported oil prices; two lags of inflation, lagged real interest rate, lagged Nixon wage-rice control dummy, and the lagged relative price of imported oil. Standard errors for all three equations were corrected for autcorrelated residuals of unspecified form using the Newey-West (1987) method.

Turning to the aggregate demand function, it is conventionally believed that demand in the US responds to movements in the long-term real interest rate.[7] Accordingly, we define the *ex ante* real interest rate, *rrl*, as the five-year government bond rate less the average inflation rate that is expected over the next five years, and compute the latter using a small-scale vector autoregression.[8] Five years is about the time period for which consumer durables and automobiles are typically financed. For the simulations to come, the duration, D, in equation (4), is set at 20 quarters in conformation with the definition of *rrl*.

The estimates of the aggregate demand function show the humped shape pattern of responses of output to demand shocks; that is, an exogenous disturbance to demand tends to overshoot initially—as determined by $\tilde{\phi}_1 = 1.21 > 1$ and then drop back, as indicated by $\tilde{\phi}_2 = -0.38 < 0$. The interest elasticity of aggregate demand is large and negative as expected.

After substituting equation (C) into equation (A) and dropping those arguments that are not of interest to us, we arrive where we began: with equations (1) through (5). The parameters of the

6 Our estimates do suggest that one ought to be wary of stretching the structural interpretation of the contract model too far. However, since we are considering changes in rules under learning rather than changes in regime, our taking the model as structural does not seem too adventurous.

7 See Fuhrer and Moore (1995a) for a extensive discussion of the linkage between monetary policy and the long-term real interest rate.

8 This is the same methodology as employed in the FRB/US macroeconometric model of the US built and maintained by the Federal Reserve Board. For more information, see Brayton and Tinsley (eds.) (1995) and Brayton *et al* (1997).

estimated model are broadly similar to estimates of other models, and are reasonable. Impulse responses of the model to exogenous shocks to the price equation and the aggregate demand function are consistent with the historical experience in the US as measured by simple vector autoregressions.[9]

The estimated equations do show some remaining residual correlation. This is a common finding in structural price equations—Roberts (1996) uncovered the same phenomenon— so as noted above, we have corrected the variance-covariance matrix for this autocorrelation using the Newey-West (1987) technique. We conclude that our model is appropriate for the questions we wish to address.

3.0 Methodology

3.1 Optimal and simple policy rules

It is useful to express the model in its first-order (state-space or canonical) form. To do this we begin

$$A_1 \begin{bmatrix} z_{t+1} \\ v_{t+1|t} \end{bmatrix} = A_0 \begin{bmatrix} z_t \\ v_t \end{bmatrix} + \begin{bmatrix} u_{z,t+1} \\ u_{v,t+1} \end{bmatrix} \qquad (6)$$

by partitioning the vector of state variables into predetermined variables and 'jumpers', and expressing the structural model as follows:

Where $z_t = [\, rs_{t-1}, \pi_t, y_t, y_{t-1} \,]$ is the vector of predetermined variables, and $v_t = [\, c_t, rrl_t \,]$ is the set of non-predetermined variables. The two non-predetermined, or 'jumper' variables are associated, respectively, with the forward-looking contracting structure equation (2) and the expectations theorem of the term structure equation (4). Henceforth, the notation $v_{t+1|t}$ shall designate the expected value of *v* for date *t + 1* conditional on information available at date *t*. Constructing the state-space representation for the model for a given policy rule is a matter of finding a matrix, $A = A_1^{-1} A_0$. In this paper, we have the added complexity of choosing the values of one row of to produce a state-space representation that is preferred from the perspective of a monetary authority.

$$\begin{bmatrix} z_{t+1} \\ v_{t+1|t} \end{bmatrix} = A \begin{bmatrix} z_t \\ v_t \end{bmatrix} + A_1^{-1} \begin{bmatrix} u_{z,t+1} \\ u_{v,t+1} \end{bmatrix} \qquad (7)$$

Equations (7) are recursive in the state variables so that manipulating the model is simple and computationally easy. However, two problems arise.

The first of these problems is a technical one having to do with the fact that is often singular. We shall return to this later on. The second problem is more interesting from an economic point of view

9 We do not show the impulse responses since they vary with the reaction function that is assumed to be in place. The appropriate reaction function is the question addressed in the rest of the paper.

and concerns finding a specific rule with the desired properties. This is an exercise in optimal control. In the forward-looking context, however, the theory is a bit more complex than the standard textbook treatments. The optimal control rule is no longer a function just of the seven state variables of the model as is the case with a backward-looking model. Rather, as Currie and Levine (1987) have shown, the optimal control rule is a function of the entire history of the predetermined state variables.[10] Even for simple models such as this one, the rule can rapidly become complex.

It may be unreasonable to expect agents to obtain the knowledge necessary to form a rational expectation of a rule that is optimal in this sense. Our experiments to date have shown that agents have great difficulty learning the parameters of rules with many conditioning variables. This is particularly so when some variables, such as contract inflation, c and price inflation π, tend to move closely together. In small samples, agents simply cannot distinguish between the two.[11] Rather than demonstrate this intuitively obvious result, in this paper, we consider instead restrictions on the globally optimal rule, or what we call simple optimal rules.[12]

Simple optimal rules are those that minimise a loss function subject to the model of the economy just as regular optimal control problems do plus a constraint on the number of arguments in the reaction function.

For our purposes, we can state the monetary authority's problem as:

$$\underset{\langle \beta_\pi \beta_y \beta_{rs} \rangle}{\arg\min} E_0 \sum_{i=0}^{\infty} \rho^i \left[\psi_\pi (\pi - \pi^*)^2 + \psi_y y^2 + \psi_{\Delta rs} (\Delta rs)^2 \right] \quad (8)$$

subject to the state-space representation of the model as in equations (6), along with the model consistency restrictions: $\pi_{t,t+1} = \pi_{t+1}$ and $c_{t,t+1} = c_{t+1}$, and the arguments of the reaction function, (5):

$$rs_t = rr_t^* + \pi_t + \beta_{rs}[rs_{t-1} - \pi_{t-1}] + \beta_\pi[\pi_{t-1} - \pi^*] + \beta_y y_{t-1} + u_{rs\,t}.$$

10 This is because the optimal control rule cannot be expressed as a function of the non-predetermined variables since these 'jump' with the selection and operation of the rule. Rather, the rule must be chosen with regard to those variables that determine the jump. In the rational expectations context, this will be given by the entire history of predetermined state variables of the model. In many cases, this history can be represented by an error-correction mechanism, as Levine (1991) shows, but not always. In any case, even when an error-correction mechanism exists, the basic point remains that the complexity of the rule is significantly enhanced by the presence of forward-looking behaviour.

11 A second concern would be that reliable data for a latent variable like contract inflation might not exist.

12 The phrase 'simple rules' is borrowed from Levine (1991) who addresses issues similar to some of the ones considered here. We adopt it and add the word 'optimal' to signify that the parameterisation of our rules is not *ad hoc*, but rather is determined from a well specified minimisation problem as described below.

The solution to this problem, which is described in some detail in the appendix, is a vector of policy rule coefficients, $\bar{\beta} = \{\beta_y, \beta_\pi, \beta_{rs}\}$ corresponding to the vector of objective function weights, $\bar{\psi} = \{\psi_y, \psi_\pi, \psi_{\Delta rs}\}$.

One cannot discuss optimal policy in a forward-looking model without discussing the issue of time inconsistency. As is well known, if the monetary authority discounts the future, then the mere passage of time may, under some circumstances, elicit a change in policy rule adopted by the monetary authority. This incentive exists not because of unforeseen events (although these may exacerbate the problem) but simply because the weight attached to an outcome at some date in the future rises as that date approaches. Thus what a policymaker was initially willing to tolerate in the distant future becomes intolerable later on, even if the world unfolds as projected. One time honoured method of avoiding this problem is to assume an undiscounted objective function for the monetary authority. However, such an authority would always ignore the transition costs of learning a new regime, an outcome that is unenlightening and implausible. The alternative strategy is to assume a commitment technology that permits the authority to fix a rule in advance and stick to it. If the authority chooses the rule that is optimal *on average*, which would be the case if the authority were prepared to commit to a rule for some time, then the optimal discounted rule and the optimal undiscounted rule will coincide for reasonable discount rates. In effect, the two "solutions" to the time inconsistency problem become one and the same. Since we shall be assessing losses on a discounted basis, we invoke the commitment technology assumption.[13]

Rules that solve the above problem will, by definition, be inferior to the globally optimal rule that could have been derived using optimal control techniques, in the presence of full information. By the same reasoning, they will be superior to even simpler rules such as the Taylor rule, which contains only two arguments, and the coefficients of which are not necessarily chosen according to optimisation criteria. In our formulation, the Taylor rule is just our equation with the added restrictions that

$$\beta_{rs} = 0, \beta_y = \beta_\pi = 0.5$$ [14]

$$rs = rr^* + \pi_t + 0.5[\pi_{t-1} - \pi^*] + 0.5 y_{t-1} + u_{rs,t} \tag{9}$$

Equations (5) and (9) are the two principal policy rules we will use in our experiments. The coefficients in these rules are what agents are assumed to learn.

[13] There will be an incentive for the authority to practice discretion in extreme states, but this incentive will be small so long as discount rates are small. Thus, small penalties against reneging, including the effect of discretion on learning, could mitigate the problem. In any case, an examination of this complex issue is beyond the scope of this paper.

