Price Stability and Monetary Policy Effectiveness when Nominal Interest Rates are Bounded at Zero

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Abstract

This paper employs stochastic simulations of a small structural rational expectations model to investigate the consequences of the zero bound on nominal interest rates. We find that if the economy is subject to stochastic shocks similar in magnitude to those experienced in the U.S. over the 1980s and 1990s, the consequences of the zero bound are negligible for target inflation rates as low as 2 percent. However, the effects of the constraint are non-linear with respect to the inflation target and produce a quantitatively significant deterioration of the performance of the economy with targets between 0 and 1 percent. The variability of output increases significantly and that of inflation also rises somewhat. The stationary distribution of output is distorted with recessions becoming somewhat more frequent and longer lasting. Our model also uncovers that the asymmetry of the policy ineffectiveness induced by the zero bound generates a non-vertical long-run Phillips curve. Output falls increasingly short of potential with lower inflation targets. At zero average inflation, the output loss is in the order of 0.1 percentage points. We also investigate the consequences of the constraint on the analysis of optimal policy based on the inflation-output variability frontier. We demonstrate that in the presence of the zero bound, the variability frontier is distorted as the inflation target approaches zero. As a result comparisons of alternative policy rules that ignore the zero bound can be seriously misleading.

KEYWORDS: Inflation targeting, price stability, monetary policy rules, liquidity trap.

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1 Introduction

There is fairly widespread consensus among macroeconomists that the primary long-term objective of monetary policy ought to be a stable currency. Studies evaluating the costs of inflation have long established the desirability of avoiding not only high but even moderate inflation.\(^1\) However, there is still a serious debate on whether the optimal average rate of inflation is low and positive, zero, or even moderately negative.\(^2\) An important issue in this debate concerns the reduced ability to conduct effective countercyclical monetary policy when inflation is low. As pointed out by Summers (1991), if the economy is faced with a recession when inflation is zero, the monetary authority is constrained in its ability to engineer a negative short-run real interest rate to damp the output loss. This constraint reflects the fact that the nominal short-term interest rate cannot be lowered below zero—the zero interest rate bound.\(^3\)

This constraint would be of no relevance in the steady state of a non-stochastic economy. In an equilibrium with zero inflation, the short-term nominal interest rate would always equal the equilibrium real rate. Stabilization of the economy in a stochastic environment, however, presupposes monetary control which leads to fluctuations in the short-run nominal interest rate. Under these circumstances, the non-negativity constraint on nominal interest rates may occasionally be binding and so may influence the performance of the economy. This bound is more likely to be reached, the lower the average rate of inflation and the

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\(^1\)Fischer and Modigliani (1978), Fischer (1981), and more recently Driffill et al. (1990) and Fischer (1994), provide a detailed accounting of the costs of inflation. An early analysis of the costs of both inflation and deflation is due to Keynes (1923).

\(^2\)The important contributions by Tobin (1965) and Friedman (1969) provided arguments in favor of inflation and deflation, respectively. But theoretical arguments alone cannot provide a resolution. The survey of the monetary growth literature by Orphanides and Solow (1990) suggests that equally plausible assumptions yield conflicting conclusions regarding the optimal rate of inflation. Similarly, recent empirical investigations suggest a lack of consensus. Cross-country studies confirm the cost of high average inflation on growth but find no robust evidence at low levels of inflation. (See Sarel, 1996, and Clark, 1997.) Judson and Orphanides (1996) find that the volatility rather than the level of inflation may be detrimental to growth at low levels of inflation. Feldstein (1997) identifies substantial benefits from zero inflation due to inefficiencies in the tax code. Akerlof, Dickens and Perry (1996), however, estimate large costs due to downward wage rigidities.

\(^3\)The argument has its roots in Hicks's (1937) interpretation of the Keynesian liquidity trap. Hicks (1967) identified the question regarding "the effectiveness of monetary policy in engineering recovery from a slump" as the key short-run concern arising from the trap. (p. 57).
greater the variability of the nominal interest rate. In this context, “inflation greases the wheels of monetary policy,” as Fischer (1996) points out. (p. 19.)

The purpose of this paper is to conduct a systematic empirical evaluation of the zero bound constraint in a stochastic environment and assess the quantitative importance of this constraint for the performance of alternative monetary policy rules. Recent quantitative evaluations of policy rules suggest that rules that are very effective in stabilizing output and inflation do indeed entail substantial variability in the short-term nominal interest rate. (Taylor, 1998.) Most often, however, the simulated models are linear and neutral to the average rate of inflation and abstract from the zero bound. Alternative policy rules are then evaluated based on their performance in terms of the variability of output and inflation they induce in such models. This approach to policy evaluation is appropriate with a high average rate of inflation when the non-negativity constraint on nominal interest rates would be unlikely to bind. However, since policy is not only concerned with stabilizing output and inflation but also with maintaining a low average inflation rate, evaluation of the impact of the zero bound on economic performance is important. To the extent that both inflation and deflation hamper economic performance and are otherwise equally undesirable, the zero bound constraint effectively renders the risks of deviating from an inflation rate of zero asymmetric. As Chairman Greenspan noted recently, “...deflation can be detrimental for reasons that go beyond those that are also associated with inflation. Nominal interest rates are bounded at zero, hence deflation raises the possibility of potentially significant increases in real interest rates.” (From Problems of Price Measurement, remarks at the Annual Meeting of the American Economic Association and the American Finance Association, Chicago, Illinois, Jan 3, 1998.)

Despite its apparent significance, efforts to evaluate the quantitative importance of the zero bound to date have been scant. The main reason is that it introduces a nonlinearity in otherwise linear models which dramatically increases computational costs. In the context of policy rule evaluations, Rotemberg and Woodford (1997) indirectly address the constraint by penalizing policies resulting in exceedingly variable nominal interest rates.
They show that such constrained optimal policies significantly differ from the optimal rules that ignore the constraint. A first assessment of the effect the zero bound that explicitly introduces this nonlinearity in a small linear model is provided by Fuhrer and Madigan (1997). Their results, based on a set of deterministic simulations, suggest that the reduced policy effectiveness at low inflation rates may have a modest effect on output in recessions.

In this paper we estimate a small rational expectations macroeconomic model of the U.S. economy in which monetary policy has temporary real effects due to sluggish adjustment in wages and prices. We then compare the stochastic properties of the economy when monetary policy follows the countercyclical policy rules suggested by Taylor (1993b) and Henderson and McKibbin (1993) under alternative inflation targets in the presence of the zero bound on nominal interest rates. Our model is linear in other respects so that, when the target rate of inflation is sufficiently high, the properties of our model are comparable to those of similar linear models that ignore the zero bound. As a consequence, comparing the stochastic distributions of output and inflation corresponding to alternative inflation targets permits the evaluation of the effect of the zero bound.

We find that if the economy is subject to stochastic shocks similar in magnitude to those experienced in the U.S. over the 1980s and 1990s, the consequences of the zero bound constraint are negligible for target inflation rates as low as 2 percent. However, the effects of the constraint are very non-linear with respect to the inflation target and become increasingly important for determining the effectiveness of policy with inflation targets of between 0 and 1 percent. We find that economic performance deteriorates significantly with such low inflation targets. The variability of output increases noticeably, while the variability of inflation also rises somewhat. The stationary distribution of output is seriously distorted with recessions becoming somewhat more frequent and longer lasting. Moreover, in our model the asymmetry of policy ineffectiveness induced by the zero bound generates a non-vertical long-run Phillips curve. Output falls increasingly short of potential, on average, as the inflation target, and therefore the average rate of inflation, becomes smaller. At zero average inflation, the output loss is in the order of 0.1 percent of potential output.
We also investigate the consequences of the constraint on the analysis of optimal policy based on the inflation-output variability frontier. In the absence of a constraint—or equivalently when the inflation target is sufficiently high—inflation-output variability frontiers are invariant to the target when policy follows a linear rule. With the zero bound, however, the frontier is significantly distorted as the inflation target approaches zero. As a result, comparisons of alternative policy rules that ignore the zero bound can be seriously misleading.

The remainder of this paper is organized as follows. The next section presents the estimated model of the U.S. economy. Section 3 illustrates the effects of the zero bound by means of a few, very specific, deterministic simulations. A quantitative assessment of the impact of the zero bound in a stochastic environment is provided in section 4, while section 5 shows the resulting distortions in the variability frontiers often employed in evaluations of monetary policy rules. In section 6 we discuss a methodological issue concerning the global stability properties of the model in the presence of the zero bound constraint and present some sensitivity results. We briefly relate our simulation analysis to the historical experience in the U.S. and recent experience in Japan in section 7 and draw some conclusions in section 8.

