PRINCIPLES OF COMPARATIVE POLITICS

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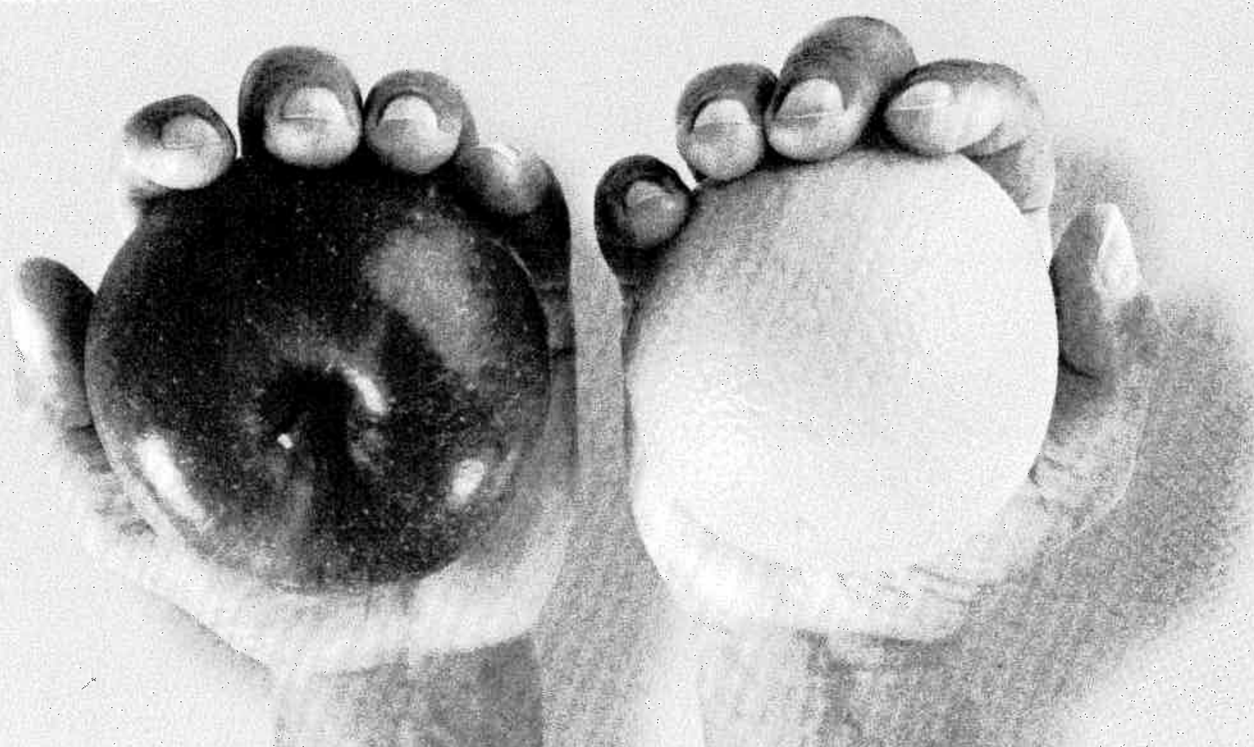


10 Democracy and Its Varieties

Democracy is the recurrent suspicion that more than half of the people are right more than half of the time.

E. B. White, *New Yorker*, July 3, 1944

OVERVIEW

- In this chapter, we look at whether the actual process by which democracies make decisions has appealing features that make it morally or normatively attractive above and beyond any material benefits that it might produce.
- At its very heart, democracy is a system in which the majority is supposed to rule. Still, making decisions that reflect the preferences of a majority can be a lot more complicated and less fair than one might think. Even if all the members of a group have rational preferences, Condorcet's Paradox shows that it might be impossible to reach a stable group decision using majority rule.
- The Median Voter Theorem indicates that stable group decisions can be achieved if we are willing to rule certain preferences out of bounds and reduce the policy space to a single-issue dimension. Unfortunately, neither of these restrictions is uncontroversial.
- Arrow's Theorem proves that no method of group decision making can guarantee a stable group decision while simultaneously satisfying minimal conditions of fairness. In effect, it proves that there is no perfect set of decision-making institutions; either fairness is compromised or there will be a potential for unstable group choices.
- The absence of an ideal decision-making mechanism means that institutional choice is an exercise in the choice of "second bests" and that trade-offs will need to be made. This helps to explain why we observe so many different types of democracy in the world.

What kinds of political institutions should we adopt when we have an opportunity to set up new institutions or change existing ones? One obvious way to begin answering this question is to see which sets of institutions produce good outcomes. For example, do democratic regimes produce better material outcomes, say, than dictatorial ones? If they do, then we should recommend adopting some set of democratic institutions. Although it is commonly believed that democracies outperform dictatorships in regard to providing material well-being, our analysis in Chapter 9 suggests that things are not this simple. As we demonstrated in that chapter, citizens living in certain types of dictatorship tend, on average, to enjoy a very high quality of life that easily rivals that of citizens in democracies, at least according to various measures of economic development and growth or the provision of government services, such as health care and education.[1] Indeed, several dictatorships regularly outperform many democracies when it comes to the measures of material outcomes that we examined.

At this point, the philosophy students among you might point out that we are overlooking some important criteria with which to evaluate sets of political institutions. Political philosophers usually use one of two broad approaches for evaluating the moral or ethical value of adopting a given set of institutions. On the one hand, those interested in **consequentialist ethics** ask whether the institutions in question produce good outcomes. This is the approach that we adopted in the previous chapter. On the other hand, those interested in **deontological ethics** attempt to evaluate institutions in a way that is independent of the outcomes that those institutions produce—they ask whether the institutions are good, fair, or just, in and of themselves. In this chapter, we consider democracy from a deontological perspective. In particular, we examine whether the actual process by which democratic governments make decisions for the entire country has appealing properties that make it morally or normatively attractive above and beyond any material benefits that it might produce. In other words, is the process by which decisions are made in democracies inherently good, fair, or just?

Consequentialist ethics evaluate actions, policies, or institutions in regard to the outcomes they produce. **Deontological ethics** evaluate the intrinsic value of actions, policies, or institutions in light of the rights, duties, or obligations of the individuals involved.

People typically assume that a dictatorial decision-making process is inherently unfair, whereas a democratic decision-making process is inherently fair. In what follows, we explore this assumption in some detail and lay out several criteria for evaluating how groups of individuals make decisions.[2] Overall, we think that the results of our analysis will surprise many

1. In Chapter 9, we focused on the average level of well-being in a society. An alternative approach would have been to see if democracies are characterized by a more equitable distribution of well-being than dictatorships. We could also have examined a whole host of other measures of government performance in addition to the ones that we focused on.

2. The material in this chapter comes from a part of the political science literature called social choice theory. Social choice theory addresses the voting rules that govern and describe how individual preferences are aggregated to form a collective group preference. Much of this literature is highly mathematical. In this chapter, we focus on providing you with the intuition behind some of the more important ideas in social choice theory. For those of you interested in examining social choice theory in more depth, we suggest starting with Sen (1970), Riker (1982), and Hinich and Munger (1997).

of you. As we demonstrate, there are no perfect decision-making processes—all institutional choices, including the decision to adopt democratic institutions, entail a set of significant trade-offs. It is the existence of these trade-offs that helps to explain, as we will see throughout Part III of this book, why there are so many different types of democracies in the world. In effect, different countries choose to make different trade-offs when they decide to adopt democratic institutions.

PROBLEMS WITH GROUP DECISION MAKING

Many people say that they like democracy because they believe it to be a fair way to make group decisions. One commonsense notion of fairness is that group decisions should reflect the preferences of the majority of group members. We believe that most people would probably agree, for example, that a fair way to decide between two options is to choose the option that is preferred by the most people. When selecting among just two options, the option preferred by the most members of a group is necessarily the option preferred by a majority of the group.[3] At its very heart, democracy is a system in which the majority rules. In this section of the chapter, we show that there are many situations in which "majority rule" is a lot more complicated and less fair than our commonsense intuition about it would suggest. In effect, we demonstrate that allowing the majority to decide can be deeply problematic on many different dimensions.

Majority Rule and Condorcet's Paradox

If a group of people needs to choose between just two options, then majority rule can be quite straightforward, as we just saw. But what if a group needs to choose between more than two options? For example, imagine a city council deciding on the level of social services it should provide.[4] The proposed options are to increase (I), decrease (D), or maintain current (C) levels of social service provision. Assume that the council is made up of three members—a left-wing councillor, a right-wing councillor, and a centrist councillor—who all rank the proposed options differently. Specifically, the left-wing councillor prefers an increase in spending to current levels of spending, and prefers current levels of spending to a decrease. The centrist councillor most prefers current levels of spending, but would prefer a decrease in spending over any increase if it came to it. The right-wing councillor most prefers a decrease in spending. Because he views current levels of spending as unsustainable, however, the right-wing councillor would prefer to "break the bank" with an increase in spending in order to spur much-needed reforms than maintain the status quo. The preference ordering for each of the council members is summarized in Table 10.1.

3. A majority of a group is defined here as more than half. If we have a group of size N, and we assume that none of the members of the group are indifferent between the two options on offer, then a majority M of the group would be any subgroup, such that $M \geq \frac{N+1}{2}$. If indifference were allowed, then it is possible for the alternative receiving the most votes to win a plurality without winning a majority of the votes.

4. This example comes from Dixit and Skeath (2004).

TABLE 10.1 City Council Preferences for the Level of Social Service Provision

Left-wing Councillors	Centrist Councillors	Right-wing Councillors
$I \succ C \succ D$	$C \succ D \succ I$	$D \succ I \succ C$

Note: I = increased social service provision; D = decreased social service provision; C =maintenance of current levels of social service provision; $\succ$ = "is strictly preferred to."

Let's assume that the council employs majority rule to make its group decisions. In this particular example, this means that any policy alternative that enjoys the support of two or more councillors will be adopted. How should the councillors vote, though? It's not obvious how they should vote given that there are more than two alternatives. One way they might proceed is to hold a **round-robin tournament** that pits each alternative against every other alternative in a set of "pair-wise votes"—*I* versus *D*, *I* versus *C*, and *C* versus *D*—and designates as the winner whichever alternative wins the most contests.[5] If we assume that the councillors all vote for their most preferred alternative in each pair-wise contest (or round), then we see that *D* defeats *I*, *I* defeats *C*, and *C* defeats *D*. The outcomes of these pair-wise contests and the majorities that produce them are summarized in Table 10.2. Notice that there is no alternative that wins most often—each alternative wins exactly one pair-wise contest. This multiplicity of "winners" does not provide the council with a clear policy direction. In other words, the council fails to reach a decision on whether to increase, decrease, or maintain current levels of social service provision.

A **round-robin tournament** pits each competing alternative against every other alternative an equal number of times in a series of pair-wise votes.

This simple example produces several interesting results that we now examine in more detail. The first is that a group of three *rational* actors (the councillors) make up a group (the council) that appears to be incapable of making a rational decision for the group as a whole.

