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# Individual forecasting and aggregate outcomes

## "Rational expectations" examined

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## Comment

JOHN B. TAYLOR

Implicit in almost all practical applications of the rational expectations method are two strong assumptions. First, it is assumed that *people know the model* of the economy used in the analysis and that they form expectations using this model. Second, it is assumed that *people know that all other people know the model* and form expectations in the same way. These two assumptions seem to restrict the range of applications

of rational expectations methods. They suggest that the methods are most realistic in situations where economic events are recurrent - such as business cycles - and where policy rules are in operation for a long time. As with most hypotheses used in economic analysis, however, these assumptions should be judged not only by their apparent realism but also by how successful they are in describing and forecasting economic behavior and by how they compare with alternative assumptions. As yet, there have been few attractive alternatives available.

In this elegant and constructive chapter, Robert Townsend proposes alternative, less restrictive assumptions that have the potential of broadening the range of economic problems to which rational expectations analysis can be applied. Moreover, he develops a methodology through which tractable results can be obtained using these alternative assumptions and shows how the methods work in some representative economic applications. In my view, these alternatives deserve careful consideration by those using rational expectations in situations where the more restrictive assumptions seem inappropriate, and, as Townsend suggests, they ought to be tried out in some practical economic policy problems. In these comments I shall discuss how the Townsend assumptions represent a generalization of existing expectational assumptions and consider the types of applications where some experimentation with the methods might be useful.

Rather than assuming that economic agents know the parameters of the model, Townsend assumes that some of these parameters are unobservable and evolve over time. For example, firms are assumed to be unaware of the intercept ($\theta$ in the chapter's notation) of the demand curve that they face. Instead, they know that this intercept moves according to the probability law

$$\theta_t = \rho\theta_{t-1} + v_t$$

and can only be observed with error $u_t = \theta_t + w_t$ [see equations (23) and (24) of the Townsend chapter]. The firms use this information structure to forecast future values of the intercept and thereby form expectations of future prices and make production decisions.

It is not difficult to imagine applications where this assumption might be more appropriate than assuming $\theta$ was known. In a commodity demand equation, the parameter $\theta$ could represent tastes that change gradually and that can be estimated with error through survey methods. In a money demand function, such an assumption could represent technological change in transactions technology that can be tracked only up to some measurement error. In a fiscal or monetary policy reaction function, such an assumption could be used to capture gradually shifting

economic policies that are never fully announced or believed. In this case, the probability law would represent how policy was evolving through time, and $u_t$ could be a current policy announcement that is only imperfectly correlated with actual policy. Note that in each of these three examples Townsend's assumptions require that agents know the model that underlies these shifts: a model of taste change, a model of technological change in financial markets, or a (political?) model of policy change.

In a number of situations the Townsend assumptions might not be appropriate as an alternative to the "agents-know-the-model" assumption. For example, an important modeling task is to describe economic behavior during a transition from one policy regime to another.[1] After a change in policy regime it would be inaccurate to assume that economic agents immediately understand the new policy. Instead, they might learn about the policy gradually as they observe policy decisions over a period of time. More generally, a structural parameter of the model might change, and agents would have to learn about this change through observation. In terms of Townsend's notation, these types of problems could be represented in terms of the parameter $\rho$ of the autoregressive process. If $\theta$ were the money supply growth rate, then a switch to $\rho=0$ could represent a fixed monetary growth rate. People would learn about $\rho$ only as they observed actual money growth rates. Because in Townsend's models people are assumed to know the process generating $\theta$, this type of problem cannot be handled.

