



# Optimal monetary policy rules in a rational expectations model of the Phillips curve<sup>1</sup>

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## Abstract

Using a rational expectations model based on a Phillips curve with persistence in inflation, we derive optimal monetary policy rules under both commitment and discretion. We assume that the central bank targets the natural rate of output, so there is no incentive generating an average inflation bias. With commitment, inflation has less persistence but more conditional variability, whereas output has more persistence and less conditional variability than with discretion. As the commitment strategy stabilizes the systematic component of inflation, it is less responsive to random shocks to inflation. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The generally poor inflation performance of industrial countries in the post-war period has led a number of central banks to adopt explicit inflation targets in an attempt to improve this performance.<sup>2</sup> How best to achieve control over inflation has been the subject of ongoing debate among economists, but one key element is the link between aggregate demand and inflation, which is embodied in the Phillips curve. Price stickiness and nominal wage rigidity are sufficiently pervasive that they need to be dealt with explicitly in analyzing the optimal monetary policy response to shocks to the economy.

Optimal monetary policy with persistence has been analyzed in some of the contributions to the discussion of time inconsistency and optimal inflation contracts for central bankers (e.g., Lockwood and Philippopoulos, 1994; Lockwood et al., 1994; Goodhart and Huang, 1995; Svensson, 1997a,b). However, in these models the persistence directly relates to output or employment rather than to prices or inflation. Such persistence is introduced by adding the lagged value of the deviation of output from potential into the standard Lucas supply function, so that market clearing still holds. Thus there is scope for exploring the implications of persistence in inflation in comparing alternative monetary policy rules.

A key element in the Phillips curve used here is a parameter that measures the degree of persistence in the inflation rate. This model therefore encompasses the assumption of no persistence as an extreme case, but is more general in that a short-run tradeoff between output and inflation lasts for more than one period. Nonetheless, it incorporates a weak form of the natural rate hypothesis, in that an increasing inflation rate is required to keep output above the natural rate. A number of the findings in the paper depend importantly on the relationship between the degree of inflation persistence and the slope of the Phillips curve. In particular, the size of the optimal adjustment in the nominal interest rate is quite sensitive to the relative magnitudes of these two parameters.

Our model is built on three equations: an objective function of the monetary authority concerned with both inflation and output stability; a Phillips curve that determines the inflation rate; and an equation for real aggregate demand which is determined by the real interest rate. The monetary instrument controlled by the authorities is the nominal interest rate, which in conjunction with the expected inflation rate determines the real interest rate. A feature of this model which distinguishes it from the related literature on time inconsistency (see Barro and Gordon, 1983; Goodhart and Huang, 1995; Svensson, 1997a,b) is that

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<sup>2</sup> The literature on this topic has grown enormously in recent years. For discussions of recent experience and relevant issues, see Ammer and Freeman (1995), Haldane (1995), and Leiderman and Svensson (1995).

the authorities do not control inflation directly, but can only influence it indirectly through the effect of the interest rate on real aggregate demand.

We derive optimal monetary policy response functions in a dynamic context both in the case of commitment as well as discretion. It is well known in the literature on time inconsistency that a commitment strategy can offset the inflation bias arising from attempts by the central bank to attain levels of output above the natural rate. As it is clear that commitment has an advantage over discretion in this case, we adopt here the assumption that the central bank targets the natural rate of output, so there is no average inflation bias, in order to ascertain what advantage remains for a policy of commitment.

We find that with persistence in inflation, the optimum monetary policy is state contingent and shock dependent, as the monetary authority adjusts the control variable – in this case the nominal interest rate – each period in response to deviations of the lagged inflation rate from the target level and to shocks in the current period to output and inflation.<sup>3</sup> This result is similar to that obtained when there is persistence in output. However, unlike the results in some other studies (e.g., Svensson, 1997a) the monetary authority cannot achieve its inflation target every period even though it can commit.

As the closed-form solution for the discretionary case is extremely complex, we characterize the relationship between the two policy regimes qualitatively. We find that in the state-contingent part of the solution the expected deviation of inflation from its target level is always smaller, but the expected deviation of output from its target level is always larger in the case of commitment than in the case of discretion. This provides a clear demonstration of a tradeoff, in terms of expectations, between the two optimal monetary policy rules.

We also compare the adjustment coefficients to the random shocks and find that both optimal rules reduce the impact of the inflation shock on inflation and output. For each optimal rule, there is always a tradeoff between the impact of the inflation shock on inflation and output; that is, a larger reduction in the impact of the inflation shock on inflation involves a smaller offset of this random shock on output, and vice versa. The comparison also shows that the discretionary rule generates a larger offset to the inflation shock on current-period inflation and a smaller offset of the shock on output. Because the commitment approach takes account of the effect of the monetary policy rule on expectations, it stabilizes the systematic, i.e., expected component of inflation and therefore is not as responsive as the discretionary rule to random shocks to inflation.

The rest of this paper is organized as follows. The rational expectations model of the Phillips curve is presented in Section 2. The optimal monetary response function under commitment is described in Section 3. This is followed by the

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<sup>3</sup> This is an example of a general result shown by Buiter (1981), which established the superiority of contingent rules over fixed rules in models with rational expectations.

derivation of the optimal monetary rule under discretion in Section 4, which includes a comparison of the two policy rules, and Section 5 concludes.

## 2. The model

We start with a standard utility or loss function for the monetary authority,

$$U_t = -(y_t - y_n)^2 - k(\pi_t - \pi^*)^2, \quad (1)$$

where  $y_t$  is the level of output and  $\pi_t$  is the inflation rate. We assume that the monetary authority is the sole relevant government decision-making unit, so that we abstract from issues arising from different preferences over output and inflation between the government and the central bank. The monetary authority has an exogenous inflation target,  $\pi^*$ , but it is also assumed that its policy actions take into account deviations of output from the natural rate,  $y_n$ , which is exogenous. The parameter  $k$ , which is between 0 and infinity, is the weight given to inflation stabilization relative to output stabilization. Finally, as noted above, we do not consider the possibility that the authorities may wish to achieve a target level of output that differs from the natural rate. The implications of this assumption have been extensively analyzed by Goodhart and Huang (1995) and Svensson (1997a,b), among others.

Unlike that of the monetary authorities, the behavior of private agents is not derived from the solution of a maximization process. This approach follows from our desire to explore the implications for optimal monetary feedback rules of a very specific kind of private sector behavior, namely, a Phillips curve that embodies persistence in inflation given by Eq. (2) below:

$$\pi_t = \lambda\pi_{t-1} + (1 - \lambda)\pi_t^e + \theta(y_t - y_n) + u_t. \quad (2)$$

In this equation  $\lambda$  and  $\theta$  are constant coefficients, the superscript  $e$  denote the rational expectation taken at the end of period  $t - 1$ , and  $u_t \in N(0, \sigma^2)$  is a random shock. For simplicity, inflation in the current period is a linear function of only the contemporaneous gap between the level of aggregate demand and the natural or capacity level of output. The parameter  $\theta$  is a positive constant which measures the sensitivity of inflation to excess demand.<sup>4</sup>

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<sup>4</sup> There are also good reasons to believe that  $\theta$  is not constant, as pointed out by Clark et al. (1996), Laxton et al. (1995), and Lipsey (1960). As we wish to obtain a closed-form solution for the optimal feedback rule in the commitment case, we retain the assumption of a linear Phillips curve in this paper. See Bean (1996) for a discussion of the implications of nonlinearity in the Phillips curve for optimal monetary policy in a simpler model than that presented here which does not embody rational expectations.