[14] We are taking a bit of license here, with our characterisation of the Taylor rule, in that inflation in the original specification of the rule appears with a four-quarter moving average of inflation and both inflation and output appear contemporaneously. We take the former simplification to reduce the size of the state matrix for computational reasons. The reasons for restricting our attention to lagged dated state variables were given previously.

3.2 Learning

Let us re-express our general policy rule, equation (5) , more compactly as:

$$rs_t = \overline{\beta}_t x_t + \hat{u}_{rs,t} \tag{10}$$

Equation (10) merely stacks the right-hand side arguments to the policy rule into a vector, $x_t = [y_{t-1}, \pi_{t-1}, rs_{t-1}]$, while allowing time variation in the *estimated* coefficients, $\hat{\beta}_t$. Let the time series of x_t be X_t ; that is, $X_t = \left[X_{t-1} \middle| x_t \right]$. We assume that agents use either least squares or discounted least squares to update their estimates of $\hat{\beta}_t$ Harvey (1993, pp. 98-99) shows that the recursive updating formula in equation (12) is equivalent to continuous repetition of an OLS regression with the addition of one observation:

$$\hat{\beta}_t = \hat{\beta}_{t-1} + P_{t-1} x_t (rs - x_t' \hat{\beta}_{t-1}) f_t^{-1} \tag{11}$$

where P_t is the ‘precision matrix’, $P_t = (X_t' X_t)^{-1}$ and $f_t = 1 + x_t' P_{t-1} x_t$ is a standardisation factor used to rescale prediction errors, $\hat{u}_{rs,t} = P_{t-1} x_t \left(rs - x_t' \hat{\beta}_{t-1} \right)$, in accordance with changes in the precision over time. The precision matrix can be shown to evolve according to:

$$P_t = \lambda P_{t-1} - P_{t-1} x_t x_t' P_{t-1} f_t^{-1} \tag{12}$$

The parameter λ is a ‘forgetting factor’. The special case of $\lambda = 1$ discussed in Harvey (1993) is standard recursive least squares (RLS). If we assume that agents have ‘less memory’ we can down-weight observations in the distant past relative to the most recent observations by allowing $0 < \lambda < 1$. This means that agents ‘forget’ the past at a geometric rate of 1 - λ percent per quarter. This is discounted recursive least squares (DRLS).

The memory of the learning system has a convenient Bayesian interpretation in that had there never been a regime shift in the past, agents would optimally place equal weight on all historical observations, as with RSL. Under such circumstances, the ‘Kalman gain’ in equation (11) goes to zero asymptotically as $P_t \to 0$ and $\hat{\beta} \to \overline{\beta}$ That is, the learning process converges on a rational expectations equilibrium. However, if there has been a history of occasional regime shifts, or if agents believe in such a possibility for other, external reasons, then the weight they discount previous observations of the rule parameters can represent the strength of their prior belief. The lower is λ, the more likely agents believe regime shifts to be. If λ is taken as an exogenously fixed parameter (as we do throughout this paper) then P_t does not reach zero in the limit, and $\hat{\beta}$ will have a tendency to fluctuate around $\overline{\beta}$ overreacting to surprises owing to the maintained belief that a regime shift has some likelihood of occurring. That agents might, in some sense, overreact to news means agents with less memory will tend to learn new rules more rapidly, but this rapid learning is not necessarily welfare improving.

Whatever the precise form, using some form of learning here is a useful step forward since it models agents on the same plane as the econometrician, having to infer the law of motion of the economy from the surrounding environment, rather than knowing the law of motion *a priori*.

3.3 Numerical issues

Choosing a policy rule to minimise a loss function, subject to a system of linear rational expectations equations, such as equations (6) plus the form of the rule give by equation (5) , and the restriction that expectations are model-consistent *from the point of view of private agents*, presents some computational difficulties. Under model consistent expectations, the solutions for current dated endogenous variables depend on expected future values of other endogenous variables, which are conditional on agents' (possibly incorrect) beliefs of what rule the monetary authority is using.

One such difficulty is that equations (7) must satisfy the Blanchard-Kahn (1980) conditions for the existence of a unique saddle-point solution to the system. The B-K conditions require that the number of eigenvalues greater than unity be equal to the number of non-predetermined variables in the system. In this model, there are two non-predetermined variables. And because the parameters of the model change over time with agent's perceptions of the rule, the B-K conditions must be checked at each date. Instability or multiple equilibria are possibilities when $\lambda < 1$. There was some minor incidence of instability in our experiments, always associated with agents perceiving the coefficient on inflation less target inflation turning negative. The incidence of instability rose with the shortening of memory in the learning process, but never occurred for full memory.[15] To keep the model stable, we restricted the perceived weight on inflation to be positive and included those trials in which this constraint was binding in our computations. However, tests on even the most egregious cases of instability revealed very close to the same results whether or not those trials that breached the B-K conditions were included.

A second numerical issue that comes up, alluded to previously, concerns the problem of the possible singularity of the matrix A_1. Without a nonsingular A_1 matrix, the state-space representation of the model cannot be constructed, meaning that a recursive representation of the forward-looking model would not be possible. The model would then have to be solved using an iterative method instead. The problem we are interested in—stochastic simulation with learning—already involves one layer of iteration associated with the time variation in perceived structural coefficients. Adding a second layer associated with the iterative solution of the model given the (perceived) structural coefficients would be prohibitively costly. Fortunately, the development in recent years of new

[15] At levels of discounting of $\lambda = 0.85$, instability was a significant problem. Discount factor this low are interesting in the same sense that any extreme case might be, but since the implied half life of memory is only about four quarters; $\lambda = 0.85$ and lower are implausible from a practical point of view. Tetlow and vo zur Muehlen (1998) note that instability is less likely to occur when monetary policy operates through the short-term interest rate. This is an intuitive result once one recognises that using the short-term interest rate eliminates a forward-looking element from the model, that associated with the expectations theorem of the term structure.

methods of constructing state-space representations of linear forward-looking models obviates these complexities, including Sims (1996), Binder and Pesaran (1995) and King and Watson (1998). However, we have found that the algorithm of Anderson and Moore (1985) is fast, efficient and reliable.[16]

Finally, the linearity of the model combined with the model consistency of expectations (from the point of view of private agents) ensures that the imposition of possibly incorrect terminal conditions is not an issue.

It is useful, at this point, to summarise our quantitative approach. Let us take the case where the monetary authority has been using the Taylor rule for some time and then shifts to a version of the simple optimal rule. The 'algorithm' for solving this problem can be summarised as follows:

1. Begin with initial estimates of the model and compute the coefficients of the simple optimal rule, by maximising equation (8) given a set of weights , $\bar{\psi}$ subject to equations (7) and the form of the policy rule, (5);
2. Initialise the matrices P, X, $\bar{\beta}$ and the variance-covariance matrix Σ associated with the residuals $\bar{u}$ at values based on historical estimates, for date *t=0*;
3. Assume agents' expectations are consistent with their beliefs of the rule that is in operation;
4. Solve the model for its state-space representation and check for satisfaction of the Blanchard-Kahn conditions (if the B-K conditions are not satisfied, set $\bar{\beta}$ = 0.01;
5. Draw random shocks for the stochastic residuals, $\bar{u}$, and set the nominal federal funds rate consistent with these shocks and the *true* policy rule, for date *t=0*;
6. Simulate the model to find endogenous solution values, for *t=0*;
7. Taking the solution values to step (6) as given, update agents' perceptions of the policy parameters;
8. Repeat steps (3) though (8) for the next *t, t={1,2,...T}.*
9. When *t=T*, stop.

The next section discusses our results.

4.0 Simulation results

Our aim in this paper is to explore the optimality of policy rules from a more general perspective than is usually the case. Before considering the implications of transitions from one rule to another, however, it is useful to fix ideas by examining the answer to the simpler question of what rule one might choose under complete information. Thus, this section contains three distinct sets of quanti-

[16] In terms of algebra, all methods for solving this problem use a singular value decomposition to get around the singularity problem. Anderson and Moore uses a QR decomposition to find a non-singular equivalent to the matrix A_1 while Sims uses a QZ decomposition.

tative results. The first subsection examines the trade-offs involved in the choice of a policy rule with complete information under a variety of preferences and degrees of rule complexity. In doing so we establish a useful benchmark for what follows.

In the second subsection, we drop the complete information assumption and consider the choices faced by a monetary authority with given preferences when it takes over for an incumbent with quite different preferences. Given the cost of learning the new rule, what rule would the new authority choose? Does the cost of learning nullify the benefit of shifting from the incumbent's rule?

Finally, in the third subsection, we consider the costs of transitions from an ad hoc (sub-optimal) rule—our version of the Taylor rule—to an optimal 2- or 3-parameter rule. We compare these costs to the cost of transition from the optimal 2-parameter rule to the optimal 3-parameter rule, for both conservative and liberal preferences. This allows us to separate the costs of *learning to be optimal*, from learning complexity. Beginning from the Taylor rule, would the authority choose to use the optimal 3-parameter rule? Or is there an advantage to using a rule that uses just the same two arguments that the Taylor rule itself? Or would it be better off to simply stick with the Taylor rule even if it is suboptimal?

