WORKING PAPER

The Performance of Simple Monetary Policy Rules in A Large Open Economy

by

Akila Weerapana

Wellesley College Working Paper 2000-05
May, 2000

Department of Economics
Wellesley College
Wellesley, Massachusetts 02481-8203

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Akila Weerapana
Wellesley College

JEL Codes: E52, E58, F41
Keywords: Open Economy Macroeconomics, Policy Rules *

May 13, 2000

Abstract

This paper examines the performance of simple monetary policy rules in a two country open economy model under both cooperative and non cooperative settings. In particular, it examines the extent to which simple policy rules with external variables and simple policy rules under policy coordination can help improve an adversely affected economy: one that faces greater persistence of inflation, higher volatility of shocks, and a greater vulnerability to exchange rate fluctuations.

We isolate the impact of individual asymmetries through simulations that introduce differences into two, otherwise symmetric, economies. The paper shows that the results vary according to the underlying cause of the adverse performance. Greater inflation persistence and higher volatility of shocks irreversibly worsen macroeconomic outcomes while greater external vulnerability affect the distribution of cooperative gains: the less-insulated economy benefits at the expense of the more-insulated economy. Furthermore, the reaction coefficients in the simple policy rules behave in an intuitive fashion: both policy cooperation and reaction to external shocks substantially attenuate the reaction coefficients on inflation and unemployment.

*Department of Economics, Wellesley College,106 Central Street, Wellesley, MA 02481. E-mail: AWEERAPANA@wellesley.edu, tel: (781) 283-2300, fax: (781) 283-2177. I am particularly grateful to John Taylor for his advice and encouragement.I have also benefited from comments by Lars Svensson, Chad Jones, Michael Kumhof and participants of the Stanford International Economics and Macroeconomics seminars. Financial support through a John M. Olin Dissertation Fellowship from the Stanford Institute for Economic and Policy Research (SIEPR) is gratefully acknowledged.

1 Introduction

Economists from Milton Friedman to Alan Blinder(1998) have claimed that countries that conduct monetary policy in a systematic, transparent manner will reap substantial economic benefits by reducing policy induced uncertainty and enhancing the ability of agents to make decisions over long horizons. As a result, there has been considerable recent interest among researchers and policy makers in identifying and analyzing the performance of simple monetary policy rules. Until recently, most researchers used closed economy macroeconomic models to analyze the performance of monetary policy rules.[1] The recent work of Ball(1997a,b) and Svensson(1998) has highlighted the potential importance of external shocks to the monetary policy maker's decision in *small* open economies.[2]

The traditional argument for using a closed economy model for analysis of U.S. monetary policy has been that its sheer size warrants such a simplifying assumption; essentially no other country is large enough for its macroeconomic fortunes to command serious consideration by the Federal Reserve. However, the East Asian economic crisis in 1997 has shown that events that affect an entire region can affect the U.S. economy, even if no individual country in the region is large enough to warrant consideration by U.S. policy makers. In the latter part of 1997 and 1998, the Federal Reserve repeatedly voiced concern about the impact of the East Asian crisis on corporate profits, on the demand for exports, and on inflation in the U.S. The recent creation of the European Monetary Union(EMU), an economic region rivaling the United States in size, also sets the stage for a renewed interest in how to conduct monetary policy in a large open economy setting.

This paper uses a two country rational expectations model that is an extension of the small open economy model of Svensson(1998), which in turn draws extensively on the open economy model of Obstfeld & Rogoff(1995) and the closed economy models of Rotemberg & Woodford(1997) and McCallum & Nelson(1998). These models, widely used in the recent literature on the performance of monetary policy rules, mostly emphasize microeconomic foundations.[3]

In other work[4], I have shown that the results derived from an open economy model of the type used by Svensson (1998) are dependent on the degree, and type, of asymmetries in the structure of the two economies. In particular, differences in the persistence of inflation, the magnitude of shocks to inflation and the vulnerability to exchange rate fluctuations affect the policy responses and the macroeconomic outcomes in the two countries.[5] By introducing these individual asymmetries into an otherwise symmetric two country model, I am able to examine the individual impact of

[1]Examples of this work include Rotemberg & Woodford(1997), McCallum & Nelson(1998), Chari, Kehoe & McGrattan(1998) and King & Wolman(1996).

[2]See also the recent survey paper by Clarida, Gali and Gertler(1999) which cites open economy models as "likely to provide new insights on the desirability of alternative policy rules"

[3]The model presented here, like other models, does include some partial adjustment terms which are not derived from microeconomic foundations. These terms appear mostly as a concession for matching the data and will be retained in this paper because they capture features, like the high degree of inflation persistence, that are significant to monetary policy makers.

[4]Weerapana(1999).

[5]In the analysis, the vulnerability to exchange rate fluctuations can come from two sources: a greater import share, and a higher exchange rate pass-through coefficient.

each asymmetry on macroeconomic performance and also examine the appropriate policy response. In particular examine whether the adversely affected country can improve its situation by following policy rules with external variables or by undertaking policy coordination.[6]

The results show that monetary policy makers are unable to compensate for the adverse impacts of greater inflation persistence or higher inflation shocks by adjusting reaction coefficients on domestic variables, by reacting to external variables or by policy coordination. This inability to find a monetary policy remedy suggests that policy makers can make things better only by attacking the root of the problem. To the extent that shocks to inflation are sometimes driven by inconsistent policy making and rigidities in inflation are driven by rigidities in expectations, a credible anti-inflationary stance can help alleviate the problem. However, the incomplete microfoundations of the sources of inflation persistence in commonly used macroeconomic models prevent a more complete answer to these problems.

The impact of an asymmetry in the vulnerability of the two economies to exchange rate fluctuations can be mitigated either through independent reaction to the real exchange rate or through policy cooperation. However, disaggregating the gains to cooperation show that, in this type of scenario, the less vulnerable country has little to gain from cooperation; any cooperative agreement requires it to be willing to make itself slightly worse off in exchange for improving the well being of the more vulnerable economy dramatically.

Overall, the results from the model are informative and accord with intuition. Furthermore, one can use these results to think about the monetary policy relationship between the Federal Reserve and the European Central Bank.[7] For example, the relationship between the U.S. and the EMU will be asymmetric as long as the EMU has higher shocks, a greater persistence of inflation and a greater vulnerability to exchange rate fluctuations, because of a larger import share and higher pass-through. As the EMU matures, better policy making by the ECB and a more dominant role for the Euro will help alleviate the performance asymmetries of the two economies. Until that time the external dimension of monetary policy is likely to be more important to the ECB: it should care about the value of the Euro much more than the Fed should care about the value of the dollar.

2 The Model

The model is a two country open economy extension of a simple, rational expectations model of the macroeconomy presented by Svensson(1998). The model is derived mostly from the behavior of optimizing consumers and firms. However, it does contain ad hoc assumptions about the partial adjustment of prices and output.

[6] To calculate the optimal policy decisions in a two country model under a game-theoretic setting, I draw on an older literature on monetary policy in a large open economy that precedes the recent heightened interest in Taylor Rules. This literature is exemplified by the work of Hamada(1979), Oudiz & Sachs(1985) and Currie & Levine(1985).

[7] In separate work (Weerapana(1999), I show that the EMU countries in general seem to have greater persistence of, and higher shocks to, inflation as well as be more vulnerable to exchange rate fluctuations.

2.1 Exchange Rates

The real exchange rate Q_t is defined as the relative price of Foreign goods:[8]

$$\ln Q_t = \ln P_t^f - \ln P_t^h \equiv \ln S_t + \ln P_t^{f^*} - \ln P_t^h.$$

where S is the nominal exchange rate expressed in terms of Home currency per unit of Foreign currency. An increase in S signifies a depreciation of the Home currency. Correspondingly, an increase in Q denotes a depreciation of the real exchange rate which raises the price of Foreign goods relative to Home goods. The real exchange rate can be written in terms of deviations from the mean as[9]

$$q_t = p_t^f - p_t^h \equiv s_t + p_t^{f^*} - p_t^h.$$

Here Home and Foreign prices are measured as percentage deviations from constant trends while the nominal exchange rate is measured as the percentage deviation from a trend which is the difference between the trend domestic and foreign price levels.

The dynamics of the nominal exchange rate are modeled, following the standard practice in the literature, as satisfying the Uncovered Interest Rate Parity(UIRP) Relationship, allowing for the existence of risk premia.[10] The behavior of the nominal exchange rate can then be expressed as

$$E_t \ln S_{t+1} - \ln S_t = I_t - I_t^* - \varphi_t$$

where φ_t is the risk premium and I is the nominal interest rate. This equation implies that arbitrageurs ensure that the return on domestic assets is higher than the risk adjusted return on foreign assets by the exact amount that they expect the nominal exchange rate to depreciate over time. The risk premium is assumed to follow a simple AR(1) form given by

$$\varphi_t = \gamma_\varphi \varphi_{t-1} + \epsilon_t^\varphi.$$

The UIRP equation can be rewritten for the real exchange rate (in terms of deviations from

[8] In order to keep the notation consistent and informative, superscript h is used for the Home country and superscript f for the Foreign country. An asterisk is used to denote the value of a variable in the Foreign currency. So P^f is the price of Foreign goods in Home currency units but P^{f^*} is the price of Foreign goods in Foreign currency units.

[9] Uppercase Roman letters are used to refer to levels of variables and lowercase Roman letters to refer to % deviations of variables from their means.

[10] Numerous studies have shown that uncovered interest parity in its purest form (i.e. not allowing for a risk premium to exist) fails to hold (see Hallwood & MacDonald(1994) for a summary). However, many of these studies do not apply to a risk premium adjusted form of UIRP, and it remains a more satisfactory way of describing exchange rate dynamics.

the mean) as the following:

$$q_t = E_t q_{t+1} - (i_t - E_t \pi^h_{t+1}) + (i^*_t - E_t \pi^{f^*}_{t+1}) + \varphi_t$$

2.2 Demand Side

The aggregate demand equations are derived from utility maximizing consumers who have an intertemporal CES utility function with intertemporal elasticity of substitution σ over a composite consumption good $\bar{C}$. This utility function can be written as

$$\sum_{\tau=0}^{\infty} \delta^\tau U(\bar{C}_{t+\tau}) \text{ where } U(\bar{C}) = \frac{(\bar{C})^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}.$$

This utility function yields a standard Euler equation (in terms of deviations from the mean) of the form[11]

$$\bar{c}_t = -\sigma \bar{r}_t + E_t \bar{c}_{t+1}.$$

The composite good is a Dixit-Stiglitz-Spence aggregate function of composite Home goods (C^h) and composite Foreign goods (C^f) with elasticity of substitution θ between the two types. The prices of these goods are given by P^h and P^f respectively. The demand for Home and Foreign goods by Home consumers (once again in deviations terms) is

$$\begin{aligned} c^h_t &= \bar{c}_t - \theta \left[p^h_t - \bar{p}_t \right] \\ c^f_t &= \bar{c}_t - \theta \left[p^f_t - \bar{p}_t \right]. \end{aligned}$$

The price of the composite consumption good $\bar{C}$ is an aggregate function of the prices of Home and Foreign goods given in deviations terms by

$$\bar{p}_t = (1-\omega) p^h_t + \omega p^f_t$$

where ω is the share of Foreign goods in the composite consumption good.

The definition of the real exchange rate $q_t = p^f_t - p^h_t$ and the relationship between the price of the composite good and the prices of Foreign and Home goods can be used to derive the demand by Home residents for Home and Foreign goods. These demands and the corresponding demands by Foreign residents for Home and Foreign goods are given by

$$\begin{aligned} c^h_t &= \bar{c}_t + \theta \omega q_t \\ c^f_t &= \bar{c}_t - \theta (1-\omega) q_t \\ c^{h^*}_t &= \bar{c}^*_t + \theta^* (1-\omega^*) q_t \\ c^{f^*}_t &= \bar{c}^*_t - \theta^* \omega^* q_t \end{aligned}$$

[11] $\bar{r}_t$ is the CPI real interest rate, calculated as the nominal interest rate less CPI inflation.

Using the Euler equations, the demand for Home and Foreign goods by Home and Foreign residents can be written as:

$$\begin{aligned} c_t^h &= -\sigma\rho_t + (\theta - \sigma)\omega q_t \\ c_t^f &= -\sigma\rho_t - [\theta(1-\omega) + \sigma\omega]\, q_t \\ c_t^{h^*} &= -\sigma^*\rho_t^* + [\theta^*(1-\omega^*) + \sigma^*\omega^*]\, q_t \\ c_t^{f^*} &= -\sigma^*\rho_t^* - (\theta^* - \sigma^*)\omega^* q_t \end{aligned}$$

In the above equations the term ρ is what Svensson(1998) refers to as an "infinite horizon market discount factor". This discount factor is defined as $\rho_t = \sum_{\tau=0}^{\infty} r_{t+\tau}^h$ where r_t^h is the short term domestic inflation real interest rate.[12] The discount factor ρ_t is the sum of current and future deviations of the short-term real interest rate from its mean. As Svensson points out, if the expectations hypothesis holds and the horizon is finite, then one can also think of it as a long-term real interest rate multiplied by the length of the term.[13]

These two equations reveal the role of the real exchange rate. A depreciation of the real exchange rate, q_t, increases the relative price of Foreign goods and leads to a higher demand for Home goods by consumers in both the Home and Foreign countries. This means that c^h and c^{h^*} both increase while c^f and c^{f^*} both decrease. There is also a secondary channel through which changes in the real exchange rate affect the demand for goods. Since q reverts back to the mean in the long run, a current depreciation results in expected future appreciation of the real exchange rate. This will lead to a decrease in expected CPI inflation, which all else equal, raises the expected future CPI real interest rate, $\bar{r}$. From the Euler equation one can see that this will reduce Home consumption of both Home and Foreign goods, i.e. c^h and c^f will both decline.[14] The net impact of these two changes is positive because the intertemporal elasticity of substitution σ (typically $<$ 1) is smaller than the elasticity of substitution between Home and Foreign goods, θ (typically ≥ 1).

The aggregate demands for Home and Foreign goods are $Y_t = C_t^h + C_t^{h^*}$ and $Y_t^* = C_t^f + C_t^{f^*}$. These aggregate demand functions can be expressed in deviations terms using Home and Foreign's demand functions for Home and Foreign goods as:

[12] r_t^h is defined as $r_t^h = i_t - E_t\pi_{t+1}^h$ where π_t^h is domestic goods inflation in the Home country.

