

Aggregate Dynamics and Staggered Contracts

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Staggered wage contracts as short as 1 year are shown to be capable of generating the type of unemployment persistence which has been observed during postwar business cycles in the United States. A contract multiplier causes business cycles to persist beyond the length of the longest contract, and a diffusion of shocks across contracts causes the persistence to increase for several periods before diminishing. A persistence of inflation is also generated by the contracts. This persistence is represented as a reduced-form distributed-lag wage equation in which the lag coefficients have a pure-expectations component and an inertia component due to the overhang of outstanding contracts. Using rational expectations to separate these components suggests that aggregate demand may have a greater impact on inflation than the simple reduced-form estimates would indicate.

A distinctive feature of recent theoretical models of macroeconomic fluctuations is the emphasis on partial rigidities, either in the form of information lags or temporary inflexibility of prices and wages. These rigidities have been remarkably successful in explaining observed correlations between such aggregates as inflation and unemployment, despite the constraints of rational expectations and a fixed natural rate of unemployment. Indeed, statistical Phillips curves are an essential property of these models.¹

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¹ Models which have stressed informational rigidities include Lucas (1973, 1975), Sargent and Wallace (1975), and Barro (1976, 1978). Models which have stressed wage or price rigidities include Fischer (1977) and Phelps and Taylor (1977). A recent paper by Hall (1977) incorporates such rigidities but leaves open whether they are informa-

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As is well known, however, these models have been unable to explain the observed serial correlation in unemployment without the imposition of additional sources of persistence—either exogenous serial correlation of shocks or capital formation lags. Neither informational rigidities nor temporary wage and price contracts, therefore, have seemed capable of independently generating the kind of serial persistence observed in most modern economies.

This paper considers a rational expectations model in which wage contracts are the only source of rigidity, yet which is capable of endogenously generating serial correlation in unemployment which significantly outlasts the duration of the longest contract. Hence, contracts which last only about 1 year can generate the degree of cyclical persistence which has been observed in the United States during the postwar period. Two key assumptions underlie the model:² (1) wage contracts are staggered, that is, not all wage decisions in the economy are made at the same time; and (2) when making wage decisions, firms (and unions) look at the wage rates which are set at other firms and which will be in effect during their own contract period. Because of the staggering, some firms will have established their wage rates prior to the current negotiations, but others will establish their wage rates in future periods. Hence, when considering relative wages, firms and unions must look both forward and backward in time to see what other workers will be paid during their own contract period. In effect, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another—a sort of “contract multiplier.” In statistical terms the overlapping contracts cause unemployment to follow a mixed autoregressive–moving-average process, rather than the relatively low-order moving-average process found in previous contract models. The mixed process implies that the impact of shocks on unemployment will first rise for several periods before decreasing toward zero—a lag shape which is characteristic of the unemployment rate in the United States.

The concern of this paper, however, is not solely with endogenous persistence of unemployment. As will be shown below, contract for-

tion or contract based. The lack of persistence in models with contract-based rigidities has not been emphasized as much as in models based on information lags. An examination of the latter three papers will indicate, however, that persistence of unemployment is very short without exogenous serial correlation. Sargent (1977) and Fischer (1979) are two recent papers which examine this persistence problem.

² The 2-period version of this contract model was suggested as a device to explain the existence of lagged inflation rates in a rational expectations Phillips curve by Taylor (1979). The implications of a multiperiod version of the contract equation for the design of guideposts to reduce inflation while maintaining full employment are discussed in Phelps (1978). Further references to similar types of models are given in Phelps (1978).

mation in this model generates an inertia of wages which parallels the persistence of unemployment. The econometric specification of this inertia is a wage equation which includes a distributed lag of past wage rates—much like an expectations-augmented Phillips curve. In contrast to other Phillips curves, however, the distributed lag of past wages incorporates not only the expectations of future wage decisions but also the overhang of previous wage decisions. Given the assumptions of the model, this lag shape has a predictable form: it declines steadily over the length of the lag and at a decreasing rate. Moreover, the lag shape depends on economic policy. A more accommodative monetary policy, for example, tends to increase the sum of the lag weights and thereby increase the serial persistence of wages.

This dependence of parameters on policy, a property which has been emphasized by Lucas (1976) in a related context, has both econometric and policy implications. It suggests that it will be difficult to estimate the structural parameters of the model without a knowledge of the aggregate-demand policy rule. It also suggests that the Phillips-curve policy trade-off will depend on expectations of policy in future periods, a point which has been discussed by Fellner (1976). In particular, because wage determination is both forward and backward looking, aggregate demand may have a greater effect on inflation than current models of the Phillips curve would suggest. One advantage of the model specified here is that it permits one to calculate the size of this expectations effect.

Section I introduces the structural model, and Section II derives the rational expectations reduced-form contract equations. Because of certain symmetries in the contract equations and the policy rule, we are able to make use of factorization theorems which frequently arise in time-series analysis to derive explicit relationships among the reduced-form parameters, the structural parameters, and the policy rule. The spectral density function of the contract wages is derived and shown to be a convenient way to describe their stochastic properties when contracts are fairly long. Section III describes the persistence effects generated by the model with 2-, 3-, or 4-quarter contracts and compares these with the actual persistence of unemployment in the United States. In Section IV the effect of aggregate demand on wages, as implied by a particular policy rule, is compared with the conventional short-run Phillips-curve approach for a certain set of parameter values. This comparison provides a way of separating the impact of wage expectations from pure wage inertia.

I. The Contract Determination Equation

As mentioned above, overlapping contracts are a key assumption behind the persistence effects generated by this model. And while

price contracts are potentially as important as wage contracts, we will focus on wage contracts in this paper. We consider an economy in which all wage contracts are N periods long and a constant fraction $1/N$ of all firms determine their wage contracts in any given time period.³ A contract is assumed to specify (implicitly or explicitly) a fixed nominal wage rate which will apply for the duration of the contract; employment is then determined by fluctuations in the demand for labor, given this nominal wage during the contract period. Let x_t be the logarithm of the wage rate specified in contracts beginning in period t . Then x_t , which applies to $1/N$ of the firms, will remain in effect until period $t + N - 1$, when it will be changed to x_{t+N} . (While for some purposes it might be useful to attach another index to x_t in order to represent a particular group of firms, such a notation would be cumbersome and does not directly relate to the results presented here. It should be noted, however, that while x_t and x_{t+N} refer to the same group of firms, x_t and x_{t+s} for $s < N$ refer to different groups of firms.)

