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Journal of Monetary Economics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jme

The term structure of policy rules

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ARTICLE INFO

Article history:
Received 29 December 2008
Received in revised form
24 August 2009
Accepted 18 September 2009

JEL classification:
E43
E44
E52
G12

Keywords:
Affine models
No arbitrage
Monetary policy rules
Term structure of interest rates

ABSTRACT

A formula is derived that links the coefficients of the monetary policy rule for the short-term interest rate to the coefficients of the implied affine equations for long-term interest rates. The formula predicts that an increase in the coefficients in the monetary policy rule will lead to an increase in the coefficients in the affine equations. Empirical evidence for such a prediction is provided. The curve of the response coefficients by maturity is also predicted by the formula. The formula's predictive accuracy and its closed form make it a useful tool for studying the policy implications of embedding no-arbitrage affine theories into macro models.

0. Introduction

One of the most debated issues in monetary economics concerns the impact of monetary policy on the term structure of interest rates.[1] Approaching this issue from the perspective of monetary policy rules seems promising because long-term interest rates depend on expectations of future short-term rates, which are determined by the response of the central bank to future developments in the economy, a response most easily captured by a policy rule. This paper takes such an approach by combining a no-arbitrage affine theory of the term structure of interest rates with a structural macro model of the impact of monetary policy rules on the economy.[2]

The paper proceeds as follows. The next section starts with a simple model in which the monetary policy rule for the short-term interest rate r_t depends only on the inflation rate π_t. That is, $r_t = \delta_\pi \pi_t$. In this case we show that the affine equations for long-term interest rates also depend on the inflation rate. That is

$$i_t^{(n)} = a_n + b_n \pi_t \tag{1}$$

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[1] The appendix and replication programs of this paper can be found on the Science Direct website.

[2] Previous work combining models of the term structure with monetary policy rules include Fuhrer (1996), who combines a pure expectations theory of the term structure, a policy rule, and a simple structural model; and Ang et al. (2005), who use the no-arbitrage affine model, a policy rule, and a vector auto-regression.

0304-3932/$ - see front matter © 2009 Published by Elsevier B.V.
doi:10.1016/j.jmoneco.2009.09.004

Please cite this article as: Smith, J.M., Taylor, J.B., The term structure of policy rules. Journal of Monetary Economics (2009), doi:10.1016/j.jmoneco.2009.09.004

for n=1, …, N, where $i_t^{(n)}$ is the yield to maturity on zero-coupon bonds with maturity n, a_n is the intercept, and b_n is the response coefficient for maturities n. The sizes of the response coefficients b_n are important for the overall behavior of the economy. For example, if b_n is large, then an increase in inflation will bring about a large increase in the yields on bonds with that maturity and thereby affect spending by firms or consumers who are borrowing funds at that maturity. In the case where the monetary policy rule for the short-term interest rate depends on the inflation rate and on the output gap y_t, so that $r_t=\delta_y y_t+\delta_\pi \pi_t$, we show that the affine equations of the term structure take the form

$$i_t^{(n)} = a_n + b_{1,n} y_t + b_{2,n}\pi_t \tag{2}$$

Observe that the affine Eqs. (1) and (2) can be interpreted as a series of implied policy rules for long-term interest rates—one policy rule for each maturity—with coefficients measuring the size of the interest rate reaction. In other words, Eqs. (1) and (2) define t*he term structure of policy rules.*

We derive analytically the formula relating the coefficients in the affine equations for long-term interest rates to the coefficients of the monetary policy rule for the short-term interest rate. In the case of Eq. (1), the formula shows how the response coefficients b_n depend on the policy rule coefficient δ for each maturity n. The formula predicts that a change in the coefficients in the monetary policy rule (δ in the simple case) will lead to a change in the coefficients (b_n in the simple case) by a specific amount. The formula takes risk into account and reveals countervailing effects of shifts in the policy rule on the affine equations for long-term interest rates.

Next, in order to assess the accuracy of this prediction, we estimate the affine Eqs. (1) and (2) using data on long-term interest rates in the United States and look for temporal shifts in the equations. The results are reported in Section 2. We find that the coefficients on inflation and real GDP in the affine equations shifted upwards in the 1980s and that this was about at the same time as a previously-documented and well-known increase in the policy rule coefficients. Thus, the prediction of the formula is confirmed by the data. Section 3 then shows graphically that the pattern of the affine response coefficients by maturity is remarkably close to what the formula derived from our model predicts.

Section 4 provides an example of how the formula can be used in practice for policy evaluation. We examine the period from 2003 to 2005 when the federal funds rate deviated significantly from what would have been predicted by the policy rule response that was typical over the period since the mid-1980s. Though this 3-year period is comparatively short for determining whether market participants interpreted this as a regime shift or a temporary deviation from the post mid-1980s regime, the interest rate response to inflation is much lower. A perception of a smaller response coefficient could have led market participants to expect smaller interest rate responses to inflation in the future, and therefore to smaller long-term interest rate responses. If so, our model provides an explanation for the puzzle—coined the "conundrum" by former Federal Reserve Chairman Alan Greenspan—that the rise in the federal funds rate starting in 2004 did not bring about an increase in long-term interest rates as would have been expected based on experience over the previous 20 years.

1. A simple model of monetary policy and the term structure

We begin with a model in which the central bank responds to inflation, but not to output. The model has the following equations:

$$r_t = \delta\pi_t \tag{3}$$

$$i_t^{(n)} = -n^{-1}\log(P_t^{(n)}) \tag{4}$$

$$P_t^{(n+1)} = E_t[m_{t+1}P_{t+1}^{(n)}] \tag{5}$$

$$m_{t+1} = \exp(-r_t - 0.5\lambda_t^2 - \lambda_t\varepsilon_{t+1}) \tag{6}$$

$$\lambda_t = -\gamma_0 - \gamma_1\pi_t \tag{7}$$

$$\pi_t = \pi_{t-1} - \phi(r_{t-1} - \pi_{t-1}) + \sigma\varepsilon_t \tag{8}$$

where the shock $\varepsilon_t \sim iid$ N(0, 1). Eq. (3) is the monetary policy rule in which the short-term nominal interest rate r_t depends on the inflation rate with a policy response coefficient $\delta > 0$ Eq. (4) gives the yield to maturity of a zero-coupon bond with a face value of 1 that matures in n periods, where $P_t^{(n)}$ is the price of the bond at time t. Eq. (5) is a no-arbitrage condition showing that the price of an n+1 period bond at time t must equal the expected present discounted value of the price of an n-period bond at time t+1, where m_t is the stochastic discount factor. Eq. (6) describes this stochastic discount factor, which has the convenient functional form used in the affine term structure literature. Eq. (7) shows that the risk term λ_t depends on two coefficients: γ_0, which represents a constant risk premium, and γ_1, which represents the time-varying risk premium attributed to changes in inflation. Finally, Eq. (8) describes how monetary policy affects inflation. It is a price adjustment equation in which the change in inflation depends on the lagged real interest rate, which we simply assume depends on the ex-post real interest rate through the parameter $\phi > 0$.