The novel feature of this specification of the Phillips curve is the explicit introduction of persistence in inflation, in the form of lagged inflation, together with inflation expectations. This type of specification is sometimes referred to as the ‘backward- and forward-looking components’ model – see Buiter and Miller (1985). The backward-looking component here reflects inertia in inflation that can be derived, for example, from overlapping wage contracts based on Fischer (1977) and Taylor (1980), as in Ireland and Wren-Lewis (1995) and Fuhrer and Moore (1995). The forward-looking component is represented by the rational expectation of the current rate of inflation, and the difference between the realized actual value and the rational expectation of inflation reflects only random disturbances. The coefficients of the two components sum to unity, so that in long-run equilibrium,  $\pi_t = \pi_{t-1} = \pi_t^e$ . The index of persistence,  $\lambda$ , therefore lies between zero (no persistence) and unity (complete persistence). Obviously, the standard Lucas surprise supply function is where  $\lambda$  is zero.

It should be clear that Phillips curve in Eq. (2) ties down only the change in the rate of inflation and not the level of inflation itself. The equilibrium level of the inflation rate obviously must be determined by a nominal anchor outside the dynamics of the inflation process. This is accomplished in the usual fashion through the loss function for the monetary authorities that includes  $\pi^*$ , the target rate of inflation. As shown below, in long-run equilibrium the control exercised by the central bank ensures that the actual inflation rate is equal to the target level.

Aggregate demand is given by Eq. (3) as a function of the real interest rate:

$$y_t - y_n = -\phi(i_t - \pi_t^e - \alpha) + v_t. \quad (3)$$

In the above equation,  $\phi$  is a positive constant coefficient,  $\alpha > 0$  is the long-run equilibrium real interest rate,  $v_t \in N(0, \sigma_v^2)$  is a random shock which is assumed to be independent of  $u_t$ , and  $i_t$  denotes the nominal interest rate, the instrument under the monetary authority’s control.<sup>5</sup> As we wish to explore the effect of persistence that arises directly in the inflation process per se, we ignore lags in the effect of the real interest rate on demand. The implications of persistence in aggregate demand have been examined in a theoretical context by Goodhart and Huang (1995) and empirically by Clark et al. (1996) and Fuhrer and Moore (1995). The real interest rate is equal to the nominal interest rate minus the expected inflation rate in the current period. The monetary authority is assumed to vary the nominal interest rate directly to achieve the level of excess demand needed to affect the inflation rate. Thus inflation control is achieved only indirectly through variations in aggregate demand via changes in the nominal interest rate relative to inflation expectations.

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<sup>5</sup> Note that it is necessary to distinguish between  $\pi_t$  and  $i_t$ . In our model  $\pi_t$  is the inflation outcome. Although it can be a target, as treated in Svensson (1997a),  $\pi_t$  cannot also be a monetary policy instrument as  $i_t$  is the only instrument in this economy.

Taking the rational expectation of Eqs. (2) and (3), using  $i_t^e = E_{t-1}[i_t]$ , and after some algebraic manipulations, we have the following reduced-form equations for inflation and output, respectively:

$$\pi_t = \pi_{t-1} - \phi\theta[(i_t - i_t^e) + (i_t^e - \pi_{t-1} - \alpha)/(\lambda - \phi\theta)] + u_t + \theta v_t, \quad (4)$$

$$y_t = y_n - \phi[(i_t - i_t^e) + \lambda(i_t^e - \pi_{t-1} - \alpha)/(\lambda - \phi\theta)] + v_t. \quad (5)$$

These two equations are useful because they show the relationship between  $\pi_t$  and  $y_t$  and both the state variable,  $\pi_{t-1}$ , and the control variable(s). In the case of discretion, the sole control variable is  $i_t$ . With commitment, by contrast, the monetary authority takes account of the effects of its actions on the expectations of the private sector. This involves minimizing the loss function not only with respect to  $i_t$  but also  $i_t^e$  as well. In this case expectations regarding the policy stance of the authorities are fully incorporated in deriving the feedback rule that determines that stance. By examining Eqs. (4) and (5) it can be seen that an increase in  $i_t$  unambiguously reduces both output and inflation, whereas the effect of the expected interest rate depends on the value of  $(\lambda - \phi\theta)$ . However, as shown below, after optimal control has been taken into account, the behavior of  $\pi_t$  and  $y_t$  does not depend on this particular relationship among the parameters.

Finally, these two equations can be expressed more compactly by using  $\pi_t^e = E_{t-1}[\pi_t]$  and  $y_t^e = E_{t-1}[y_t]$ . This can be done by taking expectations of Eqs. (4) and (5), which upon substitution yield:

$$\pi_t = \pi_t^e - \phi\theta(i_t - i_t^e) + u_t + \theta v_t, \quad (6)$$

$$y_t = y_t^e - \phi(i_t - i_t^e) + v_t. \quad (7)$$

Eq. (6) shows that in this model actual inflation is determined by expected inflation plus random disturbance terms, and similarly for output.

### 3. The optimal commitment policy rule

#### 3.1. General considerations

In the literature on optimal monetary policy with time inconsistency, a standard result is that a commitment strategy by the central bank is one way to overcome the inflation bias resulting from attempts to achieve a level of output higher than the natural rate. As shown by Svensson (1997a), in this case the commitment solution leads to a better outcome than discretion because the latter results in too high an inflation rate. Alternative ways of improving on the discretionary outcome have been suggested in the literature, e.g., delegation to a conservative central banker (Rogoff, 1985) and linear inflation contracts (Walsh, 1995). Here we have chosen a different approach to compare the two

strategies, namely, one where the central bank does not aim at an output level greater than the natural rate. As commitment dominates discretion when there is an inflation bias, it is useful to explore what advantage remains in adopting a monetary policy based on commitment in the absence of an inflation bias.

When the central bank is committed to a state-contingent rule in conducting monetary policy, this implies – as noted by Svensson (1997a) – that the monetary authority internalizes the impact of its decision rule on the expectations of the private sector. In other words, the monetary authority takes into account how its actions affect the private sector's expectations. It does this by minimizing its loss function with respect to the private sector's expectations of the interest rate under the explicit constraint that these expectations are formed rationally. With this approach to monetary policy there are in effect two decision variables or instruments: the actual *ex post* and the expected *ex ante* interest rate each period. The optimal response function is derived by taking account of how the economic system responds to both control variables. By contrast, under discretion the central bank does not take into account how its actions affect expectations, and as a result it loses one policy instrument, which is the expectation of the interest rate. As shown below, in this case the optimal rule is derived by minimizing the loss function of the monetary authority only with respect to the actual *ex post* interest rate.

With a commitment solution, therefore, the monetary authority's maximization problem involves the additional constraint that the *ex ante* expected nominal interest rate,  $i_t^e$ , is equal to its rational expectation,  $E_{t-1}[i_t]$ .<sup>6</sup> Hence, its problem is:

$$\begin{aligned} \max_{i_t, i_t^e} E_{t-1} \sum_{t=1}^{\infty} \beta^t U_t \\ \text{s.t. Eqs. (4), (5) and } i_t^e = E_{t-1}[i_t]. \end{aligned}$$

This is a dynamic programming problem with one state-variable,  $\pi_{t-1}$ , and two control variables,  $i_t$  and  $i_t^e$ , and where  $\beta$  is the discount factor. The solution

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<sup>6</sup> Note that the private sector's expectation of the nominal interest rate,  $i_t^e$ , is a variable, while  $E_{t-1}[i_t]$ , the rational expectation at  $t-1$ , is a function of  $i_t$ . One may wonder how the central bank can commit to this expectation. Notice, first, that such a rational expectation is endogenously derived from the central bank's dynamic programming problem by the private sector and the central bank. Committing to such a rule means that the monetary authority internalizes the impact of its decision rule on the expectations of the private sector. Moreover, central banks indeed take into consideration in their decision-making information on the private sector's expectations. For example, as part of its conduct of monetary policy, the Bank of England surveys financial market expectations of inflation and analyzes market expectations of future expected short-term interest rates, including their probabilities as ascertained from options markets. Committing to  $E_{t-1}[i_t]$  has been widely used in the recent literature as a commitment strategy (e.g., Lockwood et al., 1994; Goodhart and Huang, 1995; Svensson, 1997a).