TABLE 10.2 Outcomes from the Round-Robin Tournament

Round	Contest	Winner	Majority that produced victory
1	Increase vs. decrease	*D*	Centrist and right
2	Current vs. increase	*I*	Left and right
3	Current vs. decrease	*C*	Left and centrist

5. This voting method is also known as Copeland's rule (Riker 1982, 76).

What do we mean by "rational"? When political scientists use the word *rational*, they have a very specific meaning in mind. An actor is said to be **rational** if she possesses a complete and transitive preference ordering over a set of outcomes.[6] An actor has a **complete preference ordering** if she can compare each pair of elements (call them x and y) in a set of feasible outcomes in one of the following ways—either the actor prefers x to y, she prefers y to x, or she is indifferent between x and y. The assumption of completeness essentially states that an individual can always make up his mind as to whether he prefers one option or is indifferent when presented with a pair of options. What about the assumption of transitivity? Before we get to this, we need to first make a distinction between strict and weak preferences. An actor is said to "strictly prefer" x to y if x is always better than y. And he is said to "weakly prefer" x to y if x is at least as good as y. An actor has a **transitive preference ordering** if for any x, y, and z in the set of outcomes it is the case that if x is weakly preferred to y, and y is weakly preferred to z, then it must be the case that x is weakly preferred to z. Actors whose preference orderings do *not* meet these conditions—completeness and transitivity—are said to be irrational.

An actor is **rational** if she possesses a complete and transitive preference ordering over a set of outcomes. An actor has a **complete preference ordering** if she can compare each pair of elements (call them x and y) in a set of outcomes in one of the following ways—either the actor prefers x to y, y to x, or she is indifferent between them. An actor has a **transitive preference ordering** if for any x, y, and z in the set of outcomes it is the case that if x is weakly preferred to y, and y is weakly preferred to z, then it must be the case that x is weakly preferred to z.

In the example we have been examining, each of the councillors is rational because each has a complete and transitive preference ordering over the three policy alternatives. For example, the left-wing councillor prefers I to C and C to D, and also prefers I to D. The outcome of the round-robin tournament, however, reveals that this set of rational individuals becomes a group that acts like an individual with intransitive preferences. Recall that the group prefers D to I and I to C. Transitivity would require, therefore, that the group prefer D to C. Round three of the round-robin tournament reveals, however, that the group prefers C to D. This juxtaposition—of rational individuals forming a group that behaves irrationally—was first described in a paper in 1785 by Marie Jean Antoine Nicolas de Caritat, the Marquis de Condorcet, and is usually referred to as **Condorcet's Paradox.**

Condorcet's Paradox illustrates that a group composed of individuals with rational preferences does not necessarily have rational preferences as a collectivity; individual rationality is not sufficient to ensure group rationality.

A second interesting aspect of our example is that a different majority supports the winning alternative or outcome in each round. In round one, the majority that votes in favor of a decrease in social service provision is made up of the centrist and right-wing councillors. In round two, the majority that votes in favor

6. Technically, a rational individual must also have a "reflexive" preference ordering. This means that any alternative in the set of outcomes can be thought of as at least as good as itself. As the Nobel Prize-winning economist Amartya Sen (1970, 2–3) has noted, the reflexivity "requirement is so mild that it is best looked at as a condition . . . of sanity rather than of rationality." Most political scientists focus on the conditions of completeness and transitivity as we do here.

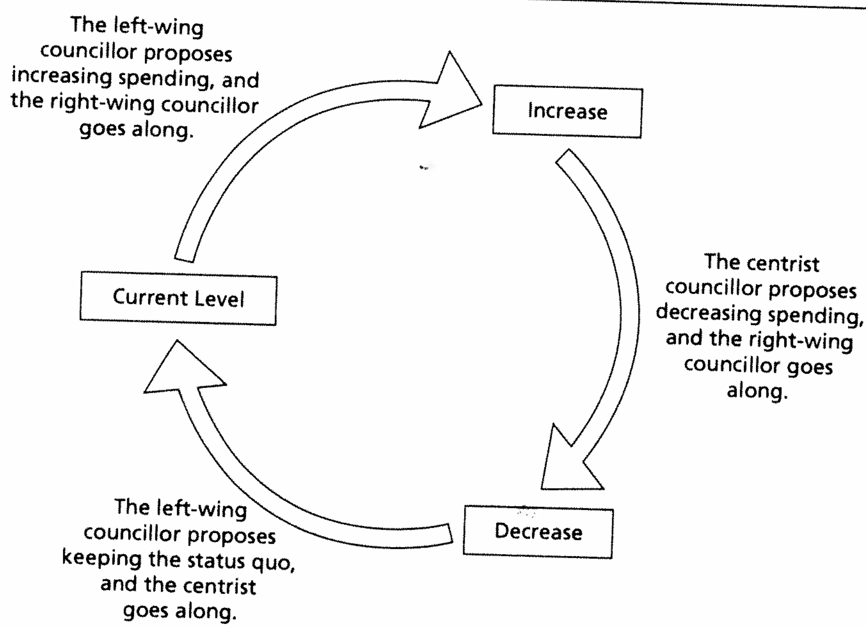
of an increase in social service provision is a coalition of "odd bedfellows" comprising the left- and right-wing councillors. Finally, in round three, the majority that votes in favor of the status quo comprises the left-wing and centrist councillors. Although "letting the majority decide" may sound fair and straightforward, our example here makes it very clear that "a majority" does not necessarily exist until the policy debate is framed in a certain way.

As we just noted, Condorcet's Paradox points out that individual rationality is not sufficient to ensure group rationality. A set of actors, each with complete and transitive preference orderings, may behave in a way that reveals group intransitivity. When this occurs, there is no "majority" to speak of; instead, there is a cycle of different majorities. For example, suppose that we start with the current level of social service provision as the status quo. A council member who is unhappy with this status quo (say, the left-wing councillor) might propose a vote like that in round two of the round-robin tournament (*C* vs. *I*). In this vote, a majority would support an increase in social service provision over the maintenance of the status quo (see Table 10.2). But as soon as this vote ends, a disgruntled council member (say, the centrist) can propose a vote like that in round one of the round-robin tournament (*I* vs. *D*). In this vote, a majority would support a decrease in social service provision over an increase. But as soon as this vote ends, a different disgruntled council member—in this case, the left-wing councillor—can propose a vote like that in round three of the round-robin tournament (*C* vs. *D*). In this vote, a majority would support maintaining the current level of social service provision rather than decreasing it. Interestingly, the centrist who just one vote ago proposed decreasing the provision of social services now votes to maintain current levels of social service provision. Having arrived back where they began (*C*), and absent any institutional mechanism to end the succession of proposals and counter-proposals, the scene is set for the cycle to begin anew—with no end in sight. This cycle through the different majorities is illustrated in Figure 10.1.

Some of you may feel that our example of cyclical majorities is unlikely ever to occur in practice. After all, we see deliberative bodies make decisions all the time and, although we may sometimes think they are far from efficient, they do not seem to be caught in the type of endless cycle suggested by our example. On the whole, there are two broad reasons for this. One has to do with preference orderings and the other has to do with decision-making rules. Let's start with preference orderings. To some extent, our current example is special in the sense that it depends on the councillors having a particular set of preference orderings. For example, if the right-wing councillor's preferences were the mirror image of those of the left-wing councillor—that is, if the right-wing council member preferred a decrease to the current level but preferred the current level to an increase—then maintaining current levels of social service provision would win both rounds of the round-robin tournament in which it competes. In political science, we call an option like this—that is, one that beats all other options in a series of pair-wise contests—a **Condorcet winner.** Given the decision-making process adopted by the council, and assuming that all councillors vote for their most preferred option, this would mean that maintaining current levels of social service provision is a stable outcome.

An option is a **Condorcet winner** if it beats all other options in a series of pair-wise contests.

FIGURE 10.1 An Example of Cyclical Majorities



To see why, imagine that maintaining current spending on social services is the status quo and ask yourself who would benefit from a change? The answer is that both the left- and right-wing councillors would like to propose a change. The right-wing council member prefers a decrease in social service provision to the status quo. If he proposed a decrease, however, both the centrist and left-wing councillors would vote against the proposal. Similarly, the left-wing council member prefers an increase in social service provision to the status quo. But if he proposed an increase, both the centrist and right-wing councillors would vote against the proposal. In other words, with this new profile of preferences in the group, there is no cycle of majorities, and, as a result, current levels of spending constitute a stable outcome. In effect, the group now behaves as if it were an individual with transitive (and complete) preferences—it prefers current levels of social service provision to a decrease, and a decrease to an increase.

The point here is that majority rule is not necessarily incompatible with rational group preferences. All that Condorcet showed was that it is *possible* for a group of individuals with transitive preferences to produce a group that behaves as if it has intransitive preferences. As a result, Condorcet's Paradox erodes our confidence in the ability of majority rule to produce stable outcomes only to the extent that we expect actors to hold the preferences that cause group intransitivity. So how likely is it that transitive individual preferences will lead to group intransitivity? Modern scholars have analyzed this problem in detail and found that the likelihood of group intransitivity increases with the number of alternatives under consideration or the number of voters, or both. In Table 10.3, we show estimates of the share of all possible strict preference orderings that fail to produce a Condorcet winner (that is, that produce group intransitivity) as the numbers of voters and alternatives increase (Riker 1982, 122).

As Table 10.3 illustrates, the example of the city council that we started with, in which a Condorcet winner fails to emerge from a contest among three alternatives and three voters, is indeed a rarity. Nearly all (94.4 percent) of the logically possible strict preference orderings produce a Condorcet winner and, hence, a stable outcome. As the number of voters increases, however, the probability of group intransitivity rises to some limit. When the number of alternatives is relatively small, this limit is still small enough that most of the logically possible preference orderings will not lead to group intransitivity. In contrast, although an increase in the number of alternatives also increases the probability of group intransitivity, this process continues until the point at which group intransitivity is certain to occur. In other words, as the number of alternatives goes to infinity, the probability of group intransitivity converges to one—even when the number of voters is small. This is an extremely important result because many political decisions involve a choice from, essentially, an infinite number of alternatives.

Imagine, for example, what would happen if we introduced a bit more realism into our example about a city council deciding on social welfare spending. Previously, we simplified the situation to one in which the councillors were deciding between three alternatives—increase, decrease, or maintain current spending. In reality, though, the councillors would normally be choosing an exact amount of money to spend on social services. In effect, they would be choosing a share of the budget to allocate to social service provision from 0 percent to 100 percent. Thus, there are an infinite number of choices that could be made in this

Table 10.3 Proportion of Possible Strict Preference Orderings without a Condorcet Winner

	Number of voters					
Number of alternatives	**3**	**5**	**7**	**9**	**11**	**→ ... Limit**
3	0.056	0.069	0.075	0.078	0.080	0.088
4	0.111	0.139	0.150	0.156	0.160	0.176
5	0.160	0.200	0.215			0.251
6	0.202					0.315
↓						↓
Limit	1.000	1.000	1.000	1.000	1.000	1.000

Source: Riker (1982, 122)

interval (0–100).[7] As a consequence, if no restrictions are placed on the councillors' preferences, then group intransitivity is all but guaranteed. Significantly, all policy decisions that involve bargaining—questions relating to things like the distribution of government resources, the allocation of the tax burden, the allocation of ministerial portfolios in the government, and the location of toxic waste—can be seen in a similar light.