For our optimal rules, we focus on the simple optimal rules described above, for two different sets of preferences regarding the penalisation of output and inflation variability. The weights applied to output variability and inflation variability are permitted to vary. We consider two alternatives: one with 'conservative' objectives, where a weight of 0.70 is applied to inflation variability and 0.25 to output variability, and one for 'liberal' preferences where the weight on the variance of inflation is 0.25 and the variability of output carries a 0.70 weight. These taste parameters and the rule coefficients they imply are summarised in table 2 below.

In addition, in all cases we consider only preferences that place a small weight on restricting the variability of the change in nominal federal funds rate. A monetary authority may be concerned with the variability of its instrument for any of a number of reasons. First, and most obviously, it may inherently prefer less variability as a matter of taste, either in its own right, or as a device to avoid criticism of excessive activism. Second, constraining the volatility of the federal funds rate might be a hedge against model misspecifications, both fundamentally, in the sense of missing variables and the like, or more broadly owing to reluctance to using point elasticities estimated over a narrow range of movement of the federal funds rate being applied over a much wider range of contemplated variation. We worked with a number of different (but small) penalties on the variability of the instrument. For all of the baseline results in this section we apply a weight of $\psi_{\Delta rs} = 0.05$. We shall, however, refer to results for a broader range of values, where applicable, as we go along.

Finally, we conduct many of these experiments using different degrees of memory. Most of the discussions that follow will focus on either the 'full memory' case ($\lambda = 1$), which uses full recursive least squares (RLS), or a particular case of discounted recursive least squares (DRLS); one with ($\lambda = 0.95$), which we will call 'limited memory.'[17] The limited memory case corresponds with a mean *lag*

[17] We also conducted extensive tests with "short memory," meaning $\lambda = 0.85$ but agents expectations frequently converged on unstable results when memory was a short as this. Accordingly, much of the discussion in this paper will concentrate on the full and limited memory cases.

of 19 quarters, a reasonable length of time for a regime. However, we shall occasionally refer to a third DRLS case, with λ = 0.85, which we call the 'short memory' case: this is an extreme case where memory is a scant 6 quarters.

4.1 The rules in the steady state

Let us build up some intuition for what follows with an examination of the optimal policy rules in steady state. These are summarised by figure 1 (overleaf) which shows the trade-offs between the variances of inflation and output in the steady state. (We ignore, for the moment, the inset diagram.) By construction, at points on these frontiers, it is not possible to reduce the variance of inflation without increasing either the variance of output or the change in the federal funds rate, given the arguments to the policy rule. Each frontier represents a different number of arguments to the rule. These curves are derived by fixing the weight on the change in the federal funds rate in the loss function, $\psi_{\Delta R}$ at 0.05 and varying the weights on output and inflation variability such that they sum to unity.

We have constrained the frontiers to show weights on inflation and output that are not less than 0.25. This constraint was arbitrarily selected so that we can define 'conservative' preferences for our experiments. The points at bottom-right end of each frontier represent the trade-offs that our conservative policymaker would choose and correspond with preferences of $\{\psi_\pi, \psi_y, \psi_{\Delta R}\}$ = {0.70,0.25,0.05} in all cases. These are labelled "CO" for the conservative optimal rule and "C3" for conservative 3-parameter rule, and so on. These points show relatively low standard deviations of inflation, in steady state, traded off against relative high standard deviations of output. The other end of each of the frontiers are points that represents our characterisation of 'liberal' preferences: $\{\psi_\pi, \psi_y, \psi_{\Delta R}\}$ = {0.25,0.70,0.05} . To some extent, these definitions are arbitrary.

We could have defined conservative as, say, ψ_π = 0.9 with ψ_y = 0.05, for example, which would extend the frontiers into steeper sections than those shown. To use such an extreme definition, however, would run the risk of setting up a straw man for the arguments that follow. Whatever the merits of our definition, we shall discuss at some length some of the implications of different definitions. The coefficient values of some of these conservative and liberal rules are shown in table 2 (overleaf).

The frontier furthest to the north east is the frontier of optimal 2-parameter rules, which uses just the output gap and inflation as arguments; our version of the Taylor rule is a (suboptimal) example of a rule of this type. The dot-dashed line, L3-C3, is the 3-parameter frontier derived from adding the lagged federal funds rate to the two arguments allowed for the 2-parameter frontier; and L4-C4 is the 4-parameter frontier.[18]

[18] Of the five possible additions to the variables that appear in the optimal 2-parameter rules, the lagged federal funds rate improves performance the most for all preferences. The argument to add to get to the 4-parameter frontier is somewhat less clear since adding second lag of output improves performance more for liberal preferences than does adding a second lag of inflation and vice versa for conservative preferences. Figure 1 shows the 4-parameter frontier with a second lag of output. The alternative specification is very similar.

Figure 1
Selected optimal policy frontiers

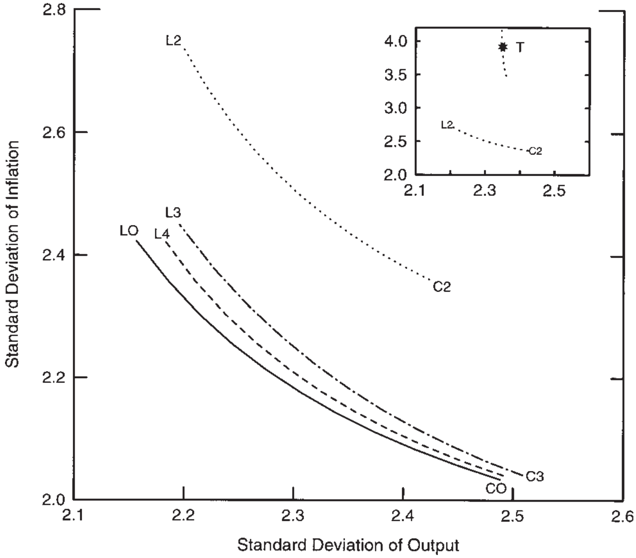


Table 2
Coefficients of simple optimal rules

Preferences		Policy rule parameters		
$\{\psi_y, \psi_\pi, \psi_{\Delta R}\}$	no of arguments	β_y	β_π	β_{r_1}
Taylor rule	2 parameter	0.5	0.5	n/a
conservative	2 parameter	-0.25	3.25	n/a
(0.25, 0.70, 0.05)	3 parameter	0.57	1.19	0.86
liberal	2 parameter	1.17	1.90	n/a
(0.70, 0.25, 0.05)	3 parameter	0.94	0.65	0.79

Notes: rule parameters are the solutions to the problem of minimising equation (8) subject to (6) and (5) under model consistent expectations, for the loss function weights shown in the first column of the table.

Finally, the solid line, labelled LO-CO, is the frontier for the globally optimal rule, which uses seven state variables.

The first and most obvious thing to note about these frontiers is that adding more parameters to a rule improves the steady-state performance: the globally-optimal (7-parameter) rule furnishes the frontier that is closest to the origin, and increasing the parsimony of the rules reduces performance.[19]

A more interesting point, however, is the diminishing returns to increasing parameterisation: Going to a 3-parameter rule from a 2-parameter one improves performance quite significantly, but moving from there to a 4-parameter rule adds comparatively little.

One might think, based on looking at figure 1 alone, that a conservative policymaker would dislike simplicity in its rules less than a liberal would. This would seem to be the case based on the relatively close proximity of the C3 point to the CO point. However, this ignores the federal funds rate volatility needed to achieve these points. In fact, the variability in the change in the federal funds rate rises so substantially with parsimonious rules that the conservative policymakers' distaste for a 2-parameter rule, relative to a 3- or 4-parameter rule, is significantly larger than the liberal policymakers'. We shall come back to this point in section 4.3 when we consider the quantitative results.[20]

A third point to be gleaned from the figure concerns the Taylor rule, shown in the inset by the point marked with a 'T'. The inset diagram is necessary because this rule performs so much worse than any optimal 3-parameter rule in terms of output and inflation variability, that it is off the appropriate scale for figure 1. However with a standard deviation of the change in the federal funds rate of 2.3, the Taylor rule produces a substantially less instrument variability than either the optimal 2- or 3-parameter rules. It follows that if one is to consider this version of the Taylor as one that is optimal for this model, it must represent preferences that place a large penalty on variability of the change in the federal funds rate. At the same time, the steepness of the frontier upon which the Taylor rule lies indicates that it is a very liberal rule. In fact, a monetary authority that would choose this point has $\{\psi_\pi, \psi_y, \psi_{\Delta R}\} = \{0.97, 0.03, 0.05\}$ and is thus willing to sacrifice a very large amount of inflation variability to reduce output variability only slightly.[21]

It is important to note that these precise numbers are only literally true for our characterisation of the Taylor rule, and only within this model. However, the same general conclusion— namely that

[19] The figures are only showing the performance in two dimensions, rather than the three that appear in the loss function. Adding the variance of the change in the federal funds rate would not change the overall impression left by figure 1.

[20] Note that the distance between frontiers of different parameterisations rises with $\psi_{\Delta R}$ for the obvious reason that the more jobs one asks a monetary authority to do, the more tools – that is, states in the rule – it needs to do its job well.

[21] For example, the same finding has been uncovered with the FRB/US model of the US economy maintained by the Federal Reserve Board, although the implied preferences behind the Taylor rule with the FRB/US model are not so heavily weighted toward output control as with the present model.

