ELSEVIER

Journal of Economic Dynamics & Control
25 (2001) 891–910

JOURNAL OF Economic Dynamics & Control

www.elsevier.nl/locate/econbase

# Optimal horizons for inflation targeting☆

Nicoletta Batini[a], Edward Nelson[b,*]

[a]*Monetary Policy Committee Unit, Bank of England, Threadneedle Street, London EC2R 8AH, UK*
[b]*Monetary Assessment and Strategy Division, Monetary Analysis, Bank of England, Threadneedle Street, London EC2R 8AH, UK*

Received 1 February 2000; accepted 12 June 2000

**Abstract**

In this paper we investigate the problem of selecting an optimal horizon for inflation targeting in the United Kingdom. Since there are two key ways of thinking about an optimal horizon, we look at optimal horizons for both of these interpretations. In addition, to see whether our results are robust in the face of model uncertainty, we compute optimal horizons for two different models with divergent structural and dynamic characteristics. © 2001 Elsevier Science B.V. All rights reserved.

*JEL classification:* E52; E37; E58

*Keywords:* Inflation targeting; Horizon; Optimal policy rules; Simple feedback rules; Target

☆We thank Claes Berg, Spencer Dale, Andrew Haldane, Peter Ireland, DeAnne Julius, Stephen Millard, Glenn Rudebusch, Frank Smets, Lars Svensson, Paul Tucker, Anders Vredin, Peter Westaway, Tony Yates, and two anonymous referees for useful comments on earlier versions of this paper. We also thank participants at Bank of England seminars, the Fifth International Conference of the Society for Computational Economics, and the Sveriges Riksbank Workshop on Inflation Targeting and Exchange Rate Fluctuations. The views expressed in this paper are our own and should not be interpreted as reflecting those of the Bank of England or the Monetary Policy Committee.

* Corresponding author. Tel.: + 44-20-7601-5692.

*E-mail addresses:* nicoletta.batini@bankofengland.co.uk (N. Batini), edward.nelson@ btinternet.com (E. Nelson).


PII: S 0 1 6 5 - 1 8 8 9 ( 0 0 ) 0 0 0 6 0 - 9

## 1. Introduction

Many central banks, including those of Australia, Canada, the Eurosystem, Israel, New Zealand, Sweden, and the United Kingdom, pursue an inflation target. In practice, this usually involves 'targeting' the conditional forecast of inflation — the inflation rate expected to prevail in the future given presently available information — rather than current inflation.

A crucial issue is how to choose the horizon, i.e. the appropriate value of $k$ when the operational target is expected inflation $k$ periods ahead. There are two key ways of thinking about an optimal horizon for inflation targeting, depending on the way that inflation targeting is modelled.

If policy is represented, for instance, by a simple feedback rule on expected future inflation, one way is to think of it as the best horizon for which the authorities should form a forecast for inflation to use in the rule. If, instead, policy is represented by an optimal rule for the instrument, the optimal horizon can be thought of as the time at which inflation should be on target in the future when the authorities aim at minimizing their loss function, and a shock occurs today. In what follows, we refer to the first kind of horizon as the 'optimal feedback horizon' and to the second kind as the 'optimal policy horizon'.

This paper calculates optimal horizons for inflation targeting, using each of the two definitions described above. Since the results may well hinge on the features of the model used for the calculations, the paper derives parallel results for two models: a vector autoregression (VAR) estimated on quarterly U.K. data; and a small-scale structural open-economy model based on Ball (1999), Batini and Haldane (1999), and McCallum and Nelson (1999a). A key difference between the two models is the importance that the second model assigns to forward-looking behavior in spending and pricing decisions.

The paper is organized as follows. In Section 2 we discuss alternative definitions of horizons for inflation targeting. In Section 3 we describe the policymakers' objective function and the macroeconomic models that we employ. In Section 4 we compute optimal policy horizons for each model, and discuss the results. In Section 5 we consider optimal feedback horizons, and Section 6 provides a brief exploration of sensitivity of the results to parameter uncertainty. Concluding remarks follow in Section 7.

## 2. Optimal horizons for inflation targeting: Two definitions

Frequently, in those countries which pursue inflation targets, the formal wording of the mandate for the central bank provides that the inflation target be achieved each year.[1] For instance, the mandate for the European Central Bank, the most recently established central bank, states: '[P]rice stability shall be defined as a year-on-year increase in the Harmonized Index of Consumer Prices for the euro area of below 2% …' (ECB, 1999, p. 46).

But central banks that have an inflation target need an operating strategy for achieving it. In practice, many do so by focussing on expected future inflation. The main reason for this is the existence of lags in the transmission of monetary policy to inflation.

The recognition of the existence of lags from monetary policy changes to inflation — and attempts to quantify these lags — have a long history. Jevons (1863), using U.K. data, concluded that 'An expansion of the currency occurs one or two years previous to a rise in prices …'. More recent empirical work, primarily using VAR analysis, has employed interest rate-based monetary policy measures. Using U.S. data, Christiano et al. (1996), for example, estimate that a monetary policy shock affects real GDP with a two-quarter lag, and the GDP deflator with a four-quarter lag.

It is possible that in more open economies, such as those currently pursuing an inflation target, the transmission lag is shorter because policy operates not only via the conventional 'output gap channel', but also via a potentially swift 'exchange rate channel'. But even in economies that are open, the exchange rate channel may not be as effective, or as quantitatively important, as the output gap channel. Neither the pass-through of exchange rate changes to import prices, nor the propagation of import price changes into aggregate price level changes, can be taken for granted.[2] In the U.K., for example, the substantial exchange rate depreciations in 1982 and 1992 failed to produce appreciable increases in the inflation rate (measured by the RPIX index). In other words, even in economies that are more open than the U.S., the time it takes for monetary policy to have its main impact on nominal demand may still be lengthy.

In the presence of transmission lags, returning inflation to target immediately after a shock may involve considerable costs. This is because offsetting immediately the inflationary consequences of a shock may require large movements in the policy instrument, with unduly large output losses as a result. One obvious way to avoid this is to try to anticipate inflationary events and react to them pre-emptively in a more gradual fashion. Acting before it is too late permits central banks to minimize those losses by reducing the extent to which the instrument has to be moved in the short run in order to control inflation.

