3. Impact of Monetary Shocks in Forward-Looking Models -- Closed Economy

John B. Taylor, May 1, 2013

Start with Simple Demand for Money Equation

- Irving Fisher's Quantity Equation:
  - "Fisher's equation plays the same foundation-stone role in monetary theory that Einstein's $E = mc^2$ does in physics"
    - Friedman *Money Mischief* (p. 39)
- Keynes' liquidity preference idea

$$M = M_1 + M_2 = L_1(Y) + L_2(r),$$

California

$$MV = PY$$

This is scanned from the *General Theory*

- Many microeconomic theories:
    - Speculative, inventory, portfolio theory, cash in advance, money in utility function, overlapping generations models
    - Medium of exchange, unit of account, store of value
- We will use "Cagan Model" (1st used for hyperinflations)

$$m_t - p_t = \lambda y_t - \beta i_t$$

m, p and y measured in logs
(so semi log from)

Make 3 simplifying assumptions to get "economy-wide" model:
Take y_t as given (=0), take $r_t = i_t - (E_t p_{t+1} - p_t)$ *as given (=0), RE*

$$m_t - p_t = -\beta(E_t p_{t+1} - p_t)$$

$\beta>0$

And rewrite it in generic form

$$y_t = \alpha E_t y_{t+1} + \delta u_t$$

$$u_t = \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i}$$

$$\alpha = \beta/(1+\beta) \text{ and } \delta = 1/(1+\beta)$$

$$y_t = p_t \text{ and } u_t = m_t$$

General stochastic process for shocks:

$$u_t = \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i},$$ where ε_t is a serially uncorrelated random variable with zero mean.

With alternative special cases:

Purely temporary: $\theta_0 = 1$ and $\theta_i = 0$ for $i > 0$

Persistent: $\theta_i = \rho^i$

Anticipated: $\theta_0 = 0,\ \theta_1 = 1,\ \theta_i = 0$ for $i > 1$

…and ways to do “thought experiments”

Method of undetermined coefficients

If you substitute this general form for the solution

$$y_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i}.$$

into the model and you will get the following deterministic difference equation (see slide 14 for the derivation)

$$\gamma_i = \alpha \gamma_{i+1} + \delta \theta_i \qquad i = 0, 1, 2, \ldots.$$

which then can be solved for different assumptions about the nature of the shocks, as shown on the next few slides…

Temporary and Unanticipated

$\theta_0 = 1$ and $\theta_i = 0$ for $i > 0$

$$\gamma_0 = \alpha\gamma_1 + \delta$$

$$\gamma_1 = \alpha\gamma_2$$

$$\gamma_2 = \alpha\gamma_3$$

$$\gamma_s = \alpha\gamma_{s+1}$$

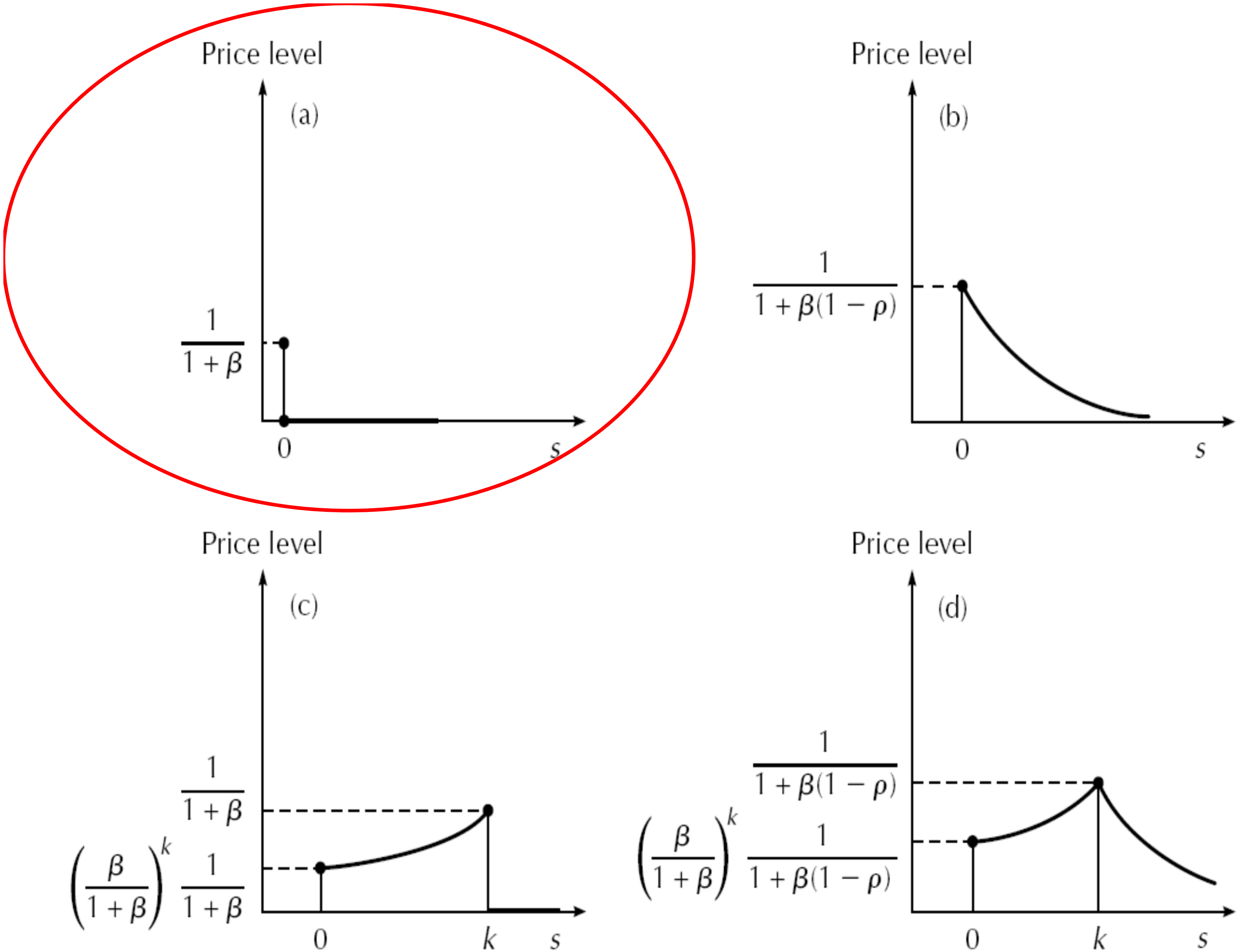
Too few equations. To get another equation we rule out explosive solutions. Recall that $\alpha = (\beta/(1+\beta) < 1$, so that

$\gamma_{s+1} = (1/\alpha)\gamma_s$ for $= 1, 2, 3, \ldots$ is unstable.

To prevent explosion must have $\gamma_1 = 0$, and thus $\gamma_s = 0$ for all $s \geq 1$.

and $\gamma_0 = \delta = (1/1+\beta)$.

Thus $p_t = \dfrac{1}{1+\beta}\varepsilon_t$ is the "solution" for the price level when $m_t = \varepsilon_t$



Price level
(a)
$\frac{1}{1+\beta}$
0
s
Price level
(b)
$\frac{1}{1+\beta(1-\rho)}$
0
s
Price level
(c)
$\frac{1}{1+\beta}$
$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta}$
0
k
s
Price level
(d)
$\frac{1}{1+\beta(1-\rho)}$
$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta(1-\rho)}$
0
k
s

Unanticipated, slow phase-out $\theta_i = \rho^i$

Set $\theta_i = \rho^i$ giving

$$\gamma_{i+1} = (1/\alpha)\gamma_i - \delta\rho^i / \alpha$$

$$\gamma_i = \gamma_i^H + \gamma_i^P$$

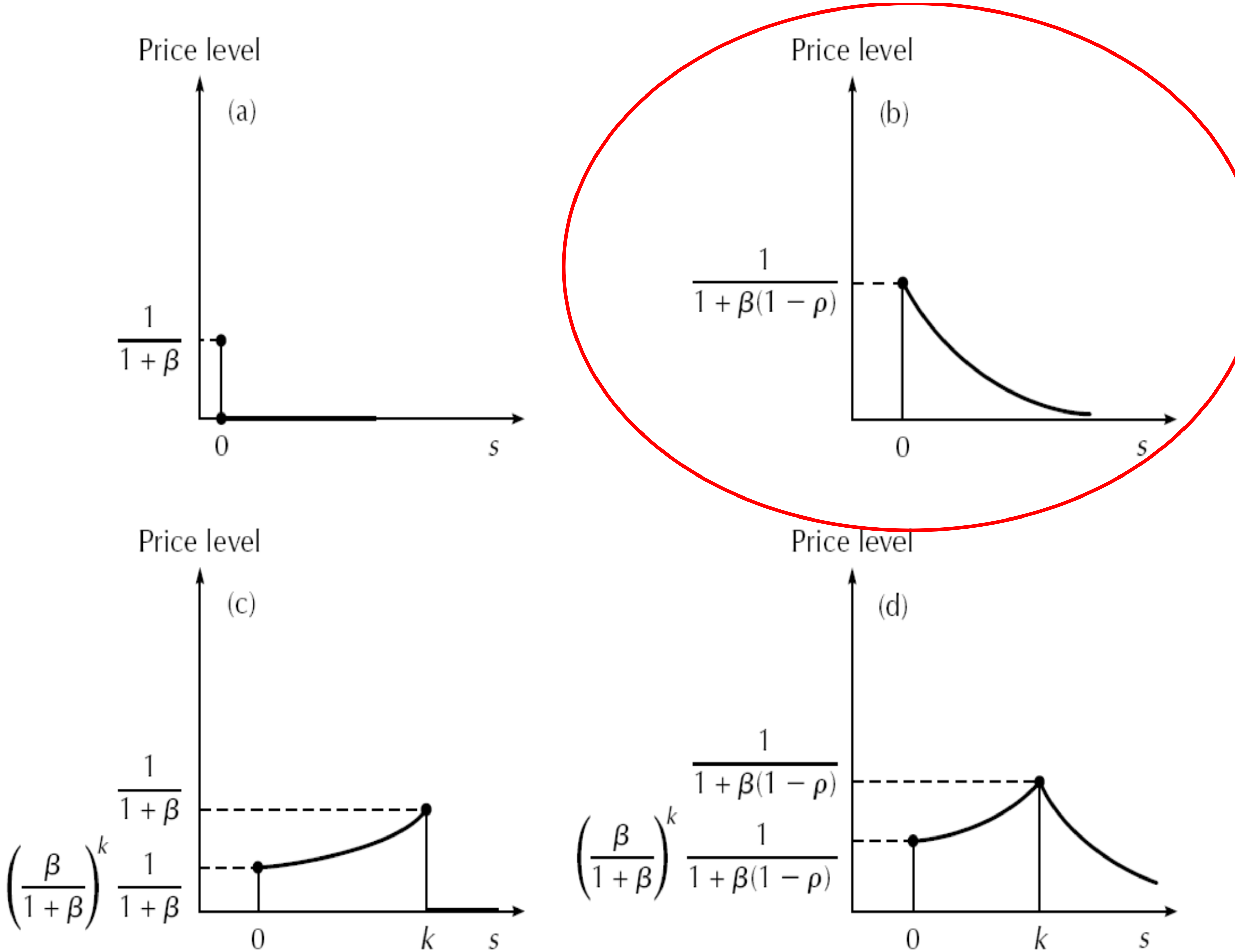
must have $\gamma_{i+1}^H = (1/\alpha)\gamma_i^H \Rightarrow \gamma_0^H = 0 \Rightarrow \gamma_i^H = 0$ for all i

now guess a form for $\gamma_i^P = hb^i$

Then $b = \rho$ and $h\rho^{i+1} = (1/\alpha)h\rho^i - \delta\rho^i / \alpha$

so that $h = \delta / (1 - \alpha\rho)$

$$\gamma_i = \frac{\delta\rho^i}{1-\alpha\rho} = \frac{\frac{1}{1+\beta}\rho^i}{1 - \frac{\beta}{1+\beta}\rho} = \frac{\rho^i}{1+\beta(1-\rho)}$$



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Price level
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$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta}$
0
k
s
Price level
(d)
$\frac{1}{1+\beta(1-\rho)}$
$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta(1-\rho)}$
0
k
s

Temporary but anticipated: $\theta_0 = 0, \ldots, \theta_{k-1} = 0, \theta_k = 1, \theta_{k+1} = 0, \ldots$

$$\gamma_i = \alpha\gamma_{i+1} \text{ for } i = 0, 1, 2, \ldots, k-1,$$

$$\gamma_{k+1} = (1/\alpha)\gamma_k - \delta/\alpha \text{ for } i = k,$$

$$\gamma_{i+1} = (1/\alpha)\gamma_i \text{ for } i = k+1, k+2, \ldots$$

$$\Rightarrow \gamma_{k+1} = 0$$

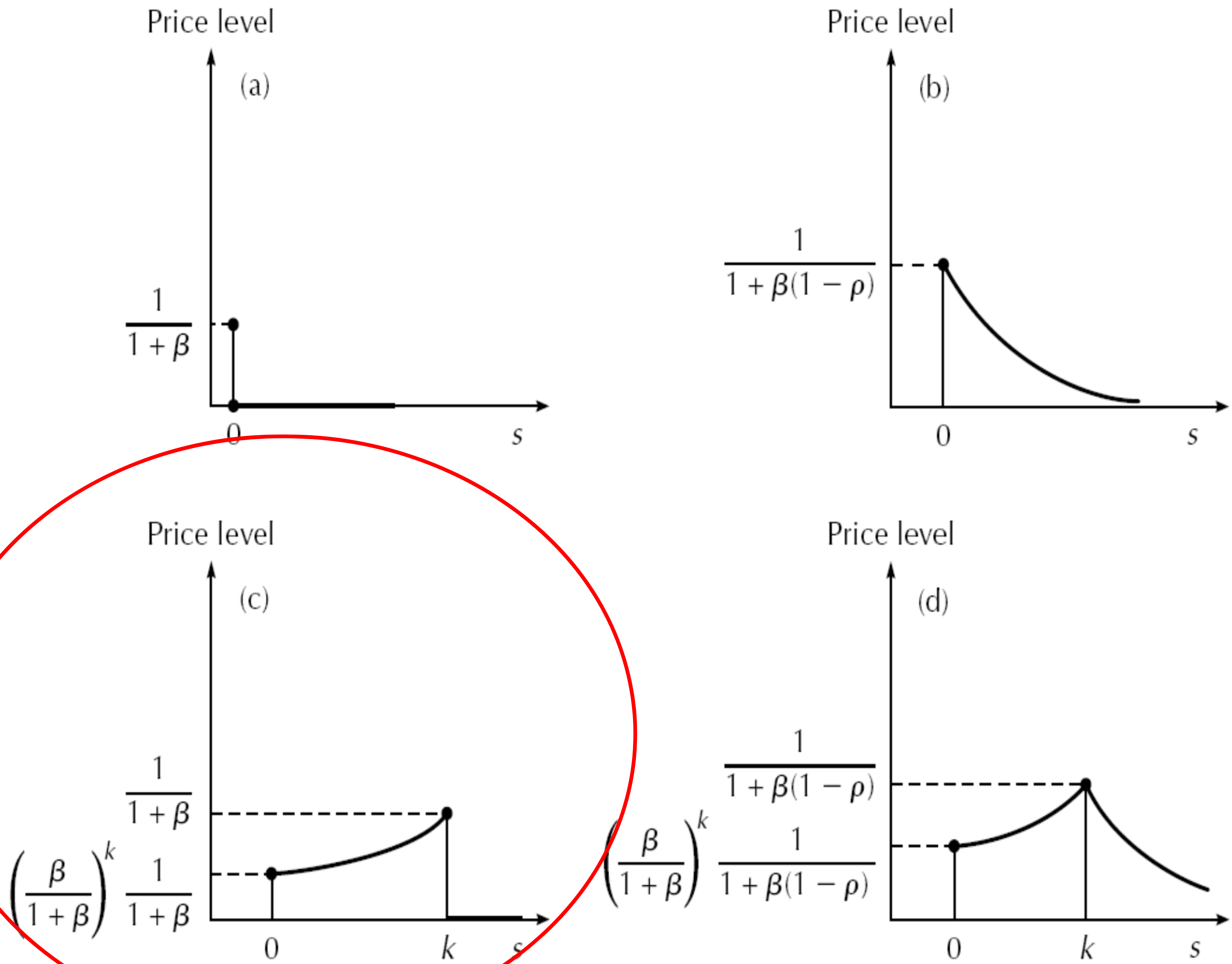
$$\Rightarrow \gamma_k = \delta$$

$$\gamma_{k-1} = \alpha\delta$$

$$\gamma_{k-2} = \alpha^2\delta$$

$$\ldots$$

$$\gamma_0 = \alpha^k\delta = \left(\frac{\beta}{1+\beta}\right)^k \left(\frac{1}{1+\beta}\right)$$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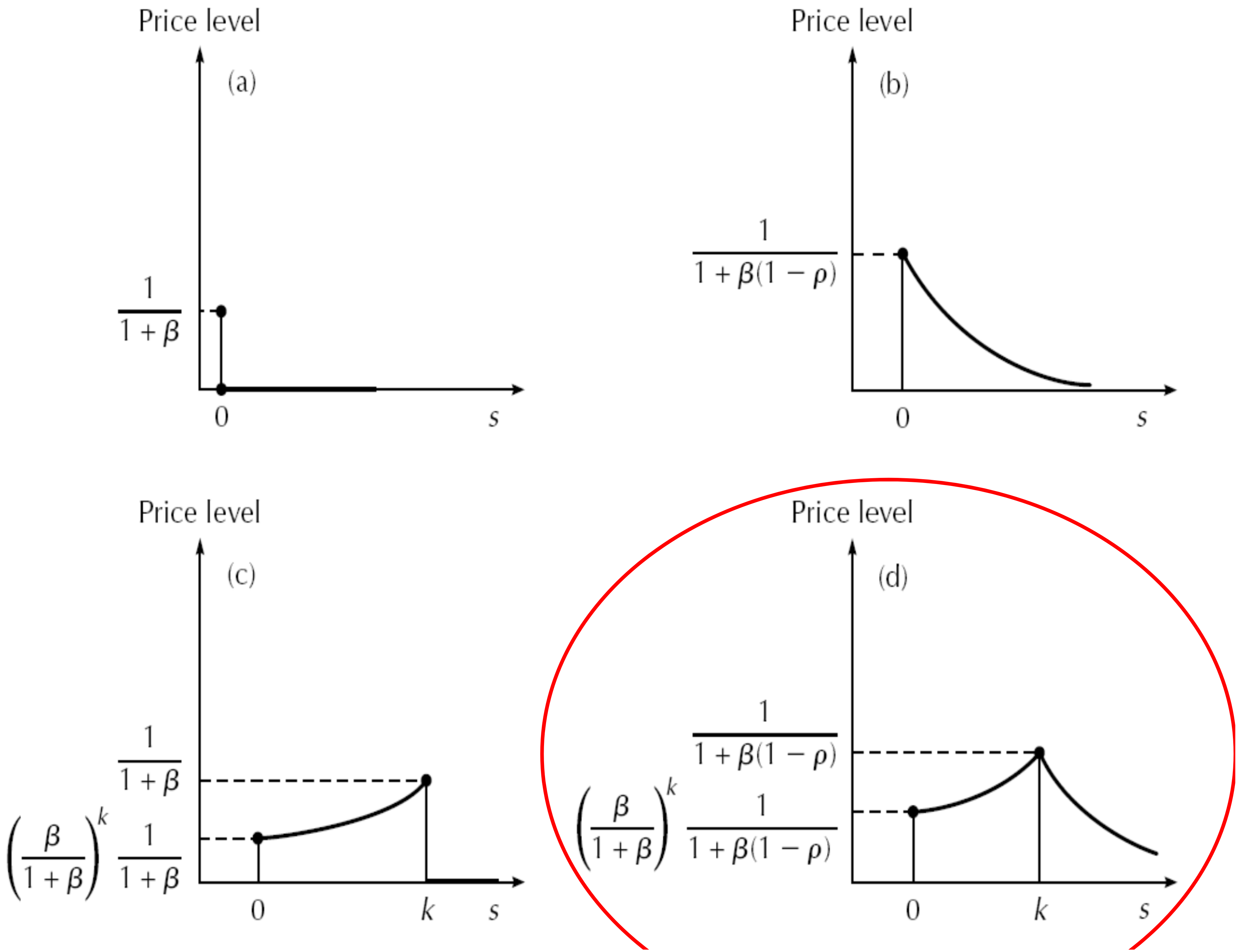
$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta}$
0
k
s
Price level
(d)
$\frac{1}{1+\beta(1-\rho)}$
$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta(1-\rho)}$
0
k
s



Price level
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s
Price level
(b)
$\frac{1}{1+\beta(1-\rho)}$
0
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Price level
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$\frac{1}{1+\beta}$
$\left(\frac{\beta}{1+\beta}\right)^k \frac{1}{1+\beta}$
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k
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k
s

Note two different ways the same mathematics is used

- To find a stochastic process for the endogenous variable y_t in terms of the stochastic process ε_t
  - Or for p_t in terms of the money supply process or "policy rule"
- To find a path for the endogenous variables for various "thought experiments" about exogenous variables.
  - What happens to the price level under different assumptions about money supply?

Derivation the of equation on slide 5

Model:

$y_t = \alpha E_t y_{t+1} + \delta u_t$

$u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots$

where $E\varepsilon_t = 0, E\varepsilon_t^2 = \sigma^2, E\varepsilon_t \varepsilon_s = 0$ for $t \neq s$

and where E_t is the conditional expectations operator based on information through period t. That is, expectations are assumed to b "rational expectations."

We want to solve the model, or in other words find a stochastic process for y_t which satisfies the model. We use the method of undetermined coefficients which leads to a deterministic difference equation which can then be solved by standard methods.

The solution will have the following linear from:

$\mathrm{y}_t = \gamma_0 \varepsilon_t + \gamma_1 \varepsilon_{t-1} + \gamma_2 \varepsilon_{t-2} + \ldots$

To find $E_t y_{t+1}$ we lead the above equation by one period and take conditional expectations:

$\mathrm{y}_{t+1} = \gamma_0 \varepsilon_{t+1} + \gamma_1 \varepsilon_t + \gamma_2 \varepsilon_{t-1} + \ldots$

$\mathrm{E}_t \mathrm{y}_{t+1} = \gamma_0 E_t \varepsilon_{t+1} + \gamma_1 E_t \varepsilon_t + \gamma_2 E_t \varepsilon_{t-1} + \ldots$

$= \gamma_1 \varepsilon_t + \gamma_2 \varepsilon_{t-1} + \ldots$ (because ε has zero mean and is uncorrelated).

Substitution into the model gives:

$\gamma_0 \varepsilon_t + \gamma_1 \varepsilon_{t-1} + \gamma_2 \varepsilon_{t-2} + \ldots = \alpha(\gamma_1 \varepsilon_t + \gamma_2 \varepsilon_{t-1} + \ldots) + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

Equatings coefficients gives

$\gamma_0 = \alpha\gamma_1 + \theta_0$ (coeffients on ε_t)

$\gamma_1 = \alpha\gamma_2 + \theta_1$ (coeffients on ε_{t-1})

.

$\gamma_s = \alpha\gamma_{s+1} + \theta_s$ (coeffients on ε_{t-s})

which is the deterministic difference equation we solved in class. (Slide 5)