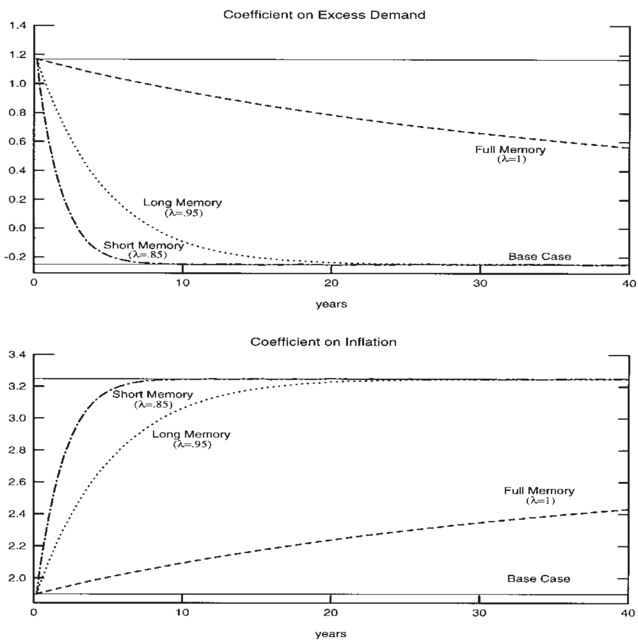
the Taylor rule is liberal in the sense defined here, and heavily penalises federal funds rate variability—can be drawn from a similar exercise using the original form of the rule from Taylor (1993) within larger, more complex models.

4.2 Changes in preferences

Having examined the steady-state performance of our rules, let us now consider transitions from one rule to another. In order to economise on space, for most of this section, we shall focus initially on changes in policy preferences using optimal 3-parameter rules. In addition, these rules perform nearly as well as more heavily parameterised rules.

The thought experiment we have in mind is of a newly installed policymaker, replacing an incumbent of quite different preferences. The new authority has to decide whether to switch to a new rule that is consistent with its preferences. The new rule will produce better steady-state perform-

Figure 2
Perceived policy rule parameters learning a 'liberal' to 'conservative' shift in policy (average of 3000 draws)



ance—from the point of view of the new authority—but presumably only after bearing some transitional costs as agents learn of the new rule. It is conceivable that the transitional costs could be so high that the new authority would prefer to continue to govern policy with the old rule, even though it is inconsistent with its steady-state preferences.[22]

Figure 2 shows the evolution over time of the parameter estimates for the transition from a liberal policy regime, to a conservative one, using the 2-parameter optimal rules.[23] The upper panel shows the perceived coefficient on excess demand and the lower panel shows the coefficient on inflation. In each panel, we also show a base case—the solid line—where agents believe the initial regime will continue, and they are correct in this expectation. For all the other cases, their prior expectations turn out to be incorrect. The dashed line is the 'full memory' case while the dotted line is 'limited memory' learning case (that is, DRLS learning with $\lambda = 0.95$). The lines in each case are the average values over 3,000 draws. Each simulation lasts 200 periods.[24]

Let us concentrate, for the moment, on the upper panel of figure 2. The first thing to note from the figure is the obvious point that agents do not get fooled in the base case. That is, if the rule is the 2-parameter optimal liberal rule, purely random shocks do not induce agents to erroneously revise their estimates of the perceived rule, when agents learn without discounting. This is not a trivial result since, as we shall see, it is not as clear when agents 'forget' past observations.

More important than this, however, is the observation that regardless of which of the three learning devices that is used, it takes a remarkably long time for agents to come to grips with the change in parameters. In particular, with full memory, agents have not learned the true rule coefficients even after the fifty years covered in the experiment. Even under the case of short memory *with* $\lambda = 0.85$ it takes more than ten years before agents reach the new parameter values. This finding, which is consistent with earlier results, such as those of Fuhrer and Hooker (1993), stems from two aspects of the learning rules. First, the speed of updating is a function of the signal-to-noise ratio in the learning mechanism. Because a considerable portion of economic variability historically has come from random sources, agents rationally infer the largest portion of surprises to the observed federal funds rate settings as being noise. Accordingly, to a large extent, they do not respond to the shock. Second, these results show how linear learning rules tolerate systematic errors; the forecast errors that agents make get increasingly smaller, but are nonetheless of the same sign for extended periods of time. A non-linear rule might react not just to the size of surprises, but also a string of errors of one sign.

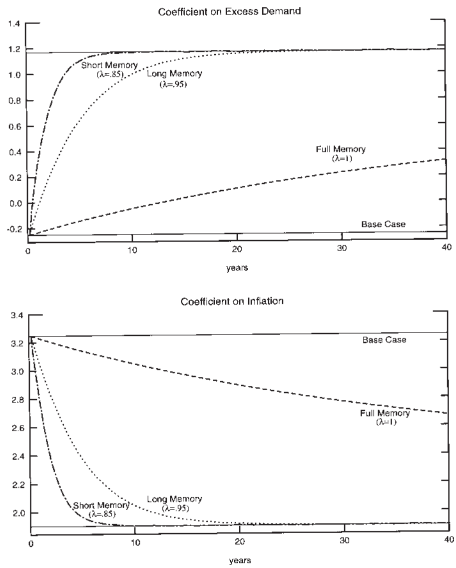
22 This experiment is broadly similar to Fuhrer and Hooker (1993) who also consider learning policy parameters. However, they do not consider optimal rules and do not examine the welfare implications of the choice of rules.

23 We use 2-parameter rules here only because there are fewer graphs involved. The results are essentially the same for more heavily parameterised rules.

24 In order to ensure the lags and the precision matrix were properly initiated according to pre-experiment conditions, 100 periods were simulated using the initial rule before shifting to the new rule. These observations were discarded, leaving 200 periods in the experimental period. All together, each of the major experiments reported in this paper involved computing 600,000 points. On an UltraSparc 166 megahertz UNIX machine, these took about 3-1/2 hours each to compute.

The bottom panel of figure 2 shows the evolution of the perceived coefficient on inflation. Figure 3 shows the analogous learning rates when a conservative regime is succeeded by a liberal one. Very much the same conclusions can be drawn from these figures as is drawn from the upper panel of figure 2. Table 3 summarises the welfare implications of this experiment. Since the other tables that follow this one are broadly similar in construction, there are dividends to be reaped from taking some time to explain in detail how to read this table. The table is divided into two panels. The top panel shows the decision a conservative policymaker would have to make after succeeding a liberal policymaker. The row in this panel labelled 'base case' shows the performance that the economy would enjoy had the conservative policy rule been in effect at the outset. This can be thought of as the 'after picture.' The second row, labelled 'liberal' shows the performance of the economy under the liberal rule; that is, the rule that was in place when the conservative policymaker took over. This is the 'before picture.' Finally, the third row shows the results for the transition case from the liberal rule to the conservative rule. The key column of the panel is the far right-hand column showing welfare loss. This is the discounted loss under the policies, *measured from the perspective of the incoming conservative policymaker*.[25] In terms of the raw numbers, the perform-

Figure 3
Perceived policy rule parameters learning a 'conservative' to 'liberal' shift in policy (average of 3000 draws)



[25] Loss figures shown are the average over the 4,000 draws conducted. A quarterly discount factor of 0.9875, or about six percent a year, is applied in computing the losses. This is a modest discount factor, in line with what might be used in financial markets. The substantive facts presented in this paper were invariant to the choice of discount factors, at least within a range we would consider to be reasonable. Political economy arguments might yield positive arguments for a substantially lower discount factor, but this is not the subject of this paper.

ance of the economy under, say, the liberal rule as shown in the second row, will not be any different than it was under the liberal regime, but the loss ascribed to this performance can differ markedly depending on the loss function of the policymaker. To aid comparison, we have normalised the loss figures for the base case to unity. By comparing the third row of the upper panel with the first row, we can see the loss associated with agents' having to learn the new rule. By comparing the third row with the second row, we can see whether the transition costs of learning the new rule are so high as to induce the new policymaker to stay with the old rule. The first three columns in the body of the table show the standard deviations of those variables that enter the loss function. The fourth column, the one marked p (π,y), shows the statistical correlation of output and inflation in the simulated data. Comparing this across experiments gives an indication of the extent to which the monetary authority is using the Phillips curve trade-off to achieve its objectives.

Table 3
Simulation results from change-in-preference learning exercises (average across 4000 draws)

Rule in use	Standard deviation of:			$\rho(\pi, y)$	Welfare loss
	π	y	ΔR		L
'conservative' preferences, full memory, 3-parameter rules					
base-case	2.0	2.5	3.4	.28	1
liberal rule	2.4	2.2	3.1	.18	1.15
lib - -> con	2.3	2.5	3.5	.26	1.16
'liberal' preferences, full memory, 3-parameter rules					
base-case	2.4	2.2	3.1	.18	1
con rule	2.0	2.5	3.4	.28	1.11
con - ->	2.1	2.2	3.1	.20	0.96

Notes: The first row of each panel contains the base-case results defined as those corresponding to the optimal 3-parameter rule for the policy preferences noted; the row immediately below each base case is the performance of, and loss to, staying with the 3-parameter policy rule inherited from the previous regime. The third row shows the performance and cost of changing from the inherited regime to the (new) optimal 3-parameter rule. Welfare losses are discounted at rate 0.9875 per quarter, the highest rate that can be justified by observations on real interest rates. Higher discount rates would tend to make successor regimes more likely to retain the former rule.