The affine term structure Eqs. (4)–(7) are simplifications of assumptions in Ang et al. (2005). These authors also assume that macroeconomic variables (π in this simple model) evolve according to an auto-regression, which does not depend on the policy rule. To answer the questions posed here about the impact of regime shifts on the term structure, it is necessary to describe how the interest rate affects inflation, and for this reason we introduce a simple structure which assumes the interest rate transmission mechanism in Eq. (8). This effect would be ignored by a vector auto-regression model with constant coefficients, leading to errors similar to those pointed out in the Lucas critique.[3] It is possible, of course, to improve on our model by introducing, for example, a forward-looking optimization model, staggered price setting, or a non-zero inflation target π^*, but the simple form of (8) allows us to obtain analytic results and focus on the term structure relations.

We now show that Eqs. (3)–(8) imply that the yields $i_t^{(n)}$ are linear functions of the inflation rate as shown in Eq. (1). For $i_t^{(1)} = r_t$ this is obvious from the policy rule Eq. (3), and so a_1=0 and b_1=δ. For $i_t^{(2)}$it is instructive to show the derivation in some detail because it easily generalizes to longer maturities and higher-order models. From Eq. (4), the price of the one-period bond at time $t+1$ is simply $P_{t+1}^{(1)} = \exp(-r_{t+1})$, which can be substituted into Eq. (5) to obtain $P_t^{(2)} = E_t[m_{t+1}\exp(-r_{t+1})]$. Now, substituting for m_{t+1} and r_{t+1} from Eqs. (3), (6), and (8) gives

$$\begin{aligned}
P_t^{(2)} &= E_t[\exp(-\delta\pi_t - 0.5\lambda_t^2 - \lambda_t\varepsilon_{t+1} - \delta(\pi_t - \phi(\delta\pi_t - \pi_t) + \sigma\varepsilon_{t+1}))] \\
&= \exp(-\delta\pi_t - 0.5\lambda_t^2 - \delta(\pi_t - \phi(\delta - 1)\pi_t)E_t[\exp(-(\delta\sigma + \lambda_t)\varepsilon_{t+1})] \\
&= \exp(-\delta\pi_t - 0.5\lambda_t^2 - \delta(\pi_t - \phi(\delta - 1)\pi_t) + 0.5\delta^2\sigma^2 + \delta\sigma\lambda_t + 0.5\lambda_t^2 \\
&= \exp(-\delta\pi_t - \delta(\pi_t - \phi(\delta - 1)\pi_t) + 0.5\delta^2\sigma^2 - \delta\sigma(\gamma_0 + \gamma_1\pi_t)) \\
&= \exp(0.5\delta^2\sigma^2 - \delta\sigma\gamma_0 - \delta(2 - \phi(\delta - 1) + \sigma\gamma_1)\pi_t)
\end{aligned} \tag{9}$$

where the normal distribution assumption for ε_{t+1} is used to evaluate the expectation. That the yield $i_t^{(2)}$ is given by $-0.5\log(P_t^{(2)})$ implies that

$$i_t^{(2)} = 0.5\delta\sigma\gamma_0 - 0.25\delta^2\sigma^2 + 0.5\delta(2 - \phi(\delta - 1) + \sigma\gamma_1)\pi_t \tag{10}$$

which is the linear form of (1) with a_2 the intercept term and b_2 the coefficient on π_t.

Proceeding in the same fashion for maturities $n>2$ (see the appendix posted on Science Direct), the response coefficient on the inflation rate for yields for any maturity is given by the formula

$$b_n = \frac{\delta\sum_{i=0}^{n-1}(1 - \phi(\delta - 1) + \sigma\gamma_1)^i}{n} \tag{11}$$

Observe that the above formula shows how δ affects b_n. Observe also that the numerator of the formula is a geometric series in which each term equals the previous term multiplied by the common ratio $(1-\phi(\delta-1)+\sigma\gamma_1)$. While it is algebraically possible for this common ratio to be negative, this would imply implausible values for the parameters, such as an enormous response coefficient δ for the central bank. Hence we will assume throughout that the common ratio is positive and focus on whether it is greater than one or not.

Note that $b_2<\delta$ if

$$\delta > 1 + \frac{\sigma\gamma_1}{\phi} \tag{12}$$

Condition (12) is closely related to the monetary policy principle that $\delta>1$ (called the "Taylor principle," by Woodford (2001) and others). Condition (12) is sufficient to prevent b_n from exploding as n increases. To see this, note that if (12) holds, then the common ratio of terms in the geometric series in the numerator of Eq. (11) is less than one. For each increase in maturity n, we calculate b_n by (i) adding a term in the numerator which is less than the previous term and (ii) adding a term in the denominator greater than the previous term. Thus we conclude that b_n cannot increase geometrically.

The close connection between condition (12) and the Taylor principle is important because the latter is usually viewed as the *sine qua non* of a good monetary policy. If that principle does not hold, then the model is not even stable; but it is possible for (12) to hold if γ_1 is negative which causes longer-term yields to have a more muted response to inflation. If the Taylor principle holds, it is possible that (12) does not hold if γ_1 is sufficiently large and positive. In the case of time-invariant risk aversion (γ_1=0), the Taylor principle and condition (12) are exactly the same. Ang and Piazzesi (2003) and Ang et al. (2005) estimated the time-varying risk parameter corresponding to inflation to be positive, though in somewhat different set-ups.

1.1. Impact of shifts in the monetary policy rule

The question to focus on now is how a change in the policy rule affects the behavior of the term structure. For b_2=$(2-\phi(\delta-1)+\sigma\gamma_1)/2$ one can easily see how the response coefficient in the policy rule has a direct impact on the response of this longer-term yield to inflation. However, there are countervailing effects. A larger reaction coefficient δ

[3] Examples where more complex structural models have been combined with affine models of the term structure are Bekaert et al. (2005), Rudebusch and Wu (2007), and Gallmeyer et al. (2007).