To summarize, Condorcet's Paradox makes it clear that restricting group decision making to sets of rational individuals is no guarantee that the group as a whole will exhibit rational tendencies. Group intransitivity is unlikely when the set of feasible options is small, but it is almost certain that majority rule applied to a pair-wise competition among alternatives will fail to produce a stable outcome when the set of feasible options gets large. As a result, it is impossible to say that the majority "decides" except in very restricted circumstances.

The analytical insight from Condorcet's Paradox suggests that group intransitivity should be common, but, as we have already noted, we observe a surprising amount of stability in group decision making in the real world. Our discussion so far suggests that this must be the result of either of two factors. Either the number of decision makers or issues is kept small *and* the kinds of preferences that produce group intransitivity are rare, or a decision-making mechanism other than a simple pair-wise comparison of alternatives is being used. We have already seen that some of the most common types of political decisions involve a great number of alternatives, so it is likely that any stability we observe in the real world results from the use of alternative decision rules. It is to these alternative decision-making rules that we now turn.

The Borda Count and the Reversal Paradox

One alternative decision-making rule—the Borda Count—was suggested by Jean-Charles de Borda, a compatriot of Condorcet, in 1770 (published in 1781).[8] The Borda Count asks individuals to rank potential alternatives from their most to least preferred and then assigns numbers to reflect this ranking.[9] For instance, if there are three alternatives as in our city council example, then the Borda Count might assign a three to each councillor's most preferred option, a two to their second-best option, and a one to their least preferred option. The weighted votes for each alternative are then summed and the alternative with the largest score wins. Using the same preferences as shown earlier in Table 10.1, the Borda Count would again be indecisive in determining whether to increase, decrease, or maintain current levels of social service provision. This is because each alternative would garner a score of 6. This is shown in Table 10.4.

7. This is true in abstract games in which players bargain over the share of a "pie." In real life, budgets are denominated in currency and so the smallest accounting increment (like a penny) can place a limit on the divisibility of the pie. What is important here, though, is that there are many, many possible outcomes, and group intransitivity is therefore nearly certain to occur. Political scientists and economists both use formal bargaining models to examine these types of situation. Interestingly, political scientists tend to bargain over a "pie," whereas economists tend to bargain over a "cake" (Morrow 1994, 346).

8. Two men other than Charles de Borda are thought to have independently come up with this decision-making mechanism. Recent evidence suggests that Ramon Llull (1232–1315) first discovered the Borda Count in the thirteenth century. The Borda Count was "reinvented" by Nicholas of Cusa (1401–1464) when he suggested (unsuccessfully) that it should be used to elect the Holy Roman Emperor in 1433.

9. We discuss the Borda Count again in Chapter 12 when we investigate electoral systems in more detail.

TABLE 10.4 Determining the Level of Social Service Provision Using the Borda Count

	Points awarded			
Alternative	Left-wing	Centrist	Right-wing	Borda Count total
Increase spending	3	1	2	6
Decrease spending	1	2	3	6
Current spending	2	3	1	6

Although the indecisiveness of the Borda Count is once again an artifact of the particular preference ordering we are examining,[10] a more troubling aspect of this decision rule can be seen if we consider the introduction of a possible fourth alternative. Let's assume, for example, that the councillors consider a new alternative: maintain current spending levels for another year (perhaps it's an election year) but commit future governments to a decrease in spending of, say, 10 percent in each successive year.[11] Suppose that the left-wing councillor likes this new option the least, the right-wing councillor prefers it to all alternatives except an immediate decrease, and the centrist councillor prefers all options except an increase to this new alternative. The preference ordering for each of the council members over the four alternatives is summarized in Table 10.5.

If we apply the Borda Count in this new situation by assigning a three to each councillor's most preferred alternative, a two to his second-best alternative, a one to his third-best alternative, and a zero to his least preferred alternative, then we find that the vote tally looks

TABLE 10.5 City Council Preferences for the Level of Social Service Provision (Four Alternatives)

Left-wing	Centrist	Right-wing
$I \succ C \succ D \succ FC$	$C \succ D \succ FC \succ I$	$D \succ FC \succ I \succ C$

Note: I = an increase in social service provision; D = a decrease in social service provision; C = a maintenance of current levels of social service provision; FC = future cuts in social service provision; $\succ$ = "is strictly preferred to."

10. We could, of course, conclude that the group actually is indifferent between these alternatives, given this aggregation of citizen preferences. Doing so, however, requires us to make what political scientists call "interpersonal comparisons of utility." For example, we would have to believe that the welfare improvement that a left-wing councillor feels when a decrease in social service provision is replaced by an increase is exactly equal to the sum of the decline in welfare experienced by the centrist and left-wing councillors when this happens. Most modern scholars are reluctant to make these types of interpersonal comparisons of utility and so would be reluctant to make normative statements about the appropriateness of this outcome.

11. This example is not as fanciful as it might sound. In fact, it shares many qualities with the "balanced budget" proposals of politicians who are all too eager to be "fiscally conservative" tomorrow (when an election is no longer looming).

like the one shown in Table 10.6. As you can see, the council now has a strict preference ordering over the alternatives. Based on their votes, the council would decrease the level of social service provision.

You will immediately notice that something very strange has happened. Despite the fact that the new alternative receives a lower score than all of the original options and that it is not the first choice of any of the councillors, its addition as an active alternative for consideration changes how the councillors, as a collectivity, rank the three original options. In doing so, it changes the outcome of the vote. Whereas the group had previously been "indifferent" between the three original options, it now possesses a strict and transitive preference ordering over them, with "decreased spending" as the group's "most preferred" outcome. Note that this is the case despite the fact that none of the councillors has changed the way that he rank orders *I*, *D*, and *C*. In effect, the choice that the council now makes has been influenced by the introduction of what might be called an "irrelevant alternative." As this example illustrates, the Borda Count does not demonstrate the property that political scientists refer to as the "independence from irrelevant alternatives."[12]

Many analysts find the susceptibility of the Borda Count to the introduction of what they consider "irrelevant alternatives" disconcerting. Note that in our city council example, there was no change in the individual preference ordering of any of the actors over the original three alternatives, and yet, the introduction of an "irrelevant alternative" had a marked effect on the outcome of the decision-making process. One reason why we might like a decision rule to be "independent from irrelevant alternatives" is that if it is not, wily politicians can more easily manipulate the outcome of a decision process in order to produce their most

TABLE 10.6 Determining the Level of Social Service Provision Using the Borda Count with a Fourth Alternative

	Points awarded			
Alternative	Left-wing	Centrist	Right-wing	Borda Count total
Increase spending	3	0	1	4
Decrease spending	1	2	3	6
Current spending	2	3	0	5
Future cuts in spending	0	1	2	3

12. Technically, the "independence from irrelevant alternatives" (IIA) property in the social choice literature refers to the independence from the "ranking" (and not the "presence") of an irrelevant alternative. This is the requirement that the ranking of an irrelevant alternative in a *fixed* set of alternatives should not affect the alternative that is chosen (Arrow 1963; Sen 1970). Our city council example can be understood in these terms too. For example, we can imagine that the city councillors all originally ranked the alternative of future spending cuts last but through some kind of deliberation process came to rank it in the way shown in Table 10-5. When the future spending cuts are ranked last, the council is indifferent between *D*, *I*, and *C*. But when the future spending cuts are ranked according to the preference orderings in Table 10-5, then the council has a strict preference ordering $D \succ C \succ I$.

preferred outcome. For example, instead of making persuasive arguments about the desirability of her preferred outcome or seeking compromise solutions that leave all parties better off, a politician might get her way by the imaginative introduction of an alternative that has no chance of winning, but that—by its sheer presence—changes the weights attached to other alternatives and, therefore, changes the alternative that is ultimately chosen.

Majority Rule with an Agenda Setter

An alternative decision-making mechanism that overcomes the potential instability of majority rule in round-robin tournaments requires actors to begin by considering only a subset of the available pair-wise alternatives. For instance, in our original city council example, we might require that both departures from the status quo (that is, increases and decreases in social service spending) first face each other in a pair-wise contest and that the winner then go on to compete in a vote against the status quo. Imposing a voting agenda such as this turns the voting process into a sequential game with three players—each player simultaneously chooses between increasing and decreasing social service provision, and then each player simultaneously chooses between the winning alternative from the first round and the maintenance of current spending levels in the second round.

Let's assume for a moment that each council member votes for her preferred option when confronted with any two choices. In other words, let's assume that each councillor casts what is known as a **sincere vote.** In the first round, the councillors choose between increasing social service spending and decreasing it. Given the preferences of the councillors in our example, we know that both the centrist and the right-wing council members prefer a decrease in spending over an increase. As a result, the vote in the first round would be 2-1 in favor of a decrease. This means that the second (and final) round of voting is a choice between decreasing social service spending and maintaining current levels of spending. Because the left-wing and centrist councillors both prefer the current level of social service spending to a decrease, the outcome of this game is the status quo; that is, current spending levels are maintained. (Stop for a moment and ask yourself which of the councillors is likely to have set this voting agenda.)

But should we expect all of the councillors to vote sincerely in our example? Consider that the councillors know that there are only two possible contests in the second round—either *D* vs. *C* or *I* vs. *C*. Given the preference orderings in our example, the councillors know that these potential second-round contests will either end up with *C* defeating *D* or *I* defeating *C*. It follows from this that the councillors know that if *D* wins in the first round against *I*, then the final outcome will be the status quo, *C*. In other words, voting for *D* in the first round is essentially equivalent to voting for the status quo in the end. As a result, the first round of voting should, in reality, be seen as a contest between *I* and *C* (even if the councillors are actually voting between *I* and *D*).

Think about how the right-wing councillor might reason through the logic of this voting procedure. Recall that her favorite outcome is a decrease in social service spending, her second-best outcome is an increase in social service spending, and her least preferred outcome is maintaining the current level of social service spending. If she casts her vote in the first round for her most preferred outcome without thinking about the consequences for the rest of the game, then we have already seen that option *D* will be victorious in the first round but will go on to lose to *C* in the second and final round. This is, of course, the right-wing councillor's worst possible outcome. As a result, she has a strong incentive to change her vote in the first round from *D* to *I* even though this new vote does not conform to her sincere preferences. If she does this and votes for an increase in social service spending in the first round, then *I* will win and be pitted against *C* in the final round. In this final round, *I* will defeat *C*. In other words, the final outcome will be an increase in social service provision. Note that by deviating from her sincere preferences in the first round, the right-wing councillor is able to alter the final outcome from her least preferred outcome to her second-best one. In this example, the right-wing councillor casts what political scientists call a **strategic, or sophisticated, vote**—a vote in which an individual votes in favor of a less preferred option because she believes doing so will ultimately produce a more preferred outcome than would otherwise be the case. Some analysts find strategic voting lamentable and would prefer decision rules that induce **sincere voting**—voting that constitutes a sincere revelation of an individual's preferences.[13]

A strategic, or sophisticated, vote is a vote in which an individual votes in favor of a less preferred option because she believes doing so will ultimately produce a more preferred outcome. A **sincere vote** is a vote for an individual's most preferred option.