[13] The corresponding definition for the discount factor in the foreign economy is $\rho_t^* = \sum_{\tau=0}^{\infty} r_{t+\tau}^{f^*}$ where $r_t^f = i_t^* - E_t\pi_{t+1}^{f^*}$ and $\pi_t^{f^*}$ is domestic goods inflation in the Foreign country.

[14] The converse is true in the Foreign economy where there is an expected future depreciation that increases expected inflation, lowers their expected real interest rate and increases their consumption of both Home and Foreign goods today, i.e. c^{h^*} and c^{f^*} both increase.

$$
\begin{aligned}
y_t &= -(1-\omega)\sigma\rho_t - \sigma^*\omega\rho_t^* + \omega[\theta^*(1-\omega^*) + \omega^*\sigma^* + (1-\omega)(\theta-\sigma)]q_t \\
y_t^* &= -(1-\omega^*)\sigma^*\rho_t^* - \sigma\omega^*\rho_t - \omega[\theta(1-\omega) + \omega\sigma + (1-\omega^*)(\theta^*-\sigma^*)]q_t
\end{aligned}
$$

An unfortunate feature of these optimization based macroeconomic models is their inability to replicate the dynamics of the data as well as a purely backward-looking model like a VAR.[15] On the other hand, as pointed out in Fuhrer(1997), a forward-looking, rational expectations model is much less vulnerable to the Lucas Critique than a purely backward-looking model. A possible compromise then is to follow Svensson(1998) and incorporate lagged adjustment terms into the model in an ad hoc manner. This can be viewed as a necessary evil required to match an optimization based model to the data.[16]

Aggregate demand is assumed to be determined one period ahead[17], and to adjust only partially from one period to the next. This leads to the following equations in which β_y denotes the degree of adjustment:

$$
\begin{aligned}
y_t &= \beta_y y_{t-1} + (1-\beta_y)\left[-(1-\omega)\sigma\rho_t - \sigma^*\omega\rho_t^*\right] + \\
&\quad (1-\beta_y)\omega\left[\theta^*(1-\omega^*) + \omega^*\sigma^* + (1-\omega)(\theta-\sigma)\right]q_t + \epsilon_t^y \\
y_t^* &= \beta_y^* y_{t-1}^* + (1-\beta_y^*)\left[-(1-\omega^*)\sigma^*\rho_t^* - \sigma\omega^*\rho_t\right] - \\
&\quad (1-\beta_y^*)\omega\left[\theta(1-\omega) + \omega\sigma + (1-\omega^*)(\theta^*-\sigma^*)\right]q_t + \epsilon_t^{y^*}
\end{aligned}
$$

These equations produce the following aggregate demand functions:

$$
y_t = \beta_y y_{t-1} - \beta_\rho E_{t-1}\rho_t - \beta_{\rho^*}E_{t-1}\rho_t^* + \beta_q E_{t-1}q_t + \epsilon_t^y \tag{1}
$$

$$
y_t^* = \beta_y^* y_{t-1}^* - \beta_\rho^* E_{t-1}\rho_t - \beta_{\rho^*}^* E_{t-1}\rho_t^* - \beta_q^* E_{t-1}q_t + \epsilon_t^{y^*} \tag{2}
$$

where the reduced form parameters in the above equations are related to the structural parameters of the model according to the following relationships:

$$
\begin{aligned}
\beta_\rho &= (1-\beta_y)(1-\omega)\sigma \qquad \beta_{\rho^*} = (1-\beta_y)\omega\sigma^* \\
\beta_\rho^* &= (1-\beta_y^*)\omega^*\sigma \qquad \beta_{\rho^*}^* = (1-\beta_y^*)(1-\omega^*)\sigma^* \\
\beta_q &= (1-\beta_y)\omega[\theta^*(1-\omega^*) + \omega^*\sigma^* + (1-\omega)(\theta-\sigma)] \\
\beta_q^* &= (1-\beta_y^*)\omega^*[\theta(1-\omega) + \omega\sigma + (1-\omega^*)(\theta^*-\sigma^*)]
\end{aligned}
$$

The reduced form of the model is written so that each of the β coefficients takes on a positive

[15] See the criticism of the forward-looking model of Rotemberg & Woodford by Fuhrer(1997). Fuhrer shows that their matching of the data is almost entirely attributable to the ad hoc "disturbance terms" that they add to their model. In the absence of these disturbances their model fits the data poorly.

[16] Even though this paper does not contain an empirical application, I feel it is still preferable to use the most practically relevant version of the model.

[17] It is assumed to take one period before changes in interest rates affect aggregate demand.

value. Therefore, the long horizon discount factors, both at home and abroad, reduce the demand for goods as the Euler equation demonstrates. Depreciation of the real exchange rate reduces the demand for foreign goods and increases the demand for home goods.[18]

2.3 Supply Side

Both Home and Foreign economies have a continuum of goods and producers that runs from zero to one.[19] Each producer, indexed by i, faces a downward sloping demand curve for their product by Home residents of $C_t^h(i) = C_t^h \left[\frac{P_t^h(i)}{P_t^h}\right]^{-\nu}$, and by Foreign residents of $C_t^{h^*}(i) = C_t^{h^*} \left[\frac{P_t^h(i)}{P_t^h}\right]^{-\nu}$. By combining the demand by Home and Foreign residents, the demand function faced by the producer of good i can be written as

$$Y_t(i) = \left[C_t^h + C_t^{h^*}\right] \left[\frac{P_t^h(i)}{P_t^h}\right]^{-\nu} \equiv Y_t \left[\frac{P_t^h(i)}{P_t^h}\right]^{-\nu}.$$

Following convention among papers in this area, prices are assumed to be set according to the staggered price-setting model of Taylor(1980). In particular, it is assumed, as in Akerlof & Yellen(1991) and Yun(1996), that firms maximize profits taking this staggered price-setting constraint into account. The model follows Calvo(1983) in that each producer is assumed to be free to change prices with probability $(1-\alpha)$ in a given period. Then a producer setting a price at period t must consider the possibility that she may never be able to change the price again. Her price setting decision is the solution to the following problem:

$$\max_{P_t^h(i)} \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \Lambda_{t+\tau} \left[P_t^h(i) y_{t+\tau}(i) - W_{t+\tau} V\left[y_{t+\tau}(i)\right]\right]$$

Here Λ_t is the marginal utility of income at time t, $V\left[y_{t+\tau}(i)\right]$ is the input requirement function (the amount of inputs needed to produce $y_{t+\tau}(i)$ units of output), and W is the per unit cost of these inputs. Embedded in this maximization problem is the assumption that the producer sets the price of her goods in domestic currency. The price of that good in the foreign market is then determined by the prevailing exchange rate. While this assumption may be realistic for U.S. producers, it is unrealistic for foreign producers; given the size of the U.S. market, foreign producers may price their goods in dollars rather than allow the dollar price to fluctuate with the exchange rate. Even though an attempt is made to control for imperfect pass-through in subsequent applications of the model, it is important to remember that the microeconomic foundations of the model do not incorporate producers who explicitly price discriminate between the domestic and foreign markets.

[18]Further details of the derivations of these aggregate demand equations are provided in the appendix.

[19]Only the derivation of the price-adjustment equation for the Home country is shown here; the derivation for the Foreign country is virtually identical.

Tedious algebra[20] results in the following first order condition in deviation terms:

$$\pi_t^h = E_t \pi_{t+1}^h + \xi \gamma q_t + \xi \tilde{v} y_t.$$

In this equation π_t^h is quarterly domestic inflation defined as $p_t^h - p_{t-1}^h$, γ is the share of Foreign inputs in the production of Home goods, $\tilde{v}$ is the elasticity of the input requirement function V(.) with respect to demand, and $\xi = \frac{(1-\alpha)(1-\alpha\delta)}{\alpha(1+\tilde{v}\nu)}$ is a function of the above described structural parameters of the model. This equation relates domestic price inflation to the demand for goods by consumers, expectations of future prices and the real exchange rate.

Domestic inflation is assumed to be predetermined two periods in advance, i.e. that there is a two period lag in the effect of policy on domestic inflation. This is a standard assumption made by both Svensson(1998) and Ball(1997), based on the empirical observation that the impact of monetary policy on domestic inflation occurs with a longer lag than does its impact on output. In this model, changes in monetary policy affect domestic inflation primarily through the aggregate demand channel. Since aggregate demand is predetermined by one period, inflation is therefore assumed to be predetermined by two periods. In addition, as in the demand equations, partial adjustment of prices is assumed for empirical estimation purposes resulting in the following price adjustment equation:

$$\pi_t^h = \alpha_\pi \pi_{t-1}^h + (1-\alpha_\pi) E_{t-2} \pi_{t+1}^h + \alpha_y E_{t-2} y_t + \alpha_q E_{t-2} q_t + \epsilon_t^\pi \qquad (3)$$

The corresponding price-adjustment equation for the Foreign country is given by

$$\pi_t^* = \alpha_\pi^* \pi_{t-1}^{f^*} + (1-\alpha_\pi^*) E_{t-2} \pi_{t+1}^{f^*} + \alpha_y^* E_{t-2} y_t^* - \alpha_q^* E_{t-2} q_t + \epsilon_t^{\pi^*} \qquad (4)$$

The relationships between the structural parameters and the reduced form parameters are as follows:

$$\alpha_y = (1-\alpha_\pi)\xi\tilde{v} \quad \alpha_y^* = (1-\alpha_\pi^*)\xi^*\tilde{v}^*$$
$$\alpha_q = (1-\alpha_\pi)\xi\gamma \quad \alpha_q^* = (1-\alpha_\pi^*)\xi^*\gamma^*$$

The general structure of the price adjustment equation follows in a straightforward fashion from the assumptions described above. Domestic inflation is affected by the previous period's inflation because of the assumed inertia of inflation. It is also affected by expectations of future inflation because of the optimization problem faced by producers who may not be able to change their own prices in the future because of the staggered price setting. As expected, higher demand for goods and services will increase the prices of goods while the expected depreciation of the real exchange rate will raise the cost of inputs and increase domestic inflation in the Home country.[21]

[20] All derivations of the supply side of the model can be found in the appendix.

[21] Note that unlike CPI inflation, the exchange rate has only a delayed impact on domestic inflation, working through the cost of inputs and the demand for goods.

This concludes the layout of the basic model. It is typical of this branch of the literature in that it has a forward-looking aggregate demand equation and an aggregate supply equation that resembles an expectations-augmented Phillips Curve. It was derived for the most part from microeconomic foundations, although there are several ad hoc assumptions about partial adjustment built in. These assumptions play an important role in any empirical matching of the model to data, thus weakening the microeconomic foundations of the model. Nevertheless, the model differs from a backward-looking model in that it does allow the expectations of agents to influence the determination of aggregate demand and the adjustment of prices.

2.4 State-Space Representation

The main equations of the model can be summarized by the following equations, rewritten in a format appropriate for a state-space representation:

$$y_{t+1} = \beta_y y_t - \beta_\rho E_t \rho_{t+1} - \beta_{\rho^*} E_t \rho^*_{t+1} + \beta_q E_t q_{t+1} + \epsilon^y_{t+1} \quad (5)$$

$$y^*_{t+1} = \beta^*_y y^*_t - \beta^*_\rho E_t \rho_{t+1} - \beta^*_{\rho^*} E_t \rho^*_{t+1} - \beta^*_q E_t q_{t+1} + \epsilon^{y^*}_{t+1} \quad (6)$$

$$E_t \pi^h_{t+3} = \frac{1}{1-\alpha_\pi}\left[E_t \pi^h_{t+2} - \alpha_\pi E_t \pi^h_{t+1} - \alpha_y E_t y_{t+2} - \alpha_q E_t q_{t+2}\right] \quad (7)$$

$$E_t \pi^{f^*}_{t+3} = \frac{1}{1-\alpha^*_\pi}\left[E_t \pi^{f^*}_{t+2} - \alpha^*_\pi E_t \pi^{f^*}_{t+1} - \alpha^*_y E_t y^*_{t+2} + \alpha^*_q E_t q_{t+2}\right] \quad (8)$$

$$E_t \rho_{t+1} = \rho_t - i_t + E_t \pi^h_{t+1} \quad (9)$$

$$E_t \rho^*_{t+1} = \rho^*_t - i^*_t + E_t \pi^{f^*}_{t+1} \quad (10)$$

$$E_t q_{t+1} = q_t + [i_t - E_t \pi^h_{t+1}] - [i^*_t - E_t \pi^{f^*}_{t+1}] - \varphi_t \quad (11)$$

$$\varphi_{t+1} = \gamma_\varphi \varphi_t + \epsilon^\varphi_{t+1}. \quad (12)$$

In the above system, equations (5) and (6) are the aggregate demand equations and equations (7) and (8) are the price adjustment equations. Equation (11) is the UIRP relationship that describes the behavior of the real exchange rate. Equation (12) describes the AR(1) representation of the risk premium. Equations (9) and (10) are recursive definitions of the infinite-horizon discount factors for the Home and Foreign economies.

The next step is to obtain a compact reduced form representation of the dynamic path of the economy. The variables for the Home economy are categorized into 2 groups, X_t, the set of predetermined state variables, and x_t, the set of forward-looking variables. The variables for the Foreign economy are categorized similarly: the two categories are labeled X^*_t and x^*_t, respectively. The forward-looking variables in this system are the real exchange rate (q_t), the infinite horizon discount rate (ρ_t) and agents' expectations at time t about inflation three periods hence ($E_t \pi^h_{t+3}$).

The actions of the policy maker at time t will affect the value of each forward-looking variable. For example, all else equal, an increase in the nominal interest rate will be associated with an immediate appreciation of the real exchange rate. In addition, the UIRP equation also implies an expected depreciation of the real exchange rate in the next period; equation (5) shows that this expected depreciation affects output in the next period and, because of the partial adjustment, also affects output two periods from now. According to equation (7) the real depreciation, working through the output gap channel, will then affect expected inflation three periods hence. Therefore, $E_t\pi^h_{t+3}$ and $E_t\pi^{f^*}_{t+3}$ are forward-looking variables as well. Finally, the discount factors, being the sum of current and expected future real interest rate deviations, are by definition directly affected by changes in the nominal interest rate today.