can be obtained by solving the following equation involving the value function,  $V(\pi_t)$ :

$$V(\pi_{t-1}) = \max_{i_t, i_t^c} E_{t-1} \{ - (y_t - y_n)^2 - k(\pi_t - \pi^*)^2 + \beta V(\pi_t) \}, \quad (8)$$

$$\text{s.t Eqs. (4), (5) and } i_t^c = E_{t-1}[i_t].$$

For the linear-quadratic problem such as ours,  $V(\pi_t)$  must also be quadratic. Without loss of generality, we can write  $V(\pi_t) = \gamma_0^c + 2\gamma_1^c \pi_t + \gamma_2^c \pi_t^2$ , so that  $V'(\pi_t) = 2(\gamma_1^c + \gamma_2^c \pi_t)$ . Using this condition together with Eqs. (4) and (5) and  $i_t^c = E_{t-1}[i_t]$ , we obtain two first-order conditions from Eq. (8) with respect to  $i_t$  and  $i_t^c$ , respectively:

$$2\phi(y_t - y_n) + 2k\phi\theta(\pi_t - \pi^*) - 2\beta\phi\theta(\gamma_1^c + \gamma_2^c \pi_t) + A_{t-1} = 0, \quad (9)$$

$$\begin{aligned} & - E_{t-1} \{ 2\phi[1 - \lambda/(\lambda - \phi\theta)](y_t - y_n) + [2k\phi\theta(\pi_t - \pi^*) \\ & - 2\beta\phi\theta(\gamma_1^c + \gamma_2^c \pi_t)][1 - 1/(\lambda - \phi\theta)] + A_{t-1} \} = 0, \end{aligned} \quad (10)$$

where  $A_{t-1}$  is the Lagrange multiplier of  $i_t^c = E_{t-1}[i_t]$ .

Eliminating  $A_{t-1}$  by adding Eqs. (9) and (10) gives us:

$$\begin{aligned} & 2\phi \{ [\lambda(y_t^c - y_n) + k\theta(\pi_t^c - \pi^*) - \beta\theta(\gamma_1^c + \gamma_2^c \pi_t^c)] / (\lambda - \phi\theta) \\ & + (y_t - y_t^c) + \theta(k - \beta\gamma_2^c)(\pi_t - \pi_t^c) \} = 0, \end{aligned}$$

which upon substituting for  $y_t - y_t^c$  and  $\pi_t - \pi_t^c$  gives

$$\begin{aligned} & 2\phi[\lambda(y_t^c - y_n) + k\theta(\pi_t^c - \pi^*) - \beta\theta(\gamma_1^c + \gamma_2^c \pi_t^c)] / (\lambda - \phi\theta) \\ & - \phi(i_t - i_t^c) + v_t + \theta(k - \beta\gamma_2^c)[ - \phi\theta(i_t - i_t^c) + u_t + \theta v_t ] = 0. \end{aligned} \quad (11)$$

Eq. (11) is the optimal feedback rule under commitment expressed as a function of the parameters of the model and two coefficients,  $\gamma_1^c$  and  $\gamma_2^c$ , which are derived below. Taking the expectation of Eq. (11) and dividing its both sides by  $2\phi/(\lambda - \phi\theta)$ , we have:

$$\lambda(y_t^c - y_n) + k\theta(\pi_t^c - \pi^*) - \beta\theta(\gamma_1^c + \gamma_2^c \pi_t^c) = 0. \quad (12)$$

One way to interpret Eqs. (11) and (12) is through specifying the information structure in our model as follows: both the monetary authority and the private sector have no information about the shocks at the beginning of each period; then the monetary authority observes the shocks and conducts its policy; and finally, near the end of each period, the private sector observes the shocks and the outcome of monetary policy. Hence, the only information asymmetry occurs in the *interim* of each period, and both the monetary authority and the private sector have exactly the same information *ex ante* and *ex post* each period. Because the private sector only observes the shocks *ex post*, it is too late for it to take any action to offset the response of the monetary authority to shocks.

However, the presence of information asymmetry is not essential for our main results, and we describe below the solution for the case where there is no information asymmetry. When there is such asymmetry and the monetary authority conducts its policy after observing the shocks  $u_t$  and  $v_t$ , Eq. (11) determines the optimal feedback rule.

It is important to note that Eq. (12) is the *expected* optimal policy rule at the beginning of period  $t$  not only from the monetary authority’s perspective, but also from the private sector’s perspective because it cannot observe the shocks until the end of period  $t$ . However, as the private sector can observe shocks at the end of each period, it can, in principle, check the monetary authority’s commitment at the end of each period.<sup>7</sup> As a result, the expected monetary authority’s optimal policy rule at the beginning of the period  $t$  is the correct conjecture on the part of the private sector because of the commitment of the monetary authority not to deviate from this rule. Thus, the commitment to the rule clearly influences the expectations of the private sector.

Since Eq. (12) always has to be satisfied, we can impose Eq. (12) on Eq. (11), which gives the result that the random shock-dependent parts imply:  $-\phi(i_t - i_t^e) + v_t + \theta(k - \beta\gamma_2^e)[-\phi\theta(i_t - i_t^e) + u_t + \theta v_t] = 0$ . This can be rewritten as:  $[1 + \theta^2(k - \beta\gamma_2^e)][-\phi(i_t - i_t^e) + v_t] + \theta(k - \beta\gamma_2^e)u_t = 0$ . We show below that  $[1 + \theta^2(k - \beta\gamma_2^e)] \neq 0$ , both with commitment ( $\gamma_2^e$ ) and discretion ( $\gamma_2^d$ ). Hence,  $-\phi(i_t - i_t^e) + v_t$  is only a function of  $u_t$ . With this property, we know that with optimum control  $v_t$  shows up neither in the equilibrium expression for  $y_t$  nor for  $\pi_t$ . Without loss of generality, we can therefore write the general form of  $\pi_t$  as

$$\pi_t = a + b\pi_{t-1} + cu_t. \tag{13}$$

From Eq. (4), we have  $\pi_t^e = \pi_{t-1} - \phi\theta(i_t^e - \pi_{t-1} - \alpha)/(\lambda - \phi\theta) = a + b\pi_{t-1}$ ; thus  $i_t^e = \alpha - (\lambda - \phi\theta)a/\phi\theta + \{[\lambda - (\lambda - \phi\theta)b]/\phi\theta\}\pi_{t-1}$ . If we write  $i_t = i_t^e + x_1u_t + x_2v_t$ , then from  $\pi_t - \pi_t^e = cu_t$ , and using Eq. (4) again, we get  $\pi_t - \pi_t^e = cu_t = \pi_{t-1} - \phi\theta[(i_t - i_t^e) + (i_t^e - \pi_{t-1} - \alpha)/(\lambda - \phi\theta)] + u_t + \theta v_t$ , which determines  $x_1 = (1 - c)/\phi\theta$  and  $x_2 = 1/\phi$ . Hence,

$$i_t = \alpha - (\lambda - \phi\theta)a/\phi\theta + \{[\lambda - (\lambda - \phi\theta)b]/\phi\theta\}\pi_{t-1} + [(1 - c)/\phi\theta]u_t + (1/\phi)v_t. \tag{14}$$

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<sup>7</sup> In practice, of course, the public finds it hard enough to observe what the shocks have been, even after the event, far less whether the authorities have abided by a complicated feed-back rule like Eq. (11) above. What actually occurs is that the authorities make a subjective estimate, based on their expectations of the probability distribution of the shocks,  $u_t$  and  $v_t$ , of the bounds on  $\pi_t - \pi^*$  and  $y_t - y_n$  that their feedback rule can deliver. Then they commit to keeping within such bounds. The public can more easily see whether the outcomes remain within the pre-commitment ranges.