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means that expected future short-term interest rates will rise by a larger amount when future inflation rises; this effect is measured by the presence of δ outside the parentheses and it depends on the risk premium parameter γ_1; but a higher δ also means that inflation is expected to increase by a smaller amount in the future for a given increase in inflation today, because the persistence in inflation declines; this effect is measured by the term $\phi(\delta-1)$.

To sort out these countervailing effects, consider the derivative of b_2 with respect to δ:

$$\frac{\partial b_2}{\partial \delta} = \frac{2+\phi+\sigma\gamma_1}{2} - \delta\phi > 0 \tag{13}$$

Note that unless δ is already very large, the derivative is likely to be positive and we will generally make this assumption, as indicated by the inequality sign in (13). Then increasing δ will raise the reaction of the two-period rate to inflation. Here, the reduced persistence has a smaller effect than the size of the reaction. For high values of δ the derivative could be negative, reflecting that reduced persistence has a larger effect.

In the general case of maturity n the derivative is

$$\frac{\partial b_n}{\partial \delta} = \frac{1}{n}\left(1+\sum_{i=0}^{n-2}(1+\phi+\sigma\gamma_1)(1-\phi(\delta-1)+\sigma\gamma_1)^i - \phi\delta\sum_{i=0}^{n-2}(i+2)(1-\phi(\delta-1)+\sigma\gamma_1)^i\right) \tag{14}$$

Observe that Eq. (14), much like Eq. (13), is composed of two countervailing terms. Both terms are strictly monotonically increasing and multiplied by $1/n$. The first term is a geometric sum with common factor equal to $(1-\phi(\delta-1)+\sigma\gamma_1)/2$ while the second term, which is subtracted from the first, is an arithmetic–geometric sum with the same common factor. Much as in Eq. (13), there are two countervailing effects: the first term is the direct effect of policy while the second term is the indirect persistence effect.

The sign of (14) is crucial to understanding how longer-term rates depend on changes in policy as shown in the following proposition:

Proposition 1. *Suppose conditions (12) and (13) hold and that* $\gamma_1>0$. *Then there exists a unique* n^* *such that for all* $n<n^*$, $\frac{\partial b_n}{\partial \delta}>0$ *and for all* $n>n*$, $\frac{\partial b_n}{\partial \delta}<0$.

Proof. See the appendix posted on Science Direct.

As explained above, the assumptions underlying Proposition 1 are empirically realistic. Hence, the model predicts that a monetary policy that reacts more aggressively against inflation implies that bond yields respond more aggressively to inflation.[4] □

1.2. The case where both inflation and output appear in the policy rule

Now, consider the following model which includes real output as well as inflation:

$$r_t = \boldsymbol{\delta}'\mathbf{z}_t \tag{15}$$

$$m_{t+1} = \exp(-r_t - 0.5\boldsymbol{\lambda}\boldsymbol{\lambda}_t' - \boldsymbol{\lambda}_t'\mathbf{v}_{t+1}) \tag{16}$$

$$\boldsymbol{\lambda}_t = \boldsymbol{\gamma} + \boldsymbol{\Gamma}\mathbf{z}_t \tag{17}$$

$$y_t = -\alpha_1(r_{t-1}-\pi_{t-1}) + \alpha_2 y_{t-1} + \sigma_\eta\eta_t \tag{18}$$

$$\pi_t = \pi_{t-1} + \phi y_t + \sigma_\varepsilon\varepsilon_t \tag{19}$$

where $\mathbf{z}_t = \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}$, $\mathbf{v}_t = \begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix}$, $\boldsymbol{\delta} = \begin{pmatrix} \delta_y \\ \delta_\pi \end{pmatrix}$, $\boldsymbol{\gamma} = \begin{pmatrix} \gamma_{01} \\ \gamma_{02} \end{pmatrix}$, $\boldsymbol{\Gamma} = \begin{pmatrix} -\gamma_{11} & \gamma_{12} \\ \gamma_{21} & -\gamma_{22} \end{pmatrix}$ and where the shocks $\eta_t \sim iid$ N(0, 1) and $\varepsilon_t \sim iid$ N(0, 1) are independent of each other. Eq. (15) is the policy rule; it replaces the simpler policy rule of Eq. (3), incorporating a measure of the real output gap in the interest rate rule of the central bank. We specify bond prices analogous to (5), but the pricing kernel m_{t+1} now has the matrix form shown in Eq. (16). Eq. (17) shows how inflation and real output affect risk aversion in the pricing kernel, a generalization of Eq. (7). The risk term λ_t is now two-dimensional. The first element corresponds to the risk term associated with real output, whereas the second element corresponds to the risk term associated with inflation. Eqs. (18) and (19) replace Eq. (8). Eq. (18) shows how real output depends on its own lag and on the ex-post real interest rate. Eq. (19) is a price adjustment equation in which inflation depends on lagged inflation and lagged real output.

[4] A critical question is how large n^* is. The empirical exercise we perform leads us to believe that n^* is sufficiently large so that it is almost surely the case that $\frac{\partial b_n}{\partial \delta}>0$.

Analogously with the first model, the yield on an n-period bond is an affine function of inflation and output, as in Eq. (2). To show this write Eq. (2) in vector form

$$i_t^{(n)} = a_n + \mathbf{b}_n' \mathbf{z}_t \tag{20}$$

with the n-period yield intercept term given by a_n and the n-period response coefficient vector given by $\mathbf{b}_n = \begin{pmatrix} b_{1,n} \\ b_{2,n} \end{pmatrix}$.

We can rewrite Eqs. (18) and (19) as a first order vector auto-regression:

$$\mathbf{z}_t = \mathbf{\Phi} \mathbf{z}_{t-1} + \mathbf{\Sigma} \mathbf{v}_t \tag{21}$$

where

$$\mathbf{\Phi} = \begin{pmatrix} \alpha_2 - \alpha_1 \delta_y & -\alpha_1(\delta_\pi - 1) \\ \phi(\alpha_2 - \alpha_1 \delta_y) & 1 - \phi\alpha_1(\delta_\pi - 1) \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} \sigma_\eta & 0 \\ \phi\sigma_\eta & \sigma_\varepsilon \end{pmatrix}$$

In the appendix we show that the n-maturity bond yield response coefficient vector is

$$\mathbf{b}_n = \frac{1}{n}\Big(\sum_{i=0}^{n-1} (\mathbf{\Phi} - \mathbf{\Sigma\Gamma})^i\Big)' \boldsymbol{\delta} \tag{22}$$

The above formula is a generalization of the formula in Eq. (11).

