7. Staggered Price Setting and New Keynesian Economics

John B. Taylor, May 8, 2013

Outline

- Why Sticky Prices in Monetary Models?
  - From Keynesian to New Classical to New Keynesian
- Original staggered contract model
  - Derivation
  - Implications
- Generalizations and special cases
  - Calvo version
- New Keynesian Phillips Curve

Sticky Prices and/or Wages: An Old Topic in Monetary Economics

- Keynes: Labor demand $L^d(w/p)$ with fixed w
  - an increase in p lowers real wage, increases quantity of labor demanded ⇨ positive relation between p and L
  - But implied real wage was countercyclical, which it wasn't
- Phillips Curve
  - Prices or wages slowly adjust to excess demand $\pi = f(y-y^*)$
- Friedman-Phelps critique and expectations augmented Phillips curve: $\pi = \pi^e + f(y-y^*)$
- Lucas supply function: prices or wages perfectly flexible
  - New classical models
  - Only unanticipated changes in money matter ($\pi^e = E_t\pi$)
  - Monetary policy ineffectiveness (Sargent)
- Thus need RE model with price stickiness: But how?

The concept of staggered price setting

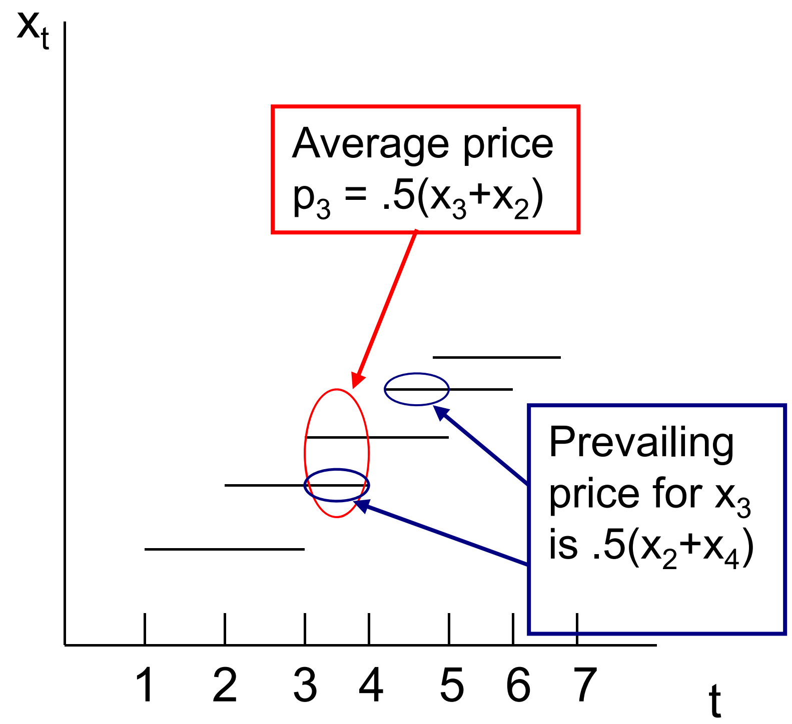
$$p_t = \frac{1}{2}(x_t + x_{t-1})$$

$$x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2}(E_{t-1}y_t + E_{t-1}y_{t+1}) + \varepsilon_t$$

Note: First term on right hand side of second equation can be written differently:

$$x_t = \frac{1}{2}(p_t + p_{t+1}) = \frac{1}{2}[\frac{1}{2}(x_t + x_{t-1}) + \frac{1}{2}(x_{t+1} + x_t)]$$

$$\Rightarrow x_t = \frac{1}{2}(x_{t-1} + x_{t+1})$$



Put equations into an economy wide model and solve

Model with money demand and policy rule

$y_t = \alpha(m_t - p_t) + v_t$ (from money demand)

$m_t = gp_t$ (monetary policy rule with $g < 1$)