The incentives to vote strategically are not the only thing that scholars find lamentable with voting agendas like the one that we just examined. Another thing that many scholars find disconcerting is that alternative agendas can produce very different outcomes even if we hold all of the actors' preferences constant. In fact, the three alternatives in our city council example can face each other in three different two-round tournaments, all of which produce a different outcome. The three different two-round tournaments and the outcomes that they produce are shown in Table 10.7. As you can see, choosing the agenda is essentially equivalent to choosing which outcome will win. For example, if you decide to have a first-round contest between *I* and *D*, you know that the eventual outcome will be a victory for *C*. If you decide to have a first-round contest between *C* and *I*, you know that the eventual outcome will be *D*. And if you decide to have a first-round contest between *C* and *D*, you know that the eventual outcome will be *I*. Consequently, if one of the councillors is given the power to choose the agenda, she is, effectively, given the power to dictate the outcome of the decision-making process. This phenomenon, in which choosing the agenda is tantamount to choosing which alternative will win, is referred to as the "power of the agenda setter" and it exists in many institutional settings. In our example, the agenda setter can obtain her most preferred outcome simply by deciding what the order of pair-wise contests should be. For example, the centrist councillor would choose agenda 1 in Table 10.7 if she were the agenda setter; the right-wing councillor would choose agenda 2; and the left-wing councillor would choose agenda 3.

13. We return to a more detailed discussion of sincere and strategic voting in Chapters 12 and 13.

Table 10.7 Pair-Wise Contests and Different Voting Agendas

Agenda	1st round	1st-round winner	2nd round	2nd-round winner	Councillor obtaining her most preferred outcome
1	*I* vs. *D*	*D*	*D* vs. *C*	*C*	Centrist councillor
2	*C* vs. *I*	*I*	*I* vs. *D*	*D*	Right-wing councillor
3	*C* vs. *D*	*C*	*C* vs. *I*	*I*	Left-wing councillor

Note: I = an increase in social service provision; *D* = a decrease in social service provision; *C* = a maintenance of current levels of social service provision.

In sum, it is possible to avoid the potential for group intransitivity that arises in majority rule round-robin tournaments by imposing an agenda—by designating which outcomes will be voted on first and which outcome will, in effect, be granted entry into a second round, in which it will compete against the winner of the first round. Unfortunately, the outcome of such a process is extremely sensitive to the agenda chosen and, consequently, either of two things is likely to happen. Either the instability of group decision making shifts from votes on outcomes to votes on the agendas expected to produce those outcomes, or some subset of actors is given power to control the agenda and, therefore, given considerable influence over the outcome likely to be produced.

Restrictions on Preferences: The Median Voter Theorem

As was the case with the Borda Count, it appears that institutional factors restricting the agenda may produce stable outcomes, but only at the expense of creating incentives for actors to attempt to manipulate the decision-making process. This obviously raises questions about our ability to design our way around the instability of majority rule in such a way that the cure is not worse than the disease. In the next section of this chapter, we discuss important work that suggests that it may be impossible to design a decision-making mechanism that ensures group transitivity while simultaneously meeting minimal criteria for fairness. But before we get there, let's pause briefly to consider the behavior of majority rule in one more special case.

Recall that group intransitivity in our original city council example seemed to stem from the fact that the right-wing councillor has a particular type of preference ordering. Specifically, the right-wing council member prefers both lower and higher levels of social service provision to the maintenance of current levels of social service provision. The right-wing councillor's most preferred option is to decrease social service spending. But if it came to it, she would rather increase spending in the hope that this would "break the bank" and force the city to adjust to lower levels of spending in the future than maintain current levels of spending (which she thinks are too high) that would slowly bleed the city dry.

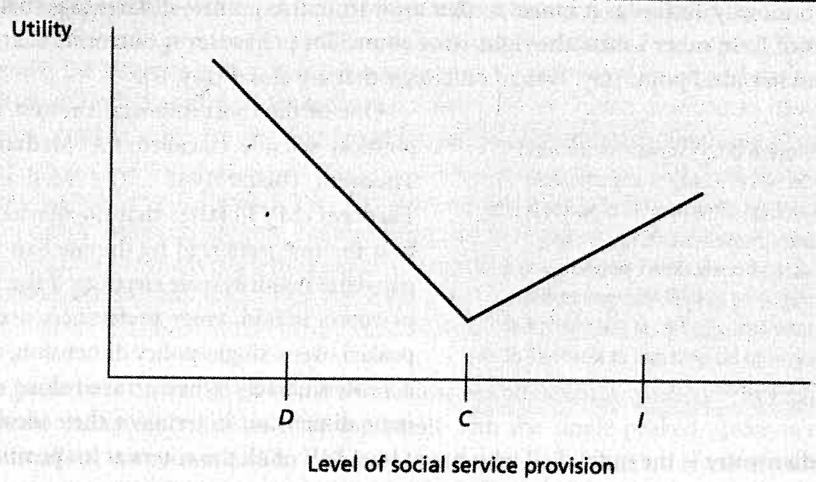
A **utility function** is essentially a numerical scaling in which higher numbers stand for higher positions in an individual's preference ordering.

It is possible to represent the right-wing councillor's preferences with what political scientists call a **utility function.** A utility function can be thought of as a numerical scale in which higher numbers stand for higher positions in an individual's preference ordering. It essentially indicates how satisfied an individual is with the available alternatives. In Figure 10.2 we display a utility function that is consistent with the preference ordering of the right-wing councillor over the level of social service provision in our example. The utility function is highest over the proposal to decrease social service provision (*D*), it is lowest over the proposal to maintain current levels of social service provision (*C*), and it is between these two extremes over the proposal to increase social service provision (*I*).

Let's now examine the utility function of the centrist councillor. This is shown in Figure 10.3. You will immediately notice that the centrist councillor's utility function looks very different from that of the right-wing councillor in Figure 10.2. In particular, the centrist councillor's utility function captures what political scientists call a **single-peaked preference ordering.** In other words, the utility function reaches a "peak" above the centrist councillor's most preferred point. In this particular example, her most preferred point, sometimes called her "ideal point," is *C*. As proposals move away from this "ideal point" in either direction, the centrist councillor experiences a decline in her utility function.[14] The basic intuition behind single peakedness is that individuals prefer outcomes that are closer to their ideal point than those that are farther away. A quick glance at Figure 10.2 again shows that the right-wing

A **single-peaked preference ordering** is characterized by a utility function that reaches a maximum at some point and slopes away from this maximum on either side.

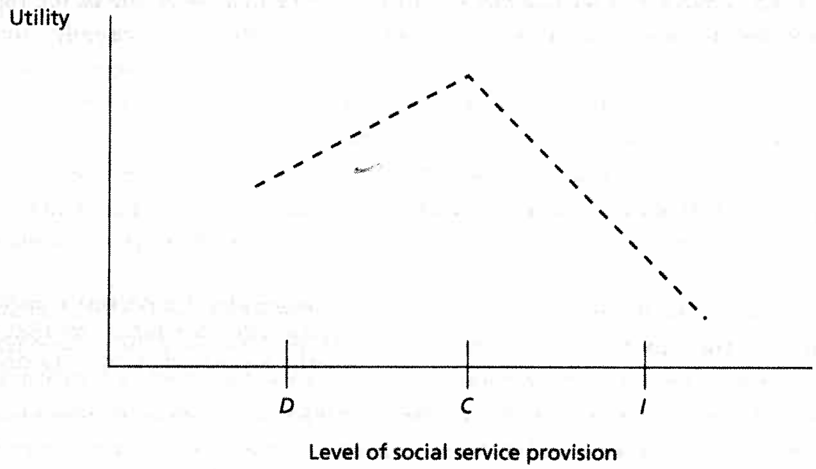
FIGURE 10.2 Right-Wing Councillor's Utility Function



Note: D = decreased social service provision; *C* = maintenance of current levels of social service provision; *I* = increased social service provision.

14. Note that the decline in utility may occur more rapidly when moving in one direction away from an individual's ideal point than the other.

FIGURE 10.3 Centrist Councillor's Utility Function



Note: D = decreased social service provision; C = maintenance of current levels of social service provision; I = increased social service provision.

councillor's utility function is not single peaked. Although there is a point at which the right-wing councillor's utility function reaches a maximum (around *D*), the utility function does not continuously decline as it moves farther away from this point—it starts to go back up to the right of *C*. In other words, the right-wing councillor prefers some outcomes that are farther from her ideal point (say, *I*) than outcomes that are closer (say, *C*).

The **Median Voter Theorem** (MVT) states that the ideal point of the median voter will win against any alternative in a pair-wise majority-rule election if the number of voters is odd, voter preferences are single-peaked over a single-policy dimension, and voters vote sincerely. When arrayed along a single-policy dimension in terms of their ideal points, the **median voter** is the individual who has at least half of all the voters at his position or to his right and at least half of all voters at his position or to his left.

One of the most important results in all of political science is called the **Median Voter Theorem** (Black 1948). The Median Voter Theorem (MVT) states that no alternative can beat the one preferred by the median voter in pair-wise majority-rule elections if the number of voters is odd, voter preferences are single-peaked over a single-policy dimension, and voters vote sincerely. When arrayed along a single-issue dimension in terms of their ideal points, the **median voter** is the individual who has at least half of all the voters at his position or to his right and at least half of all the voters at his position or to his left.

Suppose that we placed a restriction on the preferences of the councillors in our city council example such that they all had single-peaked preference orderings. For example, we could restrict the preferences of the right-wing councillor such that her most preferred proposal is *D*, her second-best proposal is *C*, and her least preferred proposal is *I*. If we did this,

10.1 THE MEDIAN VOTER THEOREM AND PARTY COMPETITION

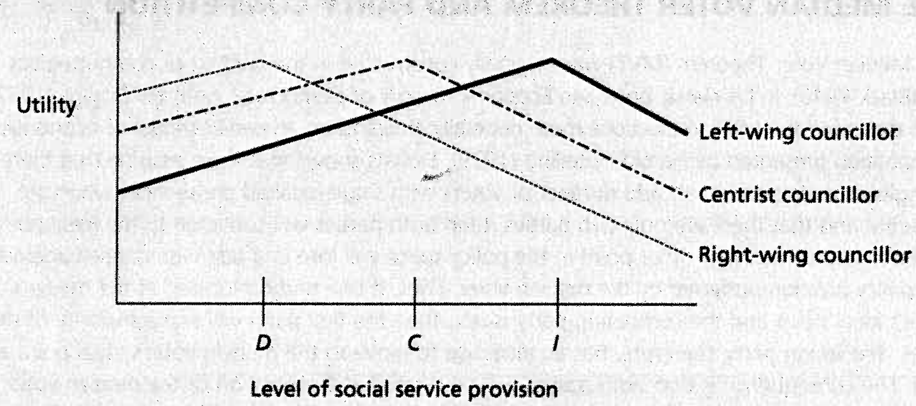
The Median Voter Theorem (MVT) was originally constructed in the context of committee voting (Black 1948). In his classic book, *An Economic Theory of Democracy,* Anthony Downs (1957) then extended the MVT to elections more generally. Building on an earlier model of economic competition presented by Harold Hotelling (1929), Downs shows that if we assume that there is a single-issue dimension, an odd number of voters with single-peaked preferences who vote sincerely, and that there are only two parties, then both parties will converge to the ideal point of the median voter. Any other point in the policy space will lose in a pair-wise contest against the policy position preferred by the median voter. Thus, if one party is located at the median voter's ideal point and the competing party is not, then the first party will win a majority of the votes. The losing party, therefore, has an incentive to move to the median voter's ideal point as well. The consequence is that both parties will be located at the position of the median voter, resulting in a tied election in which each party wins with equal probability.