In the first model, the inflation response coefficient in the simple policy rule had a direct impact on the response of longer-term yields to inflation and the direction of the reaction had two countervailing forces. The same is true in this model. To see this let $\mathrm{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \Phi - \Sigma\Gamma$. One can then show that the derivative of $\mathbf{b}_n$ with respect to each of the elements of δ is then given by

$$\frac{\partial \mathbf{b}_n}{\partial \delta_y} = \frac{1}{n} \begin{pmatrix} 1 - \alpha_1 \sum_{i=1}^{n-1} \Big(i(\delta_y h_{11}^{i-1} + \phi\delta_\pi h_{21}^{i-1})\Big) + \sum_{i=1}^{n-1} h_{22}^i \\ \sum_{i=1}^{n-1} h_{12}^i \end{pmatrix} \tag{23}$$

$$\frac{\partial \mathbf{b}_n}{\partial \delta_\pi} = \frac{1}{n} \begin{pmatrix} \sum_{i=1}^{n-1} h_{21}^i \\ 1 - \alpha_1 \sum_{i=1}^{n-1} \Big(i(\delta_y h_{12}^{i-1} + \phi\delta_\pi h_{22}^{i-1})\Big) + \sum_{i=1}^{n-1} h_{22}^i \end{pmatrix} \tag{24}$$

Now consider the derivative $\frac{\partial b_{2,n}}{\partial \delta_\pi} = \frac{1}{n}(1 + \sum_{i=1}^{n-1} h_{22}^i) - \frac{1}{n}\Big(\alpha_1 \sum_{i=1}^{n-1} i(\delta_y h_{12}^{i-1} + \phi\delta_\pi h_{22}^{i-1})\Big)$

Observe that this derivative is very similar to Eq. (14) and that countervailing forces are again at work. There are two sums, one is geometric and the other is arithmetic–geometric. The geometric term represents the direct effect of policy, while the arithmetic–geometric term is a combination of the persistence of inflation and the output gap. As in the simpler case one can use the formula to calculate how the response coefficients for the longer yields are affected by the policy response coefficients.

2. Empirical evidence of the upward shift in the term structure of policy rules

To assess the predictive power of the formula, we estimated empirical counterparts of the affine Eqs. (1) and (2) using US data. The zero-coupon bond yields are the quarterly averages of monthly CRSP data on US Treasury yields at 1–5-year maturities. These maturities are chosen because they are the only maturities in the CRSP database that have been converted to zero-coupon yields. The inflation measure is the four-quarter moving average of the percentage change of the US GDP chain-weighted price index. The GDP gap is the percentage deviation of real GDP from a Hodrick-Prescott trend using quarterly GDP data. Table 1 reports unconstrained ordinary least squares estimates of Eq. (1) and shows the response coefficients b_n for a range of maturities. Table 2 reports analogous least squares estimates of Eq. (2).

The formula derived in the previous section predicts that a shift in the monetary policy rule coefficients should cause a shift in the coefficients of the affine equations for long rates. A natural place to test for the accuracy of this prediction is around the time of a large shift in the monetary policy rule. In the United States such a shift has been previously-documented by Clarida et al. (2000), Taylor (1999), and Woodford (2003) as occurring in the early 1980s. In particular the estimated δ_y and δ_π coefficients in the policy rule shifted upwards. The policy shift occurred as the Federal Reserve, under the leadership of Paul Volcker, began to pursue a different approach to monetary policy. Though the shift in policy might be modeled in other ways—perhaps as due to a shift in the target rate of inflation or to poor measurements of the GDP gap—there is considerable agreement that the coefficients in the monetary policy rule shifted.

The question is whether the estimated $b_{1,n}$ or $b_{2,n}$ coefficients in the affine equations for long-term interest rates also shifted upward at this time. We are not aware of previous research that has found or studied such a shift, but if we

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Table 1
Estimates of the coefficients in Eq. (1) and the corresponding monetary policy rule for the overnight federal funds rate.

Maturity (years)	1960Q1–1979Q4			1984Q1–2006Q4		
	a_n	b_n	R^2	a_n	b_n	R^2
Fed funds	2.234	0.761	0.603	1.312	1.579	0.264
	(0.493)	(0.151)		(1.277)	(0.479)	
1	2.913	0.605	0.681	1.487	1.549	0.279
	(0.388)	(0.113)		(1.201)	(0.453)	
2	2.989	0.603	0.758	1.790	1.566	0.286
	(0.333)	(0.097)		(1.141)	(0.449)	
3	3.188	0.580	0.797	2.114	1.540	0.291
	(0.295)	(0.085)		(1.061)	(0.433)	
4	3.282	0.573	0.816	2.338	1.536	0.297
	(0.28)	(0.081)		(1.001)	(0.422)	
5	3.319	0.573	0.832	2.480	1.528	0.305
	(0.288)	(0.077)		(0.951)	(0.412)	

OLS regression estimation was performed for two samples: 1960Q1–1979Q4 and 1984Q1–2006Q4. Standard errors are reported in parentheses below coefficient estimates.

Table 2
Estimates of the coefficients in Eq. (2) and the corresponding monetary policy rule for the overnight federal funds rate.

Maturity (years)	1960Q1–1979Q4				1984Q1–2006Q4			
	a_n	$b_{1,n}$	$b_{2,n}$	R^2	a_n	$b_{1,n}$	$b_{2,n}$	R^2
Fed funds	2.180	0.623	0.760	0.777	2.022	1.322	1.234	0.504
	(0.278)	(0.118)	(0.080)		(0.921)	(0.265)	(0.362)	
1	2.874	0.454	0.604	0.847	2.156	1.244	1.224	0.513
	(0.194)	(0.078)	(0.051)		(0.856)	(0.252)	(0.343)	
2	2.960	0.335	0.602	0.860	2.394	1.124	1.273	0.478
	(0.186)	(0.076)	(0.053)		(0.849)	(0.257)	(0.356)	
3	3.166	0.259	0.580	0.865	2.652	1.000	1.279	0.451
	(0.181)	(0.071)	(0.052)		(0.818)	(0.254)	(0.355)	
4	3.263	0.222	0.572	0.868	2.815	0.888	1.304	0.426
	(0.183)	(0.071)	(0.053)		(0.798)	(0.254)	(0.358)	
5	3.303	0.191	0.573	0.872	2.913	0.806	1.318	0.416
	(0.186)	(0.07)	(0.054)		(0.777)	(0.244)	(0.356)	

