Appendix 3A: Estimation and Test Procedures

The generalized method of moments (GMM) estimator and a test of the overidentifying restrictions were programmed in *TSP*. The specific formulas for the estimator and the test are briefly described in this appendix, and a table is presented with the results of the tests of the overidentifying restrictions for the equations of the model.

Each equation of the model can be written in the form

$$y = Z\delta + \varepsilon,$$

where

y = $(T \times 1)$ vector of observations;
Z = $(T \times K)$ matrix of observations;
δ = $(K \times 1)$ vector of parameters to be estimated;
ε = $(T \times 1)$ vector of disturbances.

The GMM estimator of δ is given by

$$\hat{\delta} = (Z'W\hat{\Omega}^{-1}W'Z)^{-1}Z'W\hat{\Omega}^{-1}W'y \tag{1}$$

and an estimate of the covariance matrix of δ is given by

$$\hat{V} = T(Z'W\hat{\Omega}^{-1}W'Z)^{-1}, \tag{2}$$

where

W = $(T \times q)$ matrix of instrumental variables,
$\hat{\Omega}^{-1}$ = $(q \times q)$ consistent estimate of the optimal weighting matrix.

Actual computation of $\hat{\delta}$ and $\hat{V}$ requires calculation of $\hat{\Omega}^{-1}$.

Sargan's Test of Overidentifying Restrictions for Consumption and Investment

Dependent Variable	*Sargan's Test Statistic*	*Degrees of Freedom*	*P Value (%)*
*CD*0	11.23	5	4.71
*CN*0	9.40	5	9.40
*CS*0	8.99	6	17.41
*CD*1	5.32	5	37.82
*CN*1	10.32	5	6.67
*CS*1	7.95	6	24.17
*CD*2	2.89	5	71.64
*CN*2	4.23	6	64.58
*CS*2	6.70	6	34.98
*C*3	9.18	5	10.20
*C*4	6.60	5	25.20
*CD*5	1.94	5	85.75
*CN*5	5.68	6	45.96
*CS*5	6.47	6	37.28
*CD*6	9.40	6	15.22
*CN*6	5.38	5	37.16
*CS*6	8.10	6	23.10
*INE*0	7.28	5	20.07
*INS*0	9.49	5	9.10
*IR*0	9.47	5	9.18
*II*0	8.94	4	6.26
*IF*1	5.64	5	34.31
*II*1	10.06	4	3.94
*IN*2	3.57	6	73.42
*IR*2	1.44	6	96.31
*II*2	3.99	4	40.73
*IF*3	4.85	5	43.47
*II*3	6.90	4	14.13
*IF*4	9.20	5	10.13
*II*4	7.79	4	9.95
*IN*5	5.69	5	33.81
*IR*5	9.66	6	13.99
*II*5	3.24	4	51.91
*IN*6	7.51	5	18.56
*IR*6	11.43	6	7.59
*II*6	7.80	4	9.94

Hansen (1982) showed that the optimal GMM estimator of δ (the consistent estimate with the smallest asymptotic variance) is obtained when the weighting matrix is proportional to the inverse of the variance-covariance matrix

$$\Omega \equiv \mathrm{Var}(T^{-1}W'(y - Z\delta))$$

Newey and West (1987) developed a consistent, positive semidefinite estimate of Ω given by

$$\hat{\Omega} = \hat{R}_0 + \sum_{j=1}^{m} \left[1 - \frac{j}{m+1}\right]\left[\hat{R}_j + \hat{R}_j'\right],$$

where

$$\hat{R}_j = \sum_{t=j+1}^{T} w(t)e(t)e(t-j)w(t-j)',$$

and where

$w(t)$ = $(q \times 1)$ vector, the transpose of the tth row of the matrix W,
$e(t)$ = tth estimated residual from a two-stage least squares procedure,
m = order of the autocorrelation in the disturbance terms.

Note that both $\hat{R}_j$, and therefore $\hat{\Omega}$, are $q \times q$ matrices. Once $\hat{\Omega}$ is obtained, GMM estimates are produced directly from (1) and (2). As the number of intrumental variables q exceeds the number of explanatory variables K, a Sargan test can be used to test the $(q - K)$ overidentifying restrictions (Hansen, 1982). (See table on p. 100.)

Under the null hypothesis that the $(q - K)$ overidentifying restrictions are not binding, the Sargan statistic Q has a chi-square $(q - K)$ distribution, where Q is defined as

$$Q = \frac{1}{T}(y - Z\hat{\delta})'W\hat{\Omega}^{-1}W'(y - Z\hat{\delta}) \overset{A}{\sim} \chi^2_{q-K}.$$

Values for Q for the consumption and investment equations are shown in the following table. These were computed subsequent to the specification and estimation of these equations, and no attempt was made to respecify or change the instruments as a result of these tests. Note that the test fails at the 5-percent level for only two of the thirty-six equations: consumer durables in the United States and inventory investment in Canada.