4. Impact of Monetary Shocks in Forward-Looking Models -- Open Economies

John B. Taylor, May 1, 2013

Consider a very simple small open economy monetary model ("Dornbusch model")

- Two variables, but only one "jump" variable
- Exchange rate (flexible) and price level (sticky)

$$m_t - p_t = -\alpha(E_t e_{t+1} - e_t)$$

$$p_t - p_{t-1} = \beta(e_t - p_t)$$

$$i_t = i_t^* + E_t e_{t+1} - e_t$$

$$m_t = \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i}$$

$$E\varepsilon_t = 0$$

$$E\varepsilon_t \varepsilon_s = 0 \quad for\ t \neq s$$

$$E\varepsilon_t^2 = \sigma^2$$

First put model into matrix form:

$$\begin{pmatrix} E_t e_{t+1} \\ p_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} e_t \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} m_t$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = (1+\beta)^{-1} \begin{pmatrix} 1+\beta(1+1/\alpha) & 1/\alpha \\ \beta & 1 \end{pmatrix}$$

$$\mathbf{d} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} -(1/\alpha) \\ 0 \end{pmatrix}$$

$$E_t \mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{d} m_t$$

To solve, look for solutions of the form:

$$e_t = \gamma_{10}\varepsilon_t + \gamma_{11}\varepsilon_{t-1} + \gamma_{12}\varepsilon_{t-2} + \cdots$$

$$p_t = \gamma_{20}\varepsilon_t + \gamma_{21}\varepsilon_{t-1} + \gamma_{22}\varepsilon_{t-2} + \cdots$$

and plug into basic model $\begin{pmatrix} E_t e_{t+1} \\ p_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} e_t \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} m_t$

$$e_{t+1} = \gamma_{10}\varepsilon_{t+1} + \gamma_{11}\varepsilon_t + \gamma_{12}\varepsilon_{t-1} + \cdots$$

$$E_t e_{t+1} = \gamma_{11}\varepsilon_t + \gamma_{12}\varepsilon_{t-1} + \cdots$$

$$p_{t-1} = \gamma_{20}\varepsilon_{t-1} + \gamma_{21}\varepsilon_{t-2} + \gamma_{22}\varepsilon_{t-3} + \cdots$$

to get :

…a deterministic vector difference equation:

$$\begin{pmatrix} \gamma_{11} \\ \gamma_{20} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \gamma_{10} \\ 0 \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Coefficients on ε_t

$$\begin{pmatrix} \gamma_{1,i+1} \\ \gamma_{2i} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \gamma_{1i} \\ \gamma_{2i-1} \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \rho^i \quad i = 1,2,\ldots$$

Coefficients on ε_{t-i}

which can be solved for the γ's

$$\boldsymbol{\gamma}_i = \mathbf{A}\boldsymbol{\gamma}_{i-1} + \mathbf{d}\rho^i \quad i = 1,2,\ldots$$

$$where\ \boldsymbol{\gamma}_i = \begin{pmatrix} \gamma_{1,i+1} \\ \gamma_{2i} \end{pmatrix}$$

Divide into homogeneous and particular solutions

$$\boldsymbol{\gamma}_i = \boldsymbol{\gamma}_i^{(H)} + \boldsymbol{\gamma}_i^{(P)}$$

$$\boldsymbol{\gamma}_i^{(H)} = \mathbf{H}\,\boldsymbol{\Lambda}\,\mathbf{H}^{-1}\boldsymbol{\gamma}_{i-1}^{(H)}$$

Λ is a diagonal matrix composed of the eigenvalues of A.

$$\mathbf{H}^{-1}\boldsymbol{\gamma}_i^{(H)} = \boldsymbol{\Lambda}\mathbf{H}^{-1}\boldsymbol{\gamma}_{i-1}^{(H)}$$

or

$$\boldsymbol{\mu}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_{i-1} \quad \mu_{1i} = \lambda_1\mu_{1,i-1} \quad \mu_{2i} = \lambda_2\mu_{2,i-1}$$

$$\text{let } \mathbf{H}^{-1} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$$

$$\text{then} \quad \mu_{1i} = g_{11}\gamma_{1,i+1}^{(H)} + g_{12}\gamma_{2i}^{(H)}$$

$$\text{and} \quad \mu_{2\mathrm{i}} = \mathrm{g}_{21}\gamma_{1,i+1}^{(\mathrm{H})} + g_{22}\gamma_{2i}^{(H)}$$

Guess form for particular solution

$$\gamma_{1i}^{(P)} = b_1 \rho^i$$

$$\gamma_{2i}^{(P)} = b_2 \rho^i$$

and plug into γ – equations to get

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \rho^2 - a_{11}\rho & -a_{12} \\ -a_{21}\rho & \rho - a_{22} \end{pmatrix}^{-1} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \rho$$

Particular solution now found from this

For stability we set

$$\mu_{10} = 0$$

$$\Rightarrow g_{11}\gamma_{11}^{(H)} + g_{12}\gamma_{20}^{(H)} = 0$$

$$\Rightarrow g_{11}(\gamma_{11} - b_1\rho) + g_{12}(\gamma_{20} - b_2) = 0$$

from the above equation we can now calculate

$$\gamma_{10} \quad \gamma_{11} \quad \gamma_{20}$$

and finally

$$\gamma_{2,\mathrm{i}+1}^{(\mathrm{H})} = \lambda_2 \gamma_{2i}^{(H)} \quad i = 0,1,\ldots$$

$$\gamma_{1,i+1}^{(H)} = -(g_{12} / g_{11})\gamma_{2i}^{(H)} \quad i = 1,2,\ldots$$

with $\gamma_{21}^{(H)} = \lambda_2(\gamma_{20} - \gamma_{20}^{(p)})$

Complete homogeneous solution here

Example

$$\alpha = .1,\ \beta = .1,\ \rho = 1$$

γ_{1i}	γ_{2i}
6.928	.623
3.232	.858
1.840	.947
1.316	.980
1.119	.992
Overshooting	Gradual price change

Response to a unanticipated, permanent increase in the money supply

$$m_t - p_t = -\alpha(E_t e_{t+1} - e_t)$$

$$p_t - p_{t-1} = \beta(e_t - p_t)$$

Larger models can be solved numerically, though economic intuition is easy to lose

$$f_i(y_t, y_{t-1}, \ldots E_t y_{t-p}, E_t y_{t+1}, \ldots E_t y_{t+q}, x_t) = u_{it} \quad i = 1, \ldots, n$$

where

y_t is a vector of endogenous variables

(like the exchange rate or the price level),

x_t is a vector of exogenous variables

(like the money supply)

u_{it} is a vector of random variables.