OLS regression estimation was performed over two samples: 1960Q1–1979Q4 and 1984Q1–2006Q4. Standard errors are reported in parentheses below the coefficient estimates.

were able to find one, it would provide evidence in support of our theory. For this reason, we estimated the regressions in Tables 1 and 2 over two sample periods. One sample is before the early 1980s, from 1960Q1 to 1979Q4, and the other after the early 1980s, from 1984Q1 to 2006Q4.[5]

The equation in the first row of Tables 1 and 2 is the estimated monetary policy rule with the overnight federal funds rate as the dependent variable. This equation simply replicates the well-known results mentioned above. Note how the monetary policy response coefficients to inflation and output are much larger in the second sub-sample. The reaction coefficient on inflation is less than one in the first sub-sample and shifted to a value substantially greater than one in the second sub-sample. The reaction coefficient on output also shows a much greater responsiveness in the second period.

The remaining rows of Tables 1 and 2 show the estimated affine models for longer-term rates. There is little or no tendency for the coefficients on inflation to be lower for longer maturities in either model or in either sample period (tests of the null hypothesis that the inflation coefficients are equal at all maturities cannot be rejected for either period). In contrast the coefficients on output are lower for longer maturities in Table 2. More important for the test of the theory is that there is a dramatic upward shift in the coefficients in the affine equations for both models between the two sample periods. For all maturities, the response coefficients are much larger in the second period than in the first period. This shift is what our formula predicts and represents empirical confirmation of the theory.[6]

[5] The results are robust to different sample periods, including using a second sub-sample of 1982Q4–2006Q4 to remove the period of reserves targeting, as well as using a second sub-sample of 1987Q3–2006Q1 coinciding with the tenure of former Federal Reserve Chairman Alan Greenspan.

[6] We find quantitatively similar results if we use zero-coupon bond data derived in Gurkaynak et al. (2006), or if we use the core CPI as our measure of inflation. In addition, the evidence of a shift in response coefficients is not sensitive to the exact choice of sample periods. Our results are similar if we include the observations between 1980Q1 and 1984Q1 in the second sample.

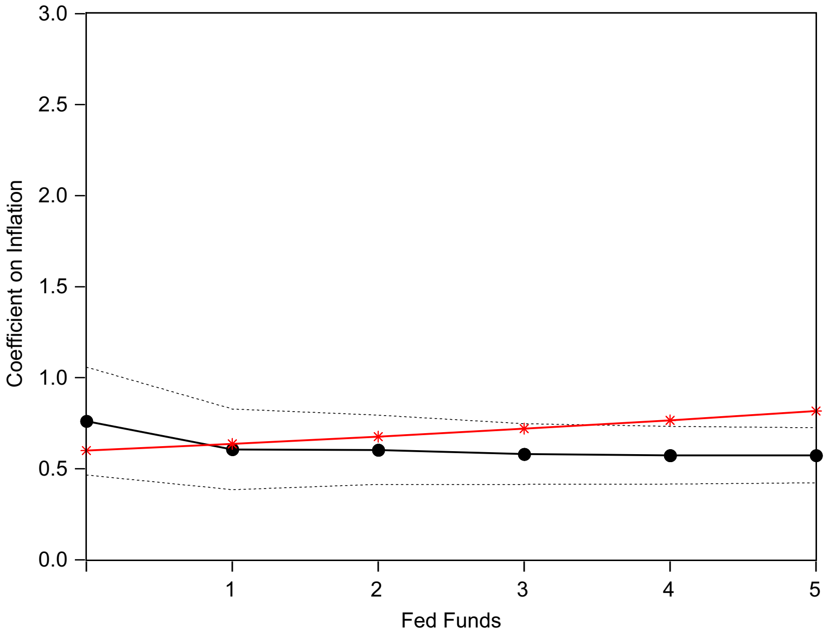

Fig. 1. The affine response coefficient (ARC) curve for inflation, derived from the model with inflation only and estimated over the period 1960Q1–1979Q4. The horizontal axis shows the maturity in years. The solid line with dots shows the estimated coefficients, the dashed lines show the 95% confidence intervals, and the solid line with stars shows the theoretical coefficients.

3. Comparing the theoretical and estimated affine response coefficients (ARC) curve

If you plot the response coefficients of the affine equations against maturity n, you get what might be called an *affine response coefficients* (ARC) curve. An ARC curve is analogous to a *yield* curve for the term structure of interest rates except that affine response coefficients are plotted rather than the yields. Such an ARC curve can be estimated, and indeed a plot with the estimated affine response coefficients from Table 1 or Table 2 on the vertical axis with maturity on the horizontal axis is such an estimate. A theoretical ARC curve can also be drawn in the same diagram using our formula in Eq. (11) or Eq. (22), and the theoretical ARC curve can be compared with the estimated ARC curve for certain parameter values. Hence, another test of our formula can be obtained by looking at how well it predicts changes in the whole shape of the ARC curve over time when the coefficients in the policy rule change.

The results of such a comparison are shown in Figs. 1–6 which show estimated and theoretical ARC curves. In each of the figures, the solid line with dots represents the empirical coefficient estimates for the specified sample period (corresponding to the estimation results reported in Tables 1 and 2), the dashed lines are the 95% confidence intervals for these estimates, and the solid line with stars represents the theoretical response coefficients for the specified macro variable given by Eqs. (11) or (22). The theoretical curves are drawn holding fixed all structural parameters of the models and changing only the policy coefficients in the monetary policy rule to approximate the kind of shift observed in the early 1980s. The values of the non-policy structural parameters are $\gamma_1 = 0.25$, $\phi = 0.15$, $\sigma = 0.25$ for the inflation only model, while for the bivariate model including inflation and the output gap, they are $\alpha_1 = 0.2$, $\alpha_2 = 0.2$, $\phi = 0.2$, $\sigma_\eta = 0.75$, $\sigma_\varepsilon = 0.36$, $\gamma_{11} = 0.1$, $\gamma_{12} = 0.15$, and $\gamma_{12} = \gamma_{21}$=0. The macro parameters are drawn from the standard macro literature, while the risk parameters are taken from the affine no-arbitrage literature.[7] For simplicity, we assume that the off-diagonal elements of the matrix γ from the bivariate model are zero, which is approximately in line with small previous estimates. In order to compare the estimated and theoretical ARC curves, we assume that policy is approximately what we estimated in Section 2. In the first model with inflation only the policy rule coefficient δ is increased from 0.6 to 1.5. In the second model with both inflation and output, the policy coefficient vector (δ_y, δ_π) is increased from (0.5, 0.6) to (1.2, 1.2).