2 A small rational expectations model of the U.S. economy

This section describes the small model that we use as a laboratory for assessing the effectiveness of monetary policy when the nominal interest rate is constrained at zero. The following four features render it a very useful tool for this purpose:

- The model incorporates forward-looking behavior by economic agents in labor markets, financial markets and goods markets. Expectations of endogenous variables are formed rationally and fully reflect the choice of monetary policy rule.

- Monetary policy has temporary real effects due to staggered wage contracts.
• The model is estimated based on US data from 1980 to 1996, thereby restricting attention to a period which was characterized by a fairly stable policy regime.

• The model is linear, save for the zero bound constraint, and is rather small relative to large structural macroeconometric models. As a result, it is well suited for large-scale stochastic simulation experiments which involve substantial computation costs.

We have used an earlier version of this model in a study of the opportunistic approach to disinflation (see Orphanides, Small, Wieland and Wilcox (1997), (OSWW)). The model is best viewed as a one-country version of the multi-country model presented and used for policy analysis in Taylor (1993a). A similar specification has been used by Taylor and Williams (1993) in a study of forecasting with rational expectations models. We alter their specification in two important ways. First, instead of using the specification of staggered wage-setting developed in Taylor (1980), we follow Fuhrer and Moore (1995a,b). The latter assume that workers and firms set the real wage in the first period of each new contract with an eye toward the real wage agreed upon in contracts signed in the recent past and expected to be signed in the near future. As Fuhrer and Moore show, models specified in this manner exhibit a greater (and hence more realistic) degree of inflation persistence than do models in which workers and firms care about relative wages in nominal terms. Secondly, we have reestimated the demand-side equations using data from 1980 to 1996 on an equation-by-equation basis. We have evaluated the fit of the model imposing the cross-equation restrictions due to rational, model-consistent expectations and found that it forecasts within-sample movements of inflation and output quite well. The model captures the degree of persistence in output and inflation that is observed in the data. The historical series of structural shocks to output and inflation, which we compute based on model-consistent expectations, show no remaining serial correlation.

\footnote{By contrast, Taylor (1980) assumed that workers and firms set the \textit{nominal} wage in the first period of each new contract with an \textit{eye} toward the \textit{nominal} wage settlements of recently signed and soon-to-be signed contracts.}
The model is a simple linear flow model of the economy. We group the various equations under three headings: interest rates, aggregate demand, and the wage-price sector.

2.1 Interest rates

Three equations determine the various interest rates in the model. A policy rule in the form of an interest rate reaction function determines the federal funds rate; a term-structure equation determines the long-term nominal rate; and a version of Fisher's equation determines the long-term real rate.

The interest rate rule is estimated from 1980 to 1996. As shown in the recent literature on policy rules, for example in Clarida, Gali and Gertler (1997), this period was characterized by a relatively stable policy regime that differed from the 1970s and 1960s. Since a stable estimated policy rule is necessary to identify the historical structural shocks and compute the associated variance-covariance matrix required for our simulations, we decided to restrict attention to this period for estimation. We obtain the following interest rate rule by means of instrumental variables techniques:

(1) estimated policy rule

\[
\frac{i_t}{i_t^*} = -0.0042 + 0.760 \frac{i_{t-1}}{i_{t-1}^*} + 0.625 \pi_t + 1.171 y_t - 0.967 y_{t-1} + u_{t,t}
\]

\[
\hat{R}^2 = 0.925 \quad SER = 0.010, \quad DW = 2.50
\]

The values in parentheses are standard errors of the coefficient estimates. The inflation rate, \(\pi_t\), reflects the rate of change of the chain-weighted GDP deflator over four quarters ending in quarter \(t\). The output gap, \(y_t\), is based on chain-weighted GDP and is constructed using estimates of potential output from the Congressional Budget Office (1997). \(u_{t,t}\) is a serially uncorrelated shock to the short-term interest rate.\(^5\) The coefficients on the output gap and its lag suggest that policy not only responded to the level of slack in the economy but also to its direction of change. As can be seen from the coefficient on the lagged federal

\(^5\)The estimated residuals likely reflect the policymaker's response to information that is not captured by the output gaps, inflation and the lagged interest rate in the policy rule. For this reason, we do not make use of these interest rate shocks in evaluating the performance of alternative policy rules in stochastic simulations later in the paper.
funds rate, this policy rule incorporates a substantial degree of interest rate smoothing. As a consequence, the longer-run response of short-term interest rates to increases in inflation or output gaps is substantially larger than the initial response. This expected response of future short rates will then be reflected in current long-term rates due to the term structure relationship. Rather than estimating the term structure explicitly, we rely on the accumulated forecasts of the short rate over the following 8 quarters which, under the expectations hypothesis, will coincide with the long rate forecast for this horizon.\(^6\)

(2) long-term nominal rate

\[
i_t^l = E_t \sum_{j=1}^{8} i_{t+j}
\]

Subtracting inflation expectations over the following 8 quarters, \(\pi_{t+8}^{(8)}\), determines the long-term real interest rate.

(3) long-term real rate

\[
r_t^l = i_t^l - E_t \pi_{t+8}^{(8)}
\]

As is common practice in the recent literature on evaluating interest-rate-based policy rules, the model contains no explicit money demand equation or variable measuring the quantity of money. With policy being fully described in terms of interest rates, monetary balances are endogenously determined with the central bank responding to money demand shocks by varying the supply of money balances as needed to achieve the interest rate prescribed by the policy rule.

2.2 Aggregate demand

Aggregate demand \(Y_t\) is broken down into its major components: aggregate consumption \(C_t\), fixed investment \(FI_t\), inventory investment \(II_t\), total (federal, state and local) government purchases \(G_t\) and net exports \(NEX_t\). Following Taylor and Williams (1993), we scale each demand component by the level of potential output, and denote the result with lower-case letters. All equations are estimated using the Generalized Method of Moments.

\(^6\)In defining the long rate in terms of the expectations hypothesis we deliberately avoid the added complexities that would be associated with modeling term and risk premia. Since our specification is invariant to the presence of a constant premium, we set it equal to zero for expositional simplicity.
Normalized consumption is modeled as a function of its own lagged value, permanent income, the expected long-term real interest rate and a serially uncorrelated shock. The lagged dependent variable can be rationalized as reflecting an adjustment cost to changing consumption. Permanent income is the annuity value of expected income in the current and next eight periods. Note that the structural shocks differ from the estimated regression residuals as the latter also reflect forecast errors.

(4) consumption

\[ c_t = 0.230 + 0.665 c_{t-1} + 0.286 y_t^p - 0.102 r_t^i + u_{c,t} \]

\[ R^2 = 0.971 \text{ SER } = 0.00356, \text{ DW } = 1.97 \]

(5) permanent income

\[ y_t^p = \frac{1 - 0.9}{1 - (0.9)^8} E_t \sum_{j=0}^{8} (0.9)^j y_{t+j} \]

The investment equations are (nearly) of the accelerator type. Fixed investment is negatively related to the real interest rate.

(6) fixed investment

\[ f_{it} = 0.0018 + 0.988 f_{it-1} + 0.171 f_{it-2} - 0.169 f_{it-3} \]
\[ + 0.134 y_t - 0.050 y_{t-1} - 0.128 y_{t-2} - 0.033 r_t^i + u_{f_{it}} \]

\[ R^2 = 0.940 \text{ SER } = 0.0024 \text{ DW } = 2.16 \]

(7) inventory investment

\[ ii_t = 0.0019 + 0.324 ii_{t-1} + 0.032 ii_{t-2} + 0.168 ii_{t-3} \]
\[ + 0.116 y_t + 0.187 y_{t-1} - 0.286 y_{t-2} + u_{ii,t} \]

\[ R^2 = 0.580 \text{ SER } = 0.0035 \text{ DW } = 1.92 \]

Net exports depend on the level of income at home and abroad \((y_t^w)\), and on the real exchange rate \((e_t)\).
(8) net exports
\[
nx_t = 0.0232 + 0.803 \, nx_{t-1} - 0.50 \, y_t + 0.099 \, y^w_t - 0.0056 \, e_t + u_{nx,t}
\]
\[(0.011) \quad (0.050) \quad (0.022) \quad (0.023) \quad (0.0024)\]
\[\hat{R}^2 = 0.962 \quad SER = 0.0026 \quad DW = 2.07\]

Government spending follows a simple autoregressive process with a near-unit root.

