

Stabilization Policies in a Growing Economy : A Comment ^{1,2}

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I. INTRODUCTION

In a recent paper,³ Professors Stein and Nagatani (SN) have examined monetary stabilization policies in a full employment nonlinear monetary growth model. Specifically, they try to give a "rigorous answer" to the question of which should be the correct control variable of monetary policy; i.e., money supply growth, interest rates, or some other variable. The model used is of the Keynes-Wicksell variety (a name used by Stein to distinguish it from the neoclassical growth models).

The method of stabilization is linear feedback control, where the money supply and/or the interest rate are regulated according to fluctuations in the proportional rate of price change. The method of analysis is to substitute expressions for government regulated money supply and interest rate into the model and see how stability is affected according to the Routh-Hurwitz criterion.⁴ Three general policies are mentioned by SN (proportional, integral, and derivative), but only derivative policy is seriously considered on the basis that no other policies are more stabilizing or less stabilizing.⁵

Using this technique SN arrive at several interesting conclusions, most notably: (1) regulation of the money supply alone cannot affect the stability of the model, (2) a combination of money supply policy and interest rate policy is necessary to affect the stability of the system and (3) using interest rate policy the monetary authorities can control the nominal rate of interest in the long run. These conclusions have obvious policy implications for the current debate over what should be the correct control variable of monetary policy.

This paper argues that these conclusions result from (1) an inconsistent model which does not consider all equilibrium conditions and which has an unrealistic specification of interest rate policy, and (2) a restrictive definition of money supply control. By correcting these deficiencies and explicitly specifying the behaviour of the monetary authorities, we will show how the money supply policy alone can stabilize the system and that a specific interest rate policy is extraneous as a stability tool.

II. THE MODEL

The SN model when regulated by money supply and interest rate policies is repeated below:

$$y = y(x) \quad \dots(27.1)$$

$$R = r(x) \quad \dots(27.2)$$

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² I wish to thank Professors Basil Moore and Bert Hickman for helpful comments, and the referee who discovered and clarified a mistake in the first version.

³ Stein, J. L. and Nagatani, K., Stabilization Policies in a Growing Economy, *Review of Economic Studies* (April 1969).

⁴ The Routh-Hurwitz criterion should be used with some caution in stabilization analysis. Because this criterion does not give qualitative results as to the degree of damping, it may not be the best method of analysis of stabilization problems where one is frequently concerned with smoothing fluctuations in a basically stable economy. The Routh-Hurwitz criterion only distinguishes between stable and unstable systems, without giving information on how effectively a given system is damped.

⁵ This conclusion is based on a comparison of each of the policies separately. However, one should remember that the main purpose of having three policies is to use them together in a complementary way. To keep their study analytically manageable SN do not consider such combinations. This study will follow the same procedure.

$$\begin{aligned}
 S/K &= S(x, w) && \dots(27.3) \\
 I/K &= n+r(x)+\pi^*-\rho && \dots(27.4) \\
 \pi &= \lambda(I/K-S/K) && \dots(27.5) \\
 \pi/\lambda h &= v-L[y(x)+\pi/\lambda, r(x)+\pi^*, \rho, w] && \dots(27.6) \\
 \pi^* &= g(\pi) && \dots(27.7) \\
 DK/K &= aI/K+(1-a)S/K; \quad 1 > a > 0 \text{ when } \pi > 0 && \dots(27.8) \\
 \rho &= \rho_0+q_1\pi && \dots(27.9) \\
 Dx/x &= n-DK/K && \dots(27.10) \\
 Dw/w &= D \ln (A/pK) = \alpha-\pi-DK/K && \dots(27.11) \\
 \alpha &= \alpha_0-\alpha_1\pi. && \dots(27.12)
 \end{aligned}$$

The definitions of variables and parameters are as follows:

- y = output per unit of capital
- x = ratio of "effective" labour N per unit of capital K
- K = stock of real capital
- R = rent per unit of capital
- r = marginal product of capital
- S = real saving
- I = rate of investment
- p = price level
- π = proportional rate of change in prices = Dp/p
- π^* = expected proportional rate of change in prices
- n = growth rate of "effective" labour
- ρ = money rate of interest
- λ = response of price change to excess demand
- L = demand for real balances per unit of capital
- M = nominal money supply
- A = net public debt = outside money + government bonds
- v = M/pK = real balances per unit of capital
- θ = A/M
- w = $\theta v = A/pK$
- h = factor of proportionality between stock excess demand and flow excess demand of real balances
- α = DA/A
- a = coefficient reflecting distribution of output when $\pi > 0$.

Equations (27.9) and (27.12) represent interest rate policy and money supply policy respectively. Both are formulated in terms of the proportional rate of price change. Thus there are twelve equations in twelve unknowns: $x, y, R, S/K, I/K, DK/K, \pi, \pi^*, \rho, w, \alpha, v$. However, as is well known from general equilibrium analysis, equation counting does not guarantee a solution to such a system of simultaneous equations.

With suitable substitutions the basic differential equations of the model can be derived:

$$\begin{aligned}
 \frac{Dx}{x} &= n - \frac{a}{\lambda} \pi(x, w) - S(x, w) = F(x, w) \\
 \frac{Dw}{w} &= \alpha_0 - \left(1 + \alpha_1 + \frac{a}{\lambda} \right) \pi(x, w) - S(x, w) = G(x, w).
 \end{aligned}$$

This is a second order nonlinear system and its local stability characteristics can be examined by Taylor series expansion around equilibrium.

Letting $x_1 = x - x_e$ and $x_2 = w - w_e$ we obtain the linear approximation:

$$\begin{aligned} Dx_1 &= x_e F_1 \cdot x_1 + x_e F_2 \cdot x_2 \\ Dx_2 &= w_e G_1 \cdot x_1 + w_e G_2 \cdot x_2 \end{aligned}$$

where

$$\begin{aligned} F_1 &= -\frac{a}{\lambda} \pi_x - S_x & F_2 &= -\frac{a}{\lambda} \pi_w - S_w \\ G_1 &= -\left(1 + \alpha_1 + \frac{a}{\lambda}\right) \pi_x - S_x & G_2 &= -\left(1 - \alpha_1 + \frac{a}{\lambda}\right) \pi_w - S_w \end{aligned}$$

The Routh-Hurwitz conditions for stability are that

$$x_e F_1 + w_e G_2 < 0 \tag{35a}$$

$$x_e w_e (F_1 G_2 - F_2 G_1) = x_e w_e (1 + \alpha_1) (\pi_w S_x - S_w \pi_x) > 0. \tag{35b}$$

This is the basic model for the SN analysis. Now let us point out where the inconsistency lies.

III. THE INCONSISTENCY AND A CORRECTION

As can be seen from (35b) if S_w is small then the stability of the system depends on the sign of $\frac{\partial \pi}{\partial w} = \pi_w$. Therefore SN examine the important relation for π_w . Substituting into (27.5) we get:

$$\pi/\lambda = n + r(x) + g(\pi) - \rho_0 - q_1 \pi - S(x, w) \tag{32}$$

and differentiating

$$\left(\frac{1}{\lambda} + q_1 - g'\right) d\pi = (r' - S_x) dx - S_w dw \tag{33}$$

which results in:

$$\frac{\partial \pi}{\partial w} = \frac{-S_w}{(1/\lambda - q_1 - g')} = \pi_w \tag{33'}$$

Thus as SN point out, if π_w happens to be negative (which implies instability), the monetary authority can simply raise q_1 until π_w becomes positive, hence stabilizing the system. This is the fundamental way in which interest rate policy affects stability (SN claim that money supply policy cannot do so).

However, in deriving $\frac{\partial \pi}{\partial w}$, SN have ignored the bond market equilibrium equation (27.6). This equation and (27.5) together determine the rate of price change and the rate of interest. After substitution into (27.5) and (27.6) we can write the two equations:

$$\begin{aligned} \frac{\pi}{\lambda} &= n + r(x) + g(\pi) - \rho - S(x, w) \\ \frac{\pi}{\lambda h} &= v - L \left[y(x) + \frac{\pi}{\lambda}, r(x) + g(\pi), \rho, w \right]. \end{aligned} \tag{1*}$$

Both expressions must be taken into account simultaneously in deriving $\frac{\partial \pi}{\partial w}$. By not considering (27.6) SN have only performed a partial analysis.

As we consider these two equations we realize that there is also a mistake in the specification of interest rate policy,

$$\rho = \rho_0 + q_1\pi. \quad \dots(27.9)$$

This seems reasonable until one asks how the monetary authority does it. Unless it has direct control over the rate of interest in the free market (as it sometimes does over the interest rate paid on time and savings deposits) then this specification is incorrect. In the free market economy of (1*) there is no such direct control. Instead, variation of θ is the indirect technique that the monetary authorities follow to affect interest rates. As SN state, "the Monetary Authorities follow a policy of engaging in open market operations; i.e., altering the composition of A (the net financial claims of the private sector upon the public sector, in nominal terms) in order to achieve a money rate of interest ρ ". However, variation of θ will affect not only the interest rate, but also the rate of inflation. Only if θ is varied according to some rule will there be a relation between ρ and π .

This incorrect specification of interest rate policy lead SN to the conclusion that the monetary authority can peg both the nominal and the real rate of interest, contrary to the views of Professor Friedman. However, this conclusion is simply a consequence of the dictatorial nature of their interest rate policy. Thus according to (27.9) in the steady state, $\rho_e = \rho_0 + q_1\pi_e$ where e represents steady state values. But since control of θ affects both π and ρ according to the equilibrium conditions of the model, the monetary authority cannot peg interest rates at a specific level.

Since (27.9) is not tenable, let us respecify an interest rate policy based on what the authorities actually do. If they alter the composition of A (as SN state) then we should write the policy as

$$\theta = \theta_0 + q_1\pi \quad \dots(2^*)$$

where

$$\theta = \frac{M_0 + B}{M_0 + M_I}$$

$M_0 =$ outside money
 $M_I =$ inside money
 $B =$ net private holdings of government bonds.

Now the monetary authority alters the composition of debt according to the rate of inflation. The ultimate effect on interest rates takes place in the free market according to portfolio preferences of individuals.

Adding (2*) to (1*) we get the static system with interest rate policy. Notice that with this formulation we maintain the distinction between v and θ . Now $\frac{\partial\pi}{\partial v}$ is the relevant derivative for stability and can be obtained by differentiating (1*) and (2*) and solving

$$\frac{\partial\pi}{\partial v} = \frac{L_4\theta - L_3S_2\theta - 1}{\left(\frac{1}{\lambda} - g'\right)L_3 - q_1(L_4v - L_3S_2v) - \frac{1}{\lambda h} - \frac{L_1}{\lambda} - L_2g'}. \quad \dots(3^*)$$

This is a simultaneous derivation of $\frac{\partial\pi}{\partial v}$ and shows how the more realistic assumption of monetary authority action eliminates the inconsistency. Note that in the case of instability $\left(\frac{\partial\pi}{\partial v} < 0\right)$ the monetary authority can raise q_1 to make $\frac{\partial\pi}{\partial v}$ positive and the economy stable if $L_4 - L_3S_2 > 0$.

IV. A RECONSIDERATION OF MONEY SUPPLY CONTROL

Within this framework consider the case of money supply control without interest rate control. SN maintain that money supply control cannot affect the sign of π_e and thus

cannot correct instability in the case of negative π_v . However, this is due to an unrealistic assumption about money supply policy. In order to "maintain the logical distinction between the two policies, it is assumed that the composition variable θ is constant when the money supply is the control variable". However, it is the contention of this paper that the assumption of a constant θ blurs rather than maintains the distinction. In order to keep θ constant when the money supply is increased, the monetary authority must take part in active open market operations or some form of debt management policy. Thus a constant θ requires a precise mix between money supply policy and θ policy (i.e., interest rate policy).

To see this consider again what the monetary authorities must do to keep θ constant when M increases. If we assume that M_0/M_1 is constant, then

$$\theta = \frac{B}{M} \left(\frac{\dot{B}}{B} - \frac{\dot{M}}{M} \right) = \frac{B}{M} (v - \mu).$$

Now, if θ is to be constant then the monetary authorities must keep the rate of increase on bonds equal to the rate of increase in money. This requires either open market sales or new issuing of bonds, either of which constitute active interest rate policy in the sense of SN. These open market operations are set up largely to counteract the usual supposed effects of money supply policy. It is small wonder that such a restricted policy does not affect stability.

An alternative hypothesis which seems to maintain the logical distinction between the two policies is that the monetary authorities do not perform any countercyclical open market operations or debt management policies. Such an assumption requires that $v = \dot{B}/B = v_0$ is exogenous, perhaps determined by other government constraints. This is more accurately a pure money supply policy. Thus we have $\mu = \mu_0 - q_2\pi$ as the money supply policy. With $v = v_0$ this results in:

$$\dot{\theta} = \frac{B}{M} (v_0 - \mu) = \frac{B}{M} (v_0 - \mu_0 + q_2\pi). \quad \dots(4^*)$$

Under this assumption the monetary authorities can affect the sign of π_v and thus can affect stability; this can be seen by examining (3*). The same expression will hold in this case if we replace

$$\theta = \theta_0 + q_1\pi \text{ with } \theta = \theta_0 + \frac{B}{M} (v_0 - \mu_0)\pi + q_2 \int_0^t \pi d\tau.$$

We can see that the monetary authority can change the sign of π_v by increasing q_2 .

An example will help to explain the economics behind this result. Suppose an increase in the rate of price change signals the monetary authority to decrease the rate of monetary expansion according to (4*). This will upset the existing desired portfolios of individuals who will react by moving from bonds and commodities into money. The result is to reduce demand. Thus, even if the effect of price expectations on demand is strong enough to make the economy unstable $\left(\frac{\partial \pi}{\partial v} < 0 \right)$, q_2 can be made large enough to offset this effect and lead to stability. Therefore, the pure money supply policy can give all the stability affects of interest rate policy.

Further, from this last statement it is easily argued that the addition of an active interest rate policy (except for $v = \mu$), will not give additional stability advantages. Whatever the combination policy can do, the money supply policy can do alone.

Thus we have shown that the main conclusions of SN do not hold and that they have not really answered the question they set out to answer; namely, what should be the control variable of monetary policy. However, rather than concluding that the monetary authorities should be indifferent between money supply policy and interest rate policy, future research might be concerned with criteria other than stability.