[1] Bernanke et al. (1999) provide a recent overview of several countries' inflation targeting arrangements.

[2] The estimates of our VAR for the U.K. (see Section 3 below) support caution about the empirical importance of the exchange rate channel for output and inflation.

In the literature on inflation targets, this forward-looking approach to policy has been represented in two ways.

Rudebusch and Svensson (1999) argue that real-world inflation targeting can be approximated by viewing the central bank as carrying out an optimization exercise, where the welfare function penalizes inflation departures from a target, and policy is thus set according to the ensuing optimal rule.[3] In their words (1999, p. 204): 'In examining policy rules that are consistent with inflation targeting, we consider two broad class of rules: instrument rules and targeting rules ... A targeting rule may be closer to the actual decision framework under inflation targeting. It is represented by the assignment of a loss function over deviations of a goal variable from a target level ...'.

Rudebusch and Svensson's definition has strengths and weaknesses. One strength is that it reflects a comprehensive approach to policy — a 'look-at-everything' approach. It also implies that information is processed in the most efficient way — which is the way central banks often visualize their policy. However, this definition implies that policy is set in a complex manner that the public may find difficult to understand, particularly when there is no consensus on either the model or the objective function.

A second definition of inflation targeting is used in work by Batini and Haldane (1999), McCallum and Nelson (1999b), and others. This views targeting expected future inflation simply as setting the policy instrument in response to deviations of future inflation from target. In other words, it defines targeting as the use of a policy rule such as

$$R_t = \text{constant} + \psi_p[E_t\pi_{t+k} - \pi^*], \tag{1}$$

where $R_t$ is the short-term nominal interest rate, $E_t\pi_{t+k}$ is the period $t$ forecast of inflation in $t+k$, $\pi^*$ is the inflation target, and $\psi_p > 1$.

Strengths of this approach are that it does not necessarily require agreement on every aspect of model specification other than in the construction of the forecast (for which there is no need for a full model). This is because rule (1) has the property that it will rein in deviations of inflation from target in a variety of models, subject to weak conditions which will be met by most standard model specifications.[4] Consequently, (1) is a rule that policymakers could agree would control inflation, even if they had no consensus on the appropriate objective function or model of the economy.

Furthermore, Eq. (1) is consistent, for $k > 0$, with the inflation forecast at some specific horizon being a key input into policymakers' decisions. By contrast, in most cases a standard optimization exercise would not give special

[3] See also Svensson (1997,1999).

[4] The condition is that the long-run response of the inflation rate to monetary policy tightenings is negative.

weight to the inflation forecast at a particular horizon, especially as that forecast may not even appear in the first order conditions for optimality.

The principal weakness of this approach is that it is based on a simple rule that uses information less efficiently than an optimal rule.

The above considerations suggest that inflation-targeting central banks need to decide how forward-looking they should be in order to bypass the transmission lags. That is, when setting policy, they must choose an 'optimal horizon' over which to pursue the goal of price stability. Since the concept of horizon means different things under the two interpretations of inflation targeting, to investigate this issue we provide two operational definitions of 'optimal horizon': the 'optimal policy horizon' and the 'optimal feedback horizon'.

We define the *optimal policy horizon* (hereafter, 'OPH') as the number of periods it takes for inflation to settle on target after a shock, under the optimal rule for the instrument. This is in line with Rudebusch and Svensson's interpretation of inflation targeting — an optimization exercise that penalizes deviations of inflation from target and of output from potential. (More specifically, this corresponds to what Svensson (1997,1999) terms 'flexible inflation targeting'.) We call this horizon the optimal *policy* horizon because of its intimate connection with the optimal policy rule; but an equally valid, and perhaps more descriptive, label would be the optimal *stabilization* horizon.

There are two things to note about this definition. First, we treat the underlying optimization exercise undertaken by the central bank as one subject to commitment. Svensson (e.g. 1999) instead generally views it as optimization subject to discretion, but Woodford (1999) demonstrates that discretion has drawbacks when (as in one of our models below) the model's structural equations contain forward-looking components.[5] More recently, Svensson and Woodford (1999) have proposed several candidate modifications to standard discretionary optimization, which — by connecting current and past policy actions — reduce the problems associated with discretionary behavior. Future work could compare our results using commitment to results based on Svensson and Woodford's modified discretionary framework. Second, the optimal policy horizon in our definition is an *output* of the optimization exercise, rather than a literal constraint on the optimization problem. See Smets (2000) for an alternative approach, in which the requirement that $E_t \pi_{t+k} = 0$ (for a specified $k \geq 0$) is a constraint on the policymaker's optimization problem (i.e., an *input* in the optimization process).

Turning to our second concept of horizon, we define the *optimal feedback horizon* ('OFH' hereafter) as the best point in the future for which the authorities should form the inflation forecast that enters their policy rule. The OFH is an

[5] These problems arise from the fact that discretionary optimization generally leads to predictable future deviations by policymakers from their presently announced plans.

optimal *response* horizon, i.e. the best horizon to focus on when designing a simple, inflation forecast-based rule. Therefore, the OFH is the $k$ associated with the minimization of the policymakers' loss function, when policy follows a simple rule such as (1).

In the next section, we define the policymakers' preferences and describe the models that we use to derive optimal horizons under each of the above definitions.

## 3. Objective function and models

### 3.1. Objective function

We assume that policymakers wish to prevent deviations of inflation from target and departures of output from potential. We also assume that policymakers dislike instrument volatility. For computational convenience, these preferences are represented by a quadratic loss function. In the optimization exercises used to derive optimal policy horizons, this is the function that is being minimized. And when we derive optimal feedback horizons by comparing the performance of rules like (1) for various $k$s, this loss function is used to compute welfare losses in all experiments. Formally, the loss function is given by

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j [\lambda_\pi (4^*\pi_{t+j} - 4^*\pi^{\mathrm{T}}_{t+j})^2 + \lambda_y (y_{t+j} - y^{\mathrm{T}}_{t+j})^2 + \lambda_{\Delta R}(4^*\Delta R_{t+j})^2], \tag{2}$$

where $\beta$ is the discount factor, $4^*\pi_t$ is annualized quarterly inflation, $\pi^{\mathrm{T}}_t$ is the inflation target, $y_t$ is log output, $y^{\mathrm{T}}_t$ is log capacity output, and where $\lambda_\pi$, $\lambda_y$ and $\lambda_{\Delta R}$ denote the weights assigned to inflation deviations from target, output deviations from potential, and volatility in the first difference of the nominal interest rate, respectively.