The bottom panel of the table is analogous to the top panel, except that the transition is from a former conservative regime to a new liberal regime.

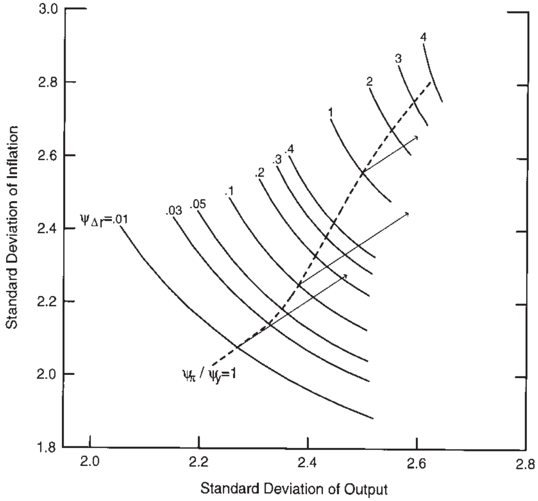
Now let us turn to the results themselves and focus, for the moment, on the top panel. In this case, where an incoming conservative policymaker inherits a liberal rule, the process of learning is costly relative to the base case, as one would expect. Not being able to simply announce a new policy regime and have private agents believe it, implies significant costs. Moreover, the comparison of rows two and three shows that the incoming conservative would be virtually indifferent to bearing the transition costs of switching rather than stick with the incumbent rule. This finding is not robust however.

As one would expect, the more different the preferences of the successor authority from the incumbent, the higher the losses associated with accepting the incumbents rule. In deciding whether to bear these costs, the successor must weigh them against the cost of learning the new rule. The cost of learning also rises with the difference in preferences, but not by as much as the steady-state costs. Thus, the 0.70-to-0.25 ratio of the weight on inflation variability to the weight on output variability that defines conservative preferences here is the borderline between accepting and refusing a change in rules: Any higher ratio of weights results in the conservative policymaker unambiguously accepting the costs of transition to the new, conservative rule.[26]

That much is logical. A somewhat less intuitive result—not shown in the table—is that the propensity of a conservative authority to accept the inherited liberal policy rule rises with $\psi_{\Delta R}$. A hint of why this is so can be gleaned from comparing interest rate volatility under the conservative and liberal policy rules. Notice that the conservative rule tolerates more variability of the federal funds rate. A more activist policy is required to control inflation than to control output, largely because the former is later in the monetary policy transmission chain: To a first approximation, policy influences inflation by first inducing movements in the output gap. It follows that for a given ratio ψ_π/ψ_y a higher $\psi_{\Delta R}$ penalises inflation control more than output control. This is illustrated in figure 4 where a family of frontiers is shown. The frontiers that are shorter in length and more to the north east of the figure are those associated with higher $\psi_{\Delta R}$. The dashed line shows the 'policy expansion path' which traces out optimal 3-parameter policy rules for a range of $\psi_{\Delta R}$ holding fixed the relative distaste for output and inflation variability. In this instance, we have traced out the policy expansion path for balanced preferences of $\psi_\pi = \psi_y = 0.5$. If increasing concern for interest-rate volatility were no more costly to inflation control than output control, this line would have a slope of unity—like the thin solid rays shown in the figure. Instead, the policy expansion path is generally steeper than this, indicating that interest-rate stabilisation and inflation stabilisation are relative substitutes in monetary control. The implications for more strident preferences are even more dramatic. To see this, observe that the locus of points formed by joining the lower-right ends of the frontiers would be the policy expansion path for conservative preferences. Note that the conservative expansion path is vertical, if not backward bending, meaning that increased concern over interest-rate volatility comes entirely at the cost of inflation variability for conservative authorities.

[26] The results shown in the table are for 3-parameter rules. Essentially the same results obtain for transitions between 2-parameter rules.

Figure 4
The policy expansion path



By contrast, the envelop formed by the upper-left ends of the frontiers has gentle positive slope indicating smooth trade-offs. This means that rising $\psi_{\Delta R}$ makes inflation control harder and thus implies that the conservative authority less likely to change rules after inheriting a liberal one.[27]

A more intriguing case is the transition from a conservative policy regime to a liberal one, shown in the lower panel. The third row of the lower panel shows that the liberal policymaker actually *benefits* from the prior belief of private agents that a conservative rule is in place: the loss under the transition, at 0.96, is lower than the normalised loss of unity in the base-case.[28] On the surface, this is surprising since by definition the base case should yield the best result possible for rules of this parameterisation. The reason why this intuition does not hold becomes clear once one recognises that the base-case rule is the (constrained) optimal rule only when expectations are rational. In this instance, expectations are not rational in the sense of Muth because the expectation of future core inflation, $E(c_{t+1}| \Omega)$ is being conditioned on incorrect information, namely that the conservative rule is in operation. This break from rational expectations is beneficial to the liberal authority because it pins down expected future inflation more substantively than would be the case under full information. This contention is supported by noting that the lower loss in the transition case comes from reduced inflation volatility, relative to the base-case.

[27] The general conclusions from figure 4 are unalterd by changing the number of parameters in the rule or to small changes in the model such as removing the term structure equation and replacing the ex ante long-term rate in the aggregate demand function with a short-term rate.

[28] As with the transition from the liberal rule to the conservative, the effects become larger as the definition of liberal and conservative becomes more extreme.

We have already seen that there is not the kind of benefit for the transition from liberal to conservative preferences shown in the upper panel of the table as there was to the liberal authority taking over from a conservative. Two reasons account for this asymmetry. The first of these relates to the fact that the monetary authority controls inflation primarily through its management of aggregate demand. Thus, to a substantial degree, the belief by private agents that the authority will manage output tightly is only beneficial to the conservative authority to the extent that inflation fluctuations originate from demand shocks. However, a large portion of inflation variability in the US economy comes from shocks to the Phillips curve—that is, from so-called supply shocks. The larger the proportion of shocks to inflation originating from supply-side sources, the more conflicts arise in the management of demand and the control of inflation. This effect is not at work when moving from the conservative to liberal preferences, because demand management comes at an earlier stage in the monetary transmission mechanism than does inflation control. The second reason is that there is no jump variable in output in this model. Because of this, there can be no effect of a perceived liberal rule pinning down future output that would then allow a conservative policymaker to pursue inflation control.[29]

This finding has some interesting implications for policy. It can explain, for example, why it is that central bankers tend to talk tough on inflation—even when the individuals that run them are selected from the ranks of people who would be considered liberal. A corollary of this finding is that to the extent that learning is slow, the liberal policymaker can indulge his inclination to manage output without bearing the costs of incipient inflation pressures, and with reduced interest rate variability as well. So while a conservative monetary authority wants to credibly announce regime changes when taking over from a liberal incumbent, the liberal successor wants to conceal his true preferences and possibly forestall private agents' learning as much as possible. This observation may also provide an explanation of why credible central bankers tend to speak ambiguously while not-so-credible ones do not. Credible central bankers (who are generally also conservatives) have the flexibility to indulge themselves in the discretionary pursuit of objectives other than inflation control to the extent that private agents continue to believe that future inflation is pinned down. Strong statements are refutable and are thereby potentially injurious to the credibility that engenders that flexibility, while ambiguous statements are not.

That liberal policymakers gain from being perceived as conservative echoes the theoretical literature on policy games. In that literature, policymakers do not find it beneficial to reveal their true preferences, as in Cukierman and Meltzer (1986) and Vickers (1986) for example. Similarly, in the 'cheap talk' literature exemplified by Stein (1989), the central bank finds that sending vague signals tends to dominate full disclosure as a policy.

[29] We believe, but cannot prove at this point, that this second factor is less important than the first. The candidate jump variable in our output is a permanent shock to the level of total factor productivity (or some similar supply shock) which, to a first approximation, should shift both actual and potential output by similar amounts, leaving the output gap more or less unchanged. If the goal of the liberal authority is to control excess demand, this should be of second-order importance.

4.3 Learning to be optimal

In the preceding section, we laid out the decisions a policymaker of given preferences needs to consider when taking over from a different regime. In doing so, we examined only rules that were optimal, *ex ante*, and we held constant the number of arguments in the policy rule. In this section we study the issue of what bearing the transitional learning costs have for the complexity of rule that is selected. We examine agents who begin with the Taylor rule but then learn that the Fed has shifted to one of our simple optimal rules laid out in table 2.

Of course we could have chosen any of a number of rules that are suboptimal in the context of our model. Our choice reflects the familiarity of the Taylor rule to a large cross-section of economists involved in monetary policy debates. In addition, as Taylor (1993) argues, the Taylor rule approximates Fed behaviour quite well over the 1980s.[30] We could also have studied transitions to rules of a wide variety of complexity, however the results for transitions to 3-parameter rules are very close to those with less parsimonious rules.

We consider first the results for conservative preferences, summarised in table 4 below. For ease of comparison we shade the base-case row—row 3—which in this case is the 3-parameter optimal rule, and normalize its loss to unity. The first panel of the table shows the performance of the economy under the three alternative steady states.[31] The information in this upper panel corresponds with figure 1 above and thus it is not surprising that the Taylor rule's performance (row 1) features substantially higher inflation variability than the conservative base-case rule, but lower variability in output and especially the federal funds rate. The Taylor rule's performance also shows substantially more persistence in inflation and output (not shown) and a higher correlation of output and inflation indicating that policy is working more through the traditional Keynesian channel, rather than through expectations, than in the base-case. From the point of view of a conservative policymaker, the Taylor rule is seen as a very poor performer. Measured in terms of discounted loss, the Taylor rule is less than half as good as the 3-parameter optimal rule. Putting the same performance a different way, the conservative policymaker using the optimal 3-parameter rule would just as soon accept an autonomous increase in the standard deviation of inflation of 1.6—or a whopping 3.2 in the standard deviation of output—as be forced to use the Taylor rule.