Using Eq. (14) in Eq. (5), we have

$$y_t = y_n + \lambda a/\theta - \{[\lambda(1-b)]/\theta\}\pi_{t-1} + [(1-c)/\theta]u_t. \quad (15)$$

Notice that Eqs. (13)–(15) are the general solutions for  $\pi_t$ ,  $i_t$  and  $y_t$ , that involve the parameters  $a$ ,  $b$  and  $c$ . Similarly, the general relationship between  $\gamma_0$ ,  $\gamma_1$ ,  $\gamma_2$  and  $a$ ,  $b$  and  $c$  can be derived using the value function:

$$\begin{aligned} V(\pi_{t-1}) &= \gamma_0 + 2\gamma_1\pi_{t-1} + \gamma_2\pi_{t-1}^2 \\ &= E_{t-1}\{- (y_t - y_n)^2 - k(\pi_t - \pi^*)^2 + \beta V(\pi_t)\}. \end{aligned}$$

After substituting  $\pi_t$  and  $y_t$  from Eqs. (13) and (15), we can identify the coefficient for  $\pi_{t-1}^2$ :

$$\gamma_2 = -[k\theta^2b^2 + \lambda^2(1-b)^2]/[\theta^2(1-\beta b^2)]. \quad (16)$$

Similarly, identifying the coefficient for  $\pi_{t-1}$ , we have

$$\gamma_1 = -[\lambda^2a(1-b) - \theta^2(ka - k\pi^* - \beta a\gamma_2)b]/[\theta^2(1-\beta b)]. \quad (17)$$

Finally, the constant is equal to

$$\begin{aligned} \gamma_0 &= [\lambda^2a^2 + (1-c)^2\sigma^2]/[\theta^2(1-\beta)] - [(k - \beta\gamma_2)(a^2 + c^2\sigma^2) \\ &\quad - 2\beta\gamma_1a]/(1-\beta), \end{aligned} \quad (18)$$

where  $\sigma^2$  is the variance of  $u_t$ .

Eqs. (16)–(18) provide the general expressions for the coefficients of the value function. However, the specific values of  $a$ ,  $b$  and  $c$  depend on whether it is the case of commitment ( $a^c$ ,  $b^c$ ,  $c^c$ ) or discretion ( $a^d$ ,  $b^d$ ,  $c^d$ ). The optimal commitment policy rule is determined by the values of  $a^c$ ,  $b^c$  and  $c^c$ , which are solved for in Appendix A. As noted below, meaningful closed-form expressions cannot be obtained for  $a^d$ ,  $b^d$ , and  $c^d$ , but Appendix B shows how these coefficients under discretion are related to those under commitment.

### 3.2. The solution for the optimal commitment policy rule

The closed-form expressions for  $a^d$ ,  $b^d$ , and  $c^d$  are given in Appendix A. What is useful in interpreting Eqs. (13)–(15) are the bounds on these coefficients that are derived in Appendices A and B, which place the following restrictions on the coefficients in the case of commitment:  $a^c \geq 0$  if  $\pi^* \geq 0$ ,  $0 < b^c < 1$ , and  $0 < c^c < 1$ . In the equation for inflation,  $\pi_t = a^c + b^c\pi_{t-1} + c^c u_t$ , it is clear that these coefficients have a plausible economic interpretation: the constant depends on the target inflation rate,  $\pi^*$ , and optimal control dampens the effect of lagged inflation and the current disturbance term,  $u_t$ , on the current inflation rate.

In addition, the results in Appendices A and B also make it straightforward to derive the following steady-state (i.e., unconditional expectation) values of the

three variables in the model:

$$E[\pi_t] = \pi^*, E[y_t] = y_n, \text{ and } E[i_t] = \alpha + \pi^*.$$

These steady-state properties confirm that there is no inflation bias and the authorities achieve, on average, the natural rate of output and the target rate of inflation.

It is useful to examine the coefficients for particular values of the parameters of the model. For example, when  $\lambda$  goes to zero, i.e., there is no persistence, it is straightforward to show using L'Hopital's Rule that  $a^c = \pi^*$ ,  $b^c = 0$ , and  $c^c = 1/(1 + k\theta^2)$ . In this special case there is no systematic tradeoff between hitting the inflation and output targets. Moreover, for certain parameter values, one target may be hit each period but not the other. Investigating the equation for  $\pi_t$  reveals that the inflation target can be hit each period when  $k$  goes to infinity, i.e., when the monetary authority is only concerned with price stability. Similarly, the equation for  $y_t$  implies that the output target can be hit each period if  $\theta = 0$  or  $k = 0$ .

However, in the more general case of persistence and all other parameter values, neither target can be hit each period. That is, with persistence, then  $i_t^c \neq \pi^* + \alpha$ ,  $\pi_t^c \neq \pi^*$ , and  $y_t^c \neq y_n$ , where the expectation is conditional at the end of period  $t - 1$ . This result implies that when there is persistence, even *in expectation* the monetary authority cannot hit its target  $\pi^*$  every period even though it can indeed fully commit to the policy rule. This is in contrast with the result from previous studies (e.g. Svensson, 1997a), where *in expectation* the monetary authority can hit its inflation target every period if it can fully commit to the policy rule, although the realized inflation rate will typically differ from its target on account of current-period shocks. The reason for this difference is that in our model, unlike previous studies, inflation also has persistence which we believe is a fundamental feature of Phillips curve.

Comparing Eqs. (13)–(15), we also notice that  $(\lambda - \phi\theta)$  only appears as a part of the numerator of the optimal feedback rule, Eq. (14), but it does not appear in the equations for  $\pi_t$  and  $y_t$ . Recall from Eqs. (4) and (5) that  $(\lambda - \phi\theta)$  does appear as a part of the denominator of  $\pi_t$  and  $y_t$ , hence it directly affects  $\pi_t$  and  $y_t$ . In particular, when  $(\lambda - \phi\theta) = 0$ , both  $\pi_t$  and  $y_t$  become infinity, i.e.,  $\pi_t$  and  $y_t$  become unstable *before* the optimal rule on the nominal interest rate is imposed. But the outcome for  $\pi_t$  and  $y_t$ , *after* imposing such an optimal rule becomes invariant to the critical condition  $(\lambda - \phi\theta)$ . Moreover, as a part of the numerator of the optimal feedback rule,  $(\lambda - \phi\theta)$  cannot affect the stability of the interest rate even if it goes to zero.

It is useful to examine first the state-contingent properties of the model alone, i.e., to ignore the effects of the random disturbances on the control variable and the two endogenous variables. This can be done by looking at how the conditional expected values of these variables change in response to changes in inflation. Because the expected optimal policy rule at time  $t$  is state contingent

on  $\pi_{t-1}$ , the change in inflation can only be measured by using the change lagged one period.