We set $\beta = 0.99$, $\lambda_\pi = 1$, $\lambda_y = 1$ and $\lambda_{\Delta R} = 0.5$, so that inflation and gap variability are penalized equally. The interest rate volatility term, which rules out extremely large movements of the instrument in response to shocks, receives a penalty half that of the other terms. These weights are similar to those used in Rudebusch and Svensson (1999).

### 3.2. Models

To explore the optimal horizon issue, we look at two models: a vector autoregression (VAR) estimated on quarterly U.K. data; and a calibrated, forward-looking small structural model. These models are described below.

### 3.3. A VAR model

Our first model is a one-lag VAR with a linear trend, estimated over 1981Q1–1998Q1.[6] There are four endogenous variables in the VAR: log output ($y_t$); the deviation of annual RPIX inflation from the inflation target ($\pi_t^{DEV}$); the log-change in the nominal exchange rate ($\Delta e_t$); and the nominal interest rate (interbank lending rate), measured as an annualized fraction ($4^*R_t$).[7]

As Rudebusch (1998) observes, the interest rate equation in a VAR has a structural interpretation as a monetary policy reaction function. As he also notes, however, there is a danger that policy regime shifts may produce nonconstant parameter estimates. In our sample period, there have been two major breaks in the U.K.'s monetary policy regime: the U.K.'s membership of the exchange rate mechanism (ERM) from 1990 to 1992; and, following its exit from the ERM, its adoption of an inflation targeting regime from 1992Q4. If we allow both the slopes and the intercepts of the interest rate equation in our VAR to vary across these regimes, the restriction of no structural change is rejected ($F(12,51) = 2.51$ [$p$ value $= 0.01$]). However, the convenient restriction that the parameter nonconstancy is isolated to the equation's intercepts is not rejected ($F(10,51) = 1.69$ [$p$ value $= 0.11$]). Hence, we proceed under that assumption, augmenting each equation of the VAR with two regime-shift intercept dummies, $DERM_t$ and $D924_t$.[8]

Estimates of this system are reported in Table 1. The lag length of one-quarter is not rejected by a $\chi^2$ test against the alternative of a two-lag VAR ($p$-value $= 0.10$).[9]

Several features of the dynamic properties of the system emerge. First, consider the output equation. Although the estimated coefficient on $4^*R_{t-1}$ in the $y_t$ regression is quite small ($-0.0908$), suggesting a minor initial impact of monetary policy on real demand, the estimated coefficient on lagged $y_t$ in the equation is large, implying that the *long-run* response to $4^*R_t$ is much greater ($-1.3373\%$). The exchange rate term in the $y_t$ equation, on the other hand, is small and insignificant.

Second, in line with economic intuition, output has a significant positive coefficient in the inflation equation; and so interest rates have a negative effect on inflation, apparently via a conventional output gap channel.

---

[6] The linear trend is included to detrend the output variable.

[7] A Data Appendix available from the authors provides a detailed description of these series and their time series properties. In our work in Sections 4 and 5 with this VAR, the estimated $y_t$ equation will serve as the output gap equation, and the $\pi_t^{DEV}$ equation, converted to quarterly units, will be used as the inflation equation.

[8] These variables take the value 1.0 for 1990Q4–1992Q3 and 1992Q4 onwards, respectively.

[9] Due to the large lagged-dependent variable coefficients, the one-lag specification is consistent with a long lag between monetary policy changes and their peak effect on output or inflation.

Table 1
VAR estimates
Sample period: 1981 Q1–1998Q1[a]

| | $y_t$ | $\pi_t^{\text{DEV}}$ | $\Delta e_t$ | $4^*R_t$ |
|---|---|---|---|---|
| $y_{t-1}$ | 0.9321 | 0.1250 | − 0.1583 | 0.0444 |
| | (0.0464) | (0.0575) | (0.3273) | (0.0782) |
| $\pi_{t-1}^{\text{DEV}}$ | − 0.0926 | 0.8559 | 0.3544 | 0.3867 |
| | (0.0731) | (0.0905) | (0.5150) | (0.1231) |
| $\Delta e_{t-1}$ | − 0.0029 | − 0.0023 | 0.1526 | − 0.0416 |
| | (0.0175) | (0.0217) | (0.1234) | (0.0295) |
| $4^*R_{t-1}$ | − 0.0908 | 0.0336 | − 0.0438 | 0.5759 |
| | (0.0529) | (0.0655) | (0.3731) | (0.0892) |
| Constant | 0.7821 | − 1.4072 | 1.7167 | − 0.4786 |
| | (0.5177) | (0.6409) | (3.6487) | (0.8722) |
| Time trend | 0.00046 | − 0.00091 | 0.002477 | 0.000287 |
| | (0.00040) | (0.00049) | (0.00280) | (0.00067) |
| $DERM_t$ | − 0.00891 | 0.0091 | − 0.0509 | − 0.0300 |
| | (0.00563) | (0.0070) | (0.0397) | (0.0095) |
| $D924_t$ | − 0.0091 | 0.0108 | − 0.0573 | − 0.0422 |
| | (0.0073) | (0.0090) | (0.0515) | (0.0123) |
| $R^2$ | 0.9984 | 0.8880 | 0.1017 | 0.9259 |
| SEE | 0.0052 | 0.0064 | 0.0364 | 0.0087 |
| SD dep. var. | 0.1212 | 0.0181 | 0.0364 | 0.0303 |

[a]Note: Standard errors in parentheses.