Row 2 of the table shows the performance of the optimal 2-parameter rule. Relative to the base-case, there is substantial loss to the conservative monetary authority associated with any requirement to use a 2-parameter rule: The conservative policymaker would be willing to accept an increase in the standard deviation of inflation of about 0.5, or a an increase in standard deviation of output of about 1.0, in order to avoid using the best 2-parameter rule. Looked at in either dimension, these sacrifices have to be considered meaningful. By contrast, the conservative monetary authority would be almost indifferent to using a 4-parameter rule instead of the 3-parameter rule—a result that is not shown in the table. Thus, a significant finding in this paper is that the gains in steady state from

[30] Our quantitative work has shown the Taylor rule both as it is written in Taylor (1993) and as we have written it here, works satisfactorily in a wide range of models.

[31] The loss in the cases where the rule is held fixed are independent of the 'memory.'

Table 4
Learning with ‘conservative’ preferences (average across 400 draws simulated for 200 periods each)

	Rule in use	Standard deviation of: π	y	ΔR	$\rho(\pi, y)$	Welfare loss L
	base-case results					
1.	Taylor rule	3.7	2.3	2.3	.37	2.43
2.	optimal 2 parameter	2.3	2.4	4.8	.28	1.29
3.	optimal 3 parameter	2.0	2.5	3.5	.28	1
	learning with full memory $(\lambda = 1)$					
4.	Taylor - -> 3 parameter	2.8	2.9	4.6	.47	1.94
5.	Taylor - -> 2 parameter	3.1	2.4	5.2	.32	2.02
6.	2 - -> 3 parameter	2.3	2.5	3.5	.29	1.17
	learning with limited memory $(\lambda = 0.95)$					
7.	Taylor - -> 3 parameter	2.2	2.6	3.7	.33	1.37
8.	Taylor - -> 2 parameter	2.5	2.4	4.9	.29	1.53
9.	2 - -> 3 parameter	2.1	2.5	3.4	.29	1.07
	learning with short memory $(\lambda = 0.85)$					
10.	Taylor - -> 3 parameter	n/a	n/a	n/a	n/a	n/a
11.	Taylor - -> 2 parameter	2.4	2.4	4.9	.28	1.43
12.	2 - -> 3 parameter	2.1	2.5	3.4	.29	1.04

Notes: The syntax “n - ->m” refers to the results from learning the transition from the n-parameter simple optimal rule to the m-parameter simple optimal rule. Losses are computed as discounted loss with a discount factor equal to 0.9875.

Notes: The syntax “*n* –>*m*” refers to the results from learning the transition from the *n*-parameter simple optimal rule to the *m*-parameter simple optimal rule. Losses are computed as discounted loss with a discount factor equal to 0.9875.

using even slightly more complex rules than a 2-parameter rule are important—at least for conservative preferences—provided that the rule’s parameters are chosen optimally and that the rule is well understood by the public.[32] The gains vanish for rules that are more complex however: As noted above in section 4.1, as well as by Williams (1999) and Tetlow and von zur Muehlen (1998), there are sharply diminishing returns to increasing complexity. The optimal 2-parameter rule has

32 Tetlow and von zur Muehlen (1996) report a similar finding with another small model as does Williams (1997) with the FRB/US model which has some 300 equations.

the same arguments as the Taylor rule, but as markedly different coefficients, as table 2 shows. Obviously there are large gains to be had from picking rule parameters judiciously.

Now let us consider the decision by a conservative policymaker, of whether to shift from a Taylor rule to the optimal 2-parameter rule as well as to the optimal 3-parameter rule. To separate the effects of optimality from complexity, we also consider moving from the optimal 2-parameter rule to the optimal 3-parameter rule. We conduct these exercises with full memory ($\lambda = 1$ in rows 4 through 6), limited memory (7 to 9), and short memory (11 to 12). The first thing to note about the results is that when agents have to learn about rule changes, short memory is a good thing—at least when the model remains stable: row 11 shows "n/a," which is our indicator that these figures could not be calculated because the model frequently became unstable when the forgetting factor was as low as 0.85. This was not an issue with large values of λ. Nor was it an issue when the initial rule was somewhat closer to the rule to be learned in terms of the underlying preferences or the number of arguments to the rule. We would argue that the lesson to be taken from this result is that profound changes in policy need to be implemented slowly in economies where agents believe that policy change is commonplace.

The loss for the short-memory case—when they are computable—are less than for the corresponding long-memory cases, which in turn are lower than the full-memory cases, regardless of which learning exercise one considers. Since the variability of β varies inversely with λ, this result is not trivial. It would stand to reason that greater variability of the perceived rule parameters in steady state would correspond with higher losses in the steady state. These higher steady-state losses would have to be netted off against the lower transitional losses as the steady state is approached. In fact, the difference in loss in the steady-state loss between the short-memory case and the full-memory case is negligible meaning that, for these cases at least, only the transitional losses matter.

A second observation from table 4 is that so long as the learning process remains stable, the conservative policymaker is always willing to bear the transitional costs of moving from the Taylor rule to either the 2- or 3-parameter optimal rules, regardless of the speed of learning. To see this, compare the loss in row 1 with the losses from any of rows 4 to 12 (except 10). More specifically, for the case of $\psi_{\Delta R} = 0.05$ shown in the table, the authority will always move to the optimal 3-parameter rule notwithstanding the added complexity of learning that rule. This result stems from the substantial difference in steady-state performance between the 2- and 3-parameter rules. As one might expect, however, as $\psi_{\Delta R}$ falls, the performance difference between the 2- and 3-parameter rules also falls and the authority is more likely to prefer a transition to the 2-parameter rule over the 3-parameter rule. Once $\psi_{\Delta R}$ reaches 0.01 or lower, the conservative authority will choose a 2-parameter rule over a 3-parameter one. However, penalties on the change in the federal funds rate this low imply standard deviations of the funds rate that are much larger than what has been seen historically: in the order of 14 versus 3.1 over the period from 1966 to 1998 in the US data. One might therefore question the meaningfulness of these special cases.

Table 5
Learning with 'liberal' preferences (average across 4000 draws simulated for 200 periods each)

#	Rule(s)	Standard deviation of: π	y	ΔR	$\rho(\pi, y)$	Loss L
		base case results				
1.	Taylor rule	3.7	2.3	2.3	.37	1.47
2.	optimal 2 parameter	2.7	2.2	4.1	.24	1.14
3.	optimal 3 parameter	2.4	2.2	3.1	.18	1
		learning with full memory ($\lambda = 1$)				
4.	Taylor - -> 3 parameter	2.8	2.4	3.6	.30	1.34
5.	Taylor - -> 2 parameter	3.2	2.3	4.3	.31	1.42
6.	2 - -> 3 parameter	2.6	2.2	3.2	.23	1.06
		learning with limited memory ($\lambda = 0.95$)				
7.	Taylor - -> 3 parameter	2.5	2.3	2.2	.21	1.13
8.	Taylor - -> 2 parameter	2.8	2.2	4.2	.25	1.23
9.	2 - -> 3 parameter	2.4	2.2	3.1	.19	1.03
		learning with short memory ($\lambda = 0.85$)				
10.	Taylor - -> 3 parameter	2.5	2.2	3.2	.20	1.13
11.	Taylor - -> 2 parameter	2.7	2.2	4.1	.24	1.19
12.	2 - -> 3 parameter	2.4	2.2	3.1	.19	1.02

Notes: The syntax "*n* - ->*m*" refers to the results from learning the transition from the *n*-parameter simple optimal rule to the *m*-parameter simple optimal rule. Losses are computed as discounted loss with a discount factor equal to 0.9875.

Now let us examine the same experiment for liberal preferences, shown in table 5. The Taylor rule is closer, in terms of preferences, to the 2-parameter liberal rule than the conservative one, and so the losses associated with its use are lower. Also, since interest-rate volatility is more a substitute for inflation control than output control, it is comparatively less costly for a liberal to use the 2-parameter rule versus the 3-parameter rule. The major point to be taken, however, is that no matter what the learning rate, the liberal authority will always accept the costs of moving away from the Taylor rule, and will always move to the 3-parameter rule rather than stopping at the 2-parameter rule. Finally, a comparison of rows 4 with 7 and then 10, and 5 with 8 and then 11, shows that the benefits of faster learning decline rapidly—a result that does not hold for the conservative preferences as shown in table 4.

Taken together, the results shown in table 4 and table 5 suggest that for reasonably parsimonious choices of policy rules, and plausible rates of learning, the complexity of the candidate successor rule is not a substantial issue. In all cases, however, the transition costs of learning are considerable and so the more important concern is the careful weighing of preferences—including preferences over instrument volatility—in order to minimise the likelihood of future changes in policy and the concomitant costs of learning.