Looking first at the response of the change in the expected nominal interest rate to lagged inflation, it is straightforward to show using Eq. (14) that the ratio between the expected adjustment in the nominal interest rate between  $t$  and  $t-1$  and the change in inflation between  $t-1$  and  $t-2$ ,  $(i_t^e - E_{t-2}i_{t-1})/(\pi_{t-1} - \pi_{t-2})$ , is greater than (less than) 1.0 if  $\lambda > \phi\theta$  ( $\lambda < \phi\theta$ ). In other words, if  $\lambda > \phi\theta$ , then the expected adjustment in the nominal interest rate is greater than the change in inflation itself, and vice versa. The condition that  $\lambda > \phi\theta$  corresponds to situations of high persistence (high  $\lambda$ ), hence high inflationary pressure is carried over to the next period; and/or low sensitivity of inflation to excess demand (low  $\theta$ ), hence the less effective is the nominal interest rate as the policy instrument to affect inflation through aggregate demand; and/or low sensitivity of aggregate demand to the real interest rate (low  $\phi$ ), hence the less effective is the nominal interest rate as the policy instrument to affect aggregate demand. With high persistence relative to the parameters affecting the monetary authorities' ability to control inflation, a more active adjustment in the nominal interest rate is called for.

Second, the ratio between the expected adjustment in inflation and the change in inflation is between (0, 1), regardless of whether  $\lambda$  is greater or less than  $\phi\theta$ . This follows directly from the fact that  $(\pi_t^e - E_{t-2}\pi_{t-1})/(\pi_{t-1} - \pi_{t-2}) = b^c$ , which is greater than zero but less than 1. Thus, the expected adjustment in inflation between  $t$  and  $t-1$  is always less than the change in prior actual inflation. Hence, inflation always converges in an expected sense; if there are no further shocks, inflation converges to its target level.

Third, the ratio between the expected adjustment in output between  $t$  and  $t-1$  and the inflation change between  $t-1$  and  $t-2$ ,  $(y_t^e - E_{t-2}y_{t-1})/(\pi_{t-1} - \pi_{t-2})$ , is between  $(-\lambda/\theta, 0)$ , regardless of whether  $\lambda$  is greater or less than  $\phi\theta$ . This also follows from  $0 < b^c < 1$ . Thus, the expected adjustment in output between  $t$  and  $t-1$  and the inflation change between  $t-1$  and  $t-2$  are always in opposite directions. The economic intuition is clear: the larger the most recent increase in inflation, the larger the expected decline in output in order to reduce inflation in the current period via the Phillips curve.

Moreover, it follows from the second and third results that the magnitude of the adjustment coefficients in front of  $(\pi_{t-1} - \pi^*)$  for  $\pi_t$  and  $y_t$  move in the opposite direction. In other words, for a given deviation of inflation from its target, the larger the adjustment of  $\pi_t$  to the target level of inflation, the larger the adjustment of expected output away from its target, and vice versa. This result is due to the effect of the Phillips curve which governs the relationship between the inflation and output adjustment; that is, the larger the adjustment in inflation in order to reach to its target, the larger the expected adjustment in output in the current period, which implies a larger deviation of output from its target.

The discussion up to this point has been focused on the state-contingent properties of the model. We now turn to investigate the effects of random shocks by examining the magnitude of the coefficients in front of  $u_t$  and  $v_t$  for  $\pi_t$ ,  $i_t$  and  $y_t$ , respectively, in Eqs. (13)–(15). Under the optimal commitment policy rule, the impact of the random shock  $u_t$  on the adjustment of the nominal interest rate is between  $(0, 1/\phi\theta)u_t$ , between  $(0, 1)u_t$  on the inflation level, and between  $(-1/\theta, 0)u_t$  on the output level. The effect of  $v_t$  on  $i_t$  is  $(1/\phi)v_t$ , but this shock to aggregate demand has no effect on either  $\pi_t$  or  $y_t$ . These coefficient values depend on the fact that  $\gamma_2^c < 0$ .

Thus, the effect of optimal control is to shrink the effect of the random shock  $u_t$  on inflation from 100% of  $u_t$  before the optimal policy is imposed to strictly less than 100% of  $u_t$ . Furthermore, optimal control completely eliminates the effect on inflation of the random shock,  $v_t$ , on aggregate demand. Similarly, the optimal control transmits an effect on output of random inflation shock  $u_t$  up to  $(1/\theta)u_t$ , and may shrink the effect of random shocks on output if  $1/\theta < 1$ . To summarize, the optimal commitment policy rule completely eliminates the effect on inflation and output of the random shock  $v_t$ . It does shrink the effect on inflation of the random shock  $u_t$ , but it may or may not shrink the effect on output of  $u_t$ .

Note that the driving force for the above results is the sluggish wage and price adjustment rather than the asymmetric information. As long as such sluggish adjustments exist – that is, so long as the private sector cannot adjust wages and prices instantly even if they observe the shocks the same time as the monetary authority – the same results hold. Therefore, our assumption regarding *interim* asymmetric information between the monetary authority and the private sector is not as restrictive as it appears.

## 4. The optimal discretionary policy rule

### 4.1. General considerations

As discussed above, in exploring the possible advantages of a policy rule based on discretion, we have chosen to examine the effects of alternative assumptions about the conduct of monetary policy where inflation bias does not arise. In this section we solve for the optimal discretionary policy rule and compare it with the optimal commitment policy rule under this assumption.

When the central bank does not commit to  $E_{t-1}[i_t]$ , it does not internalize the impact of its decision rule on the expectations of the private sector. With this approach to monetary policy there is only one decision variable or instrument: the actual ex post interest rate,  $i_t$ , each period. The central bank loses one policy instrument, which is the expectation of the interest rate, so that it is no longer

bound by the constraint of taking into account the impact of its behavior on the private sector's expectations. In this case the optimal rule is derived by minimizing the loss function of the monetary authority only with respect to the actual ex post interest rate. Consequently, the new dynamic problem in this case is

$$\begin{aligned} \max_i E_{t-1} \sum_{t=1}^{\infty} \beta^t U_t \\ \text{s.t. Eqs. (4) and (5).} \end{aligned}$$

This is a dynamic programming problem with one state-variable,  $\pi_{t-1}$ , and one control variable,  $i_t$ . As in the commitment case, the solution can be obtained by solving the following problem:

$$\begin{aligned} V(\pi_{t-1}) = \max_i E_{t-1} \{ - (y_t - y_n)^2 - k(\pi_t - \pi^*)^2 + \beta V(\pi_t) \}, \\ \text{s.t. Eqs. (4) and (5).} \end{aligned} \quad (19)$$

For the linear-quadratic problem such as ours,  $V(\pi_t)$  must also be quadratic. Without loss of generality, we can write  $V(\pi_t) = \gamma_0^d + 2\gamma_1^d \pi_t + \gamma_2^d \pi_t^2$ , so that  $V'(\pi_t) = 2(\gamma_1^d + \gamma_2^d \pi_t)$ . Using this condition together with Eqs. (4) and (5), we obtain the first-order conditions from Eq. (19) with respect to  $i_t$ :

$$(y_t - y_n) + k\theta(\pi_t - \pi^*) - \beta\theta(\gamma_1^d + \gamma_2^d \pi_t) = 0. \quad (20)$$

Taking the expectation of the above equation, we have

$$(y_t^e - y_n) + k\theta(\pi_t^e - \pi^*) - \beta\theta(\gamma_1^d + \gamma_2^d \pi_t^e) = 0. \quad (21)$$

Eq. (20) defines the optimal feedback rule and Eq. (21) defines the expected optimal feedback rule. As in the commitment case, we can assume that the only information asymmetry occurs in the *interim* of each period, and both the monetary authority and the private sector have exactly the same information ex ante and ex post in each period.