(9) government spending
\[
g_t = 0.0033 + 0.982 \, g_{t-1} + u_{g,t}
\]
\[(0.0086) \quad (0.043)\]
\[\hat{R}^2 = 0.893 \quad SER = 0.0018 \quad DW = 2.10\]

(10) output gap
\[
y_t = c_t + f_t + i_t + nx_t + g_t - 1
\]

2.3 Wages and Prices

The wage-price block consists of three equations that determine the real contract wage to be paid in the current quarter under newly-signed contracts and the price in the current period. This specification is due to Fuhrer and Moore (1995a,b) and we adopt the parameter estimates from Fuhrer (1997b).

(11) index of real contract wages
\[
v_t = 0.37045(x_t - p_t) + 0.29015(x_{t-1} - p_{t-1}) + 0.20985(x_{t-2} - p_{t-2}) + 0.12955(x_{t-3} - p_{t-3})
\]

(12) current real contract wage
\[
x_t - p_t = E_t(0.37045v_t + 0.29015v_{t+1} + 0.20985v_{t+2} + 0.12955v_{t+3})
\]
\[+ 0.0055E_t(0.37045y_t + 0.29015y_{t+1} + 0.20985y_{t+2} + 0.12955y_{t+3}) + u_{x,t}\]

(13) aggregate nominal price
\[
p_t = 0.37045x_t + 0.29015x_{t-1} + 0.20985x_{t-2} + 0.12955x_{t-3}
\]
Equations (11) and (12) specify that the real wage under contracts signed in the current period is set in reference to a centered moving average of initial-period real wages established under contracts signed as many as three quarters earlier as well as contracts to be signed as many as three quarters ahead. Furthermore, the negotiated real wage is assumed to depend also on expected excess-demand conditions. Once contracts are signed, they remain in force for up to four quarters. Thus, the aggregate wage is a weighted average of the nominal contract wages that were negotiated in the current and previous three quarters (and thus still remain in force), with the weights reflecting the proportion of contracts outstanding from each quarter. With a fixed markup from wages to prices the dynamic behavior of the aggregate price, $p_t$, is the same as that of the aggregate wage. For this reason, Fuhrer uses price data in estimation. There are two estimated parameters, the coefficient on expected future output gaps in the contract wage equation, which is crucial for the short-run inflation-output tradeoff in this model, and a coefficient determining the share of contracts negotiated in each quarter.\footnote{The distribution of contract prices is a downward-sloping linear function of contract length, with weights $\omega_i = .25 + (1.5 - i)s, i = 0, \ldots, 3$. This distribution depends on a single slope parameter, $s$, which is estimated by Fuhrer (1997b) to equal 0.0803, and is invertible.}

2.4 The steady state of the model

In the deterministic steady state of this model, output is at potential, and the sectoral allocation of GDP is constant. Because we hold rest-of-world output and the real exchange rate constant, these conditions define a unique steady-state value of the long-term real interest rate. The coefficients of the demand equations and steady state shares imply a 1 percent equilibrium long-term real rate, the value of the real rate for which output equals potential in steady state. In the absence of a term premium this estimate implies that the short-term real rate of interest is also 1 percent. The steady-state consumption share is 68.4 percent, the fixed investment share is 14.7 percent, the inventory investment share is 0.4 percent, the net exports share is \(-2\) percent, and the government spending share is 18.7 percent.
The steady-state value of inflation is determined exclusively by the policymaker’s reaction function, because the wage-price block does not impose any restriction on the steady-state inflation rate. These conditions guarantee that the steady-state inflation rate will be equal to the inflation target of the policymaker. Together with the estimated steady-state real interest rate of 1 percent, the estimated rule implies an inflation target of about 2 percent. This can be deduced from the coefficients in the rule by noting that the steady-state nominal short-term rate equals the sum of the steady-state real rate and the steady-state inflation rate.

2.5 Simulation techniques

We conduct stochastic simulations of the model to obtain the stochastic distributions of the endogenous variables under monetary policy rules with alternative (known) inflation targets. In preparation for these simulations, we first computed the structural residuals of the model based on U.S. data from 1980 to 1996. Since the non-negativity constraint for nominal interest rates was never binding during this period and our model is otherwise linear, we obtained the structural shocks by solving the model analytically for the reduced form using the AIM implementation (Anderson and Moore, 1985, and Anderson, 1997) of the Blanchard and Kahn (1980) method for solving linear rational expectations models. We calculated the covariance matrix of those structural residuals and using this covariance matrix, we generated 1000 sets of artificial normally-distributed shocks with 100 quarters of shocks in each set. We then used these shocks to conduct stochastic simulations under alternative policy rules and inflation targets, while imposing the non-negativity constraint on nominal interest rates.9

We simulate the model using an efficient algorithm that was recently implemented in

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8The process of calculating the structural residuals would be straightforward if the model in question were a purely backward-looking model. For a rational expectations model, however, structural residuals can be computed only by simulating the full model and computing the time series of model-consistent expectations with respect to historical data. The structural shocks differ from the estimated residuals to the extent of agents’ forecast errors.

9If it were not for this nonlinearity, we could use the reduced form of the model corresponding to the alternative policy rules to compute unconditional moments of the endogenous variables without having to resort to stochastic simulations.
TROLL based on work by Boucekkine (1995), Juillard (1994) and Laffargue (1990) and is related to the Fair-Taylor (1983) extended path algorithm. A limitation of the algorithm is that the model-consistent expectations of market participants are computed under the counterfactual assumption that “certainty equivalence” holds in the nonlinear model being simulated. This means, when solving for the dynamic path of the endogenous variables from a given period onwards, the algorithm sets future shocks equal to their expected value of zero. Thus the variance of future shocks has no bearing on the formation of current expectations and economic performance. This would be correct in a linear model. However once we introduce the zero bound on nominal interest rates into the model, we are able to show that the variance of future shocks ought to be expected to introduce a small bias in the average levels of various variables, including importantly, interest rates. This result is discussed in detail in section 4 of this paper. To be clear, we should emphasize that the variance of shocks has both a direct and an indirect effect on the results. The direct effect is that a greater variance of shocks implies that the zero bound on nominal interest rates binds with greater frequency, the indirect effect is that all agents should be taking this effect of the variance into account when they form their expectations. The simulation algorithm captures the direct effect but not the indirect one.

There are other solution algorithms for nonlinear rational expectations models that do not impose certainty equivalence. But these alternative algorithms would be prohibitively costly to use with our model, which has more than twenty state variables. Even with the algorithm we are using, stochastic analysis of nonlinear rational expectations models with a moderate number of state variables remains fairly costly in terms of computational effort. The typical stochastic simulation experiment in this paper involves computing the stationary distributions of endogenous variables for a given interest rate rule and inflation target based on 1000 draws of shocks. Solution of the model for one such draw which has 100 quarters of shocks requires about 10 minutes on a SPARC 20 Sun workstation. Thus, computation of the stationary distributions for one specific rule and one value of the inflation target takes about one week.
3 Some Illustrative Dynamic Simulations

This section uses simple deterministic simulations to illustrate the behavior of the model when the non-negativity constraint on nominal interest rates is binding and when it is not. While we have used the estimated policy rule for computing historical structural shocks, we replace it in the following simulations with the rules proposed by Taylor (1993) and Henderson and McKibbin (1993) respectively, which have received considerable attention in the recent literature on evaluating monetary policy rules.

(14) Taylor’s rule

\[ i_t^s = r^* + \pi_{t-1} + .5y_{t-1} + .5(\pi_{t-1} - \pi^*) \]

(15) Henderson and McKibbin’s rule

\[ i_t^s = r^* + \pi_{t-1} + 2y_{t-1} + 1(\pi_{t-1} - \pi^*) \]

Here \( \pi^* \) is the policymaker’s inflation target that determines the steady-state rate of inflation in the model and \( r^* \) is the short-term real equilibrium rate. For our simulations we set \( r^* \) equal to the estimated long-term real equilibrium rate of 1 percent, maintaining the absence of a term premium. Taylor’s rule responds less aggressively to output gaps and inflation deviations from target, than the Henderson-McKibbin rule (HM). Since current values of inflation and the output gap are not available to the policymaker within the same quarter but become available with a lag, we assume that the rules are specified in terms of one-period lagged values of output and inflation.\(^{10}\) Previous research using this and other models, as for example in Levin (1996), Williams (1997) and OSWW (1997), has shown that the HM rule is more effective in stabilizing output and inflation, but at the expense of somewhat higher interest variability than under the rule.