If contracts are set in this way, they will clearly overlap each other. At the time that a given wage contract is in the process of being set, there will be an overhang of contracts set in the last $N - 1$ periods which will still be in effect during part of the current contract period. Moreover, during the next $N - 1$ periods, contracts will be written which will also be in effect during part of the current contract period. Wage rates set in the current period should reflect the wage rates set in these previous and future contracts. They serve as a base for determining the relative wage of the current contract. A simple and plausible wage-setting procedure which weights other wage rates proportionally to the number of periods they overlap with the current contract period, and which is sensitive to excess demand in the labor market, is then given by the log-linear form

$$x_t = \sum_{s=1}^{N-1} b_s x_{t-s} + \sum_{s=1}^{N-1} b_s \hat{x}_{t+s} + \frac{h}{N} \sum_{s=0}^{N-1} \hat{e}_{t+s} + \epsilon_t, \quad (1)$$

where e_t is a measure of excess demand in the labor market ($e_t = 0$ represents full employment), h is a positive parameter, and ϵ_t is a random shock, which will be assumed to be serially uncorrelated. (The "hat" over a variable represents the mathematical conditional expectation operator, given information through time $t - 1$.) The weights on the future and lagged contract wages are given by $b_s = (N$

³ This uniform distribution assumption is made for simplicity. A nonuniform distribution (as long as it is not degenerate with all contracts set in 1 period) would introduce seasonal effects which, while important from a practical point of view, would not alter the general qualitative features of the model. In this regard it is interesting to note that the *Economist* has recently suggested that the distribution of contract negotiation in the United Kingdom be made less uniform, presumably to reduce overlapping.

$-1)^{-1}(1-s/N)$, $s = 1, 2, \dots, N-1$. They decline linearly into the past and future, and they sum to one. Contracts close to the current contract are given most weight, while contracts in the more distant past or future are given less weight. The symmetric linear decline is a result of our assumption that these past and future wages are weighted proportionally to the number of periods they overlap with x_t . For example, if contracts are N periods long, then \hat{x}_{t+s} and x_{t-s} , for $s < N$, will overlap with x_t for $N-s$ periods each. The total number of overlapping periods for all contracts is, therefore, $2 \sum_{s=1}^{N-1} (N-s) = (N-1)N$. The contracts \hat{x}_{t+s} and x_{t-s} should therefore each be weighted by $N-s$ divided by $(N-1)N$, which defines the b -weights. The proportional weighting scheme does not allow for either discounting effects which would lead to a sharper decline on future weights or forgetting effects which would lead to a sharper decline on past weights.

In addition to past and future wages, equation (1) indicates that the current contract wage will be sensitive to expected labor market conditions during the contract period; that is, x_t will respond to $\hat{\ell}_t, \hat{\ell}_{t+1}, \dots, \hat{\ell}_{t+N-1}$. Equation (1) implies that all of these periods are weighted equally and with a weight $1/N$. (Of course, $1/N$ could be incorporated in the parameter h , but we leave it explicit so that excess-demand effects can be held constant when we consider changes in the contract length N .) It should be noted that only expected and not actual excess demand in period t enters equation (1); it would be a relatively easy matter to include current e_t in equation (1) and thereby permit the wage contract to react simultaneously to actual market conditions in the current period.

Our assumption that the contract wage rate x_t is fixed for the entire length of the contract implies heavy front-end loading. That is, the entire wage adjustment occurs in the first period. Available information on explicit long-term union contracts indicates that front-end loading is not generally this extreme. However, casual observation suggests that most implicit contracts, which appear to be about 1 year in duration, are front-end loaded in this way.

In order to obtain a solution for x_t from equation (1), it is necessary to model the determinants of the excess demand for labor e_t and the relationship between contract wages and prices. A very simple model which achieves this end is given by

$$y_t + p_t = m_t + v_t, \quad (2)$$

$$p_t = \frac{1}{N} \sum_{i=0}^{N-1} x_{t-i}, \quad (3)$$

$$e_t = g_2 y_t, \quad (4)$$

$$m_t = g_3 p_t, \quad (5)$$

where p_t = log of aggregate price level, y_t = log of real output less log of full-employment output, m_t = log of nominal money balances less the log of full-employment money balances, and v_t = random velocity shock. Equation (2) is a simple quantity-theory representation of aggregate demand. It is written in deviation form; hence, $y_t = 0$ represents full-employment output. The assumption that the elasticity of real balances with respect to output is one introduces no loss of generality, as will become clear below. The log of velocity v_t is assumed to be a serially uncorrelated shock; this will highlight other persistence effects of the model.

Equation (3) states that the aggregate price level is determined by a simple proportional markup over the average wage; hence, we abstract from the important problem of real wage and productivity changes. The term $N^{-1} \sum_{i=0}^{N-1} x_{t-i}$ is the logarithm of the geometric average of the contract wages in effect at time t . Equation (4) is a simple production relationship which states that excess demand for labor is a simple proportion of the deviation of output from trend or full-employment output.

Equation (5) is the policy rule; $g_3 = 0$ corresponds to a fixed money supply while $g_3 > 0$ allows for some accommodating increase in the money supply in response to price increases. By substituting (4) into (1) and (5) into (2), we arrive at a simple two-equation representation of the model:

$$x_t = \sum_{s=1}^{N-1} b_s x_{t-s} + \sum_{s=1}^{N-1} b_s \hat{x}_{t+s} + \frac{\gamma}{N} \sum_{s=0}^{N-1} \hat{y}_{t+s} + \epsilon_t, \quad (6)$$

$$y_t = -\beta p_t + v_t, \quad (7)$$

where $\gamma = hg_2$ and $\beta = (1 - g_3)$. The parameter γ is the major structural parameter of the model. It represents the sensitivity of wages to aggregate-demand policy. The policy parameter β measures how accommodative aggregate-demand policy is to changes in the price level from its long-run equilibrium level. We now turn to the derivation of the rational expectations solution of the model.