#### 4.2. The solution for the optimal discretionary policy rule

As already noted above, Eqs. (13)–(15) describe the behavior of the three variables in both commitment and discretionary cases, and therefore we also have  $\pi_t = a^d + b^d \pi_{t-1} + c^d u_t$  under discretion. To determine the values of  $a^d$ ,  $b^d$  and  $c^d$ , we start with Eqs. (20) and (21). After substituting  $y_t^e$  and  $\pi_t^e$  into Eq. (20), we get:

$$\begin{aligned} i_t^e - \pi_{t-1} - \alpha \\ = \theta(\lambda - \phi\theta)[(k - \beta\gamma_2^d)\pi_{t-1} - k\pi^* - \beta\gamma_1^d] / \{\phi[\lambda + \theta^2(k - \beta\gamma_2^d)]\}. \end{aligned}$$

Using this in the equation for  $E_{t-1} [\pi_t]$ , we have

$$E_{t-1} [\pi_t] = \{\theta^2(k\pi^* + \beta\gamma_1^d)/[\lambda + \theta^2(k - \beta\gamma_2^d)]\} + \{\lambda/[\lambda + \theta^2(k - \beta\gamma_2^d)]\}\pi_{t-1}.$$

Thus,

$$a^d = \theta^2(k\pi^* + \beta\gamma_1^d)/[\lambda + \theta^2(k - \beta\gamma_2^d)], \tag{22}$$

$$b^d = \lambda/[\lambda + \theta^2(k - \beta\gamma_2^d)]. \tag{23}$$

As in the case of commitment, by using Eqs. (13)–(15) in Eq. (20), and imposing Eq. (21), one obtains

$$\begin{aligned} & -\phi\{[(1 - c^d)/\phi\theta]u_t + (1/\phi)v_t\} + v_t \\ & + \theta(k - \beta\gamma_2^d)\{-\phi\theta\{[(1 - c^d)/\phi\theta]u_t + (1/\phi)v_t + u_t + \theta v_t\}\} = 0, \end{aligned}$$

which leads to

$$c^d = 1/[1 + \theta^2(k - \beta\gamma_2^d)]. \tag{24}$$

To solve for  $b^d$ , we substitute Eq. (16) into Eq. (23). Unlike the case of commitment, however, we now obtain a third-degree equation. Using Mathematica, we can solve for  $b^d$  and obtain three roots, all of which are extremely complicated. Alternatively, we can take a more fruitful route, which is to characterize the qualitative relationships between the coefficient under discretion and commitment.<sup>8</sup> This is done in Appendix B, where we prove the following proposition:

*Proposition.*  $a^c \geq a^d \geq 0$ ,  $0 < b^c < b^d < 1$ ,  $0 < c^d < c^c < 1$ ,  $\gamma_1^d > \gamma_2^c > 0$ , and  $\gamma_2^d < \gamma_2^c < 0$ .

This Proposition shows that the bounds on the  $a$ ,  $b$ , and  $c$  coefficients under discretion are the same as under commitment. Consequently, the properties of the two monetary policy operating procedures are the same in many cases. In particular, the steady-state features noted above in the case of commitment carry over to discretion. When there is no persistence in inflation, it is again the case that  $a^d = \pi^*$ ,  $b^d = 0$ , and  $c^d = 1/(1 + k\theta^2)$ . It also follows that when there is persistence, *in expectation* the monetary authority cannot hit its target  $\pi^*$  every period even though it follows consistently the optimal discretionary policy rule; but it does hit the target on average, and there is again no average inflation bias.

However, the fact that the coefficients differ systematically from each other implies that the two operating procedures have certain distinct dynamic

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<sup>8</sup> The authors are indebted to Lars Svensson for his suggested approach for establishing these relationships.

features. It is useful to look at the relationship between the optimal commitment rule and the optimal discretionary rule first under the assumption that the shocks are zero. As  $b^d > b^c$ , it is clear from Eq. (13) that there is more persistence in inflation with discretion than with commitment, so that discretion can be said to have an inflation-persistence bias. Therefore, the expected deviation of inflation from its target level is always larger, while the expected deviation of output from its target level is always smaller with discretion than with commitment, as  $[\lambda(1 - b^d)]/\theta < [\lambda(1 - b^c)]/\theta$ . By contrast, commitment generates smaller expected deviations of inflation from its target but simultaneously larger expected deviations of output from its target. There is, therefore, a clear tradeoff, in the sense of ex ante expectation, between the optimal commitment and discretionary monetary policy rules. Notice that this result is obtained in our model where there is no average inflation bias and the discretionary approach is defined by the optimal discretionary monetary policy rule.

The main intuition behind this result is that with the commitment rule, the monetary authority, with its additional instrument  $i_t^e$ , internalizes the impact of its decision rule on the expectations of the private sector. In this way the monetary authority reduces the persistence of inflation compared to discretion, but at the cost of larger deviation in output from the natural rate. In effect, the choice of a commitment rule involves what can be called an anti-inflation bias in that it stabilizes the systematic component of inflation, whereas a discretionary rule involves an inflation-persistence bias.

Another interesting result is that under discretion, the adjustment in the control variable  $i_t$ ,  $[\lambda - (\lambda - \phi\theta)b]/\phi\theta$ , is smaller than in the case of commitment, if  $\lambda > \phi\theta$ , and vice versa otherwise. Thus, when  $\lambda > \phi\theta$ , the commitment strategy involves a more active use of interest rate policy to achieve smaller deviations of inflation from target but larger deviations of output from target.<sup>9</sup>

Looking now at the effects of the random shocks, as  $c^d < c^c$ , there is a larger response to current shocks to inflation with discretion. As the discretionary rule involves an inflation-persistence bias relative to the commitment rule, it is more responsive to the random shock to inflation and therefore dampens the effect of this shock on current inflation to moderate the subsequent persistence in inflation. Thus discretion can be characterized as having a conditional inflation-stabilization bias relative to commitment. This is in contrast to the 'comparative advantage' of the commitment rule in keeping inflation closer to target in the systematic or expected component of inflation, but commitment permits

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<sup>9</sup> The model considered here does not take account of possible costs in adjusting interest rates, which have been analyzed by Goodhart (1997). If the model were extended to incorporate these costs, the difference between the two strategies would probably be less sharp, but the qualitative results described above would still appear to hold.

a larger portion of the current inflation shock to pass through, thus stabilizing current-period output.

In comparing the two rules in terms of the combined responses to the systematic and the random components, we find that with the commitment rule, inflation has less persistence but more random variability, whereas output has more persistence but less random or conditional volatility. This is in accord with what one would expect regarding the differences in the two rules. Discretion entails a greater responsiveness to the random shocks to inflation; by contrast, because the commitment strategy internalizes the effect of the monetary policy rule on expectations, it stabilizes the systematic component of inflation and leads to less persistent deviations of inflation from target. Because of the stabilizing effect of the commitment rule on inflation, there is less need to respond as aggressively to contemporaneous random shocks to inflation.

Finally, it is worth noting that when the discretionary rule is optimally designed, as it should be, even in the presence of a large shock there may be no need for an ‘escape clause’ for a conservative central bank, an idea initially proposed by Flood and Isard (1989) and further extended to a ‘flexible’ central bank by Lohmann (1992). This is so because even if  $u_t$  is a large shock, there will be no obvious advantage for the monetary authority to switch from the commitment to the discretionary rule. That is, depending on the economic environment, the ‘escape clause’ may not be needed even in the presence of a large shock because the authorities are by construction responding optimally.

## 5. Conclusions

This paper has derived the optimal monetary policy feedback rules with both commitment and discretionary cases in a simplified economy that is characterized by what we believe to be a more realistic version of the Phillips curve than that used in previous analysis of this topic. As persistence in prices and inflation is a feature of all empirical versions of the Phillips curve, it is important that such persistence be captured in theoretical discussions of optimal stabilization policy. Indeed, we would argue that policy questions and issues are relatively uninteresting in a model where a short-run tradeoff between output and inflation exists only in the current period, as such an economy is essentially self-stabilizing.

Moreover, our characterization of inflation control appears to us to have captured at least some of the challenges faced by monetary authorities in achieving their objectives. While ultimately the inflation level depends on the rate of growth of the money stock, short-run stabilization of output and inflation depends on adjusting the (real) interest rate to affect the level of

aggregate demand relative to output capacity, and thereby inflation. Thus inflation control is achieved only indirectly via changes in demand.

We have two main findings in the paper. First, with inflation persistence, both commitment and discretion strategies lead to state-contingent and shock-dependent feedback rules. Thus the overall stance of policy is important and our analysis is consistent with that contained in the literature (see Goodhart and Huang, 1995; Svensson, 1997a for recent examples) that has examined the implications of output persistence on the optimal policy rule. Moreover, the form of the feedback rules are economically plausible, and both the signs and the variations in the coefficients in response to changes in parameter values are also in accord with economic intuition.

Second, the analysis shows that in the sense of *ex ante* expectation there always exists a tradeoff relationship between the two optimal monetary policy rules. A commitment rule takes account of the effects of its actions on market expectations and in this sense is forward looking, whereas a discretionary rule is myopic in the sense that it only concerned with the impact of the realized interest rate on the economy without taking into consideration how the market responds to the monetary authority's operating rule. The commitment rule leads to expected inflation that is closer to its target, but simultaneously to expected output that is further away from its target, whereas the opposite is the case for the discretionary rule. Moreover, a discretionary rule is more responsive in reducing the effect of the random shock on inflation, with the result that there is a larger effect of this random shock on output than the commitment rule. Basically, the commitment rule has a stabilizing effect on inflation expectations, so that there are less persistent deviations of inflation from target. As a result, there is less need to respond as aggressively to random inflation shocks.

It is useful to compare the optimal feedback rules derived here with that described by Taylor (1993). In what he calls a representative monetary policy rule, the nominal interest rate that is the instrument of the central bank is set equal to the lagged inflation rate (plus the real steady-state growth rate of 2.2% to give a positive real interest rate) and is specified as responding to deviations of inflation from a target of two percent and deviations of output from trend GDP. It is noteworthy that this admittedly *ad hoc* policy rule is quite similar in form to the optimal rules derived above, where the nominal interest rate is a function of the inflation target plus three other terms, namely, the steady-state real interest rate,  $\alpha$ , a coefficient times the deviation of lagged inflation from the target level (the state-contingent item), and the effect of random shocks (the shock-dependent item). As the optimal rule takes full account of preferences regarding deviations of output from target, such deviations do not appear in the policy rule itself. Taylor's policy rule can be viewed as a kind of reduced form that, in principle, combines the preferences of the policymaker embodied in the loss function as well as the behavioral parameters and structural relationships in the

model, all of which are explicitly incorporated in the coefficients in the optimal policy rules.<sup>10</sup>

Taylor describes his policy rule as having the general properties of rules that have been examined in recent research, (e.g., Bryant et al., 1993). Moreover, he argues that it explains remarkably well the actual behavior of the federal funds rate controlled by the Federal Reserve over the period 1987–1992. Clarida and Gertler (1996) estimate a modified Taylor rule over the period 1974–1992 for the short-term interest rate used by the Bundesbank as its policy instrument. They also find that it has considerable explanatory power. While it would be far too strong to conclude anything about the optimality of the policy reaction functions of these two central banks, the theoretical results in this paper suggest that a Taylor-type rule does at least embody certain aspects of an optimal feedback rule. Alternatively, one can regard this empirical evidence as providing some support for the type of theoretical analysis pursued in this paper.

The analysis in this paper could be extended in a number of ways. Allowing for lags in the effect of interest rates on aggregate demand and for persistence in output would add greater realism, but is likely to add to the complexity of the analysis without affecting the basic finding that with persistence, an optimal monetary policy must be active in the sense of being state contingent and shock dependent. In this case there would be a second state variable, lagged output, and it would be of interest to explore whether it would be possible to derive an optimal policy rule that would have the same symmetry as Taylor's rule, i.e., in which the equation for the control variable would involve adjustment coefficients for deviations of both inflation and output from their respective targets. In addition, the results in the paper could be elaborated by exploring the extent to which the inflation-persistence bias could be ameliorated by means of a conditional linear inflation contract, as described by Walsh (1995), or by a conditional inflation target discussed by Svensson (1997a). Similarly, it would appear that the conditional inflation-stabilization bias with discretion could be addressed by a lower  $k$ , i.e., a weight-liberal central bank.

## Appendix A.

Solution for  $a^c$ ,  $b^c$  and  $c^c$ . To determine the values of  $a^c$ ,  $b^c$  and  $c^c$ , we can start with Eqs. (11) and (12). After substituting  $y_t^c$  and  $\pi_t^c$  into Eq. (12),

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<sup>10</sup> Svensson (1997b) provides a framework of inflation forecast targeting in which he derives an interest-rate adjustment equation that is similar to a Taylor rule.

we get:

$$\begin{aligned} & \lambda[-\phi\lambda(i_t^c - \pi_{t-1} - \alpha)/(\lambda - \phi\theta)] \\ & + \theta(k - \beta\gamma_2^c)[\pi_{t-1} - \phi\theta(i_t^c - \pi_{t-1} - \alpha)/(\lambda - \phi\theta) - \pi^*] \\ & - \theta(k\pi + \beta\gamma_1^c) = 0, \end{aligned}$$

which gives

$$\begin{aligned} & i_t^c - \pi_{t-1} - \alpha \\ & = \{\theta(\lambda - \phi\theta)/\phi[\lambda^2 + \theta^2(k - \beta\gamma_2^c)]\}[(k - \beta\gamma_2^c)\pi_{t-1} - (k\pi + \beta\gamma_1^c)]. \end{aligned}$$

Using this in  $E_{t-1}\pi_t$ , we have

$$E_{t-1}\pi_t = \theta^2(k\pi^* + \beta\gamma_1^c)/[\lambda^2 + \theta^2(k - \beta\gamma_2^c)] + \{\lambda^2/[\lambda^2 + \theta^2(k - \beta\gamma_2^c)]\}\pi_{t-1}.$$

Thus,

$$a^c = \theta^2(k\pi^* + \beta\gamma_1^c)/[\lambda^2 + \theta^2(k - \beta\gamma_2^c)], \quad (\text{A.1})$$

$$b^c = \lambda^2/[\lambda^2 + \theta^2(k - \beta\gamma_2^c)]. \quad (\text{A.2})$$

Using Eqs. (13)–(15) in Eq. (11), and imposing Eq. (12), we get

$$\begin{aligned} & -\phi\{(1 - c^c)/\phi\theta\}u_t + (1/\phi)v_t\} + v_t \\ & + \theta(k - \beta\gamma_2^c)\{-\phi\theta\{(1 - c^c)/\phi\theta\}u_t + (1/\phi)v_t + u_t + \theta v_t\} = 0, \end{aligned}$$

which leads

$$c^c = 1/[1 + \theta^2(k - \beta\gamma_2^c)]. \quad (\text{A.3})$$

To solve for  $b^c$ , we substitute Eq. (16) into Eq. (A.2). After some simplifications, we get  $b^c = \lambda^2[1 - \beta(b^c)^2]/[(1 + \beta)\lambda^2 + k\theta^2 - 2\beta\lambda^2b^c]$ . Solving it, we get

$$\begin{aligned} b^c & = \frac{(1 + \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 + \beta)\lambda^2 + k\theta^2]^2 - 4\beta\lambda^4}}{2\beta\lambda^2} \\ & = \frac{(1 + \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 - \beta)\lambda^2 + k\theta^2]^2 + 4\beta\lambda^2k\theta^2}}{2\beta\lambda^2} < 1. \end{aligned} \quad (\text{A.4})$$

Because

$$\begin{aligned} & \frac{(1 + \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 - \beta)\lambda^2 + k\theta^2]^2 + 4\beta\lambda^2k\theta^2}}{2\beta\lambda^2} \\ & < \frac{(1 + \beta)\lambda^2 + k\theta^2 - (1 - \beta)\lambda^2 - k\theta^2}{2\beta\lambda^2} = 1, \end{aligned}$$

but

$$\frac{(1 + \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 - \beta)\lambda^2 + k\theta^2]^2 - 4\beta\lambda^4}}{2\beta\lambda^2} > \frac{(1 + \beta)\lambda^2 + k\theta^2 - (1 + \beta)\lambda^2 - k\theta^2}{2\beta\lambda^2} = 0,$$

$0 < b^c < 1$ .