The logic of the MVT indicates that political parties have an incentive to converge to the position of the median voter and adopt similar policy positions in two-party systems. The fact that observers of two-party systems frequently criticize the dominant parties in these countries for being ideologically indistinguishable on the major issues provides some evidence that policy convergence does, indeed, occur. These observers complain that the parties are giving voters an "echo" rather than a "choice" (Page 1978; Monroe 1983). A common expression for the convergence of party platforms in these countries is "Tweedledee-Tweedledum politics" (Goodin and Pettit 1993, 6). For example, while campaigning in the midwestern states in 1968, third-party presidential candidate George Wallace famously referred to the Democratic and Republican Parties in a speech as "Tweedledum and Tweedledee." He argued that both were serving the interests of the "Eastern establishment" and that since around the time of the Civil War, "both parties have looked down their noses and called us rednecks down here in this part of the country. I'm sick and tired of it, and on November 5, they're goin' to find out there are a lot of rednecks in this country" ("Neither Tweedledum Nor Tweedledee," *Time Magazine,* September 20, 1968). Similar criticisms are made by smaller parties or outsider candidates in nearly every country that has two large parties that dominate elections. Indeed, political discourse in Britain often refers explicitly to the Tweedledum-Tweedledee phrase. Non-English-speaking countries, of course, have their own expressions for this phenomenon.

then the preference orderings of all the councillors would be single peaked.[15] In Figure 10.4, we illustrate utility functions that are consistent with the single-peaked preferences of all three councillors. As we saw earlier in the chapter, maintaining current levels of social service provision (*C*) would win a round-robin tournament in which the councillors have preferences consistent with the utility functions shown in Figure 10.4.

15. We already showed in Figure 10.3 that the centrist councillor has a single-peaked preference ordering over the level of social service provision. You can check for yourself that the left-wing councillor also has a single-peaked preference ordering.

FIGURE 10.4 When All Three Councillors Have Single-Peaked Preference Orderings



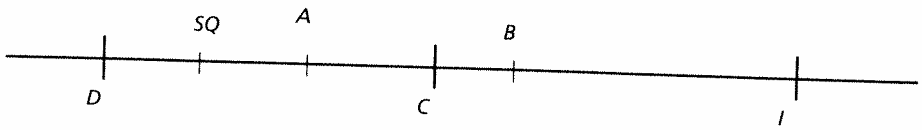
Note: *I* = the ideal point of the left-wing councillor; *C* = the ideal point of the centrist councillor; *D* = the ideal point of the right-wing councillor.

Up to this point, we have allowed the city councillors to choose between only three alternatives—increase, decrease, and maintain current levels of social service provision. We are now in a position to consider what might happen if the councillors are free to propose any level of social service spending. In other words, we can now look at the situation in which the councillors can pick any point on the x-axis of Figure 10.4 as their proposed level of social service spending.

Let's look at what happens if we assume that all the councillors vote sincerely for whichever proposal is closest to their ideal point. We will look at two kinds of scenarios. First, suppose that the status quo level of social service spending is given by point *C*—the ideal point of the centrist councillor. Clearly, the left-wing councillor would like to move social service spending to the right, toward her own ideal point *I*. Any proposal to do this, however, would be opposed by the centrist and right-wing councillors because any such proposal would be farther from their ideal points than the existing status quo. The right-wing councillor would like to move social spending to the left, toward her own ideal point *D*. Any proposal to do this would now be opposed by the centrist and the left-wing councillors because any such proposal would be farther from their ideal points than the existing status quo. As a result, if the status quo is at the centrist councillor's ideal point, then it is an equilibrium.

Second, suppose that the status quo level of social service spending is anywhere other than *C*—let's say somewhere to the left of *C*. This type of scenario is shown in Figure 10.5, with the status quo policy arbitrarily placed at *SQ* (status quo). In this type of situation, both the centrist and left-wing councillors are likely to propose moving social service spending closer to *C*. Let's suppose they propose A. Proposal A beats the *SQ* because the left-wing and

FIGURE 10.5 **Illustrating the Power of the Median Voter**



Note: *D* = the ideal point of the right-wing councillor ; *C* = the ideal point of the centrist councillor; *I* = the ideal point of the left-wing councillor; *SQ* = status quo level of social service provision; *A* and *B* = proposals for a new level of social service provision.

centrist councillors vote for it and only the right-wing councillor votes against. But is policy A an equilibrium? The answer is no. The left-wing and centrist councillors would like to move social service provision farther to the right, closer to their ideal points. Let's suppose that they now propose B. Proposal B will be adopted because it is closer to the ideal points of both the left-wing and centrist councillors than proposal A; the right-wing councillor will vote against the new proposal but will lose. Is proposal B an equilibrium? Again, the answer is no. The right-wing and centrist councillors will now want to move social service provision to the left, closer to their ideal points. Any proposal that is closer to *C* than B is will win with the support of the right-wing and centrist councillors. This process will continue until policy fully converges to the ideal point of the centrist councillor at *C*. Only then will the policy outcome be stable. A similar process of convergence to the position of the centrist councillor would occur if the status quo started off to the right of *C* instead of to the left.

Even if the centrist councillor is never given the opportunity to propose a policy change, we would still expect to see alternative offers by the left- and right-wing council members that slowly converge to the most preferred policy of the centrist candidate. In fact, if making different policy proposals was sufficiently costly, far-sighted councillors of the left and right might look to the end of this convergence process and simply propose a policy that matched the policy preferences of the centrist candidate from the very beginning. Whatever the process that produces the convergence to the centrist councillor's ideal point, once policy arrives there, there is no longer any impetus for change in the system. In other words, the policy that is most preferred by the centrist councillor is the only point on the policy continuum for which there is no policy alternative that is preferred by a majority of the councilors—it is the only equilibrium. This is so not because we have labeled the policymaker in the center a "centrist" but because the centrist happens to be the median voter.[16]

16. The Median Voter Theorem does not assert that the equilibrium policy outcome will be centrist in terms of the underlying issue dimension. All it states is that the equilibrium policy will be the ideal point of the median voter. Whether it is centrist or not will, therefore, depend on the location of the median voter in the issue space.

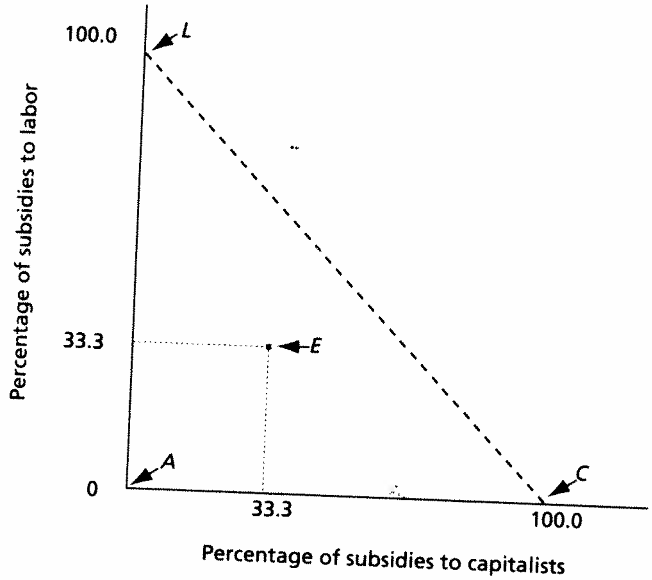
The Median Voter Theorem essentially shows that the difficulties we encountered earlier with Condorcet's Paradox, such as group intransitivity and cyclical majorities, can be avoided if we are willing to both rule certain preference orderings "out of bounds" and reduce the policy space to a single-issue dimension. Unfortunately, neither of these restrictions is uncontroversial. For example, there is nothing intrinsically troubling about individual preferences that are not single peaked. In fact, there are a whole host of issues for which voters might, like the right-wing councillor in our example, legitimately prefer a lot or a little of something to a moderate amount.[17] As a result, we might have moral objections to a decision-making procedure that prohibits individuals from holding preferences that are not single-peaked.

The restriction of politics to a single-issue dimension can also be controversial. This is because many political questions are inherently multidimensional. As an example, consider a situation in which the representatives of three constituencies—labor, capital, and agriculture—are deciding how to divide a pot of subsidies from the government's budget. This decision-making situation can be represented by a two-dimensional policy space in which the percentage of subsidies going to labor is one dimension and the percentage of subsidies going to capital owners is the other; anything left over goes to agriculture. This decision-making situation is depicted in Figure 10.6. The downward-sloping dashed line sets an upper bound on all the possible distributions of subsidies. This limit is necessary because there is a finite amount of resources that can be spent on subsidies. In what follows, we assume that the entire pot of subsidies will be distributed between the three constituencies. At point *L*, all of the subsidies go to labor. At point *C*, all of the subsidies go to capital. And at point *A*, all of the subsidies go to agriculture. Any point along the sloping dashed line between *L* and *C* is some distribution of the subsidies between labor and capital; agriculture gets nothing. Any point along the solid vertical line between *L* and *A* is some distribution of the subsidies between labor and agriculture; capital gets nothing. And any point along the solid horizontal line between *A* and *C* is some distribution of the subsidies between agriculture and capital; labor gets nothing. Finally, any point within the triangle *LAC* is some distribution of the subsidies between all three constituencies. For example, at point *E*, the subsidies are divided equally between labor, capital, and agriculture.

Imagine that each constituency wants to maximize its share of the government subsidies but has no opinion about how the portion it does not receive is divided among the other constituencies. If each constituency votes to allocate the subsidies by majority rule and can propose a change in the division at any time, then the problem of cyclical majorities that we encountered with Condorcet's Paradox will rear its ugly head again. To see why, imagine that someone, perhaps the national government, proposes to divide the subsidies equally between all three constituencies. This point can be thought of as the status quo proposal, and

17. We suspect that many of you probably have the following non-single-peaked preference ordering over coffee when the single dimension under consideration is the temperature of the coffee—you prefer both hot coffee and iced coffee to lukewarm coffee. We see nothing inherently wrong with a preference ordering like this.

FIGURE 10.6 **Two-Dimensional Voting**



Note: At *L* all the subsidies go to labor; at *C* all the subsidies go to capital; at *A* all the subsidies go to agriculture; and at *E* the subsidies are divided equally between labor, capital, and agriculture.

it is marked as *SQ* in Figure 10.7. Given the assumptions that we have made, the most preferred outcome for each constituency will be to get 100 percent of the subsidies for itself. Recall that these ideal points are given by points *L* (labor), *A* (agriculture), and *C* (capital) in Figure 10.6.

The gray sloping line through the status quo proposal in Figure 10.7 indicates all of the ways that the pot of subsidies can be divided between labor and capital such that agriculture receives one-third of the pot. Because agriculture cares only about how much it is getting, agriculture is essentially indifferent between any of the points on this line and the status quo proposal made by the national government. For this reason, the gray sloping line is called agriculture's **indifference curve** (with respect to the status quo).[18] Any point to the southwest of this indifference curve involves a division of subsidies in which agriculture receives more than one-third of the pot. As a result, agriculture strictly prefers any point to the southwest of its indifference curve to any point on its indifference curve. The vertical gray line through the status quo proposal is capital's indifference curve because it indicates all of

An **indifference curve** is a set of points such that an individual is indifferent between any two points in the set.