The categorization of the variables in the system is as follows:

$$X_t = \begin{bmatrix} y_t \\ E_t\pi^h_{t+1} \\ \varphi_t \\ q_{t-1} \\ i_{t-1} \\ \pi^h_t \end{bmatrix} \qquad X^*_t = \begin{bmatrix} y^*_t \\ E_t\pi^{f^*}_{t+1} \\ \varphi^*_t \\ q^*_{t-1} \\ i^*_{t-1} \\ \pi^{f^*}_t \end{bmatrix} \qquad x_t = \begin{bmatrix} q_t \\ \rho_t \\ E_t\pi^h_{t+3} \end{bmatrix} \qquad x^*_t = \begin{bmatrix} q^*_t \\ \rho^*_t \\ E_t\pi^{f^*}_{t+3} \end{bmatrix}.$$

By defining the following terms,

$$\hat{X}_t = \begin{bmatrix} X_t \\ X^*_t \end{bmatrix} \qquad \hat{x}_t = \begin{bmatrix} x_t \\ x^*_t \end{bmatrix} \qquad \hat{Z}_t = \begin{bmatrix} \hat{X}_t \\ \hat{x}_t \end{bmatrix},$$

a system of equations that incorporates the above equations with appropriate identities can be used to provide a notationally compact description of the dynamic path of the economy as follows:

$$\Lambda_0 \begin{bmatrix} \hat{X}_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} = \Lambda_1\hat{Z}_t + \Lambda_2\hat{U}_t + \Lambda_3 E_t\hat{U}_{t+1} + \Lambda_4\hat{W}_t + \Lambda_5\upsilon_{t+1}.$$

In this equation $\hat{U}_t = [i_t, i^*_t]'$ is the vector of choice variables and

$$\hat{W}_t = [E_t y_{t+2}, E_t y^*_{t+2}, E_t q_{t+2}, E_t q^*_{t+2}, E_t\rho_{t+2}, E_t\rho^*_{t+2}]'$$

is a vector of variables that are functions of the forward-looking variables and the state variables. The entries of the Λ matrices are functions of the reduced form coefficients of the model. This system is simplified to obtain the following representation of the dynamic path of the economy:[22]

$$\begin{bmatrix} \hat{X}_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} = A\hat{Z}_t + B\hat{U}_t + B^F E_t\hat{U}_{t+1} + \Upsilon\upsilon_{t+1}.$$

[22]Details of these derivations and the definitions of the Λ matrices and the matrices A, B, B^F and Υ are provided in the appendix.

3 Specifying the Behavior of the Monetary Policy Maker

Policy makers are assumed to be completely aware of the dynamic path of the economy when they decide upon the appropriate monetary policy for their country. Each policy maker is assumed to have a loss function of the form:[23]

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\mu_{\bar{\pi}} (\bar{\pi}_{t+\tau})^2 + \mu_y (y_{t+\tau})^2 + \mu_i (i_{t+\tau} - i_{t+\tau-1})^2 \right]$$

This is an extremely general loss function which allows for different preferences for the policy maker by varying the weights in the loss function. This loss function lends itself naturally for analyzing any economy, such as the United States in the last 15 years, where policy makers act systematically to minimize fluctuations in macroeconomic variables.

There are a few interesting features of this loss function. First, the loss function does not incorporate foreign variables; the policy maker's concern about foreign variables is for purely selfish reasons. She does not care about the welfare of the other economy or about potentially beneficial externalities of her actions, unless there are repercussions for the domestic economy. Second, as reflected by the addition of an interest rate smoothing term into the loss function, the policy maker is assumed to care about the volatility of interest rates. The smoothing term is added to capture some realism; in its absence the model will tend to generate considerable fluctuations in interest rates as the policy maker attempts to ward off the negative impacts of exchange rate fluctuations for example. The resulting scenario of low inflation and output volatility combined with a high level of interest rate volatility is unrealistic despite generating "good performance" in the context of this model. In a world with equity markets that are highly sensitive to interest rate fluctuations, excessive interest rate volatility will generate adverse macroeconomic outcomes. Given the many asset markets that are omitted from this model for analytical convenience, one way to introduce realism into the model is to directly incorporate the smoothing term into the policy maker's loss function; albeit with a lower weight than on inflation and output.[24]

The international dimension of monetary policy becomes important because of the presence of the CPI inflation term in the loss function. The definition of CPI inflation,

$$\begin{aligned} \bar{\pi}_t &= (1-\omega)\pi_t^h + \omega \pi_t^f \\ &= (1-\omega)\pi_t^h + \omega \left[\pi_t^{f^*} + (s_t - s_{t-1}) \right] \end{aligned}$$

shows how movements in the exchange rate directly affect the rate of CPI inflation. If the policy maker had the ability to directly control exchange rates then she would have a direct channel

[23]The term $\bar{\pi}_t$ refers to CPI inflation (which is distinct from domestic inflation which can be thought of as a GDP deflator for example), while the μ_t's are weights that depend on the policy maker's preferences.

[24]Woodford(1998) also addresses the inertia of central bank policies. He shows that when a central bank recognizes the forward-looking nature of the private sector's behavior, its credibility may depend on its commitment to maintaining nominal interest rate inertia. This provides some justification for incorporating a direct concern about interest rate volatility into the policy maker's loss function.

through which she could easily control inflation. According to the UIRP equation

$$q_t = E_t q_{t+1} - (i_t - E_t \pi^h_{t+1}) + (i^*_t - E_t \pi^{f^*}_{t+1}) - \varphi_t,$$

the policy maker does have some ability to influence the exchange rate by changing nominal interest rates, but this ability depends on expectations remaining static. Therefore, unlike in Ball(1998), a Monetary Conditions Index is not used as a policy tool. Instead, the nominal interest rate serves as the policy tool with exchange rate movements having a potentially important impact on the loss function of the policy maker.

3.1 Parameter Choice

The relevant equations of the model are summarized below. Values for the α and β parameters in the price adjustment and demand equations, the AR(1) coefficient in the risk premium equation and the import shares (ω and ω^*) for the two countries are chosen according to what Svensson(1998) terms "a priori not unreasonable" criteria. These parameters can then be varied in a series of simulations designed to impose different types of asymmetry on the two economies.[25]

The main equations of the model are given by the following equations:

$$\begin{aligned}
y_t &= \beta_y y_{t-1} - \beta_\rho E_{t-1}\rho_t - \beta_{\rho^*} E_{t-1}\rho^*_t + \beta_q E_{t-1} q_t + \epsilon^y_t \\
y^*_t &= \beta^*_y y^*_{t-1} - \beta^*_\rho E_{t-1}\rho_t - \beta^*_{\rho^*} E_{t-1}\rho^*_t - \beta^*_q E_{t-1} q_t + \epsilon^{y^*}_t \\
\pi^h_t &= \alpha_\pi \pi^h_{t-1} + (1-\alpha_\pi) E_{t-2}\pi^h_{t+1} + \alpha_y E_{t-2} y_t + \alpha_q E_{t-2} q_t + \epsilon^\pi_t \\
\pi^{f^*}_t &= \alpha^*_\pi \pi^{f^*}_{t-1} + (1-\alpha^*_\pi) E_{t-2}\pi^{f^*}_{t+1} + \alpha^*_y E_{t-2} y^*_t - \alpha^*_q E_{t-2} q_t + \epsilon^{\pi^*}_t \\
\varphi_t &= \gamma_\varphi \varphi_{t-1} + \epsilon^\varphi_t .
\end{aligned}$$

Values chosen for the reduced form parameters of the model are given below. Demand side parameters are selected to be consistent with an inter-temporal elasticity of substitution of 2/3 and an import share of 0.2.

Table 1: Chosen Parameter Values

Demand Side		**Supply Side**		**Other**	
Parameter	*Value*	*Parameter*	*Value*	*Parameter*	*Value*
β_y, β^*_y	0.8	α_π, α^*_π	0.6	ω, ω^*	0.20
β_ρ, $\beta^*_{\rho^*}$	0.1067	α_y, α^*_y	0.08	γ_φ	0.8
β^*_ρ, β^*_ρ	0.0267	α_q, α^*_q	0.01		
β_q, β^*_q	0.0480				

Variances of the shocks to the above system of equations are presented below; the shocks to output and the exchange rate are assumed to have a variance of 1/2 and the shocks to inflation

[25] For most of the simulations, exchange rate pass-through is assumed to be complete. The final simulation introduces incomplete pass-through combined with a greater fluctuation in exchange rates.

to have a variance of 1.[26]

$$\begin{aligned} Std.Dev(\epsilon_y), Std.Dev(\epsilon_y^*) &= 0.7071\% \text{ (Output shock)} \\ Std.Dev(\epsilon_\pi), Std.Dev(\epsilon_{y^*}) &= 1\% \text{ (Domestic Inflation shock)} \\ Std.Dev(\epsilon_\varphi) &= 0.7071\% \text{ (Exchange Rate shock)} \end{aligned}$$

4 Overview of Simulations

Three asymmetries are then introduced into the model. The first simulation exercise examines the impact of an asymmetry in the persistence of inflation: one country is forced to have relatively long lasting inflation. In addition to quantifying the negative impact on the adversely affected economy, several other issues are addressed in this exercise. Does the greater persistence cause the policy maker to increase the reaction coefficient on inflation in her policy rule? Does the policy maker in the other country gain a strategic edge by having a counterpart who is "handcuffed" by more persistent inflation?

The second exercise introduces an asymmetry in the shocks that hit the two economies: one country is hit with higher shocks to both inflation and output. The primary interest is the impact of this asymmetry on the macroeconomic performance of the affected country. Another issue explored in this exercise is the distribution of gains from cooperation. When two, otherwise symmetric, economies that have a different distribution of shocks cooperate, they are likely to transfer the impact from the adversely affected country to the other. Overall performance may improve through cooperation with the distribution of gains being in favor of the adversely affected economy.

The final set of simulations are designed to explore the impact of an asymmetry in the vulnerability of the economy to exchange rate fluctuations. Exchange rate fluctuations can then adversely affect the performance of the more open economy. The greater vulnerability takes two forms. First, an asymmetry in the openness of the two economies: one country is assumed to have a larger share of imports. Second, an asymmetry in exchange rate pass-through, combined with an increase in the volatility of the shocks to the exchange rate. The central issue addressed in this simulation is whether open economy policy rules benefit the more vulnerable economy by allowing them to react to the external, negative influences. Asymmetric gains from cooperation also become a possibility in this scenario; by internalizing the impact of its policy decisions on the exchange rate, the less open economy can become worse off in order to make the more open economy proportionally better off.

[26] In a subsequent simulation exercise, I increase the variance of the shocks to the exchange rate, but for the baseline exercise the volatility of the exchange rate is kept relatively in line with the other macroeconomic variables. Setting the variance of the exchange rate equal to the variance of inflation does not change the analysis very much.

5 Solution Techniques

5.1 Simple Policy Rules: Non-Cooperative Case

This section describes how to calculate the performance of various policy rules when the policy makers in the two countries do not cooperate. Each policy maker has an objective function defined only over her own country's variables and each controls only her own policy tools. The period loss functions for the Home and Foreign countries respectively are given by:

$$\begin{aligned} L_t &= \left[\bar{\pi}_t^2 + y_t^2 + 0.1(i_t - i_{t-1})^2\right] \\ L_t^* &= \left[\bar{\pi}_t^{*2} + y_t^{*2} + 0.1(i_t^* - i_{t-1}^*)^2\right]. \end{aligned}$$

These objective functions can be defined in terms of the goal variables Y_t and Y_t^* as $L_t = Y_t' K Y_t$ and $L_t^* = Y_t^{*\prime} K Y_t^*$ where

$$Y_t = \begin{bmatrix} \bar{\pi}_t \\ y_t \\ i_t - i_{t-1} \end{bmatrix} \quad K = \begin{bmatrix} 1\,0\;\;0 \\ 0\,1\;\;0 \\ 0\,0\,\frac{1}{10} \end{bmatrix} \quad Y_t^* = \begin{bmatrix} \bar{\pi}_t^* \\ y_t^* \\ i_t^* - i_{t-1}^* \end{bmatrix} \quad K^* = \begin{bmatrix} 1\,0\;\;0 \\ 0\,1\;\;0 \\ 0\,0\,\frac{1}{10} \end{bmatrix}.$$

The strategic interactions between the two policy makers must be factored into the calculation of the optimized simple policy rules. Each policy maker is assumed to be aware of the type of rule that the other is following. The adjustment process (possibly) leading to equilibrium is as follows. Taking the other's policy rule as given, the first policy maker searches over the coefficients of her own type of rule to find the best response. The second policy maker, anticipating this behavior, searches over the coefficients of her own rule to find the appropriate response to this new set of coefficients. This type of Cournot adjustment process is repeated until the coefficients in each feedback rule converge.

In order to search over the coefficients for the best performing rule, it is first necessary to calculate the loss function for each policy maker for a given set of feedback rules, $i_t = f\hat{X}_t$ and $i_t^* = f^*\hat{X}_t$. The policy variables at time t+1, $i_{t+1} = f\hat{X}_{t+1}$ and $i_{t+1}^* = f^*\hat{X}_{t+1}$, can then be collectively expressed as $\hat{U}_{t+1} = \hat{f}\hat{X}_{t+1} \equiv F\hat{Z}_{t+1}$ where

$$\hat{U}_{t+1} = \begin{bmatrix} i_{t+1} \\ i_{t+1}^* \end{bmatrix} \qquad \hat{f} = \begin{bmatrix} f \\ f^* \end{bmatrix}$$

and $F = \left[\hat{f}\ \hat{g}\right]$. [27]

Recall that the dynamic path of the economy is given by

$$\left[X_{t+1}', X_{t+1}^{*\prime}, E_t x_{t+1}', E_t x_{t+1}^{*\prime}\right]' \;=\; A\hat{Z}_t + B\hat{U}_t + B^F E_t \hat{U}_{t+1} + \Upsilon \upsilon_{t+1}.$$

[27] Typically, g is just a 2*6 matrix of zeros that is added to the production function. Sometimes, the policy rules considered may depend on forward-looking variables in which case the g function will be non zero.