The other root has a value greater than 1. As Lockwood and Philippopoulos (1994) and Svensson (1997a) have shown, only the smaller root with value less than one is relevant in these circumstances.

Substituting Eq. (A.4) in Eq. (16), we have

$$\begin{aligned} \gamma_2 &= -[k\theta^2b^2 + \lambda^2(1 - b^2)]/[\theta^2(1 - \beta b^2)] \\ &= \frac{(1 - \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 - \beta)\lambda^2 + k\theta^2]^2 + 4\beta\lambda^2k\theta^2}}{2\beta\lambda^2} < 0. \end{aligned} \tag{A.5}$$

Thus,

$$a^c = \frac{-(1 - \beta)\lambda^2 + k\theta^2 + \sqrt{[(1 - \beta)\lambda^2 + k\theta^2]^2 - 4\beta\lambda^2k\theta^2}}{2\beta\lambda^2} \pi^* \geq 0, \tag{A.6}$$

$$c^c = \frac{2 - (1 - \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 - \beta)\lambda^2 + k\theta^2]^2 + 4\beta\lambda^2k\theta^2}}{2(1 + k\theta^2)[1 - (1 - \beta)\lambda^2] - 2k\beta\lambda^2\theta^2}. \tag{A.7}$$

**Appendix B.**

Proof of Proposition that  $a^c \geq a^d \geq 0$ ,  $0 < b^c < b^d < 1$ ,  $0 < c^d < c^c < 1$ ,  $\gamma_1^d > \gamma_1^c$ , and  $\gamma_2^d < \gamma_2^c < 0$ .

Let  $V_t(\pi_t) = \gamma_{0,t} + 2\gamma_{1,t}\pi_t + \gamma_{2,t}\pi_t^2$ , then if we know the  $\gamma$ -coefficients for  $t$ , we can find the  $\gamma$ -coefficients for  $t - 1$  based on a recursive relationship.

Under discretion, we know that the first-order condition leads to

$$\pi_t = a_t + b_t\pi_{t-1} + c_tu_t, \tag{B.1}$$

where

$$b_t = \lambda/[\lambda + \theta^2(k - \beta\gamma_{2,t})]. \tag{B.2}$$

Substituting  $\pi_t$  and  $y_t$ , expressed in  $\gamma_t$ , into  $V_{t-1}(\pi_t)$  and identifying the coefficient for  $\pi_t^2$  leads to

$$\gamma_{2,t-1} = -[k\theta^2b_t^2 + \lambda^2(1 - b_t^2)]/[\theta^2(1 - \beta b_t^2)]. \tag{B.3}$$

By Eqs. (B.2) and (B.3),  $b^d$  and  $\gamma_2$  can be solved by iteration backward as  $t$  goes to  $-\infty$ . The iteration converges quickly, with starting values  $\gamma_{2,t-1} = 0$  and  $b_0 = \lambda/(\lambda + k\theta^2)$ .

After having established the recursive relationship under commitment and discretion, we are ready to prove  $0 < b^c < b^d < 1$ .

Based on the recursive relationship,  $b_{t-1} = f^c(b_t)$ ,  $b_{t-1} = f^d(b_t)$ , with  $b^c = f^c(b^c)$ ,  $b^d = f^d(b^d)$ . Thus, for the same value of  $b_t$ , we also know that:  $f^c(b_t) < f^d(b_t)$ , because

$$f^c(b_t) = \lambda^2/[\lambda^2 + \theta^2(k - \beta\gamma_2(b_t))] < \lambda/[\lambda + \theta^2(k - \beta\gamma_2(b_t))] = f^d(b_t),$$

due to  $0 < \lambda < 1$ . Thus,  $b^c < b^d$ .

Furthermore, with  $b^c$  and  $b^d$ , we can also show that  $\gamma_2^d < \gamma_2^c < 0$ . This is because the coefficients of  $\gamma_2$  and  $b$  are related according to Eq. (16), i.e.,

$$\gamma_2 = -\frac{k\theta^2 b^2 + \lambda^2(1 - b)^2}{\theta^2(1 - \beta b^2)}.$$

Differentiation of Eq. (16) with respect to  $b$  leads to

$$\begin{aligned} \gamma_2'(b) &= \frac{2\{\beta\lambda^2 b^2 - [k\theta^2 + (1 + \beta)\lambda^2]b + \lambda^2\}}{\theta^2(1 - \beta b^2)^2} \\ &= \frac{2\beta\lambda^2}{\theta^2(1 - \beta b^2)^2}(b - b^c)(b - \bar{b}^c), \end{aligned}$$

where  $b^c$  is the relevant root for  $b^c$  given in Eq. (A.4), while  $\bar{b}^c$  is the other root for  $b^c$ , i.e.,

$$\bar{b}^c = \frac{(1 + \beta)\lambda^2 + k\theta^2 - \sqrt{[(1 + \beta)\lambda^2 + k\theta^2]^3 - 4\beta\lambda^4}}{2\beta\lambda^2} > 1.$$

Since  $b^d$  fulfills  $b^c < b^d$ , we have  $\gamma_2'(b) < 0$  for all  $b^d \in (b^d, \bar{b}^c)$ , thus  $\gamma_2^d < \gamma_2^c < 0$ . As a result,  $b^c < b^d < 1$ , and the system stability condition is satisfied.

Moreover, with  $\gamma_2^d < \gamma_2^c < 0$ , and noticing

$$c^c = \frac{1}{1 + \beta^2(k - \beta\gamma_2^c)} \quad \text{and} \quad c^d = \frac{1}{1 + \theta^2(k - \beta\gamma_2^d)}$$

we have  $1 > c^c > c^d > 0$ .

Noting further that:  $a^c/(1 - b^c) = (k\pi^* + \beta\lambda_1^c)/(k - \beta\lambda_2^c)$  and  $a^d/(1 - b^d) = (k\pi^* + \beta\lambda_1^d)/(k - \beta\lambda_2^d)$ , we thus have the general relationship:

$$a/(1 - b) = (k\pi^* + \beta\lambda_1)/(k - \beta\lambda_2). \quad (\text{B.4})$$

Recall from Eq. (17) in the text that in the case of both commitment and discretion:  $\gamma_1 = -[\lambda^2 a(1 - b) - \theta^2(ka - k\pi^* - \beta a\gamma_2)b]/[\theta^2(1 - \beta b)]$ . Using

Eq. (B.4), and substituting  $a$  for  $[(k\pi^* + \beta\gamma_1)/(k - \beta\gamma_2)](1 - b)$  in the equation for  $\gamma_1$  above, we have

$$\gamma_1 = -\gamma_2\pi^*. \quad (\text{B.5})$$

Using this result in Eq. (B.4), it follows that

$$a/(1 - b) = \pi^*.$$

With  $0 < b^c < b^d < 1$ , we have

$$a^c = (1 - b^c)\pi^* \geq (1 - b^d)\pi^* = a^d \geq 0, \quad \text{for } \pi^* \geq 0.$$

Finally, with  $\gamma_2^d < \gamma_2^c < 0$ , we have  $\gamma_1^d = -\gamma_2^d\pi^* > -\gamma_2^c\pi^* = \gamma_1^c \geq 0$ .

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