First consider Figs. 1 and 2 which show the estimated ARC curves for the first model in the two sample periods along with the theoretical ARC curves. Fig. 1 plots the empirical estimate of the curve from Section 2 for the sample period 1960Q1–1979Q4 with its 95% confidence intervals along with the theoretical response coefficients with $\delta = 0.6$. Fig. 2 plots the same two curves except the sample is 1984Q1–2006Q4 for the estimated curve and the coefficient δ equals 1.5 for the theoretical curve which is calculated using the formula in Eq. (11). Note how the theoretical affine response curves match

[7] See Ball (1999), Rudebusch (2002), and Ang and Piazzesi (2003) for an overview of the range of parameters available in the literature.

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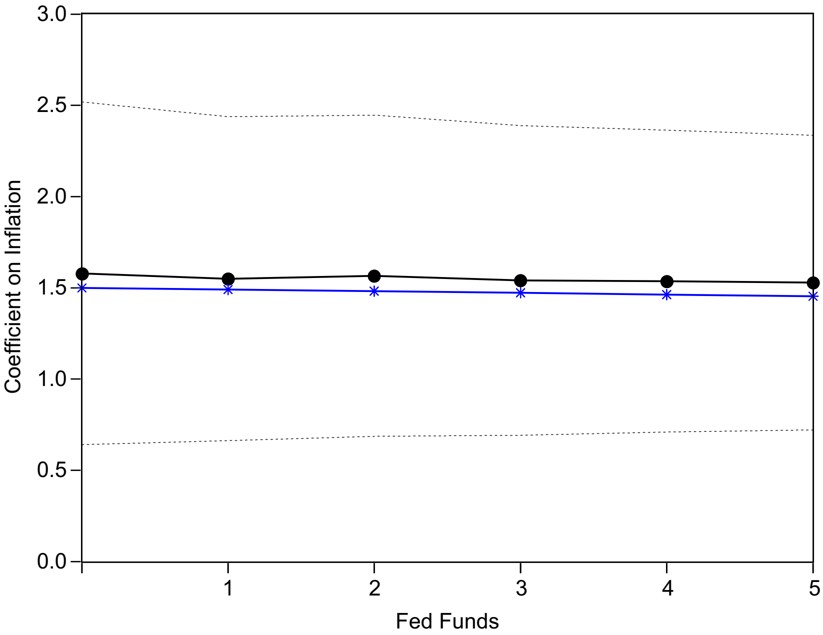

Fig. 2. The affine response coefficient (ARC) curve for inflation, derived from the model with inflation only and estimated over the period 1984Q1–2006Q4. The horizontal axis shows maturity in years. The solid line with dots shows the estimated coefficients, the dashed lines show the 95% confidence intervals, and the solid line with stars shows the theoretical coefficients.

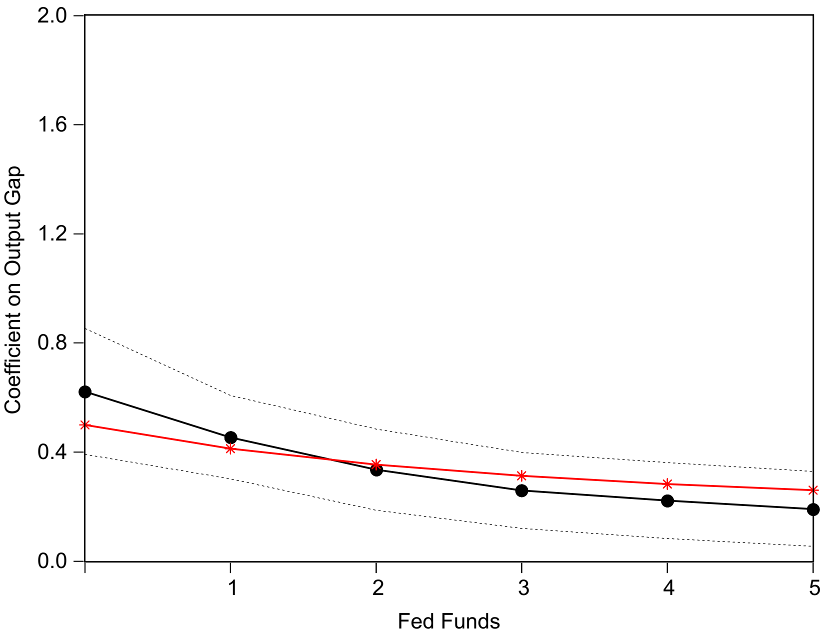

Fig. 3. The affine response coefficient (ARC) curve for output, derived from the model with both output and inflation and estimated over the period 1960Q1–1979Q4. The horizontal axis shows maturity in years. The solid line with dots shows the estimated coefficients, the dashed lines show the 95% confidence intervals, and the solid line with stars shows the theoretical coefficients.

the pronounced upward shift in the estimated curve and generally lie within the standard error bands from the estimation. Hence, a substantial portion of the increase in the affine response coefficients for long term rates is explained by the formula with these parameters and is attributable to the change in policy responsiveness.

Figs. 3–6 give estimated and theoretical ARC curves for the second model with both inflation and output. Here we use the formula in Eq. (22). The ARC curve for output (the $b_{1,n}$ coefficients) is shown in Figs. 3 and 4 for the two sample periods.

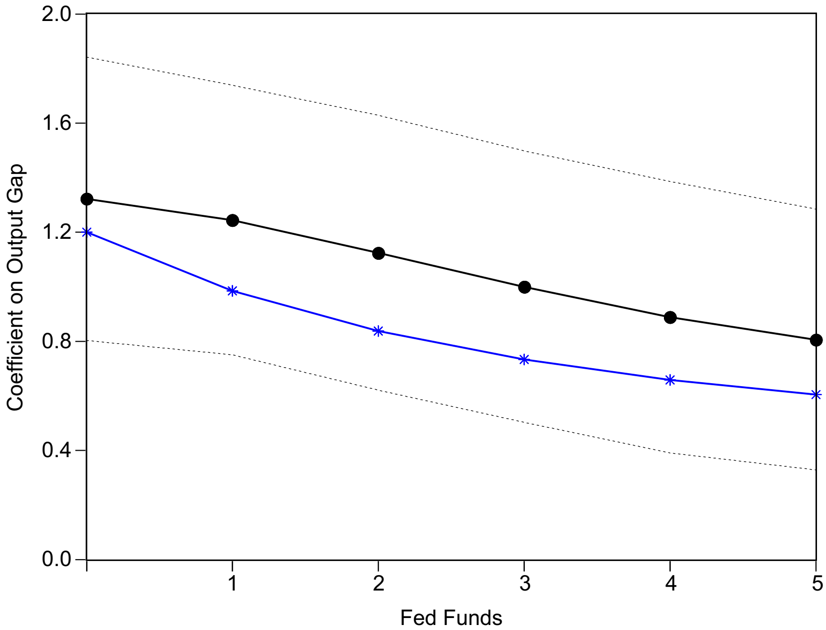

Fig. 4. The affine response coefficient (ARC) curve for output, derived from the model with both output and inflation and estimated over the period 1984Q1–2006Q4. The horizontal axis shows maturity in years. The solid line with dots shows the estimated coefficients, the dashed lines show the 95% confidence intervals, and the solid line with stars shows the theoretical coefficients.

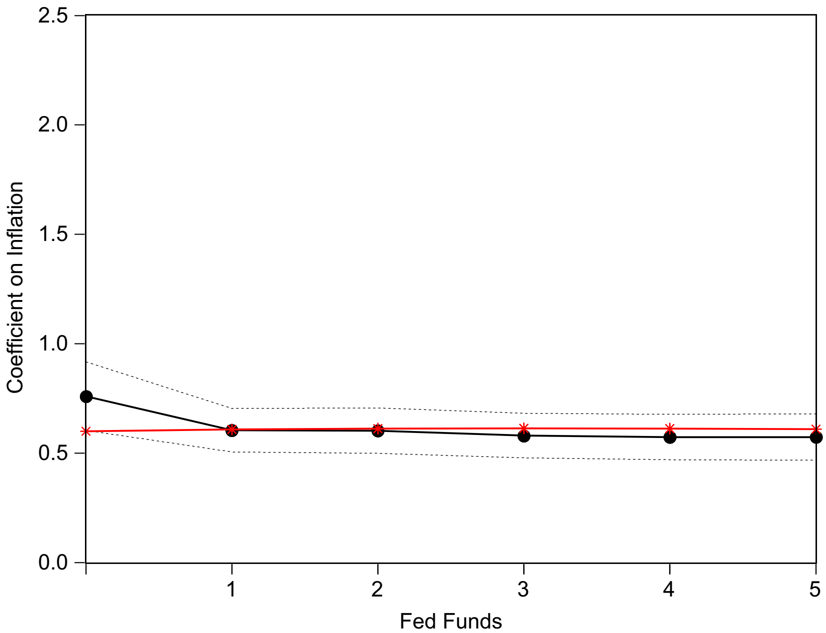

Fig. 5. The affine response coefficient (ARC) curve for inflation, derived from the model with both output and inflation and estimated over the period 1960Q1–1979Q4. The horizontal axis shows maturity in years. The solid line with dots shows the estimated coefficients, the dashed lines show the 95% confidence intervals, and the solid line with stars shows the theoretical coefficients.

In this case the formula captures not only the upward shift in the ARC curve but also its general downward slope. The ARC curve for inflation (the $b_{2,n}$ coefficients) is shown in Figs. 5 and 6 for the two sample periods. Again the formula accurately captures the upward shift and the nearly flat slope. Even though these macro models are extremely simple the results are a very encouraging confirmation of the usefulness of the formula and the overall approach.

Please cite this article as: Smith, J.M., Taylor, J.B., The term structure of policy rules. Journal of Monetary Economics (2009), doi:10.1016/j.jmoneco.2009.09.004

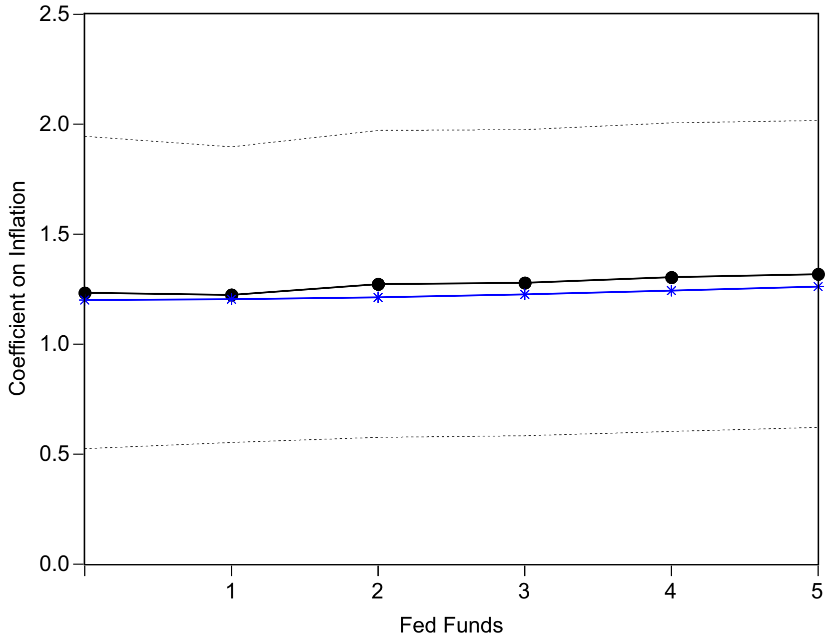

Fig. 6. The affine response coefficient (ARC) curve for inflation, derived from the model with both output and inflation and estimated over the period 1984Q1–2006Q4. The horizontal axis shows the maturity in years. The solid line with dots shows the estimated coefficients, the dashed lines show the 95% confidence intervals, and the solid line with stars shows the theoretical coefficients.