5.0 Concluding remarks

This paper has examined the implications for the design of monetary policy rules of the need for agents to learn the rule that is put in place. In particular, we took a small New Keynesian macroeconometric model and computed the optimal simple rule for two sets of preferences: 'conservative' preferences, where a substantial weight is placed on inflation control and only a comparatively small weight on output or instrument control, and 'liberal' preferences, where the same substantial weight is placed on output control, and not on inflation or instrument control. Then we compared the stochastic performance of these policies that would have been optimal within a single regime to the cases of transitions. We examined two cases. In one case, we examined the choices faced by a monetary authority that takes over from a predecessor with quite different tastes. In this case, we held constant the number of arguments to the policy rule considered. In the second case, we examined the transition from simple rules to more complex rules, holding tastes constant.

Our four basic findings are: (1) learning should be expected to be a slow process. Even when agents 'forget' the past with extraordinary haste, it takes more than ten years for agents to learn the correct parameters of a new rule. (2) The costs of these perceptual errors can vary widely, depending on the rule that is initially in force, and on the preferences of the monetary authority, but they are generally high. In particular, a conservative monetary authority tends to experience high costs associated with the need for agents to learn the new (conservative) rule. It follows that such a monetary authority should be willing to take steps to make its policy preferences transparent to private agents. Paradoxically, a liberal authority will sometimes benefit from being misperceived, posting a better economic performance than would have been the case if the optimal rule had been in place all along. The sharp contrasts in these results have to do with the multiplicity of sources of shocks to inflation, the nature of inflation in this model being a forward- looking variable, and the fact that inflation appears later in the chain of the monetary policy transmission mechanism than does output. (3) The performance, in steady state, of optimal two-parameter policy rules relative to that of optimal three-parameter policy rules depends in large part on the weight that the monetary authority places on instrument variability. The more adverse the authority is to instrument variability, the more costly it is to use a simple rule: Simply put, increasing the penalty to instrument variability increases the burden on an authority that already has fewer tools (states) at its disposal than would be optimal in a world of full information. This is particularly true for conservative policymakers because of the need for instrument variability to keep inflation variability in close check. (4) While diminishing returns to the complexity of a policy rule set in relatively early in terms of the steady-state performance of that rule, complexity is not an important issue in the context of learning—at least for reasonable preferences. That is, the authority that chooses the simplest rule capable of delivering close to fully optimal performance need not also be concerned with the ability of agents to learn that rule.

This paper has dropped one strong maintained assumption in the traditional analysis of monetary policy rules, namely that the policy rule is known and understood. Our departure from the full-information assumption is admittedly a measured, tentative one. A useful future step would be to consider broader ranges of information deficiency and wider collections of learning rules. Each of these presents challenges. Larger departures from full information run into problems of existence and uniqueness of equilibrium which, while interesting in their own right, are difficult to analyse, much less communicate to policymakers. The problem with alternative learning rules is that neither the theoretical literature nor empirical studies offer any guidance on what is a "reasonable" rule.

We began this paper by noting the wide range of alternative experiments and specifications that we examined in the process of preparing this paper, of which we have mentioned only a small portion. We also conjectured that the central lessons of the paper would be useful in the context of an economy like New Zealand's, where policy change has been a relatively frequent occurrence, and where the existing rule might not be well understood at present. However, the fact that some of the results in this paper depend on either central bank preferences, or on the specifics of model specification, suggests some gains from extensions. The obvious extension to open economies would be useful to explore whether foreign shocks bolster the case for added arguments to the constrained optimal rule—such as the exchange rate, or foreign demand. The importance of these variables to open economies is difficult to overstate. At the same time, however, the exchange rate is subject to such volatility, and subject to such idiosyncratic shocks, that it may be too costly to learn rules that contain them. By the same token, one could investigate the costs of the sort of exchange rate smoothing in which so many small open economies engage for both the steady-state costs of monetary control and for the transitional costs of learning.

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7.0 Appendix: Derivation of optimal rules

A State space representation of the model

In the main body of this paper, the following variable definitions are used: y_t is excess demand, π_t is goods inflation, rs_t is the Federal funds rate, c_t is contract inflation, and rrl_t is the long-term real rate. D is the average duration of a fictitious coupon bond with a real periodic yield, rrl_t. The control variable is denoted u. Equations (A1)-(A4) in the text may be re-written as follows:

$$\pi_t = \delta \pi_{t-1} + (1 - \delta) c_t \tag{1}$$

$$c_t = (1 - \delta)(\pi_t + \gamma y_t) + \delta c_{t+1|t} + u_{\pi,t} \tag{2}$$

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 (rs_{t-1} - \pi_{t-1}) + u_{y,t} \tag{3}$$

$$rrl_t = \left(\frac{1}{1+D}\right)(rs_t - \pi_t) + \left(\frac{D}{1+D}\right) rrl_{t+1|t} \tag{4}$$

where *t + 1,t* denotes the expectation for *t+1* , given information on period *t*. Further, replace the policy rule in the text with an equation defining the control rule,

$$u_t = rs_t - \pi_t \qquad (5)$$

All variables are measured as deviations from equilibrium, implying that their steady-states, including the inflation target, are zero. A state space representation of the above model is obtained as follows. Define the 6x1 state vector, $x_t = [\,z_t\,, q_t\,]'$ where $z_t = [\,rs_{t-1}, y_{t-1}, \pi_{t-1}, y_{t-2}\,]'$ is the 4 x 1 vector of predetermined (inertial) variables in the system, and $q_t = [\,c_t\,, rrl_t\,]'$ is the 2 x 1 vector of forward-looking jump variables. Under the assumption of rational expectations, $c_{t+1,t}$ and $rrl_{t+1,t}$ are consistent with the mathematical expectations of the model's solution obtained by the Anderson and Moore (1995) generalised saddlepath procedure. Also define the vector, $B_0 = [1,0,0,0,0,0]'$, the vector of disturbances, $\varepsilon_t = [0, u_{y,t}, 0, 0, u_{\pi,t}, 0]'$ and two 6 x 6 matrices, A_1 and A_0,

$$A_1 = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -(1-\delta) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{D}{1+D} \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1 & 0 & \phi_2 & 0 & \phi_3 \\ 0 & 0 & \delta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \gamma(1-\delta) & 1-\delta & 0 & 0 & 0 \\ \dfrac{1}{1+D} & 0 & -\dfrac{1}{1+D} & 0 & 0 & -1 \end{bmatrix},$$

Premultiplying by A_1^{-1} the conventional state-space representation of the model becomes,

$$x_{t+1} = Ax_t + Bu_t + \eta_t \qquad (6)$$

where

$$A = \begin{bmatrix} 0 & -\frac{\gamma(1-\delta)^2}{\delta} & \delta - \frac{(1-\delta)^2}{\delta} & 0 & \frac{1-\delta}{\delta} & 0 \\ \phi_s & \phi_1 & -\phi_s & \phi_2 & 0 & \phi_s \\ 0 & -\frac{\gamma(1-\delta)^2}{\delta} & \delta - \frac{(1-\delta)^2}{\delta} & 0 & \frac{1-\delta}{\delta} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-\gamma(1-\delta)}{\delta} & -\frac{1-\delta}{\delta} & 0 & \frac{1}{\delta} & 0 \\ -\frac{1}{D} & 0 & \frac{1}{D} & 0 & 0 & \frac{1+D}{D} \end{bmatrix},$$

and , $B = A_1^{-1} B_0$, $\eta_t = A_1^{-1} \varepsilon_t$. The covariance matrix of the transformed disturbance vector, η is $E\eta_t\eta_t' = \Sigma_\eta = A_1^{-1} \Sigma_\varepsilon (A_1^{-1})'$. It should be noted that there is no contradiction between equation (A6) and equation (7) in the text, where A is understood to represent the transition matrix after control.

Next, define the set of target variables as the output vector, $s_t = [rs_t - rs_{t-1}, y_t, \pi_t]'$ These can be represented as the following mapping from the state vector, x_t, and the control variable u,

$$s_t = Mx_t + M_u u_t, \tag{7}$$

where

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, M_u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Defining the diagonal 3 x 3 performance metric,

$$M = \begin{bmatrix} \psi_{\Delta rs} & 0 & 0 \\ 0 & \psi_y & 0 \\ 0 & 0 & \psi_\pi \end{bmatrix},$$

the expected loss to be minimised is,

$$EW_0 = \frac{1}{2} E \sum_{t=0}^{\infty} \rho^t s_t' \Psi_s s_t,$$

$$= \frac{1}{2(1-\rho)} tr(\Psi_s \Sigma_s) \tag{8}$$

where $0 < \rho \leq 1$ is the discount factor, and Σ_s is the unconditional covariance matrix of s, so that asymptotically, the authority is seeking to minimise a weighted sum of the unconditional variances of the three target variables.