II. The Reduced-Form Contract Equation

Derivation

In order to eliminate the expectation variables in equation (6), we make use of the aggregate-demand policy rule (eq. [7]) and the definition of the aggregate price level in equation (3). That is, we substitute the conditional expectation of output

$$\hat{y}_{t+s} = -\beta \hat{p}_{t+s} = -\frac{\beta}{N} \sum_{i=0}^n \hat{x}_{t+s-i} \quad (8)$$

into equation (6) to obtain

$$x_t = \sum_{s=1}^n b_s x_{t-s} + \sum_{s=1}^n b_s \hat{x}_{t+s} - \frac{\beta\gamma}{N^2} \sum_{s=0}^n \sum_{i=0}^n \hat{x}_{t+s-i} + \epsilon_t, \quad (9)$$

where $n = N - 1$. Equation (9) involves only the log of the contract wage x_t and its expectations. It states that the current contract will be influenced by relative wage effects (represented by the first two sums in eq. [9]) and also by the impact of the wage settlements on aggregate demand, as implied by the policy rule (this influence is represented by the double sum in eq. [9]).

Taking expectations on both sides of equation (9), conditional on information available in period $t - 1$, and noting the identity,⁴

$$\frac{1}{N^2} \sum_{s=0}^n \sum_{i=0}^n \hat{x}_{t+s-i} = \frac{n}{N} \sum_{s=1}^n b_s \hat{x}_{t-s} + \frac{\hat{x}_t}{N} + \frac{n}{N} \sum_{s=1}^n b_s \hat{x}_{t+s}, \quad (10)$$

where the b -weights are as in Section I, we have

$$\left(1 + \frac{\beta\gamma}{N}\right) \hat{x}_t = \left(1 - \frac{n\beta\gamma}{N}\right) \sum_{s=1}^n b_s \hat{x}_{t-s} + \left(1 - \frac{n\beta\gamma}{N}\right) \sum_{s=1}^n b_s \hat{x}_{t+s}. \quad (11)$$

Dividing through by $[1 - (n\beta\gamma/N)]$ and rearranging terms reduces equation (11) to

$$\sum_{s=1}^n b_s \hat{x}_{t-s} - c \hat{x}_t + \sum_{s=1}^n b_s \hat{x}_{t+s} = 0, \quad (12)$$

where $c = (N + \beta\gamma)/(N - n\beta\gamma)$. Note that c is the only parameter of equation (12) which depends on either the policy or the structural parameters of the model. We assume that $0 \leq \beta\gamma$ so that $|c| \geq 1$. Using the lag-operator notation ($L^s x_t = x_{t-s}$), and defining the polynomial

$$B(L) = \sum_{s=-n}^n b_s L^s, \quad (13)$$

where $b_0 = -c$ and $b_{-s} = b_s$, $s = 1, 2, \dots, n$, equation (12) can be rewritten

$$B(L) \hat{x}_t = 0. \quad (14)$$

Obtaining a unique rational expectations solution to equation (14) involves some technical considerations. The polynomial $B(L)$ has negative as well as positive powers of L ; however, because of its symmetry ($b_s = b_{-s}$), it can be factored into a product of a polynomial in L and the same polynomial in L^{-1} ; that is,

$$B(L) = \lambda A(L)A(L^{-1}), \quad (15)$$

⁴ This identity is easily established using induction. Note that N times eq. (10) is an average of a sum, that is, a Cesaro sum in the contract wage.

where λ is a normalization constant and

$$A(L) = \sum_{s=0}^n \alpha_s L^s, \quad (16)$$

with $\alpha_0 = 1$. The canonical representation in equation (15) follows directly from factorization theorems for symmetric polynomials (see Anderson 1971, p. 224, e.g.), which frequently arise in the theory of stochastic processes. (It may be helpful to note that the problem of obtaining the factorization in eq. [15] is identical with the problem of obtaining the moving-average representation of a stochastic process given its autocovariance function or correlogram. The polynomial [13] would be the autocovariance-generating function and the coefficient of $A[L]$ would be the coefficients of the moving-average representation.)

From the factorization (15) we can obtain a unique rational expectations solution to (14) by imposing the usual stability condition and thereby choosing the polynomial $A(L)$ which corresponds to the roots of $B(L)$ that lie outside the unit circle.⁵ (Again, the analogy from stochastic processes is helpful here: our assumption of stability corresponds to the assumption of invertibility of a moving-average process, which is enough to give a unique representation.) Having chosen $A(L)$ in this fashion, we can then divide equation (14) by $\lambda A(L^{-1})$ to obtain

$$A(L)\hat{x}_t = 0. \quad (17)$$

A comparison of (17) with (9) indicates that the rational expectations reduced-form stochastic difference equation for the contract wage is

$$A(L)x_t = \epsilon_t, \quad (18)$$

or, writing the lagged contracts on the right-hand side of the equation,

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \dots + a_n x_{t-n} + \epsilon_t, \quad (19)$$

where $a_i = -\alpha_i$, $i = 1, \dots, n$.

Note that there is an explicit set of constraints which the structural parameters γ and β , working through the parameter $b_0 = -c$, put on this reduced-form equation. These constraints can be derived by equating the coefficients of the polynomials on the left- and right-hand side of (15) to obtain:

⁵ Let λ_i , $i = 1, \dots, n$ be the roots of $B(L)$ which lie outside or on the unit circle; then λ_i^{-1} are the roots of $B(L)$ which lie within or on the unit circle. The coefficients of $A(L)$ are therefore equal to the coefficients of the corresponding powers of L in $\prod_{i=1}^n (L - \lambda_i)$.

$$\begin{aligned}
-c &= \lambda(1 + \alpha_1^2 + \alpha_2^2 + \dots + \alpha_n^2) \\
b_1 &= \lambda(\alpha_1 + \alpha_1\alpha_2 + \alpha_2\alpha_3 + \dots + \alpha_{n-1}\alpha_n) \\
b_2 &= \lambda(\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_4 + \dots + \alpha_{n-2}\alpha_n) \\
&\vdots \\
&\vdots \\
&\vdots \\
b_{n-1} &= \lambda(\alpha_{n-1} + \alpha_1\alpha_n) \\
b_n &= \lambda\alpha_n,
\end{aligned} \tag{20}$$

or, more compactly,

$$b_s = \lambda \sum_{u=0}^{n-s} \alpha_u \alpha_{u+s}, \quad s = 0, 1, 2, \dots, n.$$

There is only one structural parameter (c) in these equations; hence, regardless of the number of lag coefficients α_i , there is only 1 degree of freedom; only the product of γ and β matters. If c is known, then the entire lag distribution is known.