$\Rightarrow y_t = -\beta p_t + v_t$ where $\beta = \alpha(1-g)$

which can be substituted into the staggered price equations

$$p_t = \frac{1}{2}(x_t + x_{t-1})$$

$$x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2}(E_{t-1}y_t + E_{t-1}y_{t+1}) + \varepsilon_t$$

to get :

$$x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2}\left[-\beta\left(\frac{E_{t-1}x_t + x_{t-1}}{2}\right) - \beta\left(\frac{E_{t-1}x_{t+1} + E_{t-1}x_t}{2}\right)\right] + \varepsilon_t$$

$$= \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) - \frac{\gamma\beta}{4}[E_{t-1}x_{t+1} + 2E_{t-1}x_t + x_{t-1}] + \varepsilon_t$$

One stochastic equation in one unknown

Now need to solve the model

$$x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) - \frac{\gamma\beta}{4}[E_{t-1}x_{t+1} + 2E_{t-1}x_t + x_{t-1}] + \varepsilon_t$$

for x_t.

Guess a solution of the form

$$x_t = ax_{t-1} + \varepsilon_t$$

where a must be determined.

Then $E_{t-1}x_t = ax_{t-1}$

and $E_{t-1}x_{t+1} = a^2 x_{t-1}$

which can be substituted back into the x equation to get :

$$ax_{t-1} + \varepsilon_t = \frac{1}{2}x_{t-1} + \frac{1}{2}a^2 x_{t-1} - \frac{\beta\gamma}{4}(a^2 x_{t-1} + 2ax_{t-1} + x_{t-1}) + \varepsilon_t$$

$$\Rightarrow a = \frac{1}{2} + \frac{a^2}{2} - \frac{\beta\gamma}{4}(a^2 + 2a + 1)$$

a quadratic in a which has solution :

$a = c \pm \sqrt{c^2 - 1}$ where $c = (1 + \beta\gamma/2)/(1 - \beta\gamma/2)$.

Clearly $c > 1$, so can chose stable root for uniqueness.

Example values
--recall $\beta=\alpha(1-g)$

a	$\beta\gamma$
1.00	.00
0.87	.01
0.75	.04
0.60	.13
0.29	.60

The complete solution: a stochastic process for p_t and y_t

Use the solution for x_t to get p_t

$x_t = ax_{t-1} + \varepsilon_t$

$x_{t-1} = ax_{t-2} + \varepsilon_{t-1}$

thus

$p_t = ap_{t-1} + .5(\varepsilon_t + \varepsilon_{t-1})$ (an ARMA(1,1) model)

$y_t = -\beta p_t + v_t$ where $\beta = \alpha(1-g)$

From these we can compute the variances of y_t and p_t and stipulate an objective function such as min $\lambda \operatorname{var}(p_t) + (1-\lambda)\operatorname{var}(y_t)$ and choose the value of g to minimize it.

Note that the infinite moving average representation is :

$p_t = .5(\varepsilon_t + \psi_1\varepsilon_{t-1} + \psi_2\varepsilon_{t-2} + \psi_3\varepsilon_{t-3} + ...)$

$y_t = v_t - .5\beta(\varepsilon_t + \psi_1\varepsilon_{t-1} + \psi_2\varepsilon_{t-2} + \psi_3\varepsilon_{t-3} + ...)$

where $\psi_i = a^{i-1}(1+a),\ i = 1,2,...$

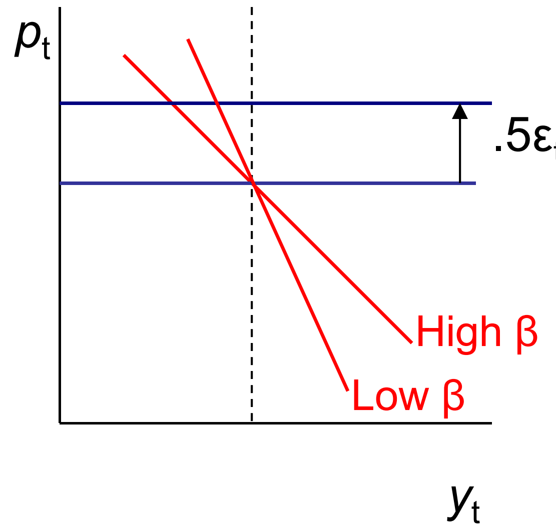
The Policy Tradeoff in a Staggered Pricing Model

An RE - Staggered Price Setting Model

$$p_t = \frac{1}{2}(x_t + x_{t-1})$$

$$x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2}(E_{t-1}y_t + E_{t-1}y_{t+1}) + \varepsilon_t$$

$y_t = -\beta p_t$ $\Leftarrow$ Can think of policy as choosing β



Solution :

$p_t = a p_{t-1} + .5(\varepsilon_t + \varepsilon_{t-1})$ (an ARMA(1,1) model in p_t)

where

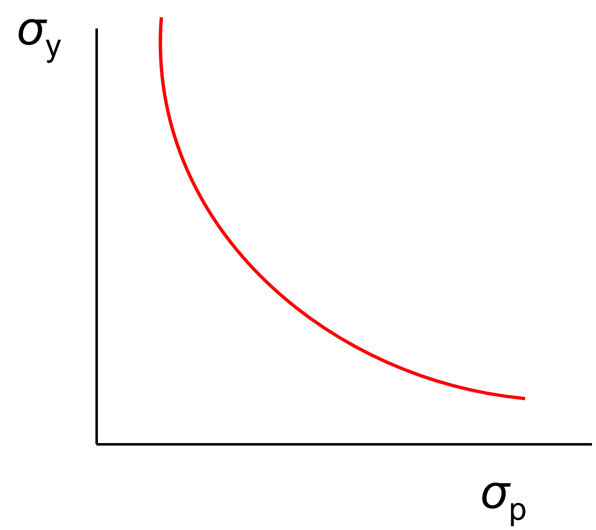
$a = c \pm \sqrt{c^2 - 1}$ with $c = (1 + \beta\gamma/2)/(1 - \beta\gamma/2)$.

Now from the formula for the variance of an ARMA (1,1) model we have

$\sigma_p^2 = .5\sigma_\varepsilon^2/(1-a)$ (See derivation on slide 13)

$\sigma_y^2 = \beta^2\sigma_p^2$

As the policy parameter β varies, the variances and standard deviations of p and y move in opposite directions.



Implications

- *Expectations of future inflation matter* for pricing decisions today.
- There is *inertia* in the inflation process
- The *inertia is longer than the length of the period during which prices are fixed*. (contract multiplier)
- The *degree of inertia or persistence depends on monetary policy*.
- The theory implies *a tradeoff curve between price stability and output stability*.

Briefly Compare with More General Model
"Aggregate Dynamics and Staggered Contracts," J.B.Taylor *JPE* 1980)

$$x_t = N^{-1}\sum_{i=0}^{N-1} E_t(p_{t+i} + \gamma y_{t+i} + \varepsilon_{t+i})$$

$$p_t = N^{-1}\sum_{i=0}^{N-1} x_{t-i}$$

- For empirical work you need to go beyond the stylized assumption

$$x_t = \sum_{i=0}^{N-1} \theta_{it} E_t(p_{t+i} + \gamma y_{t+i} + \varepsilon_{t+i})$$

$$p_t = \sum_{i=0}^{N-1} \delta_{it} x_{t-i}$$

Guillermo Calvo version of staggered price setting

$$x_t = (1-\beta\omega)\sum_{i=0}^{\infty}(\beta\omega)^i E_t(p_{t+i} + \gamma y_{t+i} + \varepsilon_t)$$

$$p_t = (1-\omega)\sum_{i=0}^{\infty}\omega^i x_{t-i}$$

Possible random price setting times, but still "time dependent," not "state dependent".

These two equations can be rewritten as

$$x_t = \beta\omega E_t x_{t+1} + (1-\beta\omega)(p_t + \gamma y_t + \varepsilon_t)$$

$$p_t = \omega p_{t-i} + (1-\omega)x_t$$

Once a model for y and the impact of monetary policy is added, you have a well-defined RE model as before.

- The two equations can also be re-written in an interesting form:

$$\pi_t = \beta E_t \pi_{t+1} + \delta\gamma y_t + \delta\varepsilon_t$$

where

$$\delta = \left[\frac{(1-\omega)(1-\beta\omega)}{\omega}\right]$$

which has become a popular way to write the model--new Keynesian Phillips Curve

Derivation of the simple aggregate equation

$$x_t = (1-\beta\omega)\sum_{i=0}^{\infty}(\beta\omega)^i E_t(p_{t+i} + \gamma y_{t+i} + \varepsilon_t)$$

$$p_t = (1-\omega)\sum_{i=0}^{\infty}\omega^i x_{t-i}$$

Note use of the lead and lag operator, L^{-1} and L

$$x_t = (1-\beta\omega)\sum_{i=0}^{\infty}(\beta\omega L^{-1})^i(p_t + \gamma y_t + \varepsilon_t)$$

$$x_t = \frac{(1-\beta\omega)}{(1-\beta\omega L^{-1})}(p_t + \gamma y_t + \varepsilon_t)$$

$$p_t = (1-\omega)\sum_{i=0}^{\infty}(\omega L^i)x_t$$

$$p_t = \frac{(1-\omega)}{(1-\omega L)}x_t$$

$$\begin{cases} x_t = \beta\omega E_t x_{t+1} + (1-\beta\omega)(p_t + \gamma y_t + \varepsilon_t) \\ p_t = \omega p_{t-i} + (1-\omega)x_t \end{cases}$$

$$p_t = \frac{(1-\omega)(1-\beta\omega)}{(1-\omega L)(1-\beta\omega L^{-1})}(p_t + \gamma y_t + \varepsilon_t)$$

$$(1-\omega L - \beta\omega L^{-1} + \beta\omega^2 - (1-\omega-\beta\omega+\beta\omega^2))\,p_t = (1-\omega)(1-\beta\omega)(\gamma y_t + \varepsilon_t)$$

$$\omega(p_t - p_{t-1}) = \beta\omega(E_t p_{t+1} - p_t) + (1-\omega)(1-\beta\omega)(\gamma y_t + \varepsilon_t)$$

Now set $\pi_t = p_t - p_{t-1}$,
divide by ω, and
define δ to get the simple form

$$\begin{cases} \pi_t = \beta E_t \pi_{t+1} + \delta\gamma y_t + \delta\varepsilon_t \\ where \\ \delta = \left[\dfrac{(1-\omega)(1-\beta\omega)}{\omega}\right] \end{cases}$$

Derivation of the expression on slide 8

To derive the variance of the price level on slide 8 of lecture 7, recall that for a general ARMA(1,1) process

$x_t = \alpha x_{t-1} + u_t - \theta u_{t-1}$ with $\mathrm{Eu_t} = 0$ and $\mathrm{var}(u_t) = \tau^2$

the variance of $\mathrm{x_t}$ is given by this formula

$$\mathrm{Var}(\mathrm{x}_t) = \frac{\tau^2(1+\theta^2 - 2\alpha\theta)}{1-\alpha^2}$$

The ARMA model derived in class on May 9 is

$p_t = ap_t + .5(\varepsilon_t + \varepsilon_{t-1})$ with $\mathrm{var}(\varepsilon_t) = \sigma_\varepsilon^2$

thus setting $\alpha = a$ and $\theta = \text{-}1$ and $\mathrm{u_t} = .5\varepsilon_t$ we get

$$Var(p_t) = \frac{.25\sigma_\varepsilon^2(1+(-1)^2 + 2a)}{1-a^2} = \frac{.5\sigma_\varepsilon^2(1+a)}{(1-a)(1+a)} = \frac{.5\sigma_\varepsilon^2}{1-a}$$

which is the expression on slide 8.