Finally, the estimated equation for $4^*R_t$ looks like a Taylor (1993)-type policy rule, with the interest rate responding positively both to lagged output (long-run coefficient = 0.10) and to inflation (long-run coefficient = 0.91, not significantly below unity).[10] In addition, the coefficient on the lagged-dependent variable (0.58) suggests a strong tendency by policymakers to smooth interest rates; and the coefficient on the exchange rate change (equal to − 0.042) suggests that the interest rate responds positively to exchange rate depreciations, as one would expect.[11]

In this paper, we subject the VAR to hypothetical policy rules different from the estimated one. This requires us to identify the VAR model's responses to shocks. We do this by means of a Cholesky decomposition, where the equation disturbances are assumed to follow the causal ordering (output

[10] Estimating U.K. policy rules over subsamples of the 1972–1997 period, Nelson (2000) finds point estimates of the long-run response to inflation below unity, with the exception of the post-1992 period. Nelson also finds output gap responses of the same magnitude as that estimated in Table 1.

[11] Note that no variables enter the exchange rate equation itself significantly, indicating that a random walk model for $e_t$ is not rejected.

innovation → inflation innovation → exchange rate innovation → interest rate innovation). Under this identification scheme, no variable beside the interest rate responds contemporaneously to the monetary policy shock.

### 3.4. A forward-looking structural model

The second model that we consider is a small-scale, forward-looking open-economy model that incorporates elements of Ball (1999), Batini and Haldane (1999), and McCallum and Nelson (1999a). The structural equations of the model are:

$$y_t = E_t y_{t+1} - \sigma(R_t - E_t \pi_{t+1}) + \delta \tilde{q}_{t-1} + e_{y_t}, \tag{3}$$

$$\pi_t = \alpha \pi_{t-1} + (1-\alpha) E_t \pi_{t+1} + \phi_y y_{t-1} + \phi_q \Delta \tilde{q}_{t-1} + e_{\pi t}, \tag{4}$$

$$E_t q_{t+1} = q_t + R_t - E_t \pi_{t+1} + \kappa_t, \tag{5}$$

where $y_t$ is log output, $R_t$ is the nominal interest rate (again, a quarterly fraction), $\pi_t$ is quarterly inflation, $q_t$ is the log real exchange rate (measured so that a rise is a depreciation), and $\tilde{q}_t = \frac{1}{4}\sum_{j=0}^{3} q_{t-j}$ is a four-quarter moving average of $q_t$. These variables are all expressed relative to steady-state values. $e_{y_t}$, $e_{\pi_t}$, and $\kappa_t$ are exogenous IS, Phillips curve, and uncovered interest parity (UIP) shocks, respectively.

Eq. (3) is the model's IS equation, giving $y_t$ as a function of its expected future value, the real interest rate, and lags of the real exchange rate. Apart from the term in $\tilde{q}_t$, this equation corresponds to the optimization-based IS function in McCallum and Nelson (1999a), and we choose parameter values based on their estimates: $\sigma = 0.2$ and an AR(1) process for $e_{y_t}$ with coefficient 0.3 and 1% innovation standard deviation. Our choice of $\delta = 0.05$ then produces the same ratio of interest rate to exchange rate coefficients in the IS curve as is used in Batini and Haldane (1999).

Eq. (4) is a quarterly version of the Ball (1999) open-economy Phillips curve, modified to allow for some forward-looking behavior. While Ball has lagged inflation appearing on the right-hand side of (4) with coefficient 1.0, we replace this with the mixed backward–forward looking term $\alpha\pi_{t-1} + (1-\alpha)E_t\pi_{t+1}$, and calibrate $\alpha$ to 0.8, close to estimates in Fuhrer (1997) and Rudebusch (1999). We calibrate the coefficient $\phi_y$ to 0.1, the quarterly counterpart of Ball's choice. We choose $\phi_q = 0.025$; this is considerably lower than Ball's 0.10, but a relatively conservative value of $\phi_q$ seems prudent in light of the failure of the VAR to pick up *any* effect of depreciation on inflation. We assume $e_{\pi_t}$ is white noise with standard deviation 1%.

The exchange rate enters both the IS and Phillips curve relationships in a backward-looking manner, as a lagged four-period average. A more forward-looking specification of the model's open-economy elements would put $\Delta q_t$ and

$E_t \Delta q_{t+1}$ in (4).[12] We found, however, that this scheme produced an implausibly tight and mechanical relationship between exchange rate change and inflation.[13] Thus, we have followed Ball (1999) by only allowing $q_t$ to enter with lags; this might be rationalized by 'gradual pass-through' of exchange rate changes to export and import prices, which might be realistic for the U.K. (Bank of England, 1999).[14] While $q_t$ enters Eqs. (3) and (4) only in a backward-looking manner, this is compensated by the fact that the exchange rate itself is a highly forward-looking variable, as Eq. (5) indicates. The shock term $\kappa_t$ that produces deviations from strict UIP in (5) is assumed to be $AR(1)$ with coefficient 0.753 and innovation standard deviation 0.92%; these choices are based on our estimates of this process using quarterly U.K. data. The shocks in (3)–(5) are assumed to be mutually uncorrelated.

Note that in this model — hereafter referred to as the FLSM (forward-looking structural model) — monetary policy has some effect on contemporaneous inflation due to the fact that $\pi_t$ responds to $E_t \pi_{t+1}$ in Eq. (4). This effect may be small in relation to the long-run effect of policy on inflation, but the presence of at least *some* effect means that the model does not have the same property as that in Svensson (1997), where there is no scope for current policy changes to affect inflation today.

## 4. Optimal policy horizons (OPHs)

In-line with our discussion in Section 2, we define the *optimal policy horizon* (OPH) as the time at which it is least costly, for a given loss function, to bring inflation back to target after a shock. More intuitively, the OPH is the horizon-analogue of the optimal speed of disinflation, i.e. the optimal time required for the dissipation of a shock. Operationally, the OPH is given by the number of periods after a shock when inflation is back on target under an optimal rule.

Below, we derive OPHs for both models. In this respect, an important question is how to interpret the idea of being 'on target'. Since, in these models, inflation tends to fluctuate around target before settling definitely on a particular number in the wake of a shock, a point target (e.g. $2\frac{1}{2}\%$) is not very

---

[12] Such a setup might also put $q_t$ and $E_t q_{t+1}$ in Eq. (3).

[13] In Bank of England (1999), it is shown that the inclusion of $\Delta q_t$ and $E_t \Delta q_{t+1}$ in Batini and Haldane's (1999) Phillips curve makes exchange rate movements the dominant determinant of inflation, and virtually removes any inflation persistence from the model.

[14] In his annual model, Ball specifies $\Delta q_t$ as entering the inflation equation with a one-period lag; in our quarterly model, we approximate this by having the prior year's average of $\Delta q_t$ entering Eq. (4).

meaningful. Instead, we consider OPHs as referring to target *ranges* — so the OPH is the time when inflation returns to a specified band around the target. This is not an argument for target ranges rather than point targets, but a device to make model experiments useful.

We use two operational definitions of an OPH: an absolute and a relative horizon concept. The first definition interprets an OPH as the number of periods ahead, $k$, at which inflation has returned permanently to within a target range of $\pm 0.1$ percentage points, following a shock today. The second definition is based on what fraction of a shock's effect policy has succeeded in eliminating. Specifically, it interprets the optimal policy horizon as the number of periods ahead, $k$, at which 90% of the peak effect of the shock on inflation has been extinguished. We denote OPHs under the absolute criterion by '$k^*_{\mathrm{A}}$', and OPHs under the relative criterion by '$k^*_{\mathrm{R}}$'. Since, typically, both $k^*_{\mathrm{A}}$ and $k^*_{\mathrm{R}}$ will vary according to the nature of the economic shock, we compute OPHs under the two criteria for different kinds of shocks. In our discussion, we focus primarily on $k^*_{\mathrm{R}}$. This is because, in these two linear models, the $k^*_{\mathrm{A}}$ value depends on the size of the initial shock, but $k^*_{\mathrm{R}}$ — defined in terms of a percentage of the effect of the shock — does not.

After solving each model of Section 3 under its optimal rule, we computed impulse responses for inflation.[15] Figs. 1(a)–(c) show the optimal paths of inflation associated with each model in the face of 1% shocks to aggregate demand, aggregate supply and the exchange rate, respectively.

Table 2 gives OPHs for each model under the two definitions ($k^*_{\mathrm{A}}$ and $k^*_{\mathrm{R}}$). We discuss these results in turn as we comment on the impulse response function of inflation for each shock.[16]

*Aggregate demand shock*: Fig. 1(a) (with units on the $x$-axis in quarters, and those on the $y$-axis in percentage point deviations from base) shows the impulse response functions of inflation for the two models described in Section 3, in the wake of a temporary positive 1% shock to aggregate demand.

In the VAR (the dashed line in Figs. 1(a)–(c)), an AD shock has no effect on inflation until period $t+1$; and by that time the policymaker (whose actions in $t$ affect inflation in $t+1$) is in a position to provide offsetting pressure on inflation through interest rates. Consequently, the initial net response of

[15] We follow King and Wolman (1999) by augmenting the model's structural equations with the policymakers first order conditions for optimality, and solving the resulting system of expectational difference equations. The Lagrange multipliers for the policymakers problem form part of the state vector in this commitment solution. A Technical Appendix available from the authors provides details.

[16] Since the OPHs are obtained from impulse response functions, the OPHs for models in previous papers, Rudebusch and Svensson (1999) for example, can be deduced provided the papers include plots of impulse responses under optimal policy (e.g. Rudebusch and Svensson's Figs. 5.3 and 5.4).

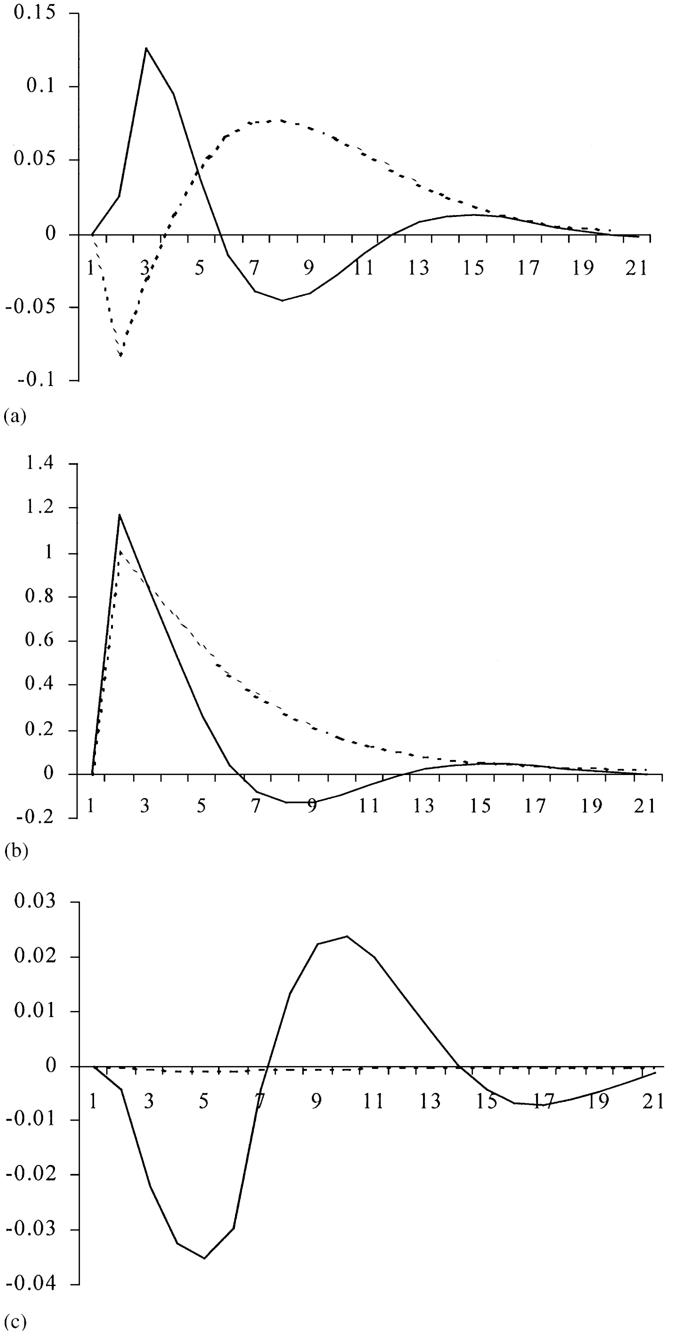


Fig. 1. Inflation impulse responses — various shocks: (a) AD shock (+1%), (b) AS shock (+1%), (c) exchange rate shock ( + 1%).