\(^{10}\)This represents a slight difference from the rules as were originally proposed. Both Taylor, and Henderson and McKibbin, assumed the policy-maker could react to same-quarter output and inflation. Our specification is the closest possible operational version of these rules. The operationality issue in the original formulation was first pointed out by McCallum (1993) in his discussion of the rules employed in the Bryant, Hooper and Mann (1993) volume which introduced the interest rate rule specification adopted by Taylor (1993b) and by Henderson and McKibbin (1993).
The exercises conducted in the remainder of this paper address the following question: suppose future policy would follow one of these two rules, to what extent would the choice of inflation target influence the effectiveness of monetary policy given the zero bound on nominal interest rates? Of course, the higher the inflation target, the higher will be the steady-state nominal interest rate, and the smaller should be the likelihood of hitting the zero bound. We enforce the zero bound directly in our simulations, rather than just restricting the variance of nominal interest rates. We do so by passing short-term interest rates through a nonlinear filter which guarantees that as long as the federal funds rate is greater than zero, it takes on the value prescribed by the respective policy rule, but if the rule prescribes negative values, the actual federal funds rate remains at zero.

3.1 Demand Shocks

As a first step, we follow the approach taken by Fuhrer and Madigan (1997) and simulate the dynamic response of the model for a set of standardized negative demand shocks. In our case these shocks include a fixed investment shock, a consumption shock, an inventory investment shock and a government spending shock. Each shock is chosen to be -1 percent of potential GDP. The advantage of choosing equal size shocks is that it is easier to compare the resulting impulse responses. These shocks are not representative of the historical shocks, but the estimated historical covariance of shocks will be properly reflected in the subsequent stochastic simulation analysis.

For each of the four shocks we compare the dynamic response under the HM rule with inflation targets of 0 percent and 3 percent. We find that while the non-negativity constraint on nominal interest rates typically binds for several periods with a zero inflation target, it never binds when the inflation target is set to 3 percent.

Figure 1 shows the dynamic response to the fixed investment shock. The top left panel

\footnote{It should be noted that the zero bound constraint has been imposed in some earlier models used for policy analysis, (for the Taylor model for example in Taylor (1993a) and Wieland (1996b), and for the IMF's MULTIMOD in Luxton and Prasad (1997)) but has not been the subject of in-depth studies based on stochastic simulations.}

\footnote{Note that in our simulations we hold net exports constant and thus consider the closed-economy version of the model.}
shows the path of the nominal interest rate for the simulations with a target of 0 percent (solid line) and a target of 3 percent (dotted line). The interest rate is initialized at its steady-state value, which coincides with the steady-state real rate of 1 percent when the inflation target is at 0 percent. In response to the negative demand shock, the policymaker lowers the federal funds rate in an attempt to boost the economy, but immediately hits the zero bound. The funds rate remains at zero for 2 years and then reverts back to its steady-state value. The dotted line is the adjustment path under an inflation target of 3 percent. In this case, the policymaker has enough room to respond to the demand shock as desired and push the nominal interest rate down to about 0.25 percent.

The top right panel in figure 1 reports the real long-term interest rate, through which monetary policy affects aggregate demand and inflation. In this model it corresponds to the average of current and expected federal funds rates over the next 2 years minus expected inflation. As is apparent from the dotted line the policymaker would like to lower the real rate by about two percentage points during the first year after the negative demand shock, but this is not possible when starting from a steady state with inflation at 0 percent and interest rates at 1 percent, because of the zero bound on nominal rates. Instead, policy only achieves a meager 50 basis points drop in the real rate. Not surprisingly then, the resulting recession is much deeper than in the case of an inflation target of 3 percent and a steady-state nominal funds rate of 4 percent. As shown in the lower right panel, the recession is almost twice as deep, with the trough at -3.5 percent (solid line) instead of -1.8 percent (dotted line). Clearly, the low average inflation and interest rates that arise under a zero inflation target turn the zero bound into a serious constraint for stabilization policy in the event of an adverse demand shock. Furthermore, the zero bound enhances the deflationary impact of the demand shock as shown in the lower-left panel of figure 1.

The dynamic response of inflation and output and the impact of the zero bound depend on the specific shock that hits the economy. The choice of a fixed investment shock may overstate the importance of the zero bound, since this shock has a fairly persistent impact on output in our model. For comparison, figures 2, 3 and 4 report the outcome under
negative shocks of the same size to consumption, inventory investment and government spending. In all cases the non-negativity constraint on interest rates binds for at least a few quarters with an inflation target of 0 percent but not with a target of 3 percent. However, in the case of a consumption shock the impact of the zero bound on output and inflation is much smaller, and with an inventory investment shock the output and inflation response is almost identical under the two different inflation targets. In both cases the reason is that the shock has a much less persistent impact on output than a shock to fixed investment. Consequently, future short rates revert more quickly to the steady-state and long real rates are less affected by the zero bound on nominal rates. The consumption shock is less persistent, because consumption demand, which responds to future permanent income, is more forward-looking than fixed investment demand and exhibits a smaller degree of partial adjustment. Inventory investment is also characterized by more rapid adjustment than fixed investment. Finally, the response to a government spending shock shows more similarities to the case of a fixed investment shock, because government spending is modeled as an autoregressive process that ascribes long-lasting effects to such shocks.

3.2 Price Shocks

In response to negative price shocks, inflation typically tends to fall and output tends to rise in our model. Technically, the price shocks we consider are shocks to the contract wage in equation (12) of section 2. Effectively, they have the same consequences as a textbook-type short-run supply shock and present the central bank with the “dilemma” of deciding whether to channel their effect more towards a temporary increase in output or towards a temporary reduction in inflation. Figure 5 shows precisely such a scenario for an inflation target of 3 percent (dotted line). Inflation falls by 2 percentage points, while output temporarily rises above potential in response to a sequence of two negative contract wage shocks of 25 basis points. Nominal interest rates as well as real interest rates decline sufficiently to reinflate the economy within five years. Of course, this type of shock is most welcome when the central bank is in the process of engineering a disinflation, and perhaps
less so when the inflation target has already been achieved.

However, once we set initial inflation and the inflation target equal to 0 percent in the simulation, the non-negativity constraint binds and we observe a much less favorable scenario. As shown by the solid lines in figure 5 the price shocks still drive down inflation and interest rates, but now nominal interest rates hit the zero bound by the second quarter and remain constrained at zero for four years. Since inflation declines rapidly during the first year, the long-term real rate first jumps up by a full percentage point, and then declines much more slowly than in the case of a 3 percent inflation target. The initial increase in the long-term real rate induces a negative output gap and further enhances the deflationary impact of the shock. It takes two years until the real rate has declined sufficiently to push output back above potential and generate inflationary pressures.

This simulation not only illustrates the importance of the zero bound, but also a mechanism which could destabilize this economy and push it towards a deflationary spiral, where the zero bound keeps the real interest rate sufficiently high so that output stays below potential and reinforces further deflation. We discuss this issue in more detail in section 6.

4 Stochastic Simulation Results

The demonstration in the previous section indicates that the zero bound on interest rates could indeed limit the potency of monetary policy and alter the dynamic response of the economy to stochastic shocks.

To evaluate whether such effects would be of quantitative significance in practice, it is necessary to assess how frequently monetary policy would be expected to be constrained if the economy were subjected to stochastic shocks with properties similar to those we anticipate to obtain in practice. To this end, we employ stochastic simulations of our model economy under monetary policy rules with alternative inflation targets. As a baseline, we assume the economy is subject to shocks drawn from a joint normal distribution with the covariance of the shocks we estimated for the 1980s and 1990s. With these simulations we construct the stationary distribution of interest rates, inflation and output that correspond
to policies with alternative inflation targets.\textsuperscript{13}

Based on the resulting stationary distributions, we investigate the extent to which the statistical properties of inflation and output are altered when the policymaker adopts inflation targets near zero. We examine the influence of the target on the means and variances of inflation and output, which would be central for welfare analysis based on a quadratic loss function. We also investigate the persistence properties of output and provide comparisons of statistics relating to the length and depth of recessions for alternative inflation targets.

An advantage of the linear structure of our model is that the stationary distributions of inflation and output that obtain in the absence of the zero-bound constraint can be computed analytically.\textsuperscript{14} But even with the zero-bound constraint in place, for any given parameterization of a policy rule’s response to inflation and output, a sufficiently high choice of inflation target would render the zero-bound constraint relevant with near zero measure. As a result, when the inflation target is sufficiently high, the stationary distributions of interest rates, inflation and output have properties identical to those that obtain when the constraint is ignored. Thus, the stationary distributions corresponding to the unconstrained case provide a useful benchmark against which the stationary distributions obtained when the zero-bound constraint is properly taken into account can be compared. To compare the influence of the constraint under rules with different response coefficients we performed stochastic simulations with monetary policy following either the Taylor rule (T) or the Henderson and McKibbin rule (HIM). For each of these rules we set the inflation target alternatively at three, two, one, one-half, one-quarter and zero percent.