The relationship between the reduced-form weights and the structural lag and lead weights is shown in figure 1 for the case of $c = 1$, the full-employment rule. Note that in this case both the b -weights and the a -weights sum to one. Since the reduced form can only be backward looking, it must capture both the backward-looking and forward-looking features of the structural form. Hence, the sum of the lag weights for the reduced form is twice as large as the sum of the lag weights for the structural form. The area between the reduced-form lag weights and the structural lag weights in figure 1 represents the component of the reduced form which is due to expectational effects, while the remaining component represents pure wage-inertia effects. As will be shown below, policy can affect the reduced-form lag weights by altering the expectations component. For example, a less accommodative policy can reduce the expectations component and even make it negative; that is, policy can push the reduced-form lag weights below the structural lag weights.

It is interesting to note that the expectations component is much larger for short lags than for longer lags. This is a property of the optimality of the rational forecasts. Hence, the reduced-form lag declines much more quickly than the structural form.

A striking characteristic of the reduced-form contract-wage equation (19) is that it does not include a measure of excess demand. The impact of excess demand on wage behavior is captured in the lag coefficients. If we had assumed *actual* excess demand influences wage behavior along with *expected* excess demand in the basic structural

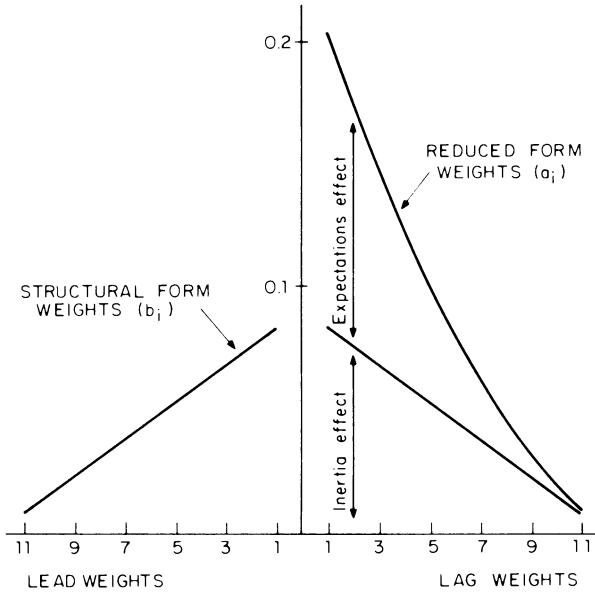


FIG. 1.—Relationship between structural contract wage equation and reduced form

equation (1), then excess-demand effects would be visible in equation (20). But even then, only unexpected excess demand would matter. Although the aggregate-demand side of this model is very simple, this feature of the wage equation is suggestive of the difficulties which are inherent in econometric attempts to estimate the impact of aggregate demand on wage behavior. If the unemployment rate were inserted on the right-hand side of a regression equation similar to (19), the estimated coefficient of the unemployment rate would have a probability limit of zero.

Shape of the Lag Distribution

The constraints in equation (20) indicate that the wage equation is sensitive to policy and the length of contracts. In order to illustrate how the lag shape depends on policy and on contract length, we have tabulated the theoretical lag distribution for 3-, 4-, and 12-period contracts and for alternative values of $\gamma\beta$. These are shown in table 1 and figure 2. Note that the lag coefficients are all positive and decline at a decreasing rate for all contract lengths and all values of $\beta\gamma$. This convexity is a result of the linearity of the structural weights. With this type of autoregressive lag shape, one would not expect to find overshooting effects; but, since the stochastic difference equation has

TABLE 1
THEORETICAL LAG DISTRIBUTIONS FOR ALTERNATIVE CONTRACT LENGTHS

$\beta\gamma =$.0	.1	.2	.3
3-Period Contracts				
a_1	.732	.464	.375	.313
a_2	.268	.188	.158	.135
4-Period Contracts				
a_1	.572	.361	.290	.240
a_2	.308	.210	.173	.146
a_3	.120	.089	.076	.065
12-Period Contracts				
a_1	.206	.128	.101	.082
a_2	.176	.113	.090	.073
a_3	.149	.098	.079	.064
a_4	.123	.083	.068	.056
a_5	.101	.070	.057	.047
a_6	.080	.057	.047	.039
a_7	.062	.045	.038	.032
a_8	.045	.034	.029	.025
a_9	.032	.024	.021	.018
a_{10}	.019	.015	.013	.011
a_{11}	.009	.007	.006	.005

complex roots, we should also examine the spectral density function for signs of cyclical behavior. This is done in the following section.

As policy gets less accommodative (β increases), or as the sensitivity of wages to excess demand increases (γ increases), the lag distribution is pushed toward zero. While the sum of the lag weights is one when $\beta\gamma = 0$, it is less than one when $\beta\gamma > 0$. Hence, a larger $\beta\gamma$ tends to reduce the serial correlation of wages.

Spectral Density of the Contract Wage

A convenient way to analyze the behavior of the contract wage is through its spectral density, which has a simple analytic representation. According to equation (18), x_t follows an n th-order autoregression. Hence, if σ^2 is the variance of ϵ_t , then the spectral density of x_t is given by

$$f(\omega) = \frac{\sigma^2}{2\pi} \left| \sum_{s=0}^n \alpha_s e^{i\omega(n-s)} \right|^{-2} \quad (-\pi < \omega < \pi). \quad (21)$$

The advantage of looking at the spectral density of x_t is that the squared modulus of the complex sum in (21) involves the same non-

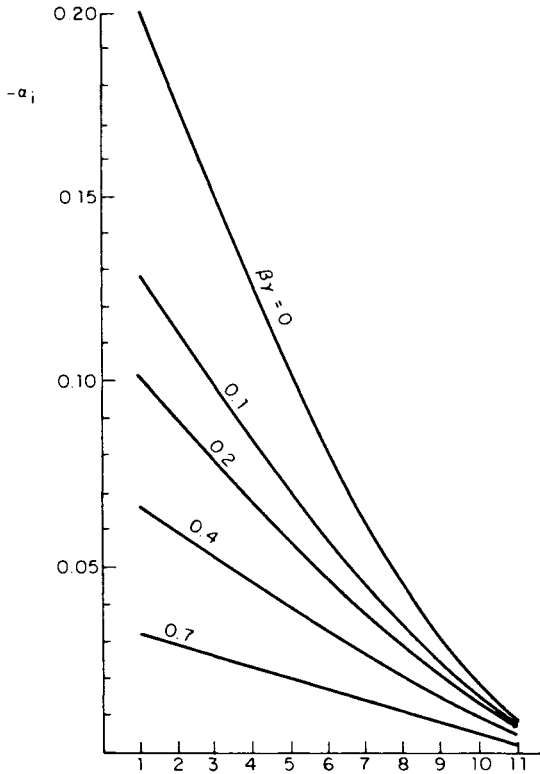


FIG. 2.—Lag coefficients of the contract equation

linear functions of the α coefficients that were found on the right-hand side of (20). Hence, we can utilize the formulas in (20) to obtain:

$$\begin{aligned}
 f(\omega) &= \frac{\sigma^2}{2\pi} \left[\frac{b_0}{\lambda} + \frac{2}{\lambda n} \sum_{s=1}^n \left(1 - \frac{s}{N} \right) \cos s\omega \right]^{-1} \\
 &= \frac{\sigma^2}{2\pi} \left[\frac{b_0}{\lambda} - \frac{1}{\lambda n} + \frac{2}{\lambda n} \left(\frac{\sin^2 \frac{1}{2}\omega N}{2N \sin^2 (\frac{1}{2}\omega)} \right) \right]^{-1}.
 \end{aligned} \tag{22}$$