4. An explanation of the term structure conundrum

As an example of how these results can be used for policy evaluation, consider the famous term structure puzzle which first arose when the Federal Reserve started raising the federal funds rate in 2004. That increase in the short-term interest rate was not associated with as large an increase in long-term interest rates as the Fed expected based on experience over the previous 20 years as explained by former Federal Reserve Chairman Alan Greenspan who called the puzzle a “conundrum.” It was a great concern for policymakers for it appeared that the tightening of monetary policy would not have had the bite that it had in previous periods of tightening. One explanation for the conundrum is that there was a global saving glut that drove down the world real interest rate, but that explanation has been challenged because world saving as a share of world GDP had actually fallen during this period.[8]

An alternative explanation naturally emerges from the theory in this paper. During this period, the federal funds rate deviated significantly from what would have been predicted by Fed’s typical response as exemplified by the empirical estimates of the policy rule reported in Table 2 of this paper for the sample period from 1984Q1 to 2006Q4. While it is difficult to determine whether this was a shift in the policy response coefficients or simply an additive deviation from the rule, there is econometric evidence that it may have been interpreted as a shift in the response coefficient. To see this, consider the following regression estimated over the 1984Q1–2006Q4 period:

$$r_t = 2.056 + 1.016y_t + 1.428\pi_t - 1.327\pi_t^D \tag{25}$$

where π_t^D is a multiplicative dummy variable in which the actual inflation rate is multiplied by a dummy variable that equals one from 2002Q4–2005Q4, and zero elsewhere. This is the period when the actual federal funds rate deviated significantly from estimated policy rules such as that in the first row of Table 2. All the coefficients in this equation are statistically significant, and in particular the inflation terms and the multiplicative dummy are highly significant. We also included an additive dummy along with the multiplicative dummy in the regression and still found a significant downward shift in the inflation response. The equation clearly suggests the possibility that the response coefficient on inflation dropped significantly during this period. The short-term interest rate response to inflation (δ_π) would seem much lower to market participants trying to assess Federal Reserve policy. Note that Davig and Leeper (2006) found a similar shift using an estimated Markov switching model.

Now, according to the theory presented in this paper, a perception of a smaller response coefficient in the policy rule could have led market participants to expect smaller interest rate responses to inflation in the future, and therefore lower

[8] See Bernanke (2005) and Taylor (2007).

long-term interest rate responses. That is, we would predict that the lower response δ_π would have lowered the response coefficient $b_{2,n}$ for inflation. Hence, our model provides a simple consistent explanation for the conundrum. While this shift was temporary when viewed from the perspective of today, it would have been difficult to assess at the time whether the Federal Reserve would have returned to the typical rule followed during the post 1984Q1 period.

5. Conclusion

In this paper, an affine no-arbitrage model of the term structure was imbedded into a simple macro model. Using this framework we derived closed-form affine equations for long-term interest rates and showed that a simple formula links the coefficients on inflation and GDP in the monetary policy rule for the short-term interest rate to comparable affine response coefficients. The formula predicts that an increase in the coefficients in the monetary policy rule will lead to an increase in the coefficients in the affine equations across a wide range of maturities, a prediction confirmed by an upward shift in the coefficients in the affine equations in the 1980s at the same time as the well-known upward shift in the policy rule coefficients. We also examined the affine response coefficient (ARC) curve which shows now the coefficients in the affine equations depend on the maturity of the bonds. Shifts in this curve are predicted remarkably well by the formula as demonstrated with simple graphs. Finally, our model helps explain the behavior of longer-term interest rates during the recent spell in 2002–2005, where standard models predicted that longer-term yields would increase more than they did. Moreover, because the formula has a closed form, it is a useful pedagogical tool.

There are several directions that future research might take. Clearly our macro model has a number of simplifying assumptions and incorporating more individual optimizing behavior would be worth exploring, though at the cost of a likely loss in analytical simplicity. Using models of stochastic regime shifting, such as Davig and Leeper (2007), along with no-arbitrage restrictions may help pin down the exact patterns of bond yields and macroeconomic variables. Including longer-term interest rates directly in the macro model would be an important check for robustness. Applying the model to policy changes in other countries—even more dramatic changes than studied here—would also be valuable.

Acknowledgments

The authors thank William English, Alan Greenspan, Refet Gurkaynak, Nir Jaimovich, Thomas Laubach, Monika Piazzesi, Ricardo Reis, Glenn Rudebusch, Ken Singleton, Johannes Stroebel, Eric Swanson, John Williams, the International Research Forum on Monetary Policy at the European Central Bank, and seminar participants from the Federal Reserve Bank of San Francisco and Stanford University for insightful comments.

Appendix. Supplementary materials

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jmoneco.2009.09.004.

References

Ang, A., Piazzesi, M., 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics 50, 745–787.
Ang, A., Dong, S., Piazzesi, M., 2005. No Arbitrage Taylor Rules. Chicago Business School.
Ball, L., 1999. Efficient rules for monetary policy. International Finance, April, pp. 63–83.
Bekaert, G., Cho, S., Moreno, A., 2005. New-Keynesian macroeconomics and the term structure. Unpublished manuscript.
Bernanke, B., 2005. The Global Saving Glut and the U.S. Current Account Deficit. Federal Reserve Speech.
Clarida, R., Gali, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. Quarterly Journal of Economics 115, 147–180.
Davig, T., Leeper, E., 2006. Fluctuating macro policies and the fiscal theory. In: Acemoglu, D., Rogoff, K., Woodford, M. (Eds.), NBER Macroeconomics Annual, pp. 247–298.
Davig, T., Leeper, E., 2007. Generalizing the Taylor principle. American Economic Review 97, 607–635.
Fuhrer, J., 1996. Monetary policy shifts and long-term interest rates. Quarterly Journal of Economics 111, 1183–1209.
Gallmeyer, M., Hollifield, B., Palomino, F., Zin, S., 2007. Arbitrage-free bond pricing with dynamic macroeconomic models. Federal Reserve Bank of St. Louis, Review, 305–326.
Gurkaynak, R., Sack, B., Wright, J., 2006. The U.S. Treasury yield curve. FEDS, Federal Reserve Board.
Rudebusch, G., 2002. Assessing the Lucas critique in monetary policy models. Federal Reserve Bank of San Francisco Working Paper.
Rudebusch, G., Wu, T., 2007. Accounting for a shift in term structure behavior with no-arbitrage and macro-finance models. Journal of Money, Credit, and Banking 39, 395–422.
Taylor, J.B., 1999. A historical analysis of monetary policy rules. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago.
Taylor, J.B., 2007. Housing and monetary policy. In: Housing, Housing Finance, and Monetary Policy, Proceedings of the Federal Reserve Bank of Kansas City Symposium, Jackson Hole.
Woodford, M., 2001. The Taylor rule and optimal monetary policy. American Economic Review 91, 232–237.
Woodford, M., 2003. In: Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.