Figure 6 shows the impact of the alternative inflation targets on the distribution of the nominal interest rate. The top panel shows the frequency with which the zero-bound constrains monetary policy, that is the frequency with which the monetary authority would

\textsuperscript{13}Typically, we estimate the stationary distributions of the endogenous variables based on 1000 independent draws of shocks. The length of each draw is 100 quarters. For each draw, we use the first twenty shocks to arrive at a stochastic initial condition and do not use them for computing the stationary distributions. We found this to be sufficient to ensure that our estimates are not affected by the deterministic choice of initial conditions.

\textsuperscript{14}This is the case because of our maintained assumption that the underlying structural shocks are jointly normally distributed.
have set the nominal rate below zero if that were feasible in that period. As can be seen, the zero bound does not represent a quantitatively important factor at inflation targets at or above two percent. An inflation target of three percent, therefore, effectively serves as the linear unconstrained model benchmark. For a policymaker following the HM rule, the constraint becomes binding with about one-tenth frequency for targets below one percent. For a policymaker following the T rule, which is less reactive to both inflation and output, the constraint becomes binding with about one-tenth frequency only for targets below one-quarter percent. However, this frequency increases considerably as the inflation target drops towards zero. For the HM rule, the constraint becomes binding with almost 30 percent frequency when $\pi^*$ equals zero.

The bottom panels of figure 6 describe the resulting distortion of the stationary distributions of the nominal interest rate. The bottom left panel shows the distortion in the average level of the nominal interest rate. This is computed as the mean of the stationary distribution of the short nominal interest rate, $i^*$, minus $r^* + \pi^*$. In the absence of the constraint, the mean nominal rate would equal the sum of the equilibrium real interest rate and the inflation target. This result is indeed confirmed in the figure with the constraint in place when the inflation target is large enough for the bind to occur very infrequently. With inflation targets near zero, however, the asymmetric nature of the constraint on policy introduces a significant bias. Since the constraint provides a lower bound on the nominal interest rate, it effectively forces policy to be tighter than it would be in the absence of the constraint under some circumstances. Since no comparable upper bound is in place, policy is tighter on average. This bias increases with the frequency with which the constraint binds that is it increases with greater activism on the part of the monetary authority. As can be seen from the chart, a policymaker following the HM rule with a zero inflation target would set the nominal interest rate almost 20 basis points higher, on average, than if the zero-bound constraint were not in place. Furthermore, since this constraint restricts the variability of interest rates, the standard deviation of both the level and the change of interest rates fall somewhat as the inflation target drops to zero. This is shown in the
bottom-right panel of figure 6, with the lower two lines plotting the standard deviation of the changes and the upper two lines the standard deviation of the levels for alternative inflation targets.

In summary, the frequency with which the zero-bound constraint binds and the distortion in the distribution of the nominal interest rate is much smaller with the less reactive T rule than with the more reactive HM rule and is not important quantitatively for inflation targets above two percent. With a zero inflation target, however, the bind introduces a noticeable bias in the stationary distribution of the nominal interest rate. Next we examine the influence of this distortion on the stationary distributions of inflation and output.

Figure 7 superimposes the probability density functions for the stationary distributions of inflation and output corresponding to the HM rule with inflation targets of zero and two percent. The distributions corresponding to the two percent inflation target are approximately normal as the zero bound constraint practically never binds. Comparison with the distributions corresponding to the zero inflation target reveals some interesting features. The right tails of the distributions for the alternative targets are essentially identical. The left tails of the distributions however, are quite different with the tails corresponding to the zero target being considerably thicker. The influence of the zero bound constraint is quite clear. When either output or inflation fall considerably below their means, policy without the constraint would engineer an easing in order to return output to potential and inflation to its target level. With the constraint binding, this is no longer feasible and consequently reflation of the economy occurs at a slower pace.

Summary information regarding the distortion of the distributions of inflation and output with the inflation target is shown in figure 8. The top panel shows the resulting bias in the means of inflation and output and the bottom panel the corresponding changes in the standard deviations. As shown in the upper-left panel, a small downward bias in average inflation (relative to the target) appears as a result of the zero-bound. Such a bias is not materially significant, however, since a small adjustment to the inflation target in the policy rule could yield any desired average level of inflation. A more significant bias materializes
with respect to the output gap. As the inflation target drops to zero, output fails to reach potential, on average, resulting in a negative average output gap. For the HM rule a zero inflation target yields an average output loss of about a tenth of a percent. As the bottom panels of the figure suggest, the variability of output and inflation also increases at near zero inflation targets.

Figure 9 presents the same information regarding the distortions of the stationary distributions of inflation and output as presented in figure 8 but shows the distortion in terms of the frequency with which the zero-bound constraint is binding instead of in terms of the inflation target. This can serve as a guide for the distortion associated with the frequency with which the non-negative interest rate constraint is violated.\textsuperscript{15} An interesting contrast appears when the HM and T rules are compared in the two figures. From figure 9, the bias corresponding to the T rule for a given frequency of binding is greater than the comparable bias in the HM rule. Based on this metric, the T rule might be considered as more prone to a bias due to the zero-bound constraint. However, a comparable bias for the T rule occurs at considerably lower inflation targets than for the HM rule so such a comparison could be misleading.

The presence of the zero-bound constraint in our model clearly invalidates the long-run superneutrality that obtains in a linear version of the model. The relationship between the average level of output and the average level of inflation that is due to the zero-bound implies the existence of a long-run Phillips curve in our model. This is shown in figure 10 which plots the upward sloping relationship between average inflation and average output. Employing Okun’s law to translate negative output gaps to positive unemployment gaps

\textsuperscript{15}The frequency with which the constraint binds, however, may be higher than the frequency with which the nominal interest rate would be negative in the absence of the constraint. For example, in the case of the HM rule with a inflation target of 0 percent, we find that the constraint is binding about 30 percent of the time. In the absence of the constraint, the nominal interest rate would be negative less than 25 percent of the time. (This can be computed using the standard deviation of the nominal interest rate in the absence of the constraint, 1.45 percentage points, and the unconditional expectation of the federal funds rate is 1 percent). To see the reason for this note that if the policy rule called for negative interest rates during a recession and this was feasible, output would revert to normal in fewer periods than if interest rates were restricted at zero. Thus, the frequency with which interest rates would turn negative in the absence of a constraint is smaller than the frequency with which the constraint actually binds, unless both equal zero.
would generate a downward sloping long-run Phillips curve in the more traditional inflation-unemployment space. To note, the slope of the long run Phillips curve generated by the zero-bound constraint is only noticeable at average inflation rates below two percent and is fairly small.\textsuperscript{16} More important, perhaps, is the non-linearity in the schedule suggesting a greater loss at the margin for additional reductions in the inflation target as the inflation target and average inflation fall towards zero.

The source of this non-neutrality can be directly traced to the interaction between the policy rules and the forward-looking nature of expectations in our model. As is well known, in models with rational expectations such as ours, the sacrifice ratio—the ratio of the cumulative output gap loss (gain) required for a given reduction (rise) in the inflation rate—is a function of the policy responsiveness to inflation and output.\textsuperscript{17} With a linear policy reaction function, as is the case when the inflation target is sufficiently high for the zero bound to be irrelevant, output losses when inflation is above the steady state and falls towards it exactly offset output gains when inflation is below the steady state and rising towards it. The responsiveness of policy to inflation and output is the same in both cases. Symmetry prevails and on average the output gap is zero. This is not the case when the zero bound becomes important. When the constraint is binding, the responsiveness of policy to marginal changes in inflation is nil—the interest rate is constrained at zero. When the constraint is not binding, the usual responsiveness of policy is restored. But the former is more likely when inflation is below its target than above its target so symmetry fails and a bias in the average output gap appears. It is worth noting that if expectations were of a backward-looking, adaptive nature, the long-run Phillips curve would be vertical as in that case the sacrifice ratio would be invariant to the policy responsiveness altogether. Of course, introducing additional non-linearities in policy might offset this bias but it would also move the policy away from its original unrestricted linear specification and distort the

\textsuperscript{16} The effect of reducing average inflation from one percent to zero on the average level of unemployment is only about a tenth of the effect identified in Akerlof, Dickens and Perry (1996), due to downward wage inflexibility.

\textsuperscript{17} See Fuhrer (1994) for a detailed analysis of the sacrifice ratio in this context.
higher moments of the stationary distributions of inflation and output.

Another effect of the zero bound constraint regards the incidence and persistence of recessions in our model. To show this we examine the frequency with which output falls below and remains below a particular level for a number of quarters. That is, we examine the properties of the left tail of the distribution of output corresponding to the policy rules with alternative inflation targets. To keep track of the effect of the zero bound, however, requires further specificity. Since the variance of output is affected by the specification of the policy rule, the frequency with which output falls below any given level (e.g. 2 percent below potential), depends on the policy rule even in the absence of the zero constraint. To make the results comparable across the different rules, we normalize our definition of low activity by the standard deviation of output corresponding to the rule in the absence of a constraint. For any particular rule (e.g. T or HM), let $\sigma_\infty$ be the standard deviation of output when the inflation target is sufficiently high for the zero bound constraint to bind with zero probability (that is as $\pi^*$ approaches infinity). Defining low activity as the state in which the output gap is at least $\kappa$ standard deviations below zero in any particular quarter we can describe the impact of the zero bound on the tail of the stationary distribution by computing the low activity frequency associated with alternative inflation targets:

$$\text{Low Activity Frequency} = \text{Prob}(y < -\kappa \sigma_\infty)$$

The top panel of figure 11 shows this measure of the incidence of recession for the T and HM rules when $\kappa = 1.5$.\(^{18}\) When the inflation target is above 2 percent, the frequency of recession corresponding to this measure is about 7 percent and equals exactly the frequency with which a normally distributed random variable falls 1.5 standard deviations below its mean. By this measure, in the absence of the zero bound constraint, the economy is in a “recession” 7 percent of the time regardless of the parameterization of the rule. As $\pi^*$ becomes smaller, however, this frequency rises somewhat, reaching 11 percent when $\pi^* = 0$

\(^{18}\)Results for alternative values of $\kappa$ are qualitatively similar. Quantitatively, the effect of the bound on the frequencies shown becomes proportionally bigger with larger values of $\kappa$ but the comparison is based on lower frequency events.
in the HM rule.

To evaluate the persistence of recessions, we investigated the frequency with which output is in the low activity state for four consecutive quarters relative to the frequency of being in this state in any one quarter. This relative frequency of persistent recession is shown in the bottom panel of the figure. Since the degree of mean reversion in the dynamics of output depends on the parameterization of the rule, the frequency of persistently low activity differs for the T and HM parameterizations even in the absence of the zero bound. As can be seen from the figure, when $\pi^*$ exceeds two percent the relative frequency of persistent recession is about 12 percent for the HM rule and 18 percent for the T rule. That is, based on this measure, the more reactive HM rule results in less persistent recessions than the less active T rule. This comparison changes when the zero bound becomes a constraining factor on monetary policy. For low inflation targets, the relative frequency of persistent recession rises, and more so for the HM rule than for the less active T rule. With a zero inflation target, the relative frequency reaches about 20 percent for the T rule and 22 percent for the HM rule. That is, by this measure, the relative frequency of persistent recession for the HM rule exceeds that of the T rule when $\pi^*$ is zero although it is smaller in a higher inflation environment when the zero bound constraint does not materially affect the conduct of monetary policy.

5 Variability Frontier Distortions

Having provided an analysis of the distortion of the stationary distributions of inflation and output that is induced by the zero bound as the inflation target approaches zero, we now focus on the variability of these variables as it relates to efficient policy frontiers.\textsuperscript{19}

In many models, including our linear economy in the absence of a zero-bound constraint, when monetary policy follows a conventional linear rule that encompasses the T and HM

\footnote{The usefulness of comparing alternative policy rules in terms of such frontiers has been highlighted in Taylor (1994), and more recently in Fuhrer (1997a), Williams (1997) and Levin, Wieland and Williams (1997).}
rules:

\[ i_t^* = r^* + \pi_{t-1} + \gamma_1 y_{t-1} + \gamma_2 (\pi_{t-1} - \pi^*), \]

then the output gap is on average zero, and inflation is on average equal to its target, \( \pi^* \), independent of the rule’s responsiveness to inflation and output, \( \gamma_1 \) and \( \gamma_2 \). It is then convenient to summarize the performance of alternative policy rules by comparing the resulting variabilities of inflation, output and interest rates that correspond to the different rules. Such comparisons are meaningful in linear models because these variabilities are invariant to the average level of inflation.

Unfortunately, in the presence of a binding non-negativity constraint on nominal interest rates, the invariance of the variability of inflation and output to the inflation target breaks down. To visualize the impact of the zero-bound effect on variability frontier analyses, in figure 12 we plot the inflation-output variance pairs corresponding to policy rules with alternative inflation targets. We use squares for the HM rule and circles for the T rule. The solid square and circle indicate the variances that correspond to inflation targets of 3 percent and which would also obtain (and be invariant to the inflation target) in the absence of the zero-bound constraint. As can be seen by following the hollow squares and circles, with inflation targets below two percent the variability pairs move northeast in the diagram indicating deteriorating performance in terms of variability. Needless to say, a policymaker with a zero inflation target should realize that the points indicating the unconstrained variability pairs do not describe the variability the economy would likely face.

To place this distortion in the context of a frontier, in figure 13 we construct an efficient frontier for the variability of output and inflation which ignores the zero-bound (a “high-inflation-target” frontier) and show its transformation in the presence of a zero-bound when the inflation target is set at zero. The frontier which ignores the zero-bound, shown with the solid line, is constructed with the restriction that the variance of the stationary distribution of interest rates changes does not exceed that of the HM rule, that is 0.86 percent. As indicated by the solid square, the HM rule is on this frontier.
A useful way to interpret points on this frontier is as solutions to the policymaker’s minimization problem:

$$\min_{\gamma_1, \gamma_2} \{ \psi \sigma_y^2 + (1 - \psi) \sigma_\pi^2 \}$$

subject to the constraint that the standard deviation of interest rate changes not exceed 0.86 percent. The parameter $\psi$ indicates the weight the policymaker puts on minimizing the unconditional variance of inflation versus the unconditional variance of the output gap. The fact that the HM rule is on the frontier indicates that the responsiveness to inflation and output embedded in the rule correspond to the optimal choices for some value $\psi$. That is, for some value of $\psi$, of the family of concentric ellipses which represent the policymaker’s indifference curves, the one indicating the best feasible outcome would be tangent to the efficient frontier drawn at exactly the point corresponding to the HM rule, the solid square.

It is important to emphasize that when a policymaker targets zero inflation, the efficient high-inflation-target frontier no longer represents feasible policy outcomes. In the figure, the dashed line illustrates the distortion of the variability frontier when the policy rules which trace it are simulated with a zero inflation target in the presence of a zero-bound on interest rates. As can be seen, the shape of the frontier is distorted with points corresponding to lower output variability in the original (high-inflation-target) frontier suffering greater distortions than points corresponding to lower inflation variability. Thus, the policy rule minimizing the objective function subject to the original frontier may be dominated by an alternative choice of response parameters, if policy is designed with a zero inflation target instead.

We conclude that because the presence of the zero-bound constraint invalidates superneutrality, welfare analysis regarding the optimal choice of inflation target can no longer be performed independently of the investigation of the stochastic properties of the model which are also deemed to influence welfare. Regarding policies geared towards price stability, the use of variability frontiers for comparisons of alternative policy rules that ignore the zero bound can easily become a haphazard enterprise.\(^{20}\)

\(^{20}\)The optimal policy rule in the presence of the zero bound on nominal interest rates is likely to be
6 Additional Considerations

6.1 An Important Methodological Issue

The zero bound constraint is the only effective non-linearity in the model. However, when the zero bound constraint is introduced, the global stability of our otherwise linear system is no longer ensured. This possibility is clearly apparent from the simulation with negative price shocks in section 2. In the presence of the zero bound on nominal interest rates, the deflationary impact of these shocks leads initially to an increase in the long real interest rate and a negative output gap which further adds deflationary impetus. In that specific simulation, despite the initial deflation, the prospect of zero interest rates for a number of periods suffices for the long-term real rate to fall and output eventually rises above potential thereby reflating the economy. Given this example, it is not difficult to imagine how a series of negative price shocks that are sufficiently large may induce a deflationary spiral. The initial increase in the real interest rate due to the zero bound will lead to further deflation, which then will lead to further increases in the real rate without bounds. This points to a limitation inherent in linear models such as this which rely on the real interest rate as the sole channel for monetary policy. But it also brings into focus the extreme limiting argument regarding the ineffectiveness of monetary policy in a liquidity trap.

To ensure the global stability of the model, when we impose the zero-bound constraint, we also introduce a second nonlinearity. We specify an exogenous policy that if deflation becomes so severe that the zero bound restricts the real interest rate at a level high enough to induce a growing aggregate demand imbalance, boosts aggregate demand until deflation returns to near zero levels. Analytically we impose this stabilizing force as a fiscal expansion rule, which operates when the economy enters a sustained deflation phase. In practice, for the variability of shocks we examine and for the inflation targets of zero or greater that we study, this non-linearity becomes relevant nonlinear and asymmetric. Given our finding of significant distortions in inflation and output distributions, it would be of interest for future research to explore the properties of an optimal policy rule in a low or zero inflation environment.
with zero frequency and consequently does not influence the results we discuss—beyond ensuring stability. If it had been the case that this global stabilization mechanism was activated with non-negligible frequency, however, a more detailed investigation of the issue would have been warranted. Yet it is important to impose this globally stabilizing force in order to be able to compute and correctly interpret the stationary distributions of the economy.