Learning about the parameters of the model in this latter sense has proved to be a quite difficult phenomenon to model adequately.[2] There are three reasons for this difficulty. First, because agents must make decisions based on estimates of parameters, their actions cannot be considered exogenous to parameter estimation. The actions form the data on which the estimates of parameters are made. Because most conventional econometric procedures require that the data be exogenous, or endogenous in particularly restrictive ways, these market interactions with data generation require different techniques for analysis. Second, there is a possibility that, as agents gradually learn about the parameters, their

[1] See, for example, J. B. Taylor (1975). "Monetary Policy during a Transition to Rational Expectations." *Journal of Political Economy,* October, pp. 1009–21, and L. H. Meyer and C. Webster, Jr. (1982). "Monetary Policy and Rational Expectations: A Comparison of Least Squares and Bayesian Learning." In: *Carnegie-Rochester Series in Public Policy,* edited by K. Brunner and A. Meltzer, Amsterdam: North Holland.

[2] Chapter 6 by Margaret Bray in this book considers such a problem in a simple one-parameter learning situation.

actions will converge to some constant value that does not generate enough new information about the parameters. In the demand-curve example, a firm might begin selling the same quantity each period based on its estimate of the expected price; this prevents quantity from varying enough to get reliable estimates of the demand curve.[3] In some instances, estimates are inconsistent, but few results are yet available. In any case, the analysis necessarily becomes quite complicated, even without the market interactions previously mentioned. The problem is much worse in a multiparameter situation, and this is one reason why many studies have focused on one-parameter examples.[4] Third, the possibility that agents might affect how much information they can obtain about the parameters changes the nature of the optimal control problems in fundamental ways. A simple example is that of a firm experimenting with its prices, temporarily deviating from its best guess of the optimal price, in order to obtain information to be used in the future. Even in one-parameter partial equilibrium problems, this "dual-control" or "joint estimation and control" problem leads to significant complications. Solutions that may have been linear in a model where the parameter was known do not even have a closed form when the parameter is unknown.[5]

Because of these computational difficulties with existing approaches to modeling learning, Townsend's approach, although assuming that the laws governing parameter movement are known, may be a satisfactory alternative. For some applications, the distinction made here between knowing parameter values ($\theta$) and knowing the probability law generating the parameter values ($\rho$) may be sufficiently fine that Townsend's more tractable approach could be used.

Thus far, I have discussed situations, as in Section 9.5 of the chapter, where expectations are assumed to be homogeneous. In Sections 9.6 and 9.7, Townsend considers ways to avoid this assumption and allow for heterogeneous, or disparate, expectations. With disparate expectations,

[3] This argument can be made more general with exogenous shifts in the functions. See T. W. Anderson and J. B. Taylor (1976). "Some Experimental Results on the Statistical Properties of Least Squares Estimates in Control Problems." *Econometrica,* November, pp. 1289–302, for a discussion of the problem and a demonstration of how this problem leads to a situation of extreme "multicollinearity" and poor parameter estimates.

[4] A proof of convergence and asymptotic normality is given for a partial equilibrium one-parameter example by J. B. Taylor (1974). "Asymptotic Properties of Multiperiod Control Rules in the Linear Regression Model." *International Economic Review,* June, pp. 472–84.

[5] See E. C. Prescott (1972). "The Multiperiod Control Problem under Uncertainty." *Econometrica,* November, pp. 1043–57.

an infinite regress problem arises in which agents must not only forecast but also forecast the forecasts of others, and so on. Townsend deals with this infinite regress problem head-on, by augmenting the state variables to include forecast of forecasts – the second-order expectation, as well as third- and higher-order expectations. There is a modeling choice, however, about where to truncate the infinite regress, or whether to truncate it at all. An element of judgment is required here, but perhaps the decision could be made empirically. As Townsend has shown in an earlier study, the regress problem has implications for the serial correlation properties of the errors in statistically estimated decision rules.[6] It would be interesting to examine whether, for example, the second- or third-order expectation truncation fits the data better than the first-order truncation that is conventionally used. But, in general, because it is impossible to know which truncation to assume, this may leave an element of arbitrariness in situations where there are other reasons for serial correlation. Clearly, some empirical work is necessary before we can say whether or not Townsend's higher-order expectations model is an improvement over the first-order methods now in use.

[6] See R. M. Townsend (1982). "Forecasting the Forecasts of Others." Unpublished manuscript, Carnegie-Mellon University (to be published in *Journal of Political Economy,* 83).