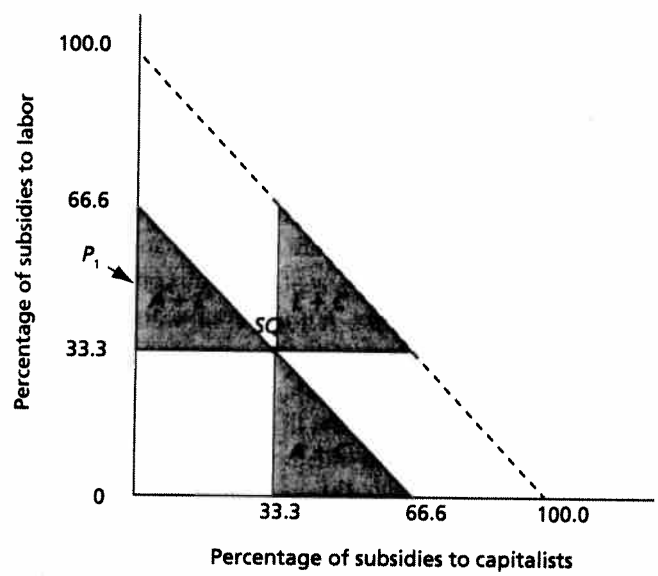
18. This is despite the fact that, in this case, it is not actually a curve but a straight line.

the ways that the pot of subsidies can be divided between labor and agriculture so that capital receives one-third of the pot. Any point to the right of this indifference curve involves a division of subsidies in which capital receives more than one-third of the pot. As a result, capital strictly prefers any point to the right of its indifference curve to any point on its indifference curve. Finally, the horizontal gray line through the status quo proposal is labor's indifference curve because it indicates all of the ways that the pot of subsidies can be divided between capital and agriculture so that labor receives one-third of the pot. Although labor is indifferent among any of the points along this line, it strictly prefers any point above this line because it will receive more than one-third of the pot.

Because labor prefers any point above the horizontal gray line to the status quo, capital prefers any point to the right of the vertical gray line, and agriculture prefers any point below the sloping gray line, each of the triangle-shaped petals radiating from the status quo represents a set of alternative divisions of the subsidies that a majority of the constituency representatives prefer to the status quo. Such sets of alternatives that would win a majority vote if pitted against the status quo in a pair-wise contest are sometimes referred to as **winsets** of the status quo. The triangle to the northwest of the status quo, labeled $A + L$, represents outcomes

The **winset** of some alternative *z* is the set of alternatives that will defeat *z* in a pair-wise contest if everyone votes sincerely according to whatever voting rules are being used.

FIGURE 10.7 **Two-Dimensional Voting with Winsets**



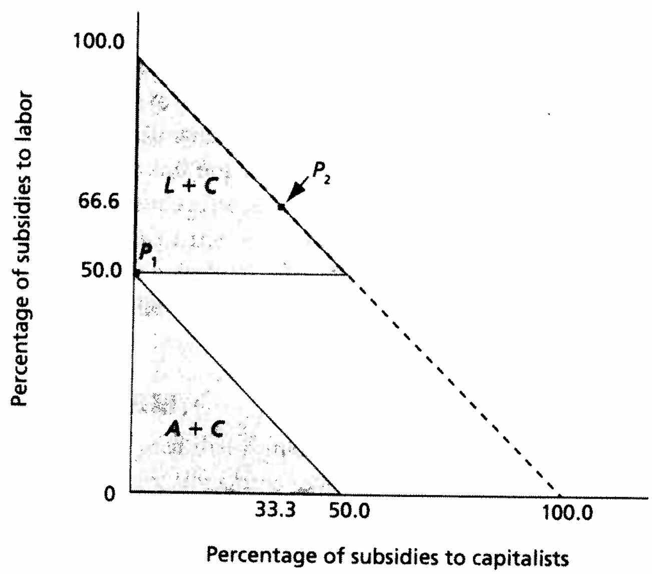
Note: The three solid gray lines going through *SQ* (status quo) are the indifference curves for labor (*L*), capital (*C*), and agriculture (*A*); P_1 = proposal 1. The shaded triangles are winsets that represent alternative divisions of the subsidies that are preferred by a majority to the status quo; the majority in question is shown in each winset.

that are preferred by labor and agriculture to the status quo. The triangle to the northeast of the status quo, labeled $L + C$, represents outcomes that are preferred by labor and capital to the status quo. Finally, the triangle to the southeast of the status quo, labeled $A + C$, represents outcomes that are preferred by capital and agriculture to the status quo.

The existence of these non-empty winsets indicates that if any of the three constituencies has an opportunity to propose a change in the division of subsidies, they will. For example, the labor representative might propose a 50-50 split of the subsidies with agriculture. This proposal is denoted by P_1 in Figure 10.7. Because this proposal leaves both agriculture and labor better off vis-à-vis the status quo, the agriculture and labor representatives will vote to accept this proposal; the capital representative will vote against the proposal because capital would be worse off. Hence, proposal P_1 will defeat the original status quo 2 to 1 and become the new status quo proposal. Are there any alternative divisions of the subsidies that a majority of representatives prefer to the new status quo proposal P_1? To answer this question, we must draw the indifference curves of the three constituencies with respect to P_1 and see if there are any non-empty winsets. We do this in Figure 10.8.

As before, the indifference curves for each constituency are shown by the gray lines going through the new status quo proposal P_1. As Figure 10.8 illustrates, there are two winsets. The

FIGURE 10.8 Two-Dimensional Voting with a New Status Quo (P_1)

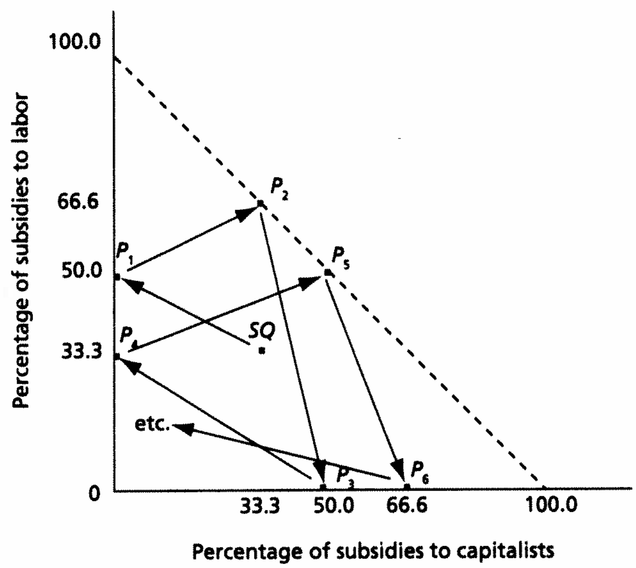


Note: The three solid gray lines going through P_1 are the indifference curves for labor (L), capital (C), and agriculture (A). The shaded triangles are winsets that represent alternative divisions of the subsidies that are preferred by a majority to the status quo; the majority in question is shown in each winset.

winset labeled $L+C$ contains alternatives that are preferred to P_1 by both labor and capital. The winset labeled $A + C$ contains alternatives that are preferred to P_1 by both agriculture and capital. In other words, there are several alternative divisions of the subsidies that are preferred by a majority to the new status quo proposal P_1. For example, the capital representative might propose to give two-thirds of the subsidies to labor and one-third of the subsidies to capital. This proposal is denoted by P_2 in Figure 10.8. Because this proposal leaves labor better off (labor receives 66.6 percent instead of 50 percent) and capital better off (capital gets 33.3 percent instead of 0 percent), the labor and capital representatives will vote to accept proposal P_2; the agriculture representative will vote against the proposal because agriculture will be worse off (agriculture receives 0 percent instead of 50 percent). Hence, proposal P_2 will defeat proposal P_1 2 to 1 and become the new status quo proposal.

Is P_2 a stable division of subsidies? The answer is no. Agriculture, which is not getting any share of the subsidies under proposal P_2, could propose a 50-50 division of the subsidies between itself and capital. This is proposal P_3 in Figure 10.9. This proposal would defeat P_2 because agriculture would vote for it (agriculture receives 50 percent instead of 0 percent), and capital would also vote for it (capital receives 50 percent instead of 33 percent). Thus, proposal P_3 would dislodge proposal P_2 as the new status quo proposal. Because there is always some division of the subsidies that gives the excluded constituency a share of the pot while giving one of the other constituencies a bigger share of the pot than they are receiving

FIGURE 10.9 Two-Dimensional Voting with Cyclical Majorities



Note: SQ = original status quo; P_1 = proposal that beats SQ; P_2 = proposal that beats P_1; P_3 = proposal that beats P_2; P_4 = proposal that beats P_3, and so on.

with the status quo proposal, this process of ever-shifting divisions of the subsidy pot can be expected to go on forever. This is illustrated in Figure 10.9.

The process of cyclical majorities highlighted in Figure 10.9 exemplifies a famously unsettling theorem about politics relating to majority rule in multidimensional settings (Plott 1967; McKelvey 1976; Schofield 1978). According to the **Chaos Theorem,** if there are two or more issue dimensions and three or more voters with preferences in the issue space who all vote sincerely, then except in the case of a rare distribution of ideal points, there will be no Condorcet winner. As a result, whoever controls the order of voting can determine the final outcome.

The **Chaos Theorem** states that if there are two or more issue dimensions and three or more voters with preferences in the issue space who all vote sincerely, then except in the case of a rare distribution of ideal points, there will be no Condorcet winner.

Like Condorcet's Paradox, the Chaos Theorem suggests that unless we are lucky enough to have a set of actors who hold preferences that do not lead to cyclical majorities, either of two things will happen: (a) the decision-making process will be indeterminate and policy outcomes hopelessly unstable; or (b) there will exist an actor —the agenda setter—with the power to determine the order of votes in such a way that she can produce her most favored outcome.

In the absence of institutions that provide an actor with agenda-setting powers, stable outcomes are even less likely to occur in the circumstances covered by the Chaos Theorem than those covered by Condorcet's Paradox. This is because the set of preferences that prevent majority cycling in two or more dimensions are extremely rare and special. Students who are interested in learning more about the conditions for stable outcomes in multidimensional policy spaces are referred to Box 1.2 titled "Stability in Two-Dimensional Majority-Rule Voting" at the end of this chapter (p. 392).

The important lesson to draw from the Chaos Theorem is that if politics cannot be reduced to a single-issue dimension, then there is a wide set of circumstances under which either (a) stable outcomes will not occur, or (b) stable outcomes will be imposed by whichever actor controls the agenda. The results of the Chaos Theorem highlight "the importance of investigating the effects of the political institutions within which collective choices are made" (McCarty and Meirowitz 2007, 80), because it is likely that it is these institutions that play a significant role in alleviating the "chaos" that might otherwise reign.