Table 2
Optimal policy horizons (OPHs)

| Shock | $k_A^*$ | | $k_R^*$ | |
|---|---|---|---|---|
| | VAR | FLSM | VAR | FLSM |
| AD | 14 | 9 | 17 | 14 |
| AS | 18 | 16 | 10 | 8 |
| Exchange rate | 0 | 5 | 16 | 19 |

inflation to the positive demand shock is negative in Fig. 1(a). Within a couple of periods, inflation has returned to zero, but inflation then overshoots for several periods — leading to a long OPH (defined by $k_R^*$) of 17 quarters. This is partly due to the presence in the loss function of a penalty for volatility in the policy instrument. If this were absent, the interest rate would be raised much more sharply in response to the shock, restraining the response of inflation, and leading to a lower OPH ($k_R = 9$).

In the FLSM (the solid line), the presence of forward-looking elements, together with a contemporaneous effect of the real interest rate on output, reduces the length of the policy transmission lag and increases the capacity of policy to offset shocks. The OPH under the relative criterion is less than that for the VAR, and suggests that it is optimal to carry out disinflation within fourteen quarters of the shock (Table 2).[17]

### 4.1. Aggregate supply shock

Fig. 1(b) shows the impulse responses of inflation for the two models, in the wake of a temporary positive 1% shock to aggregate supply. For this disturbance, both models display quite smooth inflation dynamics. In the VAR case, monetary policy cannot affect inflation in the period of the shock, so the 1% supply shock raises inflation by a full one percent in the first period. Strong inflation persistence inhibits policymakers' ability to remove the effect of the shock without unduly large output costs; it takes two and a half years for this shock to be reversed under the optimal rule ($k_R^* = 10$).

For the FLSM, the response of inflation to the supply shock is visually close to that in the VAR. This reflects some similarities between the Phillips curve in this model and the VAR's inflation equation, notably the fact that lagged inflation enters with a sizeable coefficient. As with the AD shock, the OPH is shorter in the FLSM than in the VAR ($k_R^* = 8$ instead of 10) — partly due to

[17] If the IS shock were white noise in this model, the OPH would be five quarters shorter ($k_R = 9$).

forward-looking dynamics in the IS curve (3), which make a more rapid disinflation optimal in this case.

### 4.2. Exchange rate shock

Fig. 1(c) shows the impulse responses of inflation, in the face of a temporary positive 1% shock to the exchange rate equation of each model — constructed such that it would lead to a 1% appreciation in both cases, *ceteris paribus*.

The estimated VAR contains an exchange rate equation that is essentially detached from the rest of the model, in the sense that there is virtually no feedback to other variables from the exchange rate. This explains the very flat response of inflation to such a shock in Fig. 1(c). Under the relative criterion, the OPH is long ($k^*_R = 16$) because it demands that 90% of an already negligible inflation response be eroded. In this case, the OPH as measured by the absolute criterion ($k^*_A$) provides useful auxiliary information: it is zero-quarters, and would be zero even if the initial shock were 10% or larger, rather than 1%.

The appearance of the exchange rate in both the IS and the AS functions implies that the exchange rate has an important role in the FLSM. In response to the UIP shock, the exchange rate appreciates on impact and (partly because of the shock's persistence) this appreciation is not completely reversed for over a year. The response of inflation is also protracted, as inflation depends on long lags of the exchange rate (both via the output gap channel and via the 'direct' exchange rate channel). The optimal policy response is to cut interest rates, which reduces the extent of the appreciation. This also stimulates demand, offsetting some of the contractionary effects of the appreciation, at the cost of creating a positive output gap and a rise in inflation above target for a few quarters. Overall, the combined effect on inflation of the UIP shock and the policy reaction is quite small — inflation is never more than 0.05 percentage points from target. This turns the optimal horizon measured by $k^*_R$ almost into a point target, so the OPH (at 19 quarters) exceeds the OPHs for the other two shocks.

## 5. Optimal feedback horizons (OFHs)

The previous section obtained optimal horizons assuming that the policymakers followed a complex optimal rule — a function of the entire state vector. Suppose instead that the policymakers operate via a simple rule that involves changing the policy instrument in response to deviations of expected inflation from its target value, as in Eq. (1).

By suitable choice of the feedback horizon, this rule can be designed so as to incorporate monetary transmission lags. In particular, in the case where lags hinder control of current inflation, the date of the inflation forecast in the rule can be chosen so that inflation at that date is indeed affected by monetary policy.

When inflation targeting is implemented through rules like (1), the best $k$-period-ahead forecast of inflation will be the one that minimizes the costs of inflation control according to loss function (2). As explained in Section 2 above, we define this horizon as the *optimal feedback horizon* (OFH). Two things are worth noting here. First, we consider the OFH to be the $k$ that minimizes (2) when the feedback coefficient in Eq. (1) is itself optimally chosen. That is, the choice of the optimal $k$ is conditioned on $\psi_p$ (and possibly an interest rate smoothing coefficient) being optimal. Second, in contrast with the previous section, the optimal horizon is not a concept that can be bracketed by a range. Rather, it can only be a discrete point (i.e., the best $k$ at which to form the forecast of inflation that enters the rule). The OPHs and OFHs are thus, as an anonymous referee has stressed, very distinct concepts: the first is a metric associated with an optimal rule, the second is an optimized parameter of a simple rule.

In this section we derive OFHs for our two models. We generalize rule (1) by including an interest rate smoothing term (a coefficient on the lag of the nominal interest rate). This gives Eq. (6) below, where the degree of interest rate smoothing is governed by the parameter $\rho_{\mathrm{R}} \in [0, 1)$.

$$R_t = \rho_{\mathrm{R}} R_{t-1} + \psi_p (E_t \pi_{t+k} - \pi^*) + \text{constant}. \tag{6}$$

Table 3 summarizes the results on OFHs. To obtain them, we closed the models with rule (6), where the parameters ($\psi_p, \rho_{\mathrm{R}}$, and $k$) were chosen optimally by minimizing loss function (2), evaluated using analytical formulae for the model moments. We contemplated values of $k$ of $0, 1, \ldots, 15$ — that is, up to four years ahead.

Table 3 indicates that both the VAR and the FLSM favour a positive feedback horizon. This is in line with Batini and Haldane (1999), who find that responding to expected future rather than current inflation is beneficial when there are lags in the effect of monetary policy. Comparing results on OFH for our two models, it appears as if forward-looking behavior 'brings forward' the optimal feedback horizon; forward-looking agents take into account current and prospective interest rate decisions in their spending and pricing decisions, which reduces the transmission lag (but does not eliminate it, since inflation still has an inert component). Specifically, the OFH is $k = 2$ for the FLSM, compared to 15 for the VAR.

Table 3
Optimal feedback horizons (OFHs)

| Model | $\psi_p^*$ | $\rho_{\mathrm{R}}^*$ | $k^*$ (OFH) |
|---|---|---|---|
| VAR | 35 | 0.85 | 15 |
| FLSM | 1.24 | 0.98 | 2 |

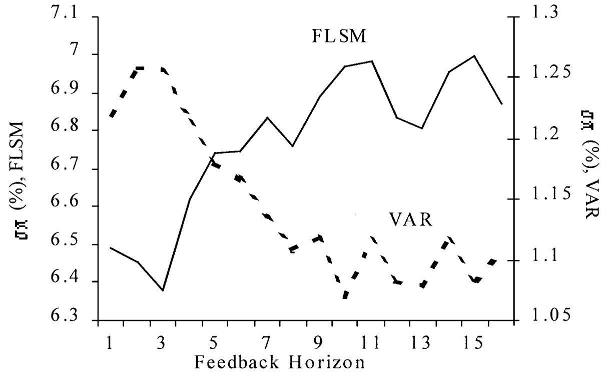


Fig. 2. Inflation SD vs. OFH.

The optimal value of $\psi_p$ is very large in the case of the VAR, whose backward-looking, data-based nature might instead have led us to expect instrument instability when policy becomes too aggressive. Such a large feedback parameter is consistent with relatively low interest rate variability because, in equilibrium, the standard deviation of the variable in the rule ($E_t\pi_{t+15}$) is very low (0.02% annualized).[18] The short-run feedback parameter is much smaller for the FLSM. In a forward-looking model, agents' actions take the expected *long-run* policy response into account. An integral component of policy in the FLSM is therefore high interest rate smoothing, which, for reasons discussed in Rotemberg and Woodford (1999), exerts a restraining effect on inflation via the forward-looking IS function.

To get an impression of the inflation cost of responding to the 'wrong' horizon, Fig. 2 plots (for both models) the standard deviation of inflation against the horizon included in rule (6).[19] For the VAR, the inflation outcome seems equally good for all reasonably long horizons (six quarters or longer); $k = 15$ is optimal in Table 3 largely because of the lower interest rate and gap volatility associated with that long horizon. By contrast, for the FLSM, there is a much sharper increase in inflation volatility from using horizons other than the OFH (both shorter and longer). The common message from both models is that inflation control is sacrificed if the chosen horizon is too short.

The optimized simple rules in Table 3 go a long way in approaching the minimum of the loss function achieved by the optimal rules. For the VAR, the

[18] Evaluating the VAR model's properties where the feedback parameter takes such large values may take the model into ranges where it ceases to be a useful approximation for policy analysis. This is a caution when interpreting our results.

[19] In each case, the coefficients in (6) were reoptimized for the fixed $k$.

optimal-rule loss function value is 0.0220 compared with a loss function value of 0.0224 for the OFH rule, a difference of only 1.8%. Similarly, for the FLSM the loss from optimal policy is 0.5945, vs. 0.6080 for the optimized simple rule, a difference of 2.2%.

For this reason, we found little welfare gain from amending rule (6) to respond to *multiple* horizons. We undertook two experiments in this regard. First, we examined optimized simple rules that responded to annual inflation 1, 2, 3, or 4 years out (e.g. responding to a moving average of $\pi_t, E_t\pi_{t+1}, E_t\pi_{t+2}$, and $E_t\pi_{t+3}$, in the case of the 1-year rule). But we found no welfare improvement for either model from any of these annual horizons, relative to the optimized simple rules in Table 3.

Second, following the suggestion of an anonymous referee, we looked at simple rules that responded to two distinct horizons. Specifically, for the FLSM with its low OFH, we examined rules that responded to horizon 3, 4, 5, or 6 in addition to horizon 2; and for the VAR, we examined rules that responded to horizon 11, 12, 13 or 14 in addition to horizon 15. Response coefficients were reoptimized, but again we found no significant gain. For the FLSM, responding to 2 and 3 quarter ahead inflation gave a welfare improvement relative to the rule that responded to 2 quarters alone, but the gain was less than 0.05%. For the VAR, we found a single horizon dominated all the multiple-horizon alternatives.

## 6. Parameter uncertainty

As our comparison of results from the VAR and FLSM shows, the specification of model structure is an important factor determining the OFH. Another issue is how sensitive the OFH is to the choice of parameter values for a *given* model structure. In this Section we briefly explore this issue for the FLSM.

We focus on the most contentious parameter in the FLSM — namely, $\sigma$ in Eq. (3).[20] In a closed-economy model, $\sigma$ can be interpreted as the intertemporal elasticity of substitution for consumption. Values of $\sigma$ in policy rule studies vary drastically. For example, the studies of Estrella and Fuhrer (1998) and McCallum and Nelson (1999a) (the source for our calibration) suggest quite low values — $\sigma = 0.2$ in the latter — while at the opposite extreme, Rotemberg and Woodford (1999) find $\sigma = 6.0$.

How then does the OFH change if we adopt a higher value of $\sigma$? Table 4 examines this issue by presenting OFHs for a version of the FLSM that uses

[20] Of course, the parameter $\alpha$ in (4) varies across studies but (unlike $\sigma$) it is at least bounded in $[0, 1]$.

Table 4
OFHs under different assumptions about $\sigma$

| | $\psi_p^*$ | $\rho_R^*$ | $k^*$ (OFH) |
|---|---|---|---|
| FLSM, $\sigma = 0.2$ | 1.24 | 0.98 | 2 |
| FLSM, $\sigma = 1.0$ | 5.02 | 0.00 | 8 |

a value of $\sigma = 1.0$ — popular in the literature because it is associated with logarithmic preferences.

Intuitively, a higher value of $\sigma$ could shorten the OFH by making spending more sensitive to current monetary policy changes. In practice, raising $\sigma$ to 1.0 lengthens the OFH from 2 to 8. The reason is that a longer horizon means responding to a lower variance variable — a far-ahead forecast of inflation. The resulting rule features low interest rate volatility and consequent low output gap volatility, producing welfare gains.[21] When $\sigma$ is 0.2 (our baseline parameterization), a given amount of interest rate volatility has less of an effect on gap volatility, than when $\sigma = 1.0$. Responding to inflation 8 periods ahead reduces interest rate volatility, but the gains in terms of reduced gap volatility are not as substantial, and policy sacrifices too much control over inflation. A shorter horizon then becomes optimal.

Thus, a policymaker that used the baseline version of our model, but was uncertain about the true value of $\sigma$ (aware that it could be too low in the baseline version), might want to respond to a longer inflation horizon than it would if it knew that $\sigma = 0.2$ with certainty.

## 7. Conclusions

In this paper we investigated the problem of selecting an optimal horizon for inflation targeting. For this purpose, we provided two operational definitions of 'optimal horizon', corresponding to two different interpretations of how inflation targeting works in practice. Results were obtained from two small-scale models with divergent dynamic properties: a VAR and a forward-looking structural model.

For the *optimal policy horizon*, a definition based on the assumption that inflation targeting involves an optimal policy (obtained in a 'flexible inflation targeting' framework that penalizes inflation, interest rate, and output gap

[21] The standard deviations of $4^*\Delta R_t$ and the output gap in the $\sigma = 1.0$ model are 0.8% and 3.2% when the optimized rule in Table 4 is used. If, on the other hand, the coefficients in that rule were used but a horizon of 2 were put in the rule instead of 8, the standard deviations of $4^*\Delta R_t$ and the gap would rise to 8.8% and 3.9%, respectively.

volatility), we found that it is optimal to remove the effects of the various shocks considered here over a period of 8–19 quarters. For the *optimal feedback horizon*, a definition based on the view that inflation targeting is well approximated by a simple forward-looking policy rule, we found that the best horizon to focus on depends crucially upon the degree of forward-looking behavior in the economy. With no forward-looking behavior (the VAR), long feedback horizons — responding to forecasts of far-ahead inflation — are desirable. With at least some forward-looking behavior (the FLSM), the appropriate feedback horizon is much shorter. Even in this case, however, it appears suboptimal to feed back on current or next-quarter inflation.

To summarize, our analysis supports the view that inflation targeting in practice should be designed so that the target is achieved over the medium term. In other words, central banks wishing to act optimally should not attempt to neutralize inflationary shocks immediately, but instead should respond gradually to those shocks. This becomes particularly important when the economy adjusts sluggishly to economic shocks. Further research on optimal horizons could investigate the robustness of our results to different models.

## References


Ball, L., 1999. Policy rules for open economies. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago, pp. 127–144.

Bank of England, 1999. Economic Models at the Bank of England. Bank of England, London.

Batini, N., Haldane, A.G., 1999. Forward-looking rules for monetary policy. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago, pp. 157–192.

Bernanke, B., Laubach, T., Mishkin, F.S., Posen, A.S., 1999. Inflation Targeting: Lessons from the International Experience. Princeton University Press, Princeton.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 1996. The effects of monetary policy shocks: some evidence from the flow of funds. Review of Economics and Statistics 78, 16–34.

Estrella, A., Fuhrer, J.C., 1998. Dynamic inconsistencies: counterfactual implications of a class of rational expectations models. Mimeo, Federal Reserve Bank of Boston.

European Central Bank, 1999. Monthly Bulletin, January.

Fuhrer, J.C., 1997. The (un)importance of forward-looking behavior in price specifications. Journal of Money, Credit, and Banking 29, 338–350.

Jevons, W.S., 1863. Serious fall in the value of gold ascertained and its social effects set forth (pamphlet). Reprinted in: W.S. Jevons, Investigations in currency and finance, London, 1884.

King, R.G., Wolman, A.L., 1999. What should the monetary authority do when prices are sticky?. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago, pp. 349–398.

McCallum, B.T., Nelson, E., 1999a. Performance of operational policy rules in an estimated semi-classical structural model. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago, pp. 15–45.

McCallum, B.T., Nelson, E., 1999b. Nominal income targeting in an open-economy optimizing model. Journal of Monetary Economics 43, 553–578.

Nelson, E., 2000. U.K. monetary policy 1972–1997: a guide using Taylor rules. Bank of England Working Paper no. 120.

Rotemberg, J.J., Woodford, M., 1999. Interest rate rules in an estimated sticky-price model. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago, pp. 57–119.

Rudebusch, G.D., 1998. Do measures of monetary policy in a VAR make sense?. International Economic Review 39, 907–931.

Rudebusch, G.D., 1999. Assessing nominal income rules for monetary policy with model and data uncertainty. Mimeo, Federal Reserve Bank of San Francisco.

Rudebusch, G.D., Svensson, L.E.O., 1999. Policy rules for inflation targeting. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press, Chicago, pp. 203–246.

Smets, F., 2000. What horizon for price stability? Mimeo, European Central Bank.

Svensson, L.E.O., 1997. Inflation targeting: some extensions. Mimeo, IIES, Stockholm University.

Svensson, L.E.O., 1999. Inflation targeting as a monetary policy rule. Journal of Monetary Economics 43, 607–654.

Svensson, L.E.O., Woodford, M., 1999. Implementing Optimal Policy through Inflation-Forecast Targeting. Mimeo, Princeton University.

Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Series on Public Policy 39, 195–214.

Woodford, M., 1999. Commentary: how should monetary policy be conducted in an era of price stability? In: New Challenges for Monetary Policy: A Symposium Sponsored by the Federal Reserve Bank of Kansas City. Federal Reserve Bank of Kansas City, pp. 277–316.