The second equality in (20) follows from a trigonometric identity which arises in the analysis of Fourier series (see Lanczos 1966, p. 56). The expression $(\sin^2 \frac{1}{2}\omega N)(2\pi N \sin^2 \frac{1}{2}\omega)^{-1}$ is known as Fejer's kernel and frequently occurs in studies of the serial correlation properties of averages.⁶

We know from the first equation of (20) that $\lambda < 0$; hence, $f(\omega)$ will have maxima and minima at the same values of ω as Fejer's kernel.

⁶ See Anderson (1971, pp. 508–9) for a description of the qualitative features of Fejer's kernel. We make use of these features in analyzing eq. (22).

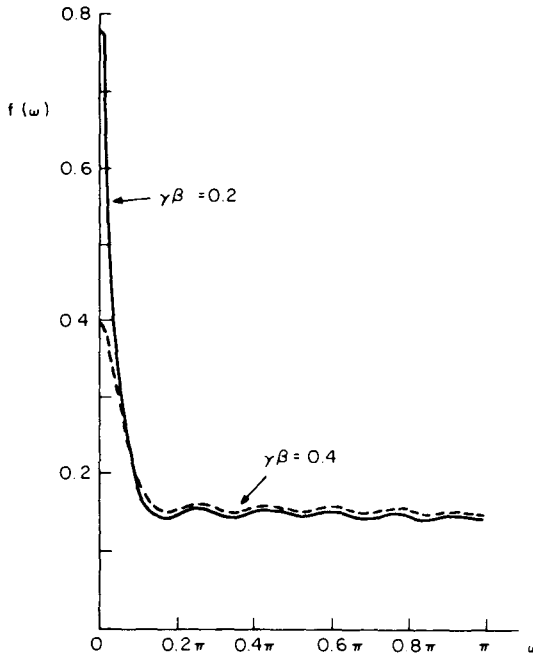


FIG. 3.—Spectral density of 12-period contract equation

Considering only the positive ω , the spectral density function has a maximum at $\omega = 0$, relative minima at $k\pi/N$, for $k = 2, 4, 6, \dots$, and relative maxima approximately at $\omega = k\pi/N$, for $k = 1, 3, 5, \dots$. If N is odd, then π is a relative maximum. The locations of these relative maxima and minima remain the same for all values of the parameter c , that is, for all policy rules. A sketch of the spectral density function for two values of $\beta\gamma$ is shown in figure 3. Note that as policy becomes less accommodative, the area under the curve is reduced, indicating the variance of the contract wage is reduced. However, the reduction in variance occurs only at low frequencies, and there is a slight increase in the variance at the higher frequencies.

The relative maxima of the spectral density in the higher frequencies are small compared with the maximum at $\omega = 0$. Hence, the contract wage has the typical spectral shape of an economic time series: no sharp peaks occur anywhere except at the zero frequency. In this sense, the persistence effects of the overlapping contracts—given the policy rule and our assumption of linearly declining weights in the structural equation—would not be expected to generate overshooting endogenously. As is shown below, however, the aggregate wage and price level follows an autoregressive–moving-average process which can have the appearance of overshooting: the impact of a

shock on the wage level can increase from its initial value for a number of periods before diminishing toward zero.

III. Aggregate Dynamics and the Persistence of Unemployment

Equations describing the macroeconomic dynamics of the model can be derived by aggregating the contract wages to determine the aggregate wage and price level. Let

$$D(L) = \frac{1}{N} \sum_{s=0}^n L^s \quad (23)$$

be the unweighted moving-average operator. Then, from the definition of the aggregate price level in equation (3) of Section I,

$$p_t = D(L)x_t. \quad (24)$$

Substitution of (24) into the contract wage equation (19) yields the aggregate price equation

$$A(L)p_t = D(L)\epsilon_t. \quad (25)$$

Hence, the price level follows an autoregressive–moving-average process, ARMA (n, n). The autoregressive part of the process is the same as that of the contract equation (19). In a deterministic perfect foresight version of this model, the aggregate equation would therefore be described by the same difference equation as the contract wages. Given equation (25), the behavior of aggregate output can easily be determined from the aggregate-demand equation (7), that is,

$$y_t = -\beta p_t + v_t. \quad (26)$$

Aggregate output, therefore, has the same basic stochastic structure as the price level. Of course, the realizations of output will look much different than those of the price level. The larger β is, the larger the output fluctuations relative to price fluctuations will be. And, except for the influence of velocity shocks, output and prices will tend to move in opposite directions. This result follows directly from our simple quantity theory of aggregate demand combined with a fixed rule for money-supply behavior. In the extreme case of the fixed and known money supply ($\beta = 1$), real output must fall by the same proportion that the price level rises.

Persistence Effects

The potential for this model to exhibit high serial correlation in unemployment is evident in equations (25) and (26). Recall that the unemployment rate is assumed to be proportional to y_t . The autore-

gressive terms in equation (25) prevent the serial correlation between y_t and y_{t+s} from hitting zero as soon as s is greater than the contract length. Instead, the correlation between y_t and y_{t+s} diminishes gradually and approaches zero only asymptotically as $s \rightarrow \infty$. If the sum of the autoregressive coefficients in (25) is large, then the serial correlation will remain high for very long lags.