ARROW'S THEOREM

Up to this point, we have seen that thinking about democracy as simply a matter of allowing the majority to make decisions runs into difficulties on several fronts. First, Condorcet's Paradox shows that a set of rational individuals can form a group that is incapable of choosing rationally in round-robin tournaments. Specifically, situations might arise in which majorities cycle indefinitely, rendering the group incapable of reaching a decision. We learned that although voting schemes, like the Borda Count, that allow voters to rank order all possible alternatives might allow clear winners to emerge under some conditions, the outcomes that are produced by such decision-making processes are not robust. In particular, altering rankings over irrelevant alternatives could change the way votes are counted and, therefore, change the outcome. Next, we learned that if round-robin tournaments are replaced by "single elimination" tournaments that form a voting agenda, then cyclical majorities may be avoided and a stable outcome achieved. Unfortunately, we also saw that whoever controls the agenda—the order by which alternatives are pitted against each other and submitted to a vote—could dictate the outcome of such procedures.

We saw, though, that the problem of instability could be overcome if the political question to be decided can be thought of as a single-issue dimension *and* if each voter has single-peaked preferences over that dimension. When these and other conditions are met, the alternative that is most preferred by the median voter is a stable outcome in the sense that no other alternative exists that can defeat it in a majority-rule pair-wise contest. Unfortunately, the Median Voter Theorem is, at best, cold comfort for a couple of reasons. First, it requires restrictions on the types of preferences that individuals can have. If some voters have non-single-peaked preferences, it is possible for a cycle of majorities to arise—just as in round-robin tournaments. Consequently, the Median Voter Theorem tells us only that we can avoid the potential for decision-making instability if certain preference orderings do not occur, either because we are lucky or because we are willing to rule them out of bounds. This is problematic because it seems reasonable that an individual's preferences should be treated as sacrosanct. It seems to us that a "democratic" decision-making process that works reliably only when we restrict participation to those with the "right kind" of preferences loses much of its ethical appeal. Second, it is likely that many crucial political questions involve more than one dimension. We presented an example of distributional bargaining between three groups as an illustration of a two-dimensional political question that is, in many ways, the very essence of politics. In this example, the decision-making process was unstable and "chaotic." As we noted at the time, the stability implied by the Median Voter Theorem occurs in situations in which there are two or more dimensions to a political problem only in exceedingly rare circumstances.

Each of these complications with majority rule raises fundamental questions about the ethical appeal of democracy—understood as majority rule—as a mechanism for making group decisions. Specifically, we have seen that it is difficult to guarantee that majority rule will produce a stable group choice without either granting someone agenda-setting power or restricting the kinds of preferences that individuals may hold. In a famous book, Kenneth Arrow (1963) put forth a theorem that shows that these problems with majority rule are, in some ways, special cases of a more fundamental problem. **Arrow's Theorem** demonstrates that it is impossible to design any decision-making system—not just majority rule—for aggregating the preferences of a set of individuals that can guarantee producing a rational outcome while simultaneously meeting what he argued was a minimal standard of fairness.[19] When deciding which set of conditions a fair decision-making procedure should

19. It is common to see Arrow's Theorem referred to as Arrow's "Impossibility" Theorem. In fact, Arrow himself labeled it the "Possibility Theorem."

meet, Arrow sought a minimal set in the sense that violations of these fairness conditions would lead to a procedure that is unfair in a way that would be evident to all.

Arrow's Fairness Conditions

Arrow presented four fairness conditions that he believed all decision-making processes should meet. Each of these four conditions is related to issues that have already arisen during our examination of majority rule and can reasonably be argued to be a part of any conception of democracy. We will discuss each of the four conditions in turn.

Non-Dictatorship

The **non-dictatorship condition** states that there must be no individual who fully determines the outcome of the group decision-making process in disregard of the preferences of the other group members. In fact, this condition is an extremely minimal fairness condition because it only says that there can be no individual who, if she prefers *x* to *y*, forces the group choice to be *x* instead of *y*, irrespective of the preferences of everyone else. A group decision-making process that pays attention to only one member of the group and disregards the preferences of all the other members is clearly not democratic. Although it is possible that a dictator would be benevolent and choose an outcome that benefits the group, it is clear that a *mechanism* that allows a single individual to determine group outcomes for everyone else is inherently unfair.

The **non-dictatorship condition** states that there must be no individual who fully determines the outcome of the group decision-making process in disregard of the preferences of the other group members.

Universal Admissibility

The **universal admissibility condition** states that any fair group decision-making rule must work with any logically possible set of individual preference orderings. This allows actors to adopt any rational preference ordering they want. This condition is closely related to the philosophical doctrine of individualism. According to this perspective, individuals should be free to formulate their own desires. Although it may be appropriate under some conditions to prohibit individuals from acting on those desires, it is inappropriate to comment on the intrinsic social desirability of another's desires per se. It is on this basis that Arrow believed it inappropriate to exclude individuals from group decision-making processes simply on the basis of the types of preferences that they happen to hold. Riker (1982, 117) defends the universal admissibility condition by stating that "if social outcomes are to be based exclusively on individual judgments—as seems implicit in any interpretation of democratic methods—then to restrict individual persons' judgments in any way means that the social outcome is based as much on the restriction as it is on individual judgments." In the context of voting, this condition states that every voter may vote as she pleases.

The **universal admissibility condition** states that individuals can adopt any rational preference ordering over the available alternatives.

Unanimity, or Pareto Optimality

The **unanimity, or pareto optimality, condition** states that if all individuals in a group prefer *x* to *y*, then the group preference must reflect a preference for *x* to *y* as well.

The **unanimity, or pareto optimality, condition** states that if all individuals in a group prefer *x* to *y*, then the group preference must reflect a preference for *x* to *y* as well. A decision-making process that fails to meet this condition is not only unfair, it is perverse. Imagine pressing the Coke button on a vending machine and having a Sprite come out. The unanimity condition is extremely minimal in that it merely states that if everybody in the group is unanimous in sharing a preference for *x* to *y*, then the group must not choose *y* when *x* is available.

Independence from Irrelevant Alternatives

The **independence from irrelevant alternatives condition** states that group choice should be unperturbed by changes in the rankings of irrelevant alternatives.

The **independence from irrelevant alternatives condition** states that when groups are choosing between alternatives in a subset, the group choice should be influenced only by the rankings of these alternatives and not by the rankings of any (irrelevant) alternatives that are not in the subset. Suppose that, when confronted with a choice between *x*, *y*, and *z*, a group prefers *x* to *y*. The independence from irrelevant alternatives condition states that if one or more individuals alter their ranking of *z*, the group must still prefer *x* to *y*. Or as Varian (1993, 535) puts it, the group's preference "between *x* and *y* should depend only on how people rank *x* versus *y*, and not on how they rank other alternatives." We saw earlier that the Borda Count can violate this condition. A decision rule respects the independence from irrelevant alternatives condition whenever the group's ranking of any two alternatives *x* and *y* depends only on the relative ranking of these alternatives by every individual in the group (Geanakoplos 2005). Some scholars have, therefore, interpreted this condition to mean that the decision rule should be reliable in the sense that it always returns the same decision if the way individuals rank relevant alternatives remains unchanged (Riker 1982). In this respect, the independence from irrelevant alternatives condition is as much a condition about the reliability of the preference aggregation technology as it is a condition about the fairness of the decision-making mechanism.

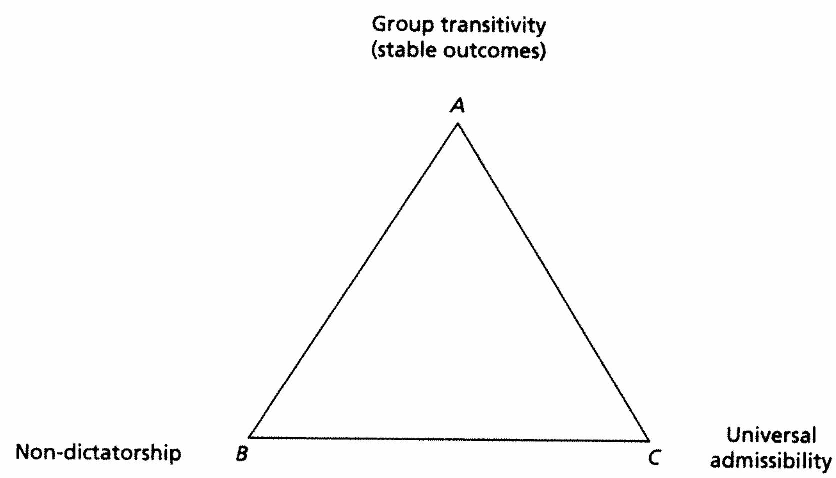
In sum, Arrow's four conditions suggest that any fair group decision-making mechanism must prevent dictatorship (non-dictatorship), must not restrict the type of preferences that individuals can hold (universal admissibility), and must link group choice, in at least some rudimentary sense, to individual preferences (unanimity and independence from irrelevant alternatives). We have already seen examples of how particular majority-rule decision-making mechanisms must violate at least one of these requirements if we wish to guarantee that the group's preference ordering will be transitive. As we have seen, group transitivity is necessary in order for the group decision-making process to produce a stable outcome. The real power of Arrow's Theorem, though, comes from demonstrating that *every* decision-making process that we could possibly design, including any majority-rule one, must sacrifice at least one of Arrow's fairness conditions if it is to guarantee group transitivity and, hence, stable outcomes. Put differently, if we insist that Arrow's four fairness conditions be met, we must accept the possibility of group intransitivity—there is no way around it.

Arrow's Theorem states that every decision-making process that we could possibly design must sacrifice at least one of Arrow's fairness conditions—non-dictatorship, universal admissibility, unanimity, or independence from irrelevant alternatives—if it is to guarantee group transitivity and, hence, stable outcomes.

The implications of Arrow's Theorem are far reaching. Suppose that we take Arrow's conditions of unanimity and independence from irrelevant alternatives as uncontroversial and given. If we do this, Arrow's Theorem tells us that we face an institutional "trilemma" between stable outcomes, universal admissibility, and non-dictatorship. In other words, we can design decision-making institutions that have at most two of these three desirable attributes. In Figure 10.10, we illustrate Arrow's institutional trilemma with the help of a triangle.

Basically, Arrow's Theorem states that when we design decision-making institutions, we can choose one and only one side of the triangle shown in Figure 10.10. If we want decision-making institutions that guarantee group transitivity and stable outcomes (*A*), then we must give up either non-dictatorship (*B*) or universal admissibility (*C*). If, on the other hand, we want to avoid dictatorship (*B*), then we must give up either transitivity (*A*) or universal admissibility (*C*). Finally, if we hold individual preferences as inviolable (*C*), then we must give up either transitivity (*A*) or non-dictatorship (*B*). To summarize, Arrow's Theorem proves that if the independence from irrelevant alternatives and the unanimity conditions

FIGURE 10.10 Arrow's Institutional Trilemma



Note: Arrow's conditions of unanimity and independence from irrelevant alternatives are assumed as given here.

are assumed, then designers of group decision-making institutions will be forced to choose their poison from the following set: restrictions on individual preferences, dictatorship, or the possibility of group intransitivity.

In addition, Arrow's Theorem shows that it is, at the very least, difficult to interpret the outcome of any group decision-making process as *necessarily* reflecting the will of the group (Shepsle 1992). When a group comes to a clear decision it *may* mean that individual preferences lined up in a way that allowed for a clear outcome that represented, in some meaningful way, the desires of a large portion of the group. But it may also mean that individuals with inconvenient preferences were excluded from the process, or that some actor exercised agenda control. In such cases, outcomes may reflect the interest of some powerful subset of the group rather than the preferences of the group as a whole, or even some majority of the individuals in the group.