### 6.2 Sensitivity Analysis

The distortions we identify with the pursuit of price stability result from the asymmetric response of policy to stochastic disturbances. In the absence of any shocks, our model is superneutral and changes in the inflation target have no influence on the deterministic steady state of the economy. For a given covariance matrix of shocks, the effect of the zero bound on nominal interest rates in our model depends importantly on the average level of nominal interest rates as well as the variability in interest rates that is influenced by the degree of policy responsiveness. So far, we have investigated the zero bound under alternative assumptions concerning the inflation target, which is one of the determinants of the average level of nominal interest rates, as well as two alternative policy rules, which differ in their responsiveness to inflation and output. Undoubtedly, our estimates are also sensitive to other underlying assumptions of the model. We discuss two potentially important sources of such sensitivity below.

First, our estimate of the long run equilibrium interest rate of 1 percent is admittedly fairly imprecise. But our results can be easily used to assess the effect of alternatives to our baseline case. The constraint regards the nominal interest rate which in steady state is the sum of the real interest rate and the rate of inflation. As a result, changes in one parameter can be offset by changes in the other. For example, our results for \( \pi^* \) equal to 1 percent with our baseline assumption of \( r^* \) equal to 1 percent also describe the outcome in an economy with \( r^* \) equal to 2 percent when \( \pi^* \) equal to 0 percent. Such an outcome may be particularly important at times of rapid productivity growth, when the equilibrium real
rate of interest may be higher than usual.

Second, in our stochastic simulations we use the covariance matrix of the shocks computed from the estimated equations for the 1980s and 1990s as a benchmark. However, a policymaker may have different expectations regarding the covariance of shocks likely to prevail in the future. Since the resulting distortions in the distributions of inflation and output associated with zero inflation are themselves a function of the covariance of the underlying shocks, the costs associated with pursuing price stability, both in terms of the variability of inflation and output and the average loss in output depend on the underlying covariance of the shocks. To evaluate the implications of alternative assumptions regarding the size of these shocks, we performed stochastic simulations of the HM rule with a zero inflation target with shocks scaled to be a constant fraction of our baseline case. Figure 14 shows the results. As can be seen from the top panels, the negative bias in inflation and the output gap is negligible when the standard deviation of the shocks is smaller than 40 percent of the estimated standard deviation we use as our baseline. This negative bias then increases quite rapidly and nonlinearly as the shocks become larger. Similarly, the two lower panels in figure 14 show that the standard deviations of inflation and output increase essentially linearly with the scale of the underlying shocks at first but much faster than the scale of the underlying shocks as the scale rises. Not unexpectedly, the impact of the zero bound on the variability of inflation and output rises more than linearly with the standard deviation of the underlying shocks to the economy. If the economy were anticipated to be more turbulent than it was over the 1980s and 1990s, the distortions we identify with a zero inflation target would be significantly higher.


In designing our experiments, we have focussed on two linear policy rules which have received much attention in the recent literature on monetary policy rules and have been analyzed in different macroeconomic models. However, policy outcomes might be improved if policy were allowed to explicitly take into account the presence of the zero bound constraint. For
example, an asymmetric policy, that is trying to prevent the economy from entering states where the zero bound constrains policy effectiveness, may dominate simple linear rules. Within the class of linear rules, one might conjecture that the effect of the zero interest rate bound could be mitigated, in part, by more forward looking behavior. By allowing the policymaker to react to an anticipated future shortfall in demand or to a deflationary shock, policy could engineer an inflation preempting the effect of the zero-bound constraint. One possibility is to respecify the policy rule as:

$$i_t^* = r^* + \pi_{t+i} + \gamma_1 y_{t+i} + \gamma_2 (\pi_{t+i} - \pi^*),$$

where $i > 0$ indicates the relevant horizon targeted by policy. We have investigated the performance of such forward-looking rules (with the same response coefficients as the HM and T rules) in the presence and absence of the zero bound on nominal interest rates.

When the inflation target is high so that the zero bound does not come into play in our model, we find that output and inflation variability deteriorate somewhat when we replace lagged inflation and output with their one to three quarter ahead forecasts. We obtain mixed results, when we only replace inflation with its forecasts. While there are marginal improvements in output variability, inflation variability deteriorates somewhat. These results are broadly consistent with the findings in Levin, Wieland and Williams (1997), who compare the performance of alternative policy rules in four different macroeconometric models of the U.S. economy with rational expectations, including the model presented in this paper.

With a zero inflation target, we find that the zero-bound on nominal interest rates induces the same type of distortions under forward-looking rules than under rules that respond to lagged variables. Furthermore, comparison of the variability of output and inflation under alternative rules in the case of high and zero inflation targets, suggests that the performance of forward-looking rules relative to rules responding to most recent outcomes is unchanged. Thus, our model does not provide any evidence that the impact of the zero-bound constraint may be alleviated by following a rule that responds to forecasts. We leave the question whether explicitly nonlinear rules are able to alleviate the effect of
the zero bound for future research.

7 Historical Relevance

Our simulation analysis based on an empirical model of the U.S. economy suggests that the zero-bound on nominal interest rates would have important effects on inflation and output if an economy in a low inflation and low interest rate environment were subjected to shocks. But has this theoretical possibility ever been of practical relevance?

The answer to this question might be no if attention were restricted to the post World War II experience of the U.S. economy, the usual laboratory for macroeconomic policy evaluations. In none of the eight recessions since 1950 did interest rates ever come close to the zero bound. The three month treasury bill rate was reduced during the course of each recession with reductions ranging from 64 to 715 basis points but with ample room remaining for additional easing in every case.\textsuperscript{21}

The experience during the 1930s and 1940s, however, was quite different. The three-month treasury bill rate fell from almost 5 percent at the beginning of the Great Depression in 1929 to just under 1 percent in 1932. The rate hovered close to zero over a prolonged period starting in 1932, not rising above one percent again until 1948. The two recessions following the Great Depression, in 1937 and 1945, provide clear examples of the inflexibility of monetary policy under such circumstances. In both cases the treasury bill rate was already under 50 basis points as the recession started, leaving virtually no room for easing. By contrast, at the beginning of the Great Depression of 1929 the room for easing exceeded five percentage points, yet it was essentially exhausted before the depression was over. One view is that monetary policy was perhaps not easy enough early enough relative to the required remedy. Writing in 1930, just a few months into the Great Depression and before its catastrophic magnitude could be known, Keynes first warned\textsuperscript{22} “I repeat that the greatest evil of the moment and the greatest danger to economic progress in the near

\textsuperscript{21}These reductions reflect the difference between the interest rates prevailing at the peak and trough quarters of each recession, using NBER dates.

\textsuperscript{22}The source is A Treatise on Money, which Keynes completed on September 14, 1930.
future are to be found in the unwillingness of the Central Banks of the world to allow the market-rate of interest to fall fast enough.” (p. 207) and then offered the remedy: “That is to say, they [the Bank of England and the Federal Reserve Board] should combine to maintain a very low level of the short-term rate of interest, and buy long-dated securities until the short-term market is saturated.” (p. 386).

By 1932 the short-term market was saturated as Keynes had forcefully called for two years earlier. Short rates were pushed virtually to zero. But economic activity remained far below reasonable estimates of its potential while monetary policy could no longer effect further short-term interest rate reductions. The economy appeared to be in a liquidity trap, the theoretical possibility of which had also been described by Keynes in 1930 and refined in his General Theory in 1936. In his famous interpretation of the General Theory, Hicks (1937) restated Keynes's argument in terms of the zero bound constraint: “If the costs of holding money can be neglected, it will always be profitable to hold money rather than lend it out, if the rate of interest is not greater than zero. Consequently the rate of interest must always be positive. In an extreme case, the shortest short-term rate may perhaps be nearly zero. But if so, the long-term rate must lie above it, for the long rate has to allow for the risk that the short rate may rise during the currency of the loan, and it should be observed that the short rate can only rise, it cannot fall.” Thus, the monetary policy experience in the United States in the midst of the economic collapse of the 1930s illustrates the practical relevance of the zero bound constraint.

A more recent case study of the limitations the zero bound may present for monetary policy is the present situation in Japan. From a monetary policy perspective, the Japanese economy during the 1990s bears an uncomfortable resemblance to the U.S. economy during the 1930s. Growth has been undeniably anemic. The Bank of Japan eased monetary policy considerably during the first half of the decade and since 1995 has maintained the discount rate at just 50 basis points. Yet the economy experienced a disinflation, which brought inflation down from between 2 and 3 percent to below zero by 1995. Arguably, the onset of deflation coupled with the zero bound may have limited the Bank of Japan's flexibility to
engineer further reductions in real short-term interest rates had it desired to do so during 1995 and 1996. Even if such reductions were not necessary during 1995 and 1996, however, two adverse developments in 1997 powerfully demonstrated the policy limitations placed by the zero bound on current Japanese monetary policy. First, an economic crisis in Southeast Asia added to deflationary pressures. Second, a previously planned increase in consumption taxes with rather unfortunate timing created a larger than anticipated drag on domestic demand. In terms of our model, such developments would have called for monetary policy to reduce interest rates. But with short-term rates already essentially at zero as the crisis unfolded, little room for additional easing was available. The outcome in terms of our model would be reflected in longer-term interest rates. Although the short-term interest rate could not be brought further down, the deterioration of the prospects for economic recovery in the face of the new crisis increased the horizon over which the short-term rate could reasonably be expected to remain close to zero. At the present, yields on Japanese government bonds with maturities up to five years are under 1 percent. Such yields are lower than even the lowest yields on U.S. government bonds with comparable maturities during the 1930s. The zero bound may be expected to remain relevant for some time indeed.

To be sure, in addition to the zero interest rate bound, other factors contributed to the poor performance of the U.S. economy in the 1930s and the present situation in Japan and other transmission channels for monetary policy may have been effective. Obviously, our brief overview is not a comprehensive treatment of either episode. Both incidents, however, illustrate situations in which the monetary policy maker may desire to lower nominal interest rates but is not able to do so due to the zero bound.

8 Conclusion

Two main results emerge from our analysis for the United States. If the economy is subject to stochastic shocks similar in magnitude to those experienced over the 1980s and 1990s, the consequences of the zero bound are negligible for target inflation rates as low as 2 percent. However, the effects of the constraint become increasingly important for determining the
effectiveness of policy with inflation targets between 0 and 1 percent. Zero average inflation is accompanied by greater inflation variability and greater output variability. Equally important, the asymmetry of the policy ineffectiveness due to the zero bound generates a non-vertical long run Phillips curve. With a zero inflation target, output falls short of potential, on average.

Although these results are suggestive, it is important to recognize that some uncertainty remains regarding the magnitude of the distortions introduced by the zero bound when targeting zero inflation. Our quantitative results are sensitive to several features of our model. Perhaps most important, since our model was estimated for the 1980s and 1990s, a relatively calm period for the U.S. economy, the variances of demand and supply shocks are relatively small and generate little volatility in inflation and output. As a result, our baseline estimates may be underestimating the importance of the zero bound. In an environment with larger disturbances, the non-negativity constraint on nominal interest rates would bind much more frequently, and result in substantially larger distortions.

Also implicit in our analysis is the assumption that policymakers have a firm understanding of the structure of the economy, as well as timely and accurate measures of the variables required for setting the policy instrument. In practice, of course, policy decisions must be made with less information. In real-time, observation of the state of the economy can be quite noisy and, as Orphanides (1997) demonstrates in the context of the policy rules we examine here, this source of noise can lead to an unintentional but considerable increase in the variability of interest rates implied by these policy rules. Uncertainty regarding key model parameters is another pervasive problem that would contribute unwanted volatility to the structure of the economy. As Wieland (1996a, 1998) points out, the presence of such uncertainty complicates substantially the policy-maker’s problem. For any given policy rule, taking these factors into account would raise the volatility of inflation and output and result in larger distortions in the presence of the zero bound.

But it is also possible that our baseline estimates may be overestimating the importance of the zero bound. By concentrating on the interest rate channel of monetary policy
transmission as the foremost stabilizing mechanism available to the monetary authority, we may be ignoring alternative channels which might, in principle, remain effective and become relatively more important when the zero bound renders the interest rate channel ineffectual.\textsuperscript{23} Lebow (1993) investigates the possible role of the credit channel and suggests that the use of non-traditional monetary policy instruments such as opening the discount window to the non-bank private sector or open market purchases of private securities could provide additional room for monetary policy. Also, our analysis is limited to linear policy rules, subject always to the zero bound. However, policy outcomes might be improved if a non-linear policy rule designed to explicitly reduce the distortions resulting from the zero bound were followed.

In summary, our results point to a fundamental difficulty associated with the evaluation of stabilization policies with a price stability objective based on simple linear models. The presence of the zero bound constraint invalidates the underlying superneutrality properties of otherwise linear models. At low rates of inflation, the zero bound distorts the stochastic properties of the economy and induces a tradeoff between the average level of inflation and the variability of inflation and output. As a result, the optimal average rate of inflation cannot be investigated independently of the variability of output and inflation. Since our results suggest that deflation potentially engenders greater dangers than inflation, it may be optimal to pursue a price stability objective that allows for a small but positive bias in the average rate of inflation. The optimal size of such a bias, however, remains an open question. Fundamentally, determination of the average rate of inflation that would promote the optimum performance of the economy over time requires careful weighing of other potential costs and benefits of inflation in addition to the effect of the zero bound that we examine here.

\textsuperscript{23}For a very useful survey of alternative channels see the symposium on the monetary policy transmission mechanism published in the Journal of Economic Perspectives and summarized by Mishkin (1995).
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Figure 1

Dynamic Response to a Negative Fixed Investment Shock
(1% of Potential GDP)

Note: The solid line denotes the dynamic response corresponding to a zero target inflation. The dotted line denotes the dynamic response corresponding to a three percent target inflation.
Figure 2

Dynamic Response to a Negative Consumption Shock
(1% of Potential GDP)

Note: The solid line denotes the dynamic response corresponding to a zero target inflation. The dotted line denotes the dynamic response corresponding to a three percent target inflation.
Figure 3

Dynamic Response to a Negative Inventory Investment Shock
(1% of Potential GDP)

Note: The solid line denotes the dynamic response corresponding to a zero target inflation. The dotted line denotes the dynamic response corresponding to a three percent target inflation.
Dynamic Response to a Negative Government Spending Shock
(1% of Potential GDP)

Note: The solid line denotes the dynamic response corresponding to a zero target inflation. The dotted line denotes the dynamic response corresponding to a three percent target inflation.
Figure 5

Dynamic Response to Two Consecutive Favorable Price Shocks (reduction of contract wage by 25 basis points each)

Note: The solid line denotes the dynamic response corresponding to a zero target inflation. The dotted line denotes the dynamic response corresponding to a three percent target inflation.
Figure 6

Impact of Zero Bound on Nominal Interest Rate

Note: The solid line corresponds to the HM rule. The dashed line corresponds to the T rule. In the lower right panel, the lines with points denote the standard deviation of the nominal interest rate. The lines with diamonds denote the standard deviation of the change in the nominal interest rate.
Figure 7
Stationary Distributions

Output Gap

\[ m^* = 0 \quad m^* = 2 \]

Inflation Gap

\[ m^* = 0 \quad m^* = 2 \]

Note: Short-term interest rates are set according to the HM rule.
Figure 8

Distortion of Stationary Distributions

Bias of Mean

Standard Deviation

Note: The solid line corresponds to the HM rule. The dashed line corresponds to the T rule.
Figure 9

Distortion of Stationary Distributions with frequency of bind

Bias of Mean

Standard Deviation

Note: The solid line corresponds to the HM rule. The dashed line corresponds to the T rule.
Note: The solid line corresponds to the HM rule. The dashed line corresponds to the T rule.
Figure 11

Incidence and Persistence of Recession

Low Activity Frequency

Persistence of Low Activity

Note: The solid line corresponds to the HM rule. The dashed line corresponds to the T rule. Low activity is defined as the state in which the output gap is at least 1.5 standard deviations below zero in any particular quarter. Persistence indicates the frequency of remaining in the low activity state for four consecutive quarters relative to the frequency of being in this state in any one quarter.
Figure 12

Variability of Output and Inflation

Note: The solid line corresponds to the HM rule. The dashed line corresponds to the T rule. Solid squares and circles denote the unconstrained variability. Hollow squares and circles denote the variability corresponding to the inflation targets shown.
Figure 13

Variability Frontier for Henderson-Mckibbin Rule

Note: The solid line denotes the unconstrained frontier. The dashed line indicates the movement of the frontier when the inflation target is zero.
Figure 14

Distortion of Stationary Distributions with Variance of Shocks

Note: Policy is based on the HM rule with an inflation target of zero.