These general features can be illustrated in the case of two-period contracts ($N = 2$). In this case, equation (25) becomes

$$p_t = a_1 p_{t-1} + \frac{\epsilon_t}{2} + \frac{\epsilon_{t-1}}{2}, \quad 0 \leq a_1 \leq 1. \quad (27)$$

From (20), $a_1 \equiv -\alpha_1 = c - \sqrt{c^2 - 1}$. The ARMA (1,1) model in (27) can be inverted in order to represent p_t as a function of the ϵ_t only. That is,

$$p_t = \frac{1}{2}[\epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \dots], \quad (28)$$

where

$$\psi_i = a_1^{i-1}(1 + a_1), \quad i = 1, 2, \dots \quad (29)$$

From equation (26), output then has the moving-average representation

$$y_t = v_t - \frac{\beta}{2}[\epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \dots]. \quad (30)$$

According to equation (29), the ψ weights increase for one lag before decreasing geometrically to zero. If a_1 were close to 1, as it would be if $\gamma\beta$ were small, then the ψ weights would decline to zero very slowly and y_t would show high correlation. On the other hand, if $\gamma\beta$ were large, the serial correlation would be weaker but would nevertheless converge to zero only asymptotically.

It is instructive to compare the type of serial persistence generated by this model with the observed serial persistence of unemployment in the United States. The ψ weights in the moving-average representation (30) are useful for this purpose. Table 2 compares the ψ weights implied by the model with the actual ψ weights for the unemployment rate for the case where velocity shocks are negligible ($v_t = 0$). The estimated ψ weights were obtained by first estimating an ARMA (2,1) process for the quarterly unemployment rate for males 20 and over (to avoid labor force composition shifts), and then by writing this process in pure moving-average form.⁷ The resulting ψ weights for the sample period 1954:1 through 1976:4 are given in

⁷ The ARMA model was $u_t = 1.39u_{t-1} - .49u_{t-2} + r_t + .27r_{t-1}$, where u_t is the quarterly unemployment rate and r_t is the serially uncorrelated error term. Nelson (1972) found this same model adequate for the 1947:1-1966:4 period.

TABLE 2
THEORETICAL AND OBSERVED SERIAL
PERSISTENCE OF UNEMPLOYMENT
(Quarterly Moving-Average Representation)

LAG	THEORETICAL ψ_i			OBSERVED ψ_i (1954:1-1976:4)
	$N = 2$	$N = 3$	$N = 4$	
0	1.00	1.00	1.00	1.00
1	1.64	1.46	1.36	1.66
2	1.05	1.86	1.7)	1.82
3	.67	1.13	1.99	1.71
4	.43	.87	1.20	1.48
5	.27	.61	1.00	1.21
6	.17	.45	.79	.96
7	.11	.32	.60	.73
8	.07	.23	.47	.55
9	.05	.17	.37	.40
10	.03	.12	.28	.28

NOTE.—The observed ψ_i are obtained by estimating an ARMA (2,1) model for the unemployment rate for males 20 and over and inverting it to obtain the pure moving-average form; the theoretical weights are described in the text. $\beta = .5$, $\gamma = .2$; contract length: $N = 2, 3$, or 4 quarters.

the fourth column of table 2. The estimated ψ weights show a tendency to rise for the first few quarters and then decline fairly steadily.

The theoretical weights are reported in the first three columns of table 2 for contract lengths of 2, 3, and 4 quarters, respectively. These have been calculated using the parametric assumption that $\beta = .5$ and $\gamma = .2$ (or, more generally, that $\gamma\beta = .1$). The results for the 3- and 4-quarter contracts appear quite similar to the observed serial correlation in unemployment. The lag weights rise for the first few quarters before beginning to diminish rather rapidly. However, the case of 2-quarter contracts appears to give substantially less serial correlation. To some extent, these results are dependent on our parameter choice; for example, a larger value for $\beta\gamma$ would reduce persistence but would not change the general humped shape of the lag distribution. In any case, the results show the capability of this type of contract model to explain the observed persistence in unemployment—even when there is no other source of persistence and contract lengths are reasonably short.

*A Statistical Phillips Curve*⁸

An essential requirement of a theoretical macromodel is that it exhibits a statistical Phillips curve. Although the model of this paper in-

⁸ This section owes much to discussions I have had with Edmund Phelps on this subject.

cludes a monetary policy rule which forces a positive correlation between the price level and the unemployment rate, there does exist a negative correlation between the change in the price level and the unemployment rate. To see this, consider the covariance between y_t and $p_{t+1} - p_t$ for the case of $N = 2$. If this covariance is positive, then there is a statistical Phillips curve. From equation (28),

$$p_{t+1} - p_t = \frac{1}{2} \left[\epsilon_{t+1} + \sum_{i=0}^{\infty} (\psi_i - \psi_{i-1}) \epsilon_{t-i} \right], \quad (31)$$

and the covariance between y_t and $p_{t+1} - p_t$ is

$$\begin{aligned} E[y_t(p_{t+1} - p_t)] &= -\beta \left[(\psi_1 - 1) + \sum_{i=2}^{\infty} (\psi_i - \psi_{i-1}) \psi_{i-1} \right] \frac{\sigma^2}{4} \\ &= -\beta \left[a_1 + \sum_{i=0}^{\infty} a_1^{2i} (a_1 - 1)(1 + a_1)^2 \right] \frac{\sigma^2}{4} \\ &= -\beta [a_1 - (1 + a_1)] \frac{\sigma^2}{4} \\ &= \beta \frac{\sigma^2}{4}, \end{aligned} \quad (32)$$

which is positive. Hence, during boom periods, when output is above the full-employment level, inflation will be higher on the average. Note that the size of this covariance depends directly on aggregate-demand policy. (Recall the assumption that price shocks and velocity shocks are uncorrelated. If there is a positive correlation between v_t and ϵ_t , then the above covariance would be larger.) The theoretical regression coefficient of $p_{t+1} - p_t$ on y_t is given by $\beta/2$. Hence, the more accommodative is aggregate-demand policy, the flatter is the statistical Phillips curve.

Stabilization Policy

The policy problem in this model is one of stabilizing the fluctuations of output about its full-employment level and fluctuations of the price level about its steady state level. The dimensions of the problem are evident in equations (25) and (26). According to (26), when the aggregate-demand reaction parameter is zero, the variance of output is at its minimum value, which is equal to the variance of v_t . However, when $\beta = 0$ the variance of the price level is infinite, for then the sum of the autoregressive weights in equation (25) is one. As the value of β is increased, the variance of the price level will fall and the variance of output will rise. The resulting trade-off between the variability of output and prices is illustrated in table 3 for the case of $N = 2$; the σ_y and σ_p columns refer to the standard deviation of output and the

TABLE 3
ALTERNATIVE POLICIES WITH 2-PERIOD CONTRACTS

$\beta\gamma$	c	a_1	σ_p	σ_y	
				$\gamma = .2$	$\gamma = .4$
.01	1.010	.868	3.89	1.02	1.004
.04	1.041	.752	2.84	1.15	1.04
.12	1.128	.606	2.25	1.68	1.21
.20	1.222	.520	2.04	2.27	1.43
.40	1.500	.382	1.78	3.70	2.04
.60	1.857	.292	1.68	5.14	2.71

price level, respectively. The standard deviation of output is given for two values of γ . Note that the larger γ is, the more favorable is the trade-off: for the same level of price variability, the variance of output can be smaller if the sensitivity of price and wages to excess demand is high. This improvement in the trade-off is uniform over its entire range and is some measure of the gain from more responsive prices.⁹

IV. Wage Expectations versus Wage Inertia

As was discussed in Section II, lagged wages appear on the right-hand side of the reduced-form wage equation in this model for two reasons: they represent the expectations of future wages and the overhang of past wages. Using the rational expectations assumption, these two components can be sorted out, as was illustrated in figure 1. One of the results of this decomposition is that the expectations component depends, in an explicit way, on the aggregate-demand policy rule. For this reason, the policy implications of this model are much different from models where either there is no expectations component—wage determination is purely backward looking—or where the expectations component is based on adaptive or extrapolative expectations schemes. The purpose of this section is to explore this difference.¹⁰

We begin by maintaining the assumption that the underlying rate

⁹ The stabilization problem considered by Taylor (1979) involves smoothing fluctuations in the inflation rate about a constant target, rather than the log of the price level about a constant trend as in this paper.

¹⁰ Similar expectational effects have been discussed by Fellner (1976, p. 117), though with more emphasis on making the aggregate-demand policy rule credible. If credibility cannot be established by direct announcement, then there will be a transition period during which the behavior of the economy might be a weighted average of the two sets of columns in table 4. Public learning during the transition would be similar to that discussed in Taylor (1975).

of inflation is zero, so that the steady-state average price level is constant. One of the objectives of demand policy is to stabilize fluctuations of prices (or wages) about this constant level. (The case of a rising-price path can be considered using similar techniques.) It will be convenient to consider the problem within the context of a single realization of a price shock which raises, say, the aggregate price level above its equilibrium value. The objective of policy, therefore, is to bring the price level back to equilibrium. The appropriate rate of return to equilibrium will depend on the relative weights of prices and output in the social welfare function: a quick return if prices have a high weight, or a slow return if output has a high weight.

Assume that $N = 2$ and that an aggregate-demand policy rule of the form discussed in this paper is in force when this price shock occurs. This rule might have been the result of decisions made within the political process and which planned for such shocks, so that when the shock occurs there is no pressure to change the rule. For illustration purposes, assume that the rule is $\beta = .5$, that $\gamma = .2$, and that $\epsilon_1 = 10$ percent ($\epsilon_s = 0, s > 1$). If $p_0 = 0$ and $\epsilon_0 = 0$ (i.e., the price level was in equilibrium before the shock), then $p_1 = 5$ and p_t follows the path

$$p_t = a_1 p_{t-1} + \frac{\epsilon_t}{2} + \frac{\epsilon_{t-1}}{2}, \quad t = 2, 3, \dots, \quad (33)$$

where $a_1 = .63$. The path for output is then given by $y_1 = -2.5$, and

$$y_t = -.5p_t, \quad t = 2, 3, \dots \quad (34)$$

For these numerical values, the two paths are given in table 4. The convergence of the price level to equilibrium is relatively quick despite the accommodating strategy. If β were greater than .5, the return would be faster, but the loss of output would be larger; conversely if β were less than .5.

To compare these results with a model which does not incorporate forward-looking wage determination or which is based on extrapolative expectations, consider equation (1) in the case of $N = 2$,

$$x_t = \frac{1}{2}(x_{t-1} + \hat{x}_{t+1}) + \frac{\gamma}{2}(\hat{y}_t + \hat{y}_{t+1}) + \epsilon_t. \quad (35)$$

A modification of equation (35) to eliminate forward looking is

$$x_t = \frac{1}{2}(x_{t-1} + x_{t-1}) + \frac{\gamma}{2}(y_{t-1} + y_{t-1}) + \epsilon_t, \quad (36)$$

or simply

$$x_t = x_{t-1} + \gamma y_{t-1} + \epsilon_t. \quad (37)$$

TABLE 4
 COMPARISON OF AGGREGATE-DEMAND EFFECTS ON
 PRICE BEHAVIOR WITH RATIONAL AND
 EXTRAPOLATIVE EXPECTATIONS
 (Percent Deviations from Trend)

<i>t</i>	RATIONAL EXPECTATIONS (Eq. [34])		EXTRAPOLATIVE EXPECTATIONS (Eq. [38])	
	<i>p_t</i>	<i>y_t</i>	<i>p_t</i>	<i>y_t</i>
0	.00	.00	.00	.00
1	5.00	-2.50	5.00	-2.50
2	8.15	-4.08	9.75	-4.08
3	5.13	-2.57	9.09	-2.57
4	3.23	-1.62	8.43	-1.62
5	2.03	-1.02	8.01	-1.02
6	1.28	-.64	7.75	-.64
7	.81	-.41	7.58	-.41
8	.51	-.26	7.48	-.26
9	.32	-.16	7.41	-.16
10	.20	-.10	7.37	-.10

Hence, either current wage negotiations ignore future developments or they simply extrapolate current developments by forecasting x_{t+1} with x_{t-1} and y_{t+1} with y_{t-1} . The aggregate price level is then given by

$$p_t = p_{t-1} + \frac{\gamma}{2}(y_{t-1} + y_{t-2}) + \frac{\epsilon_t}{2} \frac{\epsilon_{t-1}}{2}. \quad (38)$$

The policy implications of equation (38) are clearly much different from equation (34). The path of the price level using (38) is given in table 4 for the same path of output considered above (starting with $p_0 = 0$, $y_0 = 0$, and $\epsilon_0 = 0$). It is evident that the reduction in the price level is much smaller than in the case of rational forward-looking expectations; according to equation (38), a much larger loss of output would be required to bring the price level back to the target path at the same rate implied by equation (34). Price stability appears to be very costly when expectations are not rational or contracts do not look forward. The difference between the two models provides a simple measure of the "expectation bonus" which comes from rational anticipatory wage determination. The difference indicates that rational expectations matter greatly—despite the existence of contracts—for macroeconomic stabilization. It also indicates the need to determine empirically whether wage-contract decisions rationally anticipate in this way.

V. Concluding Remarks

The analysis of this paper has centered on a stationary economy with staggered overlapping wage contracts and rational expectations. The aggregate dynamics of this economy—when subject to continual demand and price shocks—were examined under alternative monetary policy rules. In order to emphasize endogenous persistence effects, these shocks were assumed to be serially uncorrelated.

The results of the analysis can be summarized as follows. Persistence of unemployment similar to that observed in the United States can be generated by the model when contracts are 3 or 4 quarters long and there are no other sources of persistence. The time shape of the dynamic impact of shocks on unemployment—rising for several quarters before tapering off—is characteristic both of the estimated process for unemployment and the theoretical process implied by the model. Moreover, the model generates a persistence of wages and prices which gives rise to a statistical Phillips curve and which presents a policy trade-off between price stability and output stability. The persistence of wages has both an expectational component and an inertia component, and these can be decomposed using the rational expectations approach. Policy affects the behavior of wages and employment by altering the expectations component of persistence. Hence, the econometric wage equations depend on the policy rule, and the estimated impact of aggregate demand on wages will be biased, unless expectations of policy in future periods are accounted for.

By viewing the logarithms of the price level, the contract wage, and the money supply as deviations from a linear trend, all the results presented here carry over to an inflationary economy. The policy problem then would be to stabilize prices as well as output about an exponentially growing target path. Given an aggregate-demand policy rule, price shocks above this path are followed by lower rates of inflation and higher unemployment, until the price level returns to the target path; conversely with negative price shocks. The statistical Phillips curve derived in Section III would be evident in the data left by these paths if the econometrician were careful to take deviations of the inflation rate from the underlying inflation rate before computing the regressions. This Phillips curve would convey little information about the sensitivity of wages or prices to excess demand, however. This sensitivity would be implicit in the reduced-form wage equations (again in deviation from trend form) derived in Section II, but it could not be extracted without knowledge of the policy rule. Finally, aggregate-demand policy makes a substantial difference for the behavior of output, wages, and prices. The choice of a target rule for aggregate-demand policy is therefore no less important than the

choice of a target unemployment rate or a target inflation rate. It should therefore be considered as carefully in the political process as the other two targets typically are.

Appendix

The Reduced-Form Contract Equation When Shocks Are Correlated

In order to emphasize the persistence generated by the staggered contracts, we have assumed that both velocity shocks and wage shocks are serially uncorrelated. In some applications, however, one might want to allow for the possibility that these shocks are correlated. The purpose of this Appendix is to show how the solution technique introduced in Section II can be used to handle the case of serially correlated shocks.

Let the wage shock ϵ_t and the velocity shock v_t in equations (6) and (7) of the text be represented as

$$\epsilon_t = \sum_{i=0}^{\infty} \delta_i u_{t-i} \quad (\text{A1})$$

and

$$v_t = \sum_{i=0}^{\infty} \theta_i \eta_{t-i}, \quad (\text{A2})$$

where (u_t, η_t) is a serially uncorrelated random vector with zero mean and where $\delta_0 = \theta_0 = 1$. Using this serial correlation structure to forecast output as in equation (8) and substituting into equation (6) gives

$$x_t = \sum_{s=1}^n b_s x_{t-s} + \sum_{s=1}^n b_s \hat{x}_{t+s} - \frac{\beta\gamma}{N^2} \sum_{s=0}^n \sum_{i=0}^n \hat{x}_{t+s-i} + \frac{\gamma}{N} \sum_{i=1}^{\infty} \rho_i \eta_{t-i} + \sum_{i=0}^{\infty} \delta_i u_{t-i}, \quad (\text{A3})$$

where $\rho_i = \sum_{s=i}^{n+i} \theta_s$. Taking expectations on both sides of (A3) and using the identity (10) yields

$$\sum_{s=1}^n b_s \hat{x}_{t-s} - c \hat{x}_t + \sum_{s=1}^n b_s \hat{x}_{t+s} = - \left(1 - \frac{n\beta\gamma}{N}\right)^{-1} \left(\frac{\gamma}{N} \sum_{i=1}^{\infty} \rho_i \hat{\eta}_{t-i} + \sum_{i=1}^{\infty} \delta_i \hat{u}_{t-i} \right). \quad (\text{A4})$$

Using the lag polynomial notation as in Section II, this can be written

$$B(L)\hat{x}_t = - \left(1 - \frac{n\beta\gamma}{N}\right)^{-1} \left[\frac{\gamma}{N} R(L)\hat{\eta}_t + \Delta(L)\hat{u}_t \right], \quad (\text{A5})$$

where $R(L) = \sum_{i=1}^{\infty} \rho_i L^i$ and $\Delta(L) = \sum_{i=1}^{\infty} \delta_i L^i$. The polynomial $B(L)$ is the same as that in equation (14). Hence, it can be factored as $B(L) = \lambda A(L)A(L^{-1})$; and by dividing both sides of (A5) by $\lambda A(L^{-1})$, we have

$$A(L)\hat{x}_t = - \lambda^{-1} \left(1 - \frac{n\beta\gamma}{N}\right)^{-1} \left[\frac{\gamma}{N} H(L)\hat{\eta}_t + G(L)\hat{u}_t \right], \quad (\text{A6})$$

where $H(L)$ is a polynomial obtained by omitting all terms involving nonpositive powers of L in the polynomial $[A(L^{-1})]^{-1}R(L)$, and $G(L)$ is a polynomial obtained by omitting all terms involving nonpositive powers of L in the polynomial $[A(L^{-1})]^{-1}\Delta(L)$. The nonpositive powers of L cancel because $L^{-s}\hat{\eta}_t = \hat{\eta}_{t+s} = 0$ for $s \geq 0$ and $L^{-s}\hat{u}_t = \hat{u}_{t+s} = 0$ for $s \geq 0$, since the expectations are conditional on information through period $t-1$ and η_t and u_t are each

serially uncorrelated. Comparing (A6) with (A3) indicates that the reduced-form contract equation is given by

$$A(L)x_t = -\lambda^{-1} \left(1 - \frac{n\beta\gamma}{N}\right)^{-1} \left[\frac{\gamma}{N} H(L)\eta_t + G(L)u_t \right] + u_t. \quad (\text{A7})$$

Equation (A7) determines the current contract in terms of past contracts, past shocks, and the current contract shock. From this relationship the behavior of the aggregate price level and aggregate output can be derived following the techniques described in Section II.

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