CONCLUSION

Most people associate democracy with majority rule. In this chapter we have examined various problems with majority-rule decision-making procedures. We have shown, for example, that there is a fundamental tension between the desire to guarantee that a group of individuals will be able to make coherent and stable choices on the one hand and the ability to guarantee the freedom of these individuals to form their own preferences and have those preferences influence group decisions on the other. Arrow's Theorem shows that these tensions extend far beyond majority rule to encompass a wide set of minimally fair group decision-making methods. Because most conceptions of democracy would probably be even more ambitious than Arrow's fairness criteria, these results have important and profound implications for democracy.

It is important to note at this point that the most direct implications of Arrow's Theorem concern particular mechanisms for group decision making. Arrow's Theorem is not in any direct sense about *collections* of decision-making procedures. Consider constitutions. A constitution does not typically stipulate *a* decision-making procedure to be used in a country but rather an entire set of decision-making procedures. For example, a constitution may stipulate how the head of the executive branch will be chosen, how legislators are chosen, how legislators choose laws, how the executive and legislative branches interact, how and if courts decide whether legislation is constitutional, which laws are under the jurisdiction of the national government and which are under local control, and the like. Arrow's Theorem applies to each and every one of these decision-making mechanisms, and the way the trilemma that Arrow's Theorem poses is resolved may vary from mechanism to mechanism within a particular constitution.[20] Some mechanisms, or institutions, may privilege group

20. Arrow's Theorem also applies to decision-making bodies we are involved with on a day-to-day basis—student organizations, faculties, labor unions, religious congregations, corporate boards, families, and groups of friends deciding which movie to see.

transitivity by reducing the number of alternatives from which people can choose—the two-party system used for electing presidents in the United States can be seen as an example of this. Other mechanisms or institutions may avoid group intransitivity by granting agenda-setting powers to an individual. A large literature on the U.S. Congress argues that committee chairs play this role in the legislative process. Cabinet ministers play a similar role in many parliamentary systems.

The key point is that *every* decision-making mechanism must grapple with the trade-offs posed by Arrow's Theorem and *every* system of government represents a collection of such decision-making mechanisms. Consequently, we can think of a system of government in terms of how its decision-making mechanisms tend to resolve the trade-offs between group transitivity and Arrow's fairness criteria. And to the extent that a set of decision-making mechanisms privileges group transitivity, it is useful to think about *which* of Arrow's fairness criteria tends to be sacrificed. For example, is stability produced because strong agenda setters are produced or because restrictions are placed on the preferences that actors hold? If it is the latter, we might ask whether this is achieved because some actors are excluded from deliberations or because strong mechanisms are in place to socialize participants so that they adopt the "right" preferences.

It should be clear by now that the most basic implication of Arrow's Theorem is this: there is no perfect set of decision-making institutions. Every set of institutions either runs the risk of group intransitivity or compromises a fairness condition. As such, democracy must in some sense be imperfect—either fairness is compromised or there will be a potential for instability. Perhaps this is what inspired former British prime minister Winston Churchill to say, "Democracy is the worst form of government, except for all those other forms that have been tried from time to time." [21] In Part II of this book, we compared democracy with dictatorship. In the following chapters, we examine the immense variety of democratic forms of government, focusing specifically on different configurations and their consequences.

In the next chapter, we examine the different ways we can choose the head of government and structure the relationship between the head of government and the legislative branch. In Chapter 12 we look at the myriad ways in which legislators are elected around the world. In Chapter 13 we investigate the ways in which group preferences interact with electoral laws to shape party systems. From the perspective of Arrow's Theorem, party systems can be thought of as a way of encouraging stability by reducing the number of alternatives to be considered in an election. In Chapter 14 we examine a whole host of other institutional mechanisms, such as the division of powers between national and subnational governments (federalism), the division of powers between different houses of the legislature (bicameralism), and the division of powers between the legislative and judicial branches (constitutional review). From the standpoint of Arrow's Theorem, these institutional mechanisms can be thought of as attempts to limit the control of powerful agenda setters by pitting them against each other.

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21. House of Commons speech, November 11, 1947.

The immense variety of democratic institutions observed in the world is itself implied by Arrow's Theorem. Because there is no ideal decision-making mechanism, institutional choice is an exercise in the choice of "second bests," and which institution (or set of institutions) is adopted in any given time or place is going to be dictated by context—some situations make instability sufficiently threatening to make the sacrifice of fairness conditions reasonable, whereas others may permit individuals to accept a certain degree of instability in exchange for protecting the fairness of the decision-making process. Of course, the suggestion that we, as a society, may "choose" optimal institutions is itself in tension with Arrow's Theorem. Perhaps the institutions we confront today are the product of choices by agenda setters in the past, or inherited from times when actors with certain preferences were restricted from participating in the constitutional deliberations.

Key Concepts

Arrow's Theorem, *383*
Chaos Theorem, *379*
complete preference ordering, *359*
Condorcet's Paradox, *359*
Condorcet winner, *360*
consequentialist ethics, *356*
deontological ethics, *356*
independence from irrelevant alternatives condition, *382*
indifference curve, *375*
median voter, *370*
Median Voter Theorem, *370*
non-dictatorship condition, *381*
rational, *359*
round-robin tournament, *358*
sincere vote, *367*
single-peaked utility function, *369*
strategic, or sophisticated, vote, *367*
transitive preference ordering, *359*
unanimity, or pareto optimality, condition, *382*
universal admissibility condition, *381*
utility function, *368*
winset, *376*

Problems

The following problems address some of the more important concepts, theories, and methods introduced in this chapter.

Individual Preferences

1. What does it mean for an individual to be "rational"? Give a brief definition of the concept in your own words. If you use terms like *complete* and *transitive* to define the concept, be sure to define those terms as well.
2. In the problems at the end of Chapter 4, you were asked to consider the children's game of Rock, Paper, Scissors. In this game, two children simultaneously choose "rock," "paper," or "scissors." Rock beats scissors, paper beats rock, and scissors beat paper. Let's say that you prefer the winner in each of these pair-wise comparisons. That is, you prefer rock to scissors, scissors to paper, and paper to rock. Is your preference ordering complete? Explain

your answer. Is your preference ordering transitive? Explain your answer. Are you rational? Explain your answer.

3. Construct a complete and transitive preference ordering between three or more alternatives that you think some people might hold (or that you yourself hold). Now construct another (reasonable) preference ordering that does *not* satisfy completeness or transitivity.
4. Choose some issue dimension. With the help of a diagram, construct one preference ordering over the issue dimension that is single peaked and one that is not. Explain what single-peaked preferences are in your own words.

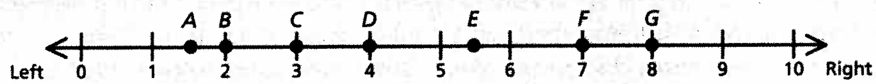
Agenda Setting

5. Imagine that you are one of three judges for a singing competition. You need to decide which of three finalists should win. You have no qualifications to evaluate actual singing ability, so you plan to make your choice based entirely on your preferences for the style of music that each performer chose: a sappy ballad, a traditional Irish folk song, and a heavy metal song. Luckily for you, you happened to see the notes written by the other judges, and so you know how your fellow judges ranked the finalists. One judge had the following preference ordering: ballad $\succ$ Irish folk song $\succ$ heavy metal. The other judge had the following preference ordering: heavy metal $\succ$ ballad $\succ$ Irish folk song. Your preference ordering, however, is the following: Irish folk song $\succ$ heavy metal $\succ$ ballad. The rules of the competition do not specify how the judges are to reach their decision.
   a. Let's suppose that you suggest a round-robin tournament in which everyone votes on the finalists in a series of pair-wise contests. How many pair-wise contests does each of the finalists win? Is there a Condorcet winner? Explain. Does this decision-making process identify a clear winner? Explain.
   b. Now let's suppose that you propose a decision-making procedure by which all of the judges begin by considering only a subset of the available pair-wise contests. The specific decision-making procedure that you propose is that two finalists should compete in a pair-wise contest with the winner competing in a second and final round against the remaining finalist. Given your preference ordering, which finalist do you want to win? If you were in charge of setting the voting agenda and could determine the order in which the pair-wise contests took place, what order would you pick and why?

Median Voter Theorem and Party Competition

6. In Figure 10.11, we illustrate an election in which there are seven voters (*A, B, C, D, E, F, G*) arrayed along a single left-right issue dimension that runs from 0 (most left) to 10 (most right). Each voter is assumed to have single-peaked utility functions and to vote for the party that is located closest to her ideal point. The voters are participating in a majority-rule election in which there are two parties, P_1 and P_2, competing for office. These parties can be thought of as "office-seeking" parties because they care only about winning the election and getting into office.

FIGURE 10.11 Illustrating the Median Voter Theorem



a. Which voter is the median voter? What is her ideological position?

b. Let's suppose that P_1 locates at position 2 on the left-right issue dimension and that P_2, locates at position 7. How many votes does P_1 win? How many votes does P_2 win? Who wins the election? Where does the winner implement policy on the left-right issue dimension? Will P_1 and P_2 want to stay at these policy positions for the next election? If not, what policy positions do you think they will adopt and why?

c. Now let's suppose that P_1 locates at position 4 and P_2 locates at position 4. What is the outcome of this election? Does P_1 or P_2 want to change policy positions given where the other party is located? If so, why? If not, why not?

Instead of office-seeking parties in our example, let's now assume that we have two "policy-seeking" parties, *L* and *R*. *L* is a left-wing party whose ideal point is 2, and *R* is a right-wing party whose ideal point is 7. Policy-seeking parties care about where policy is implemented.

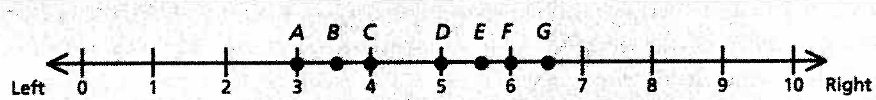
d. Let's suppose that *L* locates at its ideal point (2) on the left-right issue dimension and that *R* locates at its ideal point (7). How many votes does *L* win? How many votes does *R* win? Who wins the election? Where does the winner implement policy on the left-right issue dimension? Will *L* and *R* want to stay at their ideal points for the next election? If not, what policy positions do you think they will adopt and why?

e. Now let's suppose that *L* locates at position 4 and *R* locates at position 4. What is the outcome of this election? Where will policy be implemented on the left-right issue dimension? Does *L* or *R* want to change policy positions given where the other party is located? If so, why? If not, why not?

f. Based on your answers so far, does the result from the Median Voter Theorem stating that parties will converge to the position of the median voter depend on whether political parties are office seeking or policy seeking? Explain.

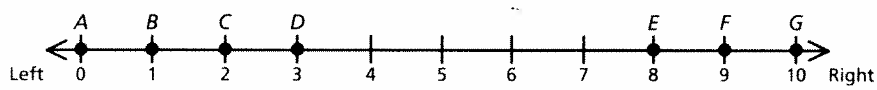
g. Suppose that some event occurs that causes several voters to adopt more centrist positions on the left-right issue dimension. The new distribution of voters is shown in Figure 10.12. Where will parties P_1 and P_2 locate in the left-right space, given the centrist nature of the electorate? Why?

FIGