7. Staggered Price Setting and New Keynesian Economics

John B. Taylor, May 8, 2013
Outline

• Why Sticky Prices in Monetary Models?
  – From Keynesian to New Classical to New Keynesian

• Original staggered contract model
  – Derivation
  – Implications

• Generalizations and special cases
  – Calvo version

• New Keynesian Phillips Curve
Sticky Prices and/or Wages: An Old Topic in Monetary Economics

- Keynes: Labor demand $L^d(w/p)$ with fixed $w$
  - an increase in $p$ lowers real wage, increases quantity of labor demanded, positive relation between $p$ and $L$
  - But implied real wage was countercyclical, which it wasn’t
- Phillips Curve
  - Prices or wages slowly adjust to excess demand $\pi = f(y-y^*)$
- Friedman-Phelps critique and expectations augmented Phillips curve: $\pi = \pi^e + f(y-y^*)$
- Lucas supply function: prices or wages perfectly flexible
  - New classical models
  - Only unanticipated changes in money matter ($\pi^e = E_t\pi$)
  - Monetary policy ineffectiveness (Sargent)
- Thus need RE model with price stickiness: But how?
The concept of staggered price setting

\[ p_t = \frac{1}{2} (x_t + x_{t-1}) \]

\[ x_t = \frac{1}{2} (x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2} (E_{t-1}y_t + E_{t-1}y_{t+1}) + \varepsilon_t \]

Note: First term on right hand side of second equation can be written differently:

\[ x_t = \frac{1}{2} (p_t + p_{t+1}) = \frac{1}{2} \left[ \frac{1}{2} (x_t + x_{t-1}) + \frac{1}{2} (x_{t+1} + x_t) \right] \]

\[ \Rightarrow x_t = \frac{1}{2} (x_{t-1} + x_{t+1}) \]
Put equations into an economy wide model and solve

Model with money demand and policy rule

\[ y_t = \alpha(m_t - p_t) + v_t \quad \text{(from money demand)} \]

\[ m_t = gp_t \quad \text{(monetary policy rule with } g < 1) \]

\[ \Rightarrow y_t = -\beta p_t + v_t \quad \text{where } \beta = \alpha(1 - g) \]

which can be substituted into the staggered price equations

\[ p_t = \frac{1}{2}(x_t + x_{t-1}) \]

\[ x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2}(E_{t-1}y_t + E_{t-1}y_{t+1}) + \varepsilon_t \]

to get:

\[ x_t = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2}\left[-\beta\left(\frac{E_{t-1}x_t + x_{t-1}}{2}\right) - \beta\left(\frac{E_{t-1}x_{t+1} + E_{t-1}x_t}{2}\right)\right] + \varepsilon_t \]

\[ = \frac{1}{2}(x_{t-1} + E_{t-1}x_{t+1}) - \frac{\gamma\beta}{4}\left[E_{t-1}x_{t+1} + 2E_{t-1}x_t + x_{t-1}\right] + \varepsilon_t \]
One stochastic equation in one unknown

Now need to solve the model

\[ x_t = \frac{1}{2} (x_{t-1} + E_{t-1}x_{t+1}) - \frac{\gamma \beta}{4} [E_{t-1}x_{t+1} + 2E_{t-1}x_t + x_{t-1}] + \varepsilon_t \]

for \( x_t \).

Guess a solution of the form

\[ x_t = ax_{t-1} + \varepsilon_t \]

where \( a \) must be determined.

Then \( E_{t-1}x_t = ax_{t-1} \)

and \( E_{t-1}x_{t+1} = a^2 x_{t-1} \)

which can be substituted back into the \( x \) equation to get:

\[ ax_{t-1} + \varepsilon_t = \frac{1}{2} x_{t-1} + \frac{1}{2} a^2 x_{t-1} - \frac{\beta \gamma}{4} (a^2 x_{t-1} + 2ax_{t-1} + x_{t-1}) + \varepsilon_t \]

\[ \Rightarrow a = \frac{1}{2} + \frac{a^2}{2} - \frac{\beta \gamma}{4} (a^2 + 2a + 1) \]

a quadratic in \( a \) which has solution:

\[ a = c \pm \sqrt{c^2 - 1} \text{ where } c = \left(1 + \frac{\beta \gamma}{2}\right)/(1 - \frac{\beta \gamma}{2}). \]

Clearly \( c > 1 \), so can chose stable root for uniqueness.

Example values --recall \( \beta = a(1-g) \)

<table>
<thead>
<tr>
<th>( a )</th>
<th>( \beta \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>.00</td>
</tr>
<tr>
<td>0.87</td>
<td>.01</td>
</tr>
<tr>
<td>0.75</td>
<td>.04</td>
</tr>
<tr>
<td>0.60</td>
<td>.13</td>
</tr>
<tr>
<td>0.29</td>
<td>.60</td>
</tr>
</tbody>
</table>
The complete solution: a stochastic process for $p_t$ and $y_t$

Use the solution for $x_t$ to get $p_t$

$$x_t = ax_{t-1} + \varepsilon_t$$

$$x_{t-1} = ax_{t-2} + \varepsilon_{t-1}$$

thus

$$p_t = ap_{t-1} + .5(\varepsilon_t + \varepsilon_{t-1}) \quad \text{(an ARMA(1,1) model)}$$

$$y_t = -\beta p_t + \nu_t \quad \text{where } \beta = \alpha(1 - g)$$

From these we can compute the variances of $y_t$ and $p_t$ and stipulate an objective function such as $\min \lambda \text{ var}(p_t) + (1 - \lambda) \text{ var}(y_t)$ and choose the value of $g$ to minimize it.

Note that the infinite moving average representation is:

$$p_t = .5(\varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \psi_3 \varepsilon_{t-3} + ...)$$

$$y_t = \nu_t - .5 \beta(\varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \psi_3 \varepsilon_{t-3} + ...)$$

where $\psi_i = a^{i-1}(1 + a)$, $i = 1, 2, ...$
The Policy Tradeoff in a Staggered Pricing Model

An RE - Staggered Price Setting Model

\[ p_t = \frac{1}{2} (x_t + x_{t-1}) \]

\[ x_t = \frac{1}{2} (x_{t-1} + E_{t-1}x_{t+1}) + \frac{\gamma}{2} (E_{t-1}y_t + E_{t-1}y_{t+1}) + \varepsilon_t \]

\[ y_t = -\beta p_t \quad \text{Can think of policy as choosing } \beta \]

Solution:

\[ p_t = ap_{t-1} + .5(\varepsilon_t + \varepsilon_{t-1}) \quad \text{(an ARMA}(1,1)\text{ model in } p_t) \]

where

\[ a = c \pm \sqrt{c^2 - 1} \text{ with } c = (1 + \beta \gamma / 2) / (1 - \beta \gamma / 2). \]

Now from the formula for the variance of an ARMA (1,1) model we have

\[ \sigma_p^2 = .5\sigma_{\varepsilon}^2 / (1 - a) \]

(See derivation on slide 13)

\[ \sigma_y^2 = \beta^2 \sigma_p^2 \]

As the policy parameter \( \beta \) varies, the variances and standard deviations of \( p \) and \( y \) move in opposite directions.
Implications

- *Expectations of future inflation matter* for pricing decisions today.
- There is *inertia* in the inflation process.
- The *inertia is longer than the length of the period during which prices are fixed.* (contract multiplier)
- The *degree of inertia or persistence depends on monetary policy.*
- The theory implies a *tradeoff curve between price stability and output stability.*
Briefly Compare with More General Model

\[ x_t = N^{-1} \sum_{i=0}^{N-1} E_t(p_{t+i} + \gamma y_{t+i} + \varepsilon_{t+i}) \]

\[ p_t = N^{-1} \sum_{i=0}^{N-1} x_{t-i} \]

- For empirical work you need to go beyond the stylized assumption

\[ x_t = \sum_{i=0}^{N-1} \theta_{it} E_t(p_{t+i} + \gamma y_{t+i} + \varepsilon_{t+i}) \]

\[ p_t = \sum_{i=0}^{N-1} \delta_{it} x_{t-i} \]
Guillermo Calvo version of staggered price setting

\[ x_t = (1 - \beta \omega) \sum_{i=0}^{\infty} (\beta \omega)^i E_t (p_{t+i} + \gamma y_{t+i} + \varepsilon_i) \]

\[ p_t = (1 - \omega) \sum_{i=0}^{\infty} \omega^i x_{t-i} \]

These two equations can be rewritten as

\[ x_t = \beta \omega E_t x_{t+1} + (1 - \beta \omega) (p_t + \gamma y_t + \varepsilon) \]

\[ p_t = \omega p_{t-i} + (1 - \omega) x_t \]

Once a model for \( y \) and the impact of monetary policy is added, you have a well-defined RE model as before.

- The two equations can also be rewritten in an interesting form:

\[ \pi_t = \beta E_t \pi_{t+1} + \delta \gamma y_t + \delta \varepsilon, \]

where

\[ \delta = \left[ \frac{(1 - \omega)(1 - \beta \omega)}{\omega} \right] \]

which has become a popular way to write the model - new Keynesian Phillips Curve
Derivation of the simple aggregate equation

\[ x_t = (1 - \beta \omega) \sum_{i=0}^{\infty} (\beta \omega)^i E_t \left( p_{t+i} + \gamma y_{t+i} + \epsilon_t \right) \]

\[ p_t = (1 - \omega) \sum_{i=0}^{\infty} \omega^i x_{t-i} \]

\[ x_t = (1 - \beta \omega) \sum_{i=0}^{\infty} (\beta \omega L^{-1})^i \left( p_t + \gamma y_t + \epsilon_t \right) \]

\[ x_t = \frac{(1 - \beta \omega)}{(1 - \beta \omega L^{-1})} \left( p_t + \gamma y_t + \epsilon_t \right) \]

\[ p_t = (1 - \omega) \sum_{i=0}^{\infty} (\omega L^i) x_t \]

\[ p_t = \frac{(1 - \omega)}{(1 - \omega L)} x_t \]

\[ p_t = \frac{(1 - \omega)(1 - \beta \omega)}{(1 - \omega L)(1 - \beta \omega L^{-1})} \left( p_t + \gamma y_t + \epsilon_t \right) \]

\[ (1 - \omega L - \beta \omega L^{-1} + \beta \omega^2 - (1 - \omega - \beta \omega + \beta \omega^2)) p_t = (1 - \omega)(1 - \beta \omega)(\gamma y_t + \epsilon_t) \]

\[ \omega(p_t - p_{t-1}) = \beta \omega (E_t p_{t+1} - p_t) + (1 - \omega)(1 - \beta \omega)(\gamma y_t + \epsilon_t) \]

Now set \( \pi_t = p_t - p_{t-1} \), divide by \( \omega \), and define \( \delta \) to get the simple form

\[ \pi_t = \beta E_t \pi_{t+1} + \delta \gamma y_t + \delta \epsilon_t \]

where

\[ \delta = \left[ \frac{(1 - \omega)(1 - \beta \omega)}{\omega} \right] \]
Derivation of the expression on slide 8

To derive the variance of the price level on slide 8 of lecture 7, recall that for a general ARMA(1,1) process

\[ x_t = \alpha x_{t-1} + u_t - \theta u_{t-1} \text{ with } \mathbb{E}u_t = 0 \text{ and } \text{var}(u_t) = \tau^2 \]

the variance of \( x_t \) is given by this formula

\[ \text{Var}(x_t) = \frac{\tau^2(1 + \theta^2 - 2\alpha\theta)}{1 - \alpha^2} \]

The ARMA model derived in class on May 9 is

\[ p_t = ap_t + .5(\varepsilon_t + \varepsilon_{t-1}) \text{ with } \text{var}(\varepsilon_t) = \sigma^2_{\varepsilon} \]

thus setting \( \alpha = a \) and \( \theta = -1 \) and \( u_t = .5\varepsilon_t \) we get

\[ \text{Var}(p_t) = \frac{.25\sigma^2_{\varepsilon}(1 + (-1)^2 + 2a)}{1 - a^2} = \frac{.5\sigma^2_{\varepsilon}(1 + a)}{(1 - a)(1 + a)} = \frac{.5\sigma^2_{\varepsilon}}{1 - a} \]

which is the expression on slide 8.