In light of (A7), the expected loss can be re-expressed as a function of the entire state vector, x_t, and the control variable, u_t.

$$EW_0 = \frac{1}{2} \sum_{t=0}^{\infty} \rho^t \left[x_t' \Psi x_t + 2 x_t' U u_t + u_t' R u_t \right] \tag{9}$$

where

$$\Psi = M' \Psi_s M$$
$$U = M' \Psi_s M_u$$
$$R = M_u' \Psi_s M_u$$

Standard optimal control packages assume no discounting, $\rho = 1$, and no crossproducts, $U = 0$. However, a simple transformation of the variables allows us to translate the problem with cross-products and discounting into a conventional optimal control problem. To this end, define,

$$\hat{u}_t = (1-\rho)^{1/2}\rho^{t/2}(u_t + R^{-1}U' x_t)$$

$$\hat{x}_t = (1-\rho)^{1/2}\rho^{t/2} x_t,$$

and observe that

$$\hat{u}_{t'} R \hat{u}_t = (1-\rho)\rho^t [x_{t'} u_{t'}] \begin{bmatrix} U R^{-1} U' & U \\ U' & R \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix},$$

so that

$$(1-\rho)\rho^t [x_{t'} u_{t'}] \begin{bmatrix} \Psi & U' \\ U & R \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix} = \hat{x}_t \overline{\Psi} \hat{x}_t + \hat{u}_{t'} R \hat{u}_t,$$

where $\overline{\Psi} = \Psi - U R^{-1} U'$.

Further defining $\overline{A} = \rho^{1/2}(A - B R^{-1} U')$

$$\overline{B} = \rho^{1/2} B$$

we may rewrite (9)

$$EW_0 = \frac{1}{2}\sum_{t=0}^{\infty}\left[\hat{x}_t' \overline{\Psi} \hat{x}_t + \hat{u}_t' R \hat{u}_t\right] \qquad (10)$$

subject to

$$\hat{x}_{t+1} = \overline{A}\hat{x}_t + \overline{B}\hat{u}_t + \hat{\eta}_t, \qquad (11)$$

where $\hat{\eta}_t = (1-\rho)^{1/2}\rho^{\frac{t+1}{2}}\eta_t$.

B Optimal control

In optimal control, we seek a vector, $\overline{F}$, satisfying

$$\hat{u}_t = -\overline{F}\,\hat{x}_t, \tag{12}$$

that minimises the asymptotic loss (A10) subject to (A11). Substituting (A12) for $\hat{u}_t$ in (A10) and (A11), the expected loss is, equivalently,

$$EW_0 = \frac{1}{2}tr[(\overline{\Psi} + \overline{F}'R\overline{F})\Sigma_x] + tr\left\{S\left[\Sigma_\eta - \Sigma_x + (\overline{A} - \overline{BF})\Sigma_x(\overline{A} - \overline{BF})'\right]\right\}$$

where Σ_x is the asymptotic covariance of x[33]

$$\Sigma_x = \Sigma_\eta + (\overline{A} - \overline{BF})\Sigma_x(\overline{A} - \overline{BF})' \tag{14}$$

and S is the 6 X 6 matrix of Lagrangian variables associated with the constraint (14). Differentiating (13) with respect to $\overline{F}$ and Σ_x, we determine the two equations familiar from the control literature,[34]

$$\overline{F} = [R + \overline{B}'S\overline{B}]^{-1}\overline{B}'S\overline{A}$$

$$S = \hat{\Psi} + (\overline{A} - \overline{BF})'S(\overline{A} - \overline{BF}) + \overline{F}'R\overline{F}.$$

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33 Let $C = \bar{A} - \bar{B}\bar{F}$, so that (11) becomes $\tilde{x}_{t+1} = C\tilde{x}_t + \tilde{\eta}_t$.

Recursively substituting $\tilde{x}$ into itself and utilising the definition of $\tilde{\eta}$,

$$\tilde{x}_{t+k} = C^k\tilde{x}_t + (1-\rho)^{1/2}\sum_{i=1}^{k}\rho^{\frac{i+k-1}{2}}C^{k-i}\eta_{t+i}$$

Thus, in the limit, as k becomes large, the covariance of $\tilde{x}_t$ and x_t are the same:

$$\Sigma_{\tilde{x}} = (1-\rho)\lim_{k\to\infty}\sum_{i=0}^{k}\rho^i C^i\Sigma_\eta(C^i)' = \Sigma_x$$

34 Here we have exploited the facts that

$$\frac{\partial}{\partial B}tr(AB) = \frac{\partial}{\partial B}tr(BA) = A'$$

$$\frac{\partial}{\partial B}tr(B'ABC) = ABC + A'BC'$$

A feedback law for the original state variables, x_t, is retrieved by observing that,

$$u_t = -(\overline{F} + R^{-1}U')x_t \equiv -Fx_t \qquad (15)$$

Formulation of an operational feedback rule is complicated by the fact that the optimal control rule, (A12), as solved, contains the expectational variables, c_t and rrl_t which themselves jump with the selection of the rule. Based on a solution due to Backus and Driffill (1986), one can express the optimal policy as a function of solely the predetermined variables, z, by writing it first as a function of z and the costate variables, p, associated with q,

$$u_t = F H^{-1}\begin{bmatrix} z_t \\ p_t \end{bmatrix},$$

$$\equiv G_1 z_t + G_2 p_t \qquad (16)$$

where

$$H = \begin{bmatrix} I & 0 \\ S_{21} & S_{22} \end{bmatrix},$$

and S_{21} and S_{22} are appropriately-dimensioned partitioned submatrices of S. The indices, '1' and '2' correspond to the predetermined and non-predetermined variables, respectively. Let $T = H(A - BF)H^{-1}$. Then the matrix transition equation for z and p is,

$$\begin{bmatrix} z_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} z_t \\ p_t \end{bmatrix},$$

Accordingly, p can be expressed as a difference equation driven by the predetermined variables, z_t

$$p_{t+1} = T_{22} p_t + T_{21} z_t.$$

Given the policy rule (A16), this can be written,

$$p_t = T_{21} z_{t-1} + T_{22} G_2^{-1}(u_t - G_1 z_t),$$

so that, solving for u_t we obtain

$$u_t = G_2(G_2T_{22}^{-1})^{-1}u_{t-1} + G_1 z_t + G_2[T_{21} - (G_2T_{22}^{-1})^{-1}G_1]z_{t-1},$$

$$= \alpha_u u_{t-1} + \alpha_0 z_t + \alpha_1 z_{t-1} \qquad (17)$$

where α_0 and α_1 are 1x4 vectors corresponding to the dimension of z_t. Note that $(G_2T_{22}^{-1})^{-1}$ is a pseudo-inverse if the number of jumpers, m, exceeds the number of instruments, k, in the model.[35] In the case of this paper, there are two non-predetermined variables and one control variable, so $(G_2T_{22}^{-1})^{-1}$ is a 2x1 vector. Observe also that the above expression posits that in each period, the control setting, u_t is determined by the entire history of the z_t's. Levine (1991) has pointed out that, in this sense, (A17) is, indeed, equivalent to an error-correction rule. The dependence of current policy on the history of the economy, ie, on initial conditions, implies that optimal control for the kind of model used here is time inconsistent.

Since some lags of rs, π, and y appear more than once, we simplify (A17) to obtain the seven-parameter optimal rule referred to in the text,

$$rs_t - \pi_t = \beta_{rr_{t-1}}(rs_{t-1} - \pi_{t-1}) + \beta_{rr_{t-2}}(rs_{t-2} - \pi_{t-2}) + \beta_{\pi_{t-1}}\pi_{t-1} + \beta_{\pi_{t-2}}\pi_{t-2} +$$
$$\beta_{y_{t-1}}y_{t-1} + \beta_{y_{t-2}}y_{t-2} + \beta_{y_{t-3}}y_{t-3},$$

$$= Dw_t \qquad (18)$$

where $w_t = [rs_{t-1} - \pi_{t-1}, rs_{t-2} - \pi_{t-2}, \pi_{t-1}, \pi_{t-2}, y_{t-1}, y_{t-2}, y_{t-3}]$, and D is the vector,

$$\beta_{rr_{t-1}} = \alpha_u + \alpha_{01},$$

[35] A pseudo-inverse of a non-square matrix requires a singular-value decomposition of the matrix to be inverted and can be obtained, for example, with the MATLAB function *pinv.m*.

$$\beta_{rr_{t-2}} = \alpha_{11},$$

$$\beta_{\pi_{t-2}} = \alpha_{11} + \alpha_{13},$$

$$\beta_{y_{t-1}} = \alpha_{02},$$

$$\beta_{y_{t-2}} = \alpha_{04} + \alpha_{12},$$

$$\beta_{y_{t-3}} = \alpha_{14},$$

The notation, *rr*, in the subscripts to the first two parameters indicates that the variables referred to are the "real rate."

C Simple optimal policy

The 'simple' policy rules considered in this paper are versions of (A18), $u_t = \bar{D}w_t = \bar{D}Jx_t$ where J is a 7x6 matrix that maps x_t into w_t and $\bar{D}$ has the same dimension as D in (A18) but may contain elements that are restricted to zero. Define the transfer function, G(L), mapping the disturbances, η_t onto the output vector, s_t : $G(L) = (M + M_u\bar{D}J)[I - (A + B\bar{D}J)L]^{-1}$, where L is the lag operator: $Lx_t = x_{t-1}$. Then, given a selection of k elements in $\bar{D}$ that are allowed to change, an optimal k-parameter rule is determined by constrained optimisation such that $\bar{D}$ satisfies,

$$EW_0 = \min_{\bar{D}} tr\left[G(1)'\Psi_s G(1)\Sigma_\eta\right],$$

subject to

$$s_{t+1} = G(L)\eta_t.$$

The minimum is determined iteratively, using MATLAB's constrained optimisation function, constr.m, where, with each *i*-th trial $\bar{D}^i$, the model is solved backward, using the Anderson and Moore (1985) generalised saddlepath procedure, until a minimum is determined.