By substituting in $\hat{U}_{t+1} = F\hat{Z}_{t+1}$ the path of the economy can be rewritten as

$$\begin{bmatrix} \hat{X}_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} = \tilde{A}^t\hat{Z}_t + \tilde{B}^t\hat{U}_t + \Upsilon\upsilon_{t+1},$$

where $\tilde{A}^t = [I - B^F F]^{-1}A$, $\tilde{B}^t = [I - B^F F]^{-1}B$.

The dynamic path of the two economies can be separated out into two matrix equations, one for the forward-looking and one for the predetermined variables, by appropriately partitioning the matrices of reduced-form parameters so that

$$\begin{aligned} \hat{X}_{t+1} &= \tilde{A}^t_{11}\hat{X}_t + \tilde{A}^t_{12}\hat{x}_t + \tilde{B}^t_1\hat{U}_t + \Upsilon\upsilon_{t+1} \\ E_t\hat{x}_{t+1} &= \tilde{A}^t_{21}\hat{X}_t + \tilde{A}^t_{22}\hat{x}_t + \tilde{B}^t_2\hat{U}_t. \end{aligned}$$

A feature of the solution to these types of models is that the forward-looking variables are endogenously determined functions of the state variables. So the forward-looking variables at time t+1 can be written as $x_{t+1} = h_{t+1}\hat{X}_{t+1}$ and $x^*_{t+1} = h^*_{t+1}\hat{X}_{t+1}$. This relationship can collectively be expressed as $\hat{x}_{t+1} = H_{t+1}\hat{X}_{t+1}$ where

$$H_{t+1} = \begin{bmatrix} h_{t+1} \\ h^*_{t+1} \end{bmatrix}.$$

By substituting $\hat{x}_{t+1} = H_{t+1}\hat{X}_{t+1}$ and $\hat{U}_t = \hat{f}\hat{X}_t$ into the dynamic path of the economy, the relationship between the forward-looking variables and the predetermined variables for the two economies can be written as:

$$\begin{aligned} \hat{x}_t &= D_t\hat{X}_t + G_t\hat{U}_t \equiv H_t\hat{X}_t \\ D_t &= \left[\tilde{A}^t_{22} - H_{t+1}\tilde{A}^t_{12}\right]^{-1}\left[H_{t+1}\tilde{A}^t_{11} - \tilde{A}^t_{21}\right] \\ G_t &= \left[\tilde{A}^t_{22} - H_{t+1}\tilde{A}^t_{12}\right]^{-1}\left[H_{t+1}\tilde{B}^t_1 - \tilde{B}^t_2\right] \\ H_t &= D_t + G_t\hat{f} \end{aligned}$$

Given an initial value for H_{t+1}, the above equations can be iterated to solve for H_t which describes the relationship between the forward-looking variables and the predetermined variables given the policy rules f and f^*.

The remaining task is to show how to calculate the value of the loss function after calculating the values of h, h^* that correspond to the given f and f^*.[28] The dynamic path of the economy can now be written as

$$\begin{aligned} \left[X'_{t+1}, X^{*'}_{t+1}, E_t x'_{t+1}, E_t x^{*'}_{t+1}\right]' &= (A + BF)\hat{Z}_t + B^F F E_t\hat{Z}_{t+1} + \Upsilon\upsilon_{t+1} \\ &= [I - B^F F]^{-1}[A + BF]\hat{Z}_t + \Upsilon\upsilon_{t+1} \equiv M\hat{Z}_t + \Upsilon\upsilon_{t+1}. \end{aligned}$$

[28]Note that H can be appropriately partitioned to obtain the h matrices so that $h_t = H^t_1$ and $h^*_t = H^t_2$

By partitioning the M matrix appropriately and using the fact that $\hat{x}_t = H\hat{X}_t$, the dynamic path of the predetermined variables can be expressed as

$$\hat{X}_{t+1} = [M_{11} + M_{12}H]\,\hat{X}_t + \Upsilon\upsilon_{t+1} = \bar{M}\hat{X}_t + \Upsilon\upsilon_{t+1}.$$

Let $\Sigma_{\upsilon\upsilon}$ denote the variance-covariance matrix of $\Upsilon\upsilon_t$, which can be calculated from the data. Then the variance-covariance matrix of the predetermined variables is given by[29]

$$\text{vec}\left(\Sigma_{\hat{X}\hat{X}}\right) = \left[I - \bar{M} \otimes \bar{M}\right]^{-1} \text{vec}\left(\Sigma_{\upsilon\upsilon}.\right)$$

The goal variables can be expressed as a function of the state variables in the system:[30] $Y_t = C_z\hat{Z}_t + C_u\hat{U}_t$ and $Y_t^* = C_z^*\hat{Z}_t + C_u^*\hat{U}_t$. These goal variables can be expressed as functions of the predetermined variables alone by using the fact that $\hat{x}_t = H\hat{X}_t$ and $\hat{U}_t = f\hat{X}_t$ and by appropriately partitioning the matrices C_z and C_z^* so that

$$\begin{aligned} Y_t &= \left[C_{z1} + C_{z2}H + C_u\hat{f}\right]\hat{X}_t \equiv C\hat{X}_t \\ Y_t^* &= \left[C_{z1}^* + C_{z2}^*H + C_u^*\hat{f}\right]\hat{X}_t \equiv C^*\hat{X}_t. \end{aligned}$$

Variances of the goal variables for the Home country and the Foreign country are given by

$$\begin{aligned} \text{vec}\left(\Sigma_{YY}\right) &= (C \otimes C)\,\text{vec}\left(\Sigma_{\hat{X}\hat{X}}\right) \\ \text{vec}\left(\Sigma_{Y^*Y^*}\right) &= (C^* \otimes C^*)\,\text{vec}\left(\Sigma_{\hat{X}\hat{X}}\right) \end{aligned}$$

The value of the loss functions can then be easily calculated by appropriately weighting the variances of the goal variables.

Calculating the performance of the case where both countries are following a simple policy rule is a straightforward application of the above technique for a given policy rule f and f^*. To find the optimized simple policy rules, each policy maker searches over the coefficients of a particular type of policy rule taking the other policy maker's actions as given; each iteration of the search requires an application of the above solution technique. Once the best response rule has been found, the entire process is repeated to find the best response of the other policy maker to this new rule. This Cournot type adjustment process, where each policy maker calculates their best response to a given policy rule, is repeated until the two policy makers converge to their best response rule.

Four different simple policy rules are analyzed for each of the simulations in this paper. The first is the well-known Taylor Rule with reaction coefficients 0.5 and 1.5 on output and inflation respectively

$$i_t = 1.5\bar{\pi}_t + 0.5y_t$$

[29]See Hamilton(1994).

[30]The matrices C_z, C_z^*, C_u and C_u^* are defined and derived in the appendix

The remaining three are optimized simple monetary policy rules, i.e. rules where the policy maker is assumed to be limited in what she can react to but allowed to pick the optimal magnitude of reaction. The first is an optimized Taylor rule which is obtained by searching over different reaction coefficients for inflation and output for the combination that minimizes the loss function. The optimal coefficients of two open economy policy rules: Open Economy Rule 1, where the policy maker also reacts to foreign output, and Open Economy Rule 2, where she instead reacts to the real exchange rate are also calculated. These optimized rules can be expressed in the following form:

$$\begin{aligned} i_t &= g_{\bar{\pi}}\bar{\pi}_t + g_y y_t + g_{y^*} y_t^* \Rightarrow \text{ OER1} \\ i_t &= g_{\bar{\pi}}\bar{\pi}_t + g_y y_t + g_q q_t \Rightarrow \text{ OER2.} \end{aligned}$$

5.2 Simple Policy Rules: Cooperative Case

The previous section described how to calculate the performance of simple policy rules in a non-cooperative setting. This section describes how this solution technique can be modified to analyze the case where the two policy makers act cooperatively to minimize a joint loss function. The period loss function of the joint policy makers is assumed to be of the form:

$$\hat{L}_t = \left[\bar{\pi}_t^2 + \bar{\pi}_t^{*2} + y_t^2 + y_t^{*2}\right] + \frac{1}{10}\left[(i_t - i_{t-1})^2 + (i_t^* - i_{t-1}^*)^2\right].$$

As before, the weights on the volatility of output and inflation in the loss function are chosen to be equal with a smaller weight placed on the interest rate smoothing term. Given the roughly equal size of the populations and the economies in the EMU 11 and the United States, an egalitarian stance about the nature of the cooperative relationship is taken in keeping the weights on each country's macroeconomic variables equal.

As before, the period loss function can be rewritten in terms of set of goal variables $\hat{Y}_t$ as $\hat{L}_t = \hat{Y}_t' K \hat{Y}_t$ where

$$\begin{aligned} \hat{Y}_t &= \left[\bar{\pi}_t, \bar{\pi}_t^*, y_t, y_t^*, (i_t - i_{t-1}), (i_t^* - i_{t-1}^*)\right]' \\ K_t &= \text{Diag}\left[1, 1, 1, 1, 1/10, 1/10\right]. \end{aligned}$$

where the goal variables can be expressed as a function of the state variables in the system: $\hat{Y}_t = C_z \hat{Z}_t + C_u \hat{U}_t$. The period loss function of the joint policy makers an also be written as

$$\hat{L}_t = \left[\hat{Z}_t' Q \hat{Z}_t + 2\hat{Z}_t' S \hat{U}_t + \hat{U}_t' R \hat{U}_t\right].$$

Recall also that the dynamic path of the economy is

$$\left[X_{t+1}', X_{t+1}^{*\prime}, E_t x_{t+1}', E_t x_{t+1}^{*\prime}\right]' = A\hat{Z}_t + B\hat{U}_t + B^F E_t \hat{U}_{t+1} + \Upsilon \upsilon_{t+1}.$$

The calculation of the performance of a simple policy rule is almost identical to the non-cooperative case. The policy makers BOTH commit to following a simple rule of the form $\hat{U}_t = \hat{f}\hat{X}_t \equiv F\hat{Z}_t$ where $F = [\hat{f} \ \oslash_{2*6}]$.[31] As before, I first show how to calculate the loss for a given simple rule, i.e. a given F. The optimized simple rule of a particular type can be found by searching over the reaction coefficients of that type of rule to find values that minimize the loss function.

The forward-looking variables at time t+1 are also endogenously determined functions of the state variables $\hat{x}_{t+1} = H_{t+1}\hat{X}_{t+1}$. By substituting $\hat{x}_{t+1} = H_{t+1}\hat{X}_{t+1}$ and $\hat{U}_{t+1} = \hat{f}\hat{X}_{t+1}$ into the dynamic path of the economy, the following relationship emerges between the forward-looking variables and the predetermined variables for the two economies:

$$\begin{aligned}
\hat{x}_t &= D_t\hat{X}_t + G_t\hat{U}_t \equiv H_t\hat{X}_t \\
D_t &= \left[\tilde{A}^t_{22} - H_{t+1}\tilde{A}^t_{12}\right]^{-1}\left[H_{t+1}\tilde{A}^t_{11} - \tilde{A}^t_{21}\right] \\
G_t &= \left[\tilde{A}^t_{22} - H_{t+1}\tilde{A}^t_{12}\right]^{-1}\left[H_{t+1}\tilde{B}^t_{1} - \tilde{B}^t_{2}\right] \\
H_t &= D_t + G_t\hat{f}_t
\end{aligned}$$

Given an initial value for H_{t+1}, the above equations are iterated on to solve for H_t, which describes the relationship between the forward-looking variables and the predetermined variables given the policy rule F_t.

Once H_t is calculated, calculating the value of the loss function is relatively straightforward. Note that the following method is also used to find the value of the loss function in the discretionary case. For a given F_t, the dynamic path of the economy is given by

$$\begin{aligned}
\begin{bmatrix} \hat{X}_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} &= (A + BF)\hat{Z}_t + B^F F E_t\hat{Z}_{t+1} + \Upsilon\upsilon_{t+1} \\
&= [I - B^F F]^{-1}[A + BF]Z_t + \Upsilon\upsilon_{t+1} \equiv MZ_t + \Upsilon\upsilon_{t+1}.
\end{aligned}$$

By partitioning the M matrix appropriately and using the fact that $\hat{x}_t = H\hat{X}_t$, the dynamic path of the predetermined variables can be written as

$$\hat{X}_{t+1} = [M_{11} + M_{12}H]\,\hat{X}_t + \Upsilon\upsilon_{t+1} = \bar{M}\hat{X}_t + \Upsilon\upsilon_{t+1}.$$

Let $\Sigma_{\upsilon\upsilon}$ denote the variance-covariance matrix of $\Upsilon\upsilon_t$; the values of this variance-covariance matrix can be calculated from the data. Then the variance-covariance matrix of the predetermined variables is given by

$$\text{vec}\left(\Sigma_{\hat{X}\hat{X}}\right) = \left[I - \bar{M} \otimes \bar{M}\right]^{-1}\text{vec}\left(\Sigma_{\upsilon\upsilon}\right).$$

Similarly, the goal variables can be expressed as a function of the predetermined variables alone:

$$\hat{Y}_t = [C_z + C_U F]\,\hat{Z}_t$$

[31] Note that some of the policy rules may also depend on forward-looking variables so that F=[f g] where g is a matrix that may not be all zeros.

$$\begin{aligned}\hat{Y}_t &= C\hat{Z}_t \equiv C_1\hat{X}_t + C_2\hat{x}_t \\ &= [C_1 + C_2H]\,\hat{X}_t \equiv \bar{C}X_t.\end{aligned}$$

The variance of the goal variables is given by

$$\text{vec}\left(\Sigma_{\hat{Y}\hat{Y}}\right) = \left[\bar{C} \otimes \bar{C}\right]\ \text{vec}\left(\Sigma_{\upsilon\upsilon}\right).$$

Given the variances of the goal variables, the value of the loss function can be calculated by weighting these variances appropriately.

6 Simulation Results

6.1 Simulation I: Persistence Asymmetry

The first set of simulations introduces an asymmetry in the persistence of inflation between the two countries: the persistence of inflation in the Foreign economy, α_p^*, is increased from 0.6 to 0.8. A priori, we expect that this asymmetry will worsen the performance of the country faced with a higher persistence of inflation. In addition to the negative impact on macroeconomic performance, there may be reasons to believe that the greater persistence also requires a high reaction coefficient on inflation in the affected country's policy rule. The intuition is that an increase in inflation, because it is long lasting, should be treated more seriously than a corresponding increase in a variable that may dissipate faster. There is also reason to believe that the policy maker in the Home country gains a strategic edge in the non-cooperative setting by having a counterpart who is "handcuffed" by more persistent inflation.

6.1.1 Non-Cooperative Case

The table below shows the results for the simulation, relative to a baseline case, in which the two economies are symmetric. The numbers fail to back up the hypothesis that the greater persistence should bring about a stronger reaction to inflation by the Foreign policy maker. In fact, the results seems to indicate the opposite: the reaction coefficient on inflation for Foreign decreases while the coefficient for Home increases.[32]

The next table, which outlines the macroeconomic performance of the two economies, shows that this relative similarity in reaction functions hides a substantial asymmetry in macroeconomic performance. The increase in asymmetry does in fact substantially worsen the macroeconomic performance of the Foreign economy, while the performance of the Home economy is barely changed from the baseline. Furthermore, the Foreign country can not reverse this deterioration in performance by following open economy policy rules.

[32] To facilitate the analysis, I have reported the reaction functions using CPI inflation instead of the combination of reactions to domestic inflation and exchange rate fluctuations implicit in a CPI reaction coefficient.

Table 2: Home Reaction Coefficients: Simulation I, Non-Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*
y_t	0.925	0.879	0.842	0.823	0.798	0.758
$\bar{\pi}_t$	2.021	2.104	1.988	1.989	2.061	2.017
y_t^*	0	0	-0.225	-0.139	0	0
q_t	0	0	0	0	0.183	0.140

Table 3: Foreign Reaction Coefficients: Simulation I, Non-Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*
y_t^*	0.925	0.890	0.842	0.866	0.798	0.865
$\bar{\pi}_t^*$	2.021	1.941	1.988	1.901	2.061	2.137
y_t	0	0	-0.225	-0.290	0	0
q_t	0	0	0	0	-0.183	-0.200

One interpretation of the relative lack of movement in the reaction coefficients, coupled with the dramatic movement in the macroeconomic performances of the two economies, is that countries have to resign themselves to the adverse outcomes. In this model, there does not seem to be much that policy makers can do in terms of varying the reaction coefficients of their policy rule to reverse the situation. The only way of halting the adverse impact on the Foreign country is to change the source of the problem itself, namely reduce the persistence of inflation. This exposes a weakness of this type of hybrid model where the microeconomic foundations of the partial adjustment process are left unspecified. There aren't many lessons to draw from this type of model on how to deal with a scenario such as a greater persistence of inflation in one country.

Table 4: Loss Functions: Simulation I, Non Cooperative

Policy	*SD($\bar{\pi}$)*	*SD($\bar{\pi}^*$)*	*SD(y)*	*SD(y^*)*	*SD(Δi)*	*SD(Δi^*)*	*Loss:Home*	*Foreign*
Taylor	1.472	2.535	1.165	2.213	1.502	1.453	**3.751**	**11.536**
OTR	1.358	2.459	1.205	2.271	1.945	1.854	**3.673**	**11.548**
OER 1	1.386	2.457	1.179	2.270	1.872	1.752	**3.662**	**11.496**
OER 2	1.320	2.397	1.164	1.508	2.262	1.819	**3.441**	**11.195**

6.1.2 Cooperative Case

The final question that needs to be explored here is whether the higher persistence of inflation "handcuffs" the Foreign policy maker, i.e. is the adverse impact on the Foreign economy worse in the non-cooperative case. A priori there are not many reasons to believe that cooperation can ameliorate the adverse situation brought about by the increase in persistence.

The reaction coefficients for the cooperative case are presented in the following tables.

Table 5: Home Reaction Coefficients: Simulation I, Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*
y_t	0.664	0.676	0.631	0.669	0.560	0.601
$\bar{\pi}_t$	1.730	1.801	1.729	1.758	1.770	1.774
y_t^*	0	0	-0.225	-0.105	0	0
q_t	0	0	0	0	0.183	0.129

Table 6: Foreign Reaction Coefficients: Simulation I, Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*	*Baseline*	*Sim. 1*
y_t^*	0.664	0.665	0.631	0.651	0.560	0.629
$\bar{\pi}_t^*$	1.730	1.701	1.729	1.696	1.770	1.850
y_t	0	0	-0.225	-0.054	0	0
q_t	0	0	0	0	-0.183	-0.174

Given the similarities between the reaction functions in the cooperative and non-cooperative environments it is not surprising to find that the performance in the cooperative case is equally asymmetric. A policy maker who is faced with a more persistent inflation series cannot gain any solace from policy coordination; she is as badly off as she was under non-cooperation. The gains from cooperation, as the table below shows, are minimal.

Table 7: The Distribution of Gains From Cooperation : Simulation I

	Cooperative		Non-Cooperative	
Policy Rule	*Loss Home*	*Loss Foreign*	*Loss Home*	*Loss Foreign*
Optimal TR	3.639	11.495	3.673	11.548
Open Econ. R1	3.586	11.485	3.661	11.496
Open Econ. R2	3.410	11.171	3.442	11.195

In summary, the results from this exercise show that a small increase in the persistence of inflation can have a significantly negative impact on the macroecomic performance of the country. Furthermore, this adverse impact is immune to different choices of simple policy rules; monetary policy makers can improve matters only by attacking the root of the problem. One solution may be to increase the credibility of the anti inflationary stance of the central bank. This will help reduce the persistence of expected inflation, for example. Finally, results show that the country with the higher persistence of inflation is not adversely affected by the strategic interactions with a counterpart facing a more favorable economic climate. The additional negative impacts of the strategic interaction, or conversely, the gains from cooperation, are fairly small and evenly distributed.

6.2 Simulation II: Asymmetry in the Volatility of Shocks

The second set of simulations introduces an asymmetry in the volatility of the shocks to inflation and output that hit the two economies: the variances of shocks to output and inflation in the foreign economy are doubled.[33] The primary interest is in examining the adverse impact this change has on macroeconomic performance in the affected country, especially in comparison to the drastic consequences exhibited by the first simulation exercise.

The second issue analyzed in this section is the distribution of gains from cooperation. When two mostly symmetric economies that have a different distribution of shocks act jointly, they may try to transfer the impact from the adversely affected country to the other. The overall performance may improve under cooperation with the underlying distribution of gains likely to be in favor of the adversely affected economy.[34]

6.2.1 Non-Cooperative Case

The following table outlines the macroeconomic performance of the two economies: it shows that the doubling of the volatility of shocks, as expected, results in almost a doubling of the loss function in the Foreign economy. However, the adverse impact on the Foreign economy is not as significant as caused by the (relatively smaller) change in the persistence term. One interesting result is that the simple Taylor rule actually yields better performance than both the optimized Taylor Rule and Open Economy Rule 1! It seems oxymoronic for the "optimized" Taylor Rule to perform worse than a simple Taylor Rule. However, this Prisoner's Dilemma can emerge because the case where both policy makers follow simple Taylor Rules is not a valid equilibrium in this case.[35] Nevertheless, the adverse impact on the Home policy maker remains somewhat of a puzzle why the Home policy maker; one would expect the adverse impact to mainly hit the economy that is faced with the more volatile shocks.

Table 8: Loss Functions: Simulation II, Non Cooperative

Policy	*SD*($\bar{\pi}$)	*SD*($\bar{\pi}^*$)	*SD*(y)	*SD*(y^*)	*SD*(Δi)	*SD*(Δi^*)	*Loss:Home*	*Foreign*
Taylor	1.481	1.941	1.164	1.604	1.642	1.905	**3.816**	**6.706**
Opt. TR	1.433	1.873	1.163	1.632	2.180	2.351	**3.880**	**6.725**
OE Rule 1	1.441	1.889	1.159	1.626	2.073	2.298	**3.849**	**6.740**
OE Rule 2	1.352	1.839	1.143	1.607	2.079	2.406	**3.566**	**6.540**

[33] The variance of output is increased from 0.5 to 1 and the variance of inflation from 1 to 2.

[34] Given the lack of interesting hypotheses about the impact of this change on the reaction functions, details of the reaction functions are not presented here. It suffices to say that the reaction functions do not dramatically differ from the baseline.

[35] Consider the case where both policy makers start with a simple Taylor Rule. One of the policy makers would find it to be in their best interest to deviate from the 0.5 and 1.5 coefficients on output and inflation. In response to this deviation, her counterpart searches for her own best response which then leads the first policy maker to search for a new best response and so on. Soon both policy makers have deviated from the Taylor Rule and adversely affected themselves creating a Prisoner's Dilemma.

6.2.2 Cooperative Case

The remaining issue is whether cooperation can transfer some of the negative impact from the adversely affected country to the other. A priori, the Prisoner's Dilemma scenario outlined in the previous analysis indicates potential benefits from cooperation; the two policy makers could choose to each follow a Taylor Rule. Given the superior performance when both policy makers follow a Taylor Rule, we would also expect the reaction coefficients in the cooperative case to be close to the Taylor Rule coefficients.

Table 9: Home Reaction Coefficients: Simulation II, Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Baseline*	*Sim. II*	*Baseline*	*Sim. II*	*Baseline*	*Sim. II*
y_t	0.664	0.676	0.631	0.643	0.560	0.501
$\bar{\pi}_t$	1.730	1.726	1.729	1.723	1.770	1.690
y_t^*	0	0	-0.225	-0.061	0	0
q_t	0	0	0	0	0.183	0.126

Table 10: Foreign Reaction Coefficients: Simulation II, Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Baseline*	*Sim. II*	*Baseline*	*Sim. II*	*Baseline*	*Sim. II*
y_t^*	0.664	0.541	0.631	0.518	0.560	0.581
$\bar{\pi}_t^*$	1.730	1.611	1.729	1.610	1.770	1.783
y_t	0	0	-0.225	-0.054	0	0
q_t	0	0	0	0	-0.183	-0.224

In the cooperative case, the optimized policy rules must perform at least as well as the simple Taylor Rule. However, as the table below shows, the gains are small; the performances of the optimized Taylor Rule and Open Economy Rule 1 are only marginally better than the Taylor Rule.

Table 11: Loss Functions: Simulation II, Cooperative

Policy	*SD*($\bar{\pi}$)	*SD*($\bar{\pi}^*$)	*SD*(y)	*SD*(y^*)	*SD*(Δi_t)	*SD*(Δi_t^*)	*Total Loss*
Taylor Rule	1.481	1.942	1.164	1.604	1.642	1.905	**10.521**
Optimized TR	1.437	1.903	1.165	1.629	1.825	2.034	**10.442**
Open Econ. R 1	1.436	1.903	1.167	1.630	1.807	2.025	**10.435**
Open Econ. R 2	1.373	1.860	1.147	1.612	1.738	2.070	**9.988**

Dissecting the gains from cooperation reveals that the gains do NOT accrue disproportionately to the Foreign economy. There is a definite lack of support in the model for the hypothesis that, under policy cooperation, the country facing the higher shocks is able to pass a significant portion of the burden on to its more fortunate counterpart and increase total welfare.

Table 12: The Distribution of Gains From Cooperation : Simulation II

	Cooperative		Non-Cooperative	
Policy Rule	*Loss Home*	*Loss Foreign*	*Loss Home*	*Loss Foreign*
Optimal TR	3.756	6.686	3.880	6.725
Open Econ. R1	3.747	6.688	3.849	6.740
Open Econ. R2	3.502	6.485	3.567	6.540

In summary, the results from the second set of simulations show that the higher volatility of shocks does have the expected negative impact on macroeconomic performance. The higher volatility of shocks also creates an interesting Prisoner's Dilemma whereby two policy makers who pick the best response coefficients to inflation and output end up at a worse outcome than could have been achieved by a simple Taylor Rule. Finally, the results show that the gains to cooperation continue to be uniform, as in Simulation I. There is little support for the idea that the adversely affected country can disproportionately help itself by passing on some of the burden to its more fortunate counterpart.

6.3 Simulations IIIa and IIIb: Asymmetry in Openness

The first two simulations had relatively few interesting explanations for differences in reaction functions, in the gains from reacting to external variables and in the gains from cooperation across countries. In both cases the only significant deviation from the baseline case was the worse performance of the adversely affected economy. The third set of simulations allows for a more significant role for open economy policy rules and for policy cooperation by introducing an asymmetry in the openness of the two economies.

In Simulation IIIa, the import share of the Foreign country is increased from 0.2 to 0.4 thereby increasing the impact of exchange rate fluctuations on the economy. In order to make the open economy dimension more important, we also increase the variance of the shocks to the exchange rate by a factor of 10, from 0.5 to 5.

Simulation IIIb also looks at the impact of high exchange rate volatility on an economy that is more vulnerable to fluctuations; however, vulnerability is measured along a different dimension, namely the impact of exchange rate fluctuations on the CPI. The variance of the shocks to the exchange rate is increased to 5 and we make the Home economy have incomplete exchange rate pass through: the Foreign country then is more vulnerable to exchange rate fluctuations.[36]

The central question of interest in Simulation III is whether the open economy policy rules have a greater beneficial impact for the more open economy: policy makers can benefit from reacting to external forces that have a more significant impact on the economy. A priori, the gains from the

[36]Details are given later in the paper. Note that the import shares are set to be equal in the two countries in Simulation IIIb.

open economy rules should be more significant in Simulation III than in the first two simulations. The second issue of interest is the possibility of asymmetric gains from cooperation. When a country is affected more by exchange rate fluctuations, there is a possibility that it can be substantially better off under policy cooperation. This can occur if the less open economy, operating under a joint loss function, internalizes the impact of its policy decisions on the exchange rate; it becomes more cautious in raising rates. For example, an interest rate increase that reduces CPI inflation also causes an exchange rate appreciation and therefore increases inflation in the Foreign country: the overall impact may be to increase the joint loss function. Such interest rate increases will be avoided under policy cooperation.

6.3.1 Non-Cooperative Case: Simulation IIIa

The following tables confirm that greater openness causes interesting deviations of the reaction functions from the baseline case.

Table 13: Home Reaction Coefficients, Simulations IIIa: Non-Cooperative

Variable	*Opt. Taylor Rule*		*OE Rule 1*		*OE Rule 2*	
	Base.	*Sim 3a*	*Base.*	*Sim 3a*	*Base.*	*Sim 3a*
y_t	0.925	0.898	0.842	0.850	0.798	0.597
$\bar{\pi}_t$	2.021	2.140	1.988	2.178	2.061	1.894
y_t^*	0	0	-0.225	-0.258	0	0
q_t	0	0	0	0	0.183	0.222

Table 14: Foreign Reaction Coefficients, Simulations IIIa: Non-Cooperative

Variable	*Opt. Taylor Rule*		*OE Rule 1*		*OE Rule 2*	
	Base.	*Sim 3a*	*Base.*	*Sim 3a*	*Base.*	*Sim 3a*
y_t^*	0.925	3.693	0.842	3.387	0.798	1.194
$\bar{\pi}_t^*$	2.021	5.489	1.988	5.169	2.061	2.780
y_t	0	0	-0.225	-0.776	0	0
q_t	0	0	0	0	-0.183	-0.552

The most striking features are the extremely high reaction coefficients on inflation and output in the optimized Taylor Rule for the Foreign economy. The greater openness of the Foreign economy leaves it more vulnerable: with complete pass-through, exchange rate fluctuations cause fluctuations in the CPI. To keep inflation under control, policy makers have to react extremely strongly and raise rates; the impact, and therefore the policy maker's reaction, is more pronounced when the volatility of the exchange rate increases.

Furthermore, reaction coefficients in the Optimized Taylor Rule and in Open Economy Rule 1 are much larger than in Open Economy Rule 2. Policy makers desire reaction to the variable that is adversely affecting them, the real exchange rate. When their hands are tied on that dimension they compensate by reacting more strongly to domestic variables. All these effects, are much more pronounced for the Foreign, the adversely affected economy.

The last issue to be addressed is the macroeconomic performance of the two economies. Specifically, examining whether the open economy policy rules, or even the optimized Taylor Rule are able to improve the performance of the Foreign economy following the initial negative impact of the shocks. The performance of the economies is presented in the following tables.

Table 15: Loss Functions: Simulation IIIa, Non-Cooperative

Policy	$SD(\bar{\pi})$	$SD(\bar{\pi}^*)$	$SD(y)$	$SD(y^*)$	$SD(\Delta i)$	$SD(\Delta i^*)$	*Home*	*Foreign*
Opt. Taylor	1.487	1.474	1.207	1.457	2.542	3.827	**4.311**	**5.760**
OE Rule 1	1.493	1.526	1.194	1.434	2.469	3.803	**4.264**	**5.833**
OE Rule 2	1.388	1.345	1.153	1.209	2.044	2.820	**3.671**	**4.067**

Several interesting results can be gleaned from this table. First, the Foreign economy performs worse than the Home economy under any policy rule that involves only domestic variables. Second, the Foreign economy is able to improve its performance substantially by following an open economy policy rule that includes the real exchange rate: it is able to directly react to the variable that affects it most. Third, the optimized policy rules perform substantially better than the simple Taylor Rule: the high reaction coefficients on inflation and output in the Foreign economy's policy rule enable it to improve its performance.

6.3.2 Cooperative Case: Simulation IIIa

What about the gains from cooperation? The non-cooperative results indicate that the economy adversely affected by exchange rate fluctuations can compensate by reacting to the real exchange rate or, to a lesser extent, by reacting more strongly to changes in domestic variables. In the cooperative case, the adversely affected economy can compensate through cooperation: Foreign can rely on Home to desist actions that cause currency appreciation and put inflationary pressure on Foreign's economy. A priori, the gains from cooperation are likely to accrue to the Foreign economy; perhaps even to the extent of making the Home economy worse off. The results for the cooperative case are summarized in the tables below.

Table 16: Loss Functions: Simulation IIIa, Cooperative

Policy	$SD(\bar{\pi})$	$SD(\bar{\pi}^*)$	$SD(y)$	$SD(y^*)$	$SD(\Delta i)$	$SD(\Delta i^*)$	*Total Loss*
Opt. TR	1.494	1.478	1.219	1.469	2.413	3.753	**10.048**
OE Rule 1	1.502	1.453	1.242	1.452	2.446	2.669	**9.971**
OE Rule 2	1.410	1.361	1.159	1.192	1.919	2.647	**7.673**

Dissecting the gains from cooperation for Simulation IIIa confirms the hypothesis that Foreign has more to gain from cooperation. Home makes itself somewhat worse off, perhaps by being more vigilant about the impact of their policies on the real exchange rate than they would have been in the non-cooperative case. This internalization of the adverse impact on the Foreign country helps to make the Foreign economy proportionally better off, reducing the aggregate loss function. The overall gains remain small, regardless of the type of rule that is followed.

Table 17: The Distribution of Gains From Cooperation : Simulation IIIa

	Cooperative		Non-Cooperative	
Policy Rule	*Loss: Home*	*Loss: Foreign*	*Loss: Home*	*Loss: Foreign*
Optimal TR	4.298	5.750	4.311	5.760
Open Econ. R1	4.398	5.573	4.264	5.833
Open Econ. R2	3.700	3.973	3.671	4.067

In summary, three results emerge from this simulation. First, policy makers in the more open economy try to compensate for the adverse impact of exchange rate fluctuations by dramatically increasing the reaction coefficients on domestic variables in their policy rule. Second, policy makers in the adversely affected economy reap substantial benefits from using open economy policy rules, these benefits are more substantial than what can be achieved through stronger reaction to domestic variables. Finally, the gains from cooperation are shown to be asymmetric: by internalizing the impact of its policy decisions on the exchange rate, Home policy makers make themselves worse off and make the Foreign economy proportionally better off. The net impact is to improve the overall performance of the two economies, albeit marginally. These results provide the basis for the next set of simulations which combine high exchange rate fluctuations with incomplete pass-through.

6.4 Incomplete Pass-Through

Given the problems associated with estimating the UIRP equation, and the volatility of exchange rates in general, the magnitude of the "shocks" to the exchange rate are likely to be substantial. One modeling issue is whether these fluctuations in exchange rates will automatically translate into fluctuations in import prices (and by extension in CPI inflation). If so, open economy policy rules, in which the policy maker reacts to the real exchange rate, are likely to yield substantial improvement in performance over closed economy policy rules. In the United States, in particular, empirical evidence suggests that pass-through is less than complete[37]: the U.S. CPI is much more stable in the face of exchange rate fluctuations than the CPI in the model would be under complete pass-through. While pass-through is likely to be higher in Europe, the extent of pass-through may diminish as more goods are priced in terms of the Euro. A correction to reduce the impact of exchange rate fluctuations on CPI inflation seems necessary before proceeding with some applications of the model.[38]

[37] See Knetter(1993).

[38] The first best solution is to build pricing to market into the microfoundations of the model.

This simulation tries to incorporate a workable treatment of imperfect exchange rate pass-through. This allows for better analysis of the policy decisions of a central bank, like the U.S. Federal Reserve, which sets monetary policy for a country that is relatively isolated from exchange rate fluctuations because of the special role of the dollar in international financial markets. Under incomplete pass-through not all of the shocks to exchange rates cause fluctuations in the domestic prices of imports. The definition of the CPI is changed so that

$$\begin{aligned}\bar{\pi}_t &= (1-\omega)\pi_t^h + \omega\pi_t^f \\ &= (1-\omega)\pi_t^h + \omega\left[\pi_t^{f^*} + \psi\Delta s_t\right] \\ &= \pi_t^h[1-\omega(1-\psi)] + \pi_t^{f^*}[\omega(1-\psi)] + \omega\psi[q_t - q_{t-1}].\end{aligned}$$

Similarly the CPI in the foreign country is being re-expressed as

$$\begin{aligned}\bar{\pi}_t^* &= (1-\omega^*)\pi_t^{f^*} + \omega^*\pi_t^{h^*} \\ &= (1-\omega^*)\pi_t^{f^*} + \omega^*\left[\pi_t^h - \psi^*\Delta s_t\right] \\ &= \pi_t^{f^*}[1-\omega^*(1-\psi^*)] + \pi_t^h[\omega^*(1-\psi^*)] - \omega^*\psi^*[q_t - q_{t-1}].\end{aligned}$$

Note that in the case of complete exchange rate pass-through, $\psi = 1$, the CPI equations collapse down to the standard cases

$$\begin{aligned}\bar{\pi}_t &= \pi_t^h + \omega(q_t - q_{t-1}) \\ \bar{\pi}_t^* &= \pi_t^{f^*} - \omega^*(q_t - q_{t-1}).\end{aligned}$$

Values of ψ, which measure the impact of a change in the exchange rate on the domestic currency price of imports, vary widely across countries. These values also vary depending on the definition of a period: long horizon values (longer than a year) are close to one while short horizon values (a quarter or two) are near zero.[39] Note that this adjustment of the CPI to account for incomplete pass-through reduces the impact of exchange rates on CPI inflation and therefore makes it less obvious that a policy rule that involves reacting to exchange rate fluctuations would necessarily yield better performance.

6.5 Simulation IIIb: Asymmetry in Pass-through Coefficients

This simulation exercise also looks at the impact of high exchange rate volatility on an economy that is more vulnerable to fluctuations; however, vulnerability is measured along a different dimension, namely the impact of exchange rate fluctuations on the CPI. The variance of the shocks to the exchange rate is increased to 5 and the pass-through coefficient in the Home country is set to 0.8

[39] I am grateful to Phong Trinh for providing me with a very useful summary of the empirical estimates of pass-through coefficients found in the literature.

instead of 1; the Foreign country continues to be more vulnerable to exchange rate fluctuations.[40]

One interesting issue is the improvement in macroeconomic performance achieved by the more insulated (i.e. lower pass-through) economy in the face of exchange rate fluctuations. Some of the issues raised in Simulation IIIa such as the distribution of the gains from cooperation and the relative importance of open economy policy rules are also addressed here. A priori, the open economy policy rules should be more important to the less insulated economy.

6.5.1 Non-Cooperative Case: Simulation IIIb

The following table outlines the reaction coefficients of the optimized policy rules for the two economies. The reaction to inflation and output in the optimized policy rules are not as strong as in the previous simulation; nevertheless, reaction coefficients on output are higher than in the baseline case. Given that fluctuations in the exchange rate have a larger impact on the Foreign economy, it makes sense for Foreign to have higher reaction coefficients on external variables as well as have higher reaction coefficients on domestic variables when prohibited from reacting to external variables. As in Simulation IIIa, the reaction coefficients on domestic variables fall dramatically when the policy maker is allowed to react to exchange rate fluctuations: the higher reaction coefficients are driven by the need to react to the detrimental effects of exchange rate fluctuations.

Table 18: Reaction Coefficients: Simulation IIIb, Non-Cooperative

	Opt. Taylor		*OE Rule 1*		*OE Rule 2*	
Variable	*Home*	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*
y_t	1.332	1.559	1.242	1.428	0.632	0.654
$\bar{\pi}_t$	2.486	2.762	2.495	2.763	1.862	1.972
y_t^*	0	0	-0.260	-0.437	0	0
q_t	0	0	0	0	0.146	-0.239

The next table outlines the macroeconomic performance of the two economies. As expected the Foreign economy performs worse but is able to improve its performance dramatically through independent reaction to fluctuations in the real exchange rate. Such dramatic gains from independent reaction to the exchange rate are not surprising because, pass-through for the Foreign economy is assumed to be complete. The bottom line on the results for Simulation IIIb is that exchange rate fluctuations have less adverse an impact on the economy than in Simulation IIIa where an asymmetry in the import shares was introduced. However, monetary policy makers in the more open economy continue to reap greater benefits from having an external dimension to their policy decisions.

[40]Note however that the import shares are now equal in the two countries.

Table 19: Loss Functions: Simulation IIIb, Non-Cooperative

Policy	*SD($\bar{\pi}$)*	*SD($\bar{\pi}^*$)*	*SD(y)*	*SD(y^*)*	*SD(Δi)*	*SD(Δi^*)*	*Loss:Home*	*Foreign*
Taylor	2.023	2.146	1.389	1.410	2.275	2.679	**6.566**	**7.310**
OTR	1.537	1.638	1.231	1.255	2.999	3.448	**4.777**	**5.448**
OER 1	1.548	1.650	1.227	1.256	2.941	3.360	**4.768**	**5.428**
OER 2	1.367	1.420	1.183	1.202	2.197	2.408	**3.754**	**4.041**

6.5.2 Cooperative Case: Simulation IIIb

The qualitative similarity in results between Simulations IIIa and IIIb for the non-cooperative case indicate that gains from policy cooperation would continue to accrue mostly to the Foreign economy. The results given in the table below confirm this suspicion although the overall benefits from cooperation are almost nonexistent in this case.

Table 20: Distribution of Gains From Cooperation: Simulation IIIb

	Cooperative		Non-Cooperative	
Policy Rule	*Loss: Home*	*Loss: Foreign*	*Loss: Home*	*Loss: Foreign*
Optimal TR	4.776	5.447	4.777	5.448
Open Econ. R1	4.775	5.406	4.768	5.428
Open Econ. R2	3.757	4.005	3.754	4.041

In summary, the impact of an asymmetry in pass-through coefficients seems to be substantially less than the impact of an asymmetry in import shares, even in the face of high exchange rate volatility. Once again, policy makers in the more vulnerable economy suffer adverse outcomes; they can help themselves by increasing their reaction to domestic macroeconomic variables or more substantially by reacting to the real exchange rate. Overall gains to cooperation are infinitesimal; the slight improvement that the Foreign country is able to achieve is almost completely offset by the slight worsening of the Home policy maker's loss function.

7 Conclusion

This paper performs a set of simulation exercises to examine the impact of asymmetries on the choice of, and performance of, simple monetary policy rules in a two country rational expectations model.

The first simulation shows that the symmetry of macroeconomic performance can be distorted by introducing a moderate degree of asymmetry in the persistence of inflation. The second shows that a lesser distortion occurs with the introduction of a moderate degree of asymmetry in the shocks that hit the economies. Furthermore, monetary policy makers are unable to compensate for the adverse impacts of either of these changes by adjusting reaction coefficients. This inability to find a monetary policy remedy suggests that policy makers can make things better only by attacking the root of the problem, namely the greater persistence of inflation or the higher shocks to inflation. To the extent that shocks to inflation are sometimes driven by inconsistent policy making and rigidities in inflation are driven by rigidities in expectations, a credible anti-inflationary stance can help alleviate the problem. However, the incomplete microfoundations of the model prevent a more complete answer to the problems caused by these asymmetries: this highlights the need for building open economy models with good microeconomic foundations.

The final set of simulations explored the impact of an asymmetry in the vulnerability of the two economies to exchange rate fluctuations. In the case, where the vulnerability comes from a greater share of imports, policy makers are shown to resort to extremely strong reactions to inflation and output fluctuations to counter the adverse impact on the economy. Policy makers are also able to substantially improve the macroeconomic performance through independent reaction to the troublesome variable, the real exchange rate. Disaggregating the gains to cooperation show that, in this type of scenario, the less vulnerable country has little to gain from cooperation; any cooperative agreement requires them to be willing to make itself slightly worse off in exchange for improving the well being of the more vulnerable economy dramatically.

The qualitative nature of these results holds up when the greater vulnerability is introduced in the form of a higher pass-through coefficient rather than a greater share of imports. However, the overall adverse impact of exchange rate fluctuations is more muted in this case relative to the simulations performed with a moderately higher import share in one country.

These simulation exercises can potentially be helpful in understanding the evolution of the relationship between policy makers in the U.S and the EMU as the EMU matures. The relationship between the U.S. and the EMU will be asymmetric as long as the EMU has higher shocks, a greater persistence of inflation and a greater vulnerability to exchange rate fluctuations, because of a larger import share and higher pass-through.

The results may provide grounds for thinking that better policy making by the ECB and a more dominant role for the Euro will help alleviate the performance asymmetries of the two economies. Until that time the external dimension of monetary policy is likely to be more important to the ECB: it should care about the value of the Euro much more than the Fed should care about the value of the dollar.

A Derivation of the Demand Side

The Euler equation, in deviations terms, for the consumption of the composite good is of the standard form

$$\bar{c}_t = -\sigma \bar{r}_t + E_t \bar{c}_{t+1}.$$

In order to find the allocation of the composite consumption between Home and Foreign goods we have to solve the following problem.

$$\min_{C_t^h, C_t^f} P_t^h C_t^h + P_t^f C_t^f \text{ s.t. } \bar{C} = \left[(C_t^h)^{1-\frac{1}{\theta}} + (C_t^f)^{1-\frac{1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

The first order conditions from this, rewritten in deviations terms are

$$\begin{aligned} c_t^h &= \bar{c}_t - \theta \left[p_t^h - \bar{p}_t \right] \\ c_t^f &= \bar{c}_t - \theta \left[p_t^f - \bar{p}_t \right] \end{aligned}$$

The composite price is itself a Dixit-Stiglitz aggregate of the price of Home and Foreign goods.

$$\bar{P}_t = \left[(P_t^h)^{1-\theta} + (P_t^f)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

When written in deviations terms this becomes $\bar{p}_t = (1-\omega)p_t^h + \omega p_t^f$ where ω is the share of Foreign goods in the composite consumption good, defined as $\omega = \left[\frac{(P^h)^{1-\theta}}{(P^h)^{1-\theta} + (P^f)^{1-\theta}} \right]$. We can use the definition of the composite price and that of the real exchange rate $q_t = p_t^f - p_t^h$ and rewrite the above FOC as:

$$\begin{aligned} c_t^h &= \bar{c}_t + \theta \omega q_t \\ c_t^f &= \bar{c}_t - \theta (1-\omega) q_t \end{aligned}$$

The corresponding equations for the Foreign country can be derived in a similar way to obtain

$$\begin{aligned} c_t^{f^*} &= \bar{c}_t^* - \theta^* \omega^* q_t \\ c_t^{h^*} &= \bar{c}_t^* + \theta^* (1-\omega^*) q_t \end{aligned}$$

These demands are then plugged into the Euler equation to obtain the following set of equations.

$$\begin{aligned} c_t^h &= E_t c_{t+1}^h - \sigma[i_t - E_t \bar{\pi}_{t+1}] - \theta\omega[E_t q_{t+1} - q_t] \\ c_t^f &= E_t c_{t+1}^f - \sigma[i_t - E_t \bar{\pi}_{t+1}] + \theta(1-\omega)[E_t q_{t+1} - q_t] \\ c_t^{h^*} &= E_t c_{t+1}^{h^*} - \sigma^*[i_t^* - E_t \bar{\pi}_{t+1}^*] - \theta^*(1-\omega^*)[E_t q_{t+1} - q_t] \\ c_t^{f^*} &= E_t c_{t+1}^{f^*} - \sigma^*[i_t^* - E_t \bar{\pi}_{t+1}^*] + \theta^*\omega^*[E_t q_{t+1} - q_t] \end{aligned}$$

These equations are re-written so as to leave domestic inflation on the right hand side by substituting in $\bar{\pi}_t = \pi_t^h + \omega[q_t - q_{t-1}]$ and $\bar{\pi}_t^* = \pi_t^{f^*} - \omega^*[q_t - q_{t-1}]$. This yields the following system of demands.

$$\begin{aligned} c_t^h &= E_t c_{t+1}^h - \sigma[i_t - E_t \pi_{t+1}^h] - \omega(\theta - \sigma)[E_t q_{t+1} - q_t] \\ c_t^f &= E_t c_{t+1}^f - \sigma[i_t - E_t \bar{\pi}_{t+1}] + [\theta(1-\omega) + \sigma\omega][E_t q_{t+1} - q_t] \\ c_t^{h^*} &= E_t c_{t+1}^{h^*} - \sigma^*[i_t^* - E_t \pi_{t+1}^{f^*}] - [\theta^*(1-\omega^*) + \omega^*\sigma^*][E_t q_{t+1} - q_t] \\ c_t^{f^*} &= E_t c_{t+1}^{f^*} - \sigma^*[i_t^* - E_t \bar{\pi}_{t+1}^*] + \omega^*(\theta^* - \sigma^*)[E_t q_{t+1} - q_t] \end{aligned}$$

By iterating these equations forward and assuming that the deviations of the real exchange rate and consumption converge to zero in the limit we can finally derive the Home and Foreign consumption of Home and Foreign goods.

$$\begin{aligned} c_t^h &= -\sigma \rho_t + (\theta - \sigma)\omega q_t \\ c_t^f &= -\sigma \rho_t - [\theta(1-\omega) + \sigma\omega] q_t \\ c_t^{h^*} &= -\sigma^* \rho_t^* + [\theta^*(1-\omega^*) + \sigma^*\omega^*] q_t \\ c_t^{f^*} &= -\sigma^* \rho_t^* - (\theta^* - \sigma^*)\omega^* q_t \end{aligned}$$

where $\rho_t = \sum_{\tau=0}^{\infty} r_{t+\tau}^h$, $\rho_t^* = \sum_{\tau=0}^{\infty} r_{t+\tau}^{f^*}$, $r_t^h = i_t - E_t \pi_{t+1}^h$, and $r_t^f = i_t^* - E_t \pi_{t+1}^{f^*}$.

The aggregate demand for Home and Foreign goods is $Y_t = C_t^h + C_t^{h^*}$ and $Y_t^* = C_t^f + C_t^{f^*}$. These aggregate demand functions can be rewritten in deviations terms as

$$\begin{aligned} y_t &= (1-\omega)c_t^h + \omega c_t^{h^*} \\ y_t^* &= (1-\omega^*)c_t^{f^*} + \omega^* c_t^f \end{aligned}$$

where ω is the share of Foreign goods in Home consumption and ω^* is the share of Home goods in Foreign consumption. By substituting in the expressions for Home and Foreign consumption of Home and Foreign goods we can obtain the following aggregate demand equations.

$$\begin{aligned} y_t &= -(1-\omega)\sigma\rho_t - \sigma^*\omega\rho_t^* + \omega\left[\theta^*(1-\omega^*) + \omega^*\sigma^* + (1-\omega)(\theta-\sigma)\right] q_t \\ y_t^* &= -(1-\omega^*)\sigma^*\rho_t^* - \sigma\omega^*\rho_t - \omega^*\left[\theta(1-\omega) + \omega\sigma + (1-\omega^*)(\theta^*-\sigma^*)\right] q_t \end{aligned}$$

Assume that demand is pre-determined one period ahead and there is only partial adjustment of demand from one period to the next. Then the equation for aggregate demand is a weighted average of last period's output and the right hand side of the expression given above.

$$\begin{aligned} y_{t+1} &= \beta_y y_t + (1-\beta_y)\left[-(1-\omega)\sigma\rho_t - \sigma^*\omega\rho_t^*\right] + \\ &\quad (1-\beta_y)\omega\left[\theta^*(1-\omega^*) + \omega^*\sigma^* + (1-\omega)(\theta-\sigma)\right] q_t + \epsilon_{t+1}^y \\ y_{t+1}^* &= \beta_y^* y_t^* + (1-\beta_y^*)\left[-(1-\omega^*)\sigma^*\rho_t^* - \sigma\omega^*\rho_t\right] - \\ &\quad (1-\beta_y^*)\omega^*\left[\theta(1-\omega) + \omega\sigma + (1-\omega^*)(\theta^*-\sigma^*)\right] q_t + \epsilon_{t+1}^{y^*} \end{aligned}$$

This gives us the aggregate demand functions that were presented in equations (1) and (2), where the reduced form coefficients are related to the structural coefficients in the following way.

$$\begin{aligned} \beta_\rho &= (1-\beta_y)(1-\omega)\sigma \\ \beta_{\rho^*} &= (1-\beta_y)\omega\sigma^* \\ \beta_q &= (1-\beta_y)\omega\left[\theta^*(1-\omega^*) + \omega^*\sigma^* + (1-\omega)(\theta-\sigma)\right] \\ \beta_\rho^* &= (1-\beta_y^*)\omega^*\sigma \\ \beta_{\rho^*}^* &= (1-\beta_y^*)(1-\omega^*)\sigma^* \\ \beta_q^* &= (1-\beta_y^*)\omega^*\left[\theta(1-\omega) + \omega\sigma + (1-\omega^*)(\theta^*-\sigma^*)\right] \end{aligned}$$

B Derivation of the Supply Side

The decision faced by each producer is given by

$$\max_{P_t^h(i)} E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \Lambda_{t+\tau}\left[P_t^h(i)Y_{t+\tau}(i) - W_{t+\tau}V\left(Y_{t+\tau}(i)\right)\right]$$

where $V\left[Y_{t+\tau}(i)\right]$ is the amount of inputs needed to produce $Y_{t+\tau}(i)$ units of output and W is the per-unit cost of inputs. Λ is the marginal utility of income, it basically serves as a discount factor for future profits, $\frac{\Lambda_t}{\delta E_t \Lambda_{t+1}} = (1+r_t)$.

The first order condition for this problem is

$$(1-\nu)E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \Lambda_{t+\tau} Y_{t+\tau}(i) \quad = \quad -\nu E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \Lambda_{t+\tau} Y_{t+\tau}(i) \frac{W_{t+\tau}}{P_t^h(i)}$$

Log-linearize this equation, around a stationary equilibrium where $P_t^h(i) = P^h$, $\Lambda_{t+\tau} = \Lambda$, $Y_{t+\tau}(j) = Y$, $W_{t+\tau} = W$, to obtain

$$\begin{aligned} &-\frac{\nu W}{P^h} V'(Y) Y \Lambda E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[1 + w_{t+\tau} - p_t^h(i) + v' + y_{t+\tau}(i) + \Lambda_{t+\tau}\right] \\ = \quad &(1-\nu) Y \Lambda E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[1 + y_{t+\tau}(i) + \Lambda_{t+\tau}\right] \end{aligned}$$

where all the lower case variables are expressed in deviations terms. Evaluate the log-linearized FOC at the steady-state so that the following relationship will hold. $(1-\nu) = -\nu \frac{W}{P^h} V'(y)$. Therefore the FOC can be simplified to the following.

$$E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[w_{t+\tau} - (p_t^h(i) - p_t^h) + \sum_{s=1}^{\tau} \pi_{t+s}^h - p_{t+\tau}^h + v' \right] \quad = \quad 0$$

Since only a subset of producers are able to change prices in a given period, it follows that

$$P_t^h = \left[\alpha (P_{t-1}^h)^{1-\nu} + (1-\alpha)(P_t^h(i))^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

which in deviations terms implies that

$$\begin{aligned}
p_t^h &= \alpha p_{t-1}^h + (1-\alpha) p_t^h(i) \\
p_t^h(i) &= \frac{1}{1-\alpha} \left[p_t^h - \alpha p_{t-1}^h \right] \\
p_t^h(i) - p_t^h &= \frac{1}{1-\alpha} \left[p_t^h - \alpha p_{t-1}^h - (1-\alpha) p_t^h \right] \\
&\equiv \frac{\alpha}{1-\alpha} \pi_t^h
\end{aligned}$$

Substituting this into the FOC yields

$$E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[w_{t+\tau} - \frac{\alpha}{1-\alpha} \pi_t^h + \sum_{s=1}^{\tau} \pi_{t+s}^h - p_{t+\tau}^h + v' \right] = 0$$

Substitute in for $v' = \tilde{v} y_{t+\tau}(i)$ where $\tilde{v} = \frac{V''(y)}{V'(y)} y$, is the elasticity of the input requirement function with respect to output. Also substitute in for

$$y_{t+\tau}(i) = y_{t+\tau} + \nu [p_{t+\tau}^h - p_t^h(i)] \equiv y_{t+\tau} + \nu \left[\sum_{s=1}^{\tau} \pi_{t+s}^h - \frac{\alpha}{1-\alpha} \pi_t^h \right]$$

Then the FOC condition becomes

$$E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[\left(w_{t+\tau} - p_{t+\tau}^h + \tilde{v} y_{t+\tau} \right) - (1 + \tilde{v}\nu) \left(\frac{\alpha}{1-\alpha} \pi_t^h - \sum_{s=1}^{\tau} \pi_{t+s}^h \right) \right] = 0$$

$$E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[w_{t+\tau} - p_{t+\tau}^h + \tilde{v} y_{t+\tau} \right] = E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[(1 + \tilde{v}\nu) \left(\frac{\alpha}{1-\alpha} \pi_t^h - \sum_{s=1}^{\tau} \pi_{t+s}^h \right) \right]$$

This can be simplified to get

$$\pi_t^h = \delta E_t \pi_{t+1}^h + \xi \left[w_t - p_t^h + \tilde{v} y_t \right] + \text{ where } \xi = \frac{(1-\alpha)(1-\alpha\delta)}{\alpha(1+\tilde{v}\nu)}$$

Complete the tedious derivation with two final steps. Since the inputs are a Dixit-Stiglitz composite of Home and Foreign goods, the price of inputs can be written in deviations terms as

$$w_t = (1-\gamma) p_t^h + \gamma p_t^f = \gamma q_t + p_t^h$$

where γ is the share of Foreign inputs in production. Then

$$\left[w_t - p_t^h + \tilde{v} y_t \right] = \gamma q_t + \tilde{v} y_t \text{ and } \pi_t^h = E_t \pi_{t+1}^h + \xi \gamma q_t + \xi \tilde{v} y_t$$

Assume that domestic inflation is pre-determined two periods ahead and adjusts only partially. Then domestic inflation will be a linear combination of lagged domestic inflation and the RHS of the above expression.

$$\begin{aligned}
\pi_{t+2}^h &= \alpha_\pi \pi_{t+1}^h + (1-\alpha_\pi) \left[E_t \pi_{t+2}^h + \xi \gamma q_t + \xi \tilde{v} y_t \right] + \epsilon_{t+2}^\pi \\
\pi_{t+2}^h &= \alpha_\pi \pi_{t+1}^h + (1-\alpha_\pi) E_t \pi_{t+3}^h + \alpha_y E_t y_{t+2} + \alpha_q E_t q_{t+2} + \epsilon_{t+2}^\pi
\end{aligned}$$

which is the price adjustment equation presented in (3) where the reduced form coefficients are related in the following way to the structural coefficients.

$$\begin{aligned}\alpha_y &= (1-\alpha_\pi)\tilde{\upsilon}\xi \\ \alpha_q &= (1-\alpha_\pi)\gamma\xi\end{aligned}$$

Similarly, the price adjustment equation for the foreign economy will be

$$\pi^{f^*}_{t+2} = \alpha^*_\pi \pi^{f^*}_{t+1} + (1-\alpha^*_\pi)E_t\pi^{f^*}_{t+3} + \alpha^*_y E_t y^*_{t+2} - \alpha^*_q E_t q_{t+2} + \epsilon^{\pi^*}_{t+2}$$

where

$$\begin{aligned}\alpha^*_y &= (1-\alpha^*_\pi)\tilde{\upsilon}^*\xi^* \\ \alpha^*_q &= (1-\alpha^*_\pi)\gamma^*\xi^*\end{aligned}$$

C Derivation of State-Space Representation

The 18 equations that describe the dynamic path of the economy can be written in a more compact matrix notation using e_i to denote a 1*18 row vector which is all zeros except for a 1 in the ith position. Λ_0, Λ_1 and Λ_5 are all 18*18 matrices, Λ_2 and Λ_3 are both 18*2 matrices and Λ_4 is an 18*6 matrix.

$$\Lambda_0 \begin{bmatrix} \hat{X}_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} = \Lambda_1\hat{Z}_t + \Lambda_2\hat{U}_t + \Lambda_3 E_t\hat{U}_{t+1} + \Lambda_4\hat{W}_t + \Lambda_5\upsilon_{t+1}$$

where

$$\hat{Z}_t = \begin{bmatrix} \hat{X}_t \\ \hat{x}_t \end{bmatrix} = \begin{bmatrix} y_t \\ E_t\pi^h_{t+1} \\ \varphi_t \\ q_{t-1} \\ i_{t-1} \\ \pi^h_t \\ y^*_t \\ E_t\pi^{*f}_{t+1} \\ \varphi^*_t \\ q^*_{t-1} \\ i^*_{t-1} \\ \pi^{f^*}_t \\ q_t \\ \rho_t \\ E_t\pi^h_{t+3} \\ q^*_t \\ \rho^*_t \\ E_t\pi^{f^*}_{t+3} \end{bmatrix} \qquad \upsilon_{t+1} = \begin{bmatrix} \epsilon^y_{t+1} \\ \epsilon^\pi_{t+1} \\ \epsilon^\varphi_{t+1} \\ \epsilon^{y^*}_{t+1} \\ \epsilon^{\pi^*}_{t+1} \\ \epsilon^{\varphi^*}_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\epsilon^{\varphi^*} = -\epsilon^\varphi$, $\varphi^* = -\varphi$ and $q^* = -q$.

The matrices of coefficients in the above equation can be expressed as

$$\Lambda_0 = \begin{bmatrix} e_1 + \beta_\rho e_{14} + \beta_{\rho^*} e_{17} - \beta_q e_{13} \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 + \beta_\rho^* e_{14} + \beta_{\rho^*}^* e_{17} + \beta_q^* e_{16} \\ e_8 \\ e_9 + e_3 \\ e_{10} + e_4 \\ e_{11} \\ e_{12} \\ e_{13} - e_4 - e_5 + e_{11} \\ e_{14} + e_5 \\ e_{15} \\ e_{16} + e_{13} \\ e_{17} + e_{11} \\ e_{18} \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} \beta_y e_1 \\ e_{15} \\ \gamma_\varphi e_3 \\ e_{13} \\ e_0 \\ e_2 \\ \beta_y^* e_7 \\ e_{18} \\ e_0 \\ e_0 \\ e_0 \\ e_8 \\ e_8 - e_2 - e_3 \\ e_{14} + e_2 \\ \frac{1}{1-\alpha_\pi} e_{15} - \frac{\alpha_\pi}{1-\alpha_\pi} e_2 - e_8 - e_9 \\ e_{17} + e_8 \\ \frac{1}{1-\alpha_\pi^*} e_{18} - \frac{\alpha_\pi^*}{1-\alpha_\pi^*} e_8 \end{bmatrix} \quad \Lambda_5 = \begin{bmatrix} e_1 \\ \alpha_\pi e_2 \\ e_3 \\ e_0 \\ e_0 \\ e_2 \\ e_4 \\ \alpha_\pi^* e_5 \\ e_6 \\ e_0 \\ e_0 \\ e_5 \\ e_0 \\ e_0 \\ e_0 \\ e_0 \\ e_0 \\ e_0 \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} e_5 \\ e_{11} \end{bmatrix}' \quad \Lambda_3 = \begin{bmatrix} e_0 \\ e_0 \end{bmatrix}' \quad \Lambda_4 = \begin{bmatrix} -\frac{\alpha_y}{1-\alpha_\pi} e_{15} \\ -\frac{\alpha_y^*}{1-\alpha_\pi^*} e_{18} \\ -\frac{\alpha_q}{1-\alpha_\pi} e_{15} \\ -\frac{\alpha_q^*}{1-\alpha_\pi^*} e_{18} \\ e_0 \\ e_0 \end{bmatrix}'$$

The vector of variables

$$\hat{W}_t = [E_t y_{t+2}, E_t y_{t+2}^*, E_t q_{t+2}, E_t q_{t+2}^*, E_t \rho_{t+2}, E_t \rho_{t+2}^*]'$$

can be expressed as a function of the forward looking variables and state variables, X_t and x_t as follows:

$$\Gamma_0 W_t = \Gamma_1 \begin{bmatrix} \hat{X}_{t+1} \\ E_t \hat{x}_{t+1} \end{bmatrix} + \Gamma_2 \hat{Z}_t + \Gamma_3 \hat{U}_t + \Gamma_4 \hat{U}_{t+1} + \Gamma_5 \upsilon_t$$

where

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & -\beta_q & 0 & \beta_\rho & \beta_{\rho^*} \\ 0 & 1 & 0 & -\beta_q^* & \beta_\rho^* & \beta_{\rho^*}^* \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \Gamma_1 = \begin{bmatrix} \beta_y e_1 \\ \beta_y^* e_7 \\ e_{13} \\ e_0 \\ e_{14} \\ e_{17} \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} e_0 \\ e_0 \\ -e_{15} + e_{18} - \gamma_\varphi e_3 \\ e_0 \\ e_{15} \\ e_{18} \end{bmatrix} \quad \Gamma_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \Gamma_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \Gamma_5 = \begin{bmatrix} -\beta_y e_1 \\ -\beta_y^* e_4 \\ e_0 \\ e_0 \\ e_0 \\ e_0 \end{bmatrix}$$

Rewrite the vector of variables $\hat{W}_t$ as

$$W_t = \Lambda_{40} \begin{bmatrix} \hat{X}_{t+1} \\ E_t \hat{x}_{t+1} \end{bmatrix} + \Lambda_{41} \hat{Z}_t + \Lambda_{42} \hat{U}_t + \Lambda_{43} E_t \hat{U}_{t+1} + \Lambda_{44} \upsilon_t$$

where

$\Lambda_{40} = \Gamma_0^{-1}\Gamma_1$, $\Lambda_{41} = \Gamma_0^{-1}\Gamma_2$, $\Lambda_{42} = \Gamma_0^{-1}\Gamma_3$, $\Lambda_{43} = \Gamma_0^{-1}\Gamma_4$, and $\Lambda_{44} = \Gamma_0^{-1}\Gamma_5$.

Complete the system of equations describing the dynamic path of the economy as

$$\begin{bmatrix} \hat{X}_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} = A\hat{Z}_t + B\hat{U}_t + B^F E_t\hat{U}_{t+1} + \Upsilon v_{t+1}$$

where the reduced form coefficients are related to the structural coefficients in the following manner.

$$\begin{aligned} A &= [\Lambda_0 - \Lambda_4\Lambda_{40}]^{-1}[\Lambda_1 + \Lambda_4\Lambda_{41}] \\ B &= [\Lambda_0 - \Lambda_4\Lambda_{40}]^{-1}[\Lambda_2 + \Lambda_4\Lambda_{42}] \\ B^F &= [\Lambda_0 - \Lambda_4\Lambda_{40}]^{-1}[\Lambda_3 + \Lambda_4\Lambda_{43}] \\ \Upsilon &= [\Lambda_0 - \Lambda_4\Lambda_{40}]^{-1}[\Lambda_4 + \Lambda_5\Lambda_{44}] \end{aligned}$$

D Derivation of the Period Loss Functions in the Non Cooperative Case

The period loss function for the Home country can be expressed in terms of a set of goal variables Y_t as: $L_t = Y_t'KY_t$ where

$$Y_t = \begin{bmatrix} \bar{\pi}_t \\ y_t \\ i_t - i_{t-1} \end{bmatrix}$$

Similarly, the period loss function for the Foreign country can be expressed as a function of their goal variables Y_t^* as: $L_t^* = Y_t^{*'}KY_t^*$ where

$$Y_t^* = \begin{bmatrix} \bar{\pi}_t^* \\ y_t^* \\ i_t^* - i_{t-1}^* \end{bmatrix}$$

Y_t and Y_t^* can be expressed as functions of the state variables and the choice variables, $Y_t = C_z\hat{Z}_t + C_u\hat{U}_t$ and $Y_t^* = C_z^*\hat{Z}_t + C_u^*\hat{U}_t$ The matrices C_Z and C_Z^* are both 3*18 matrices while C_u and C_u^* are both 3*2 matrices. These matrices can be defined as:

$$C_z = \begin{bmatrix} e_6 + \omega(e_{13} - e_4) \\ e_1 \\ -e_5 \end{bmatrix} \quad C_z^* = \begin{bmatrix} e_{12} + \omega^*(e_{16} - e_{10}) \\ e_7 \\ -e_{11} \end{bmatrix} \quad C_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \quad C_u^* = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The period loss function of the Home policy maker, rewritten in terms of the state variables and the choice variables are

$$\begin{aligned} L_t &= \left[C_z\hat{Z}_t + C_u\hat{U}_t\right]' K \left[C_z\hat{Z}_t + C_u\hat{U}_t\right] \\ L_t &= \left[\hat{Z}_t'Q\hat{Z}_t + 2\hat{Z}_t'S\hat{U}_t + \hat{U}_t'R\hat{U}_t\right] \\ Q &= C_z'KC_z \\ R &= C_u'KC_u \\ S &= C_z'KC_u \end{aligned}$$

Similarly, the period loss function of the Foreign policy maker is

$$\begin{aligned} L_t^* &= \left[C_z^*\hat{Z}_t + C_u^*\hat{U}_t\right]' K \left[C_z^*\hat{Z}_t + C_u^*\hat{U}_t\right] \\ L_t^* &= \left[\hat{Z}_t'Q^*\hat{Z}_t + 2\hat{Z}_t'S^*\hat{U}_t + \hat{U}_t'R^*\hat{U}_t\right] \\ Q^* &= C_z^{*'}KC_z^* \\ R^* &= C_u^{*'}KC_u^* \\ S^* &= C_z^{*'}KC_u^* \end{aligned}$$

E Derivation of the Period Loss Function in the Cooperative Case

The period loss function can be expressed in terms of a set of goal variables $\hat{Y}_t$ is: $\hat{L}_t = \hat{Y}_t' K \hat{Y}_t$ where

$$\hat{Y}_t = \begin{bmatrix} \pi_t \\ \bar{\pi}_t^* \\ y_t \\ y_t^* \\ i_t - i_{t-1} \\ i_t^* - i_{t-1}^* \end{bmatrix}$$

Y_t can be expressed as a function of the state variables and the choice variables, $\hat{Y}_t = C_z \hat{Z}_t + C_u \hat{U}_t$. The matrix C_Z is a 6*18 matrix and the matrix C_u is a 6*2 matrix defined as:

$$C_z = \begin{bmatrix} e_6 + \omega(e_{13} - e_4) \\ e_{12} + \omega^*(e_{16} - e_{10}) \\ e_1 \\ e_7 \\ -e_5 \\ -e_{11} \end{bmatrix} \quad C_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We can also rewrite the period loss function of the policy maker in terms of the state variables and the choice variables as

$$\begin{aligned} \hat{L}_t &= \left[C_z \hat{Z}_t + C_u \hat{U}_t\right]' K \left[C_z \hat{Z}_t + C_u \hat{U}_t\right] \\ \hat{L}_t &= \left[\hat{Z}_t' Q \hat{Z}_t + 2\hat{Z}_t' S \hat{U}_t + \hat{U}_t' R \hat{U}_t\right] \\ Q &= C_z' K C_z \\ R &= C_u' K C_u \\ S &= C_z' K C_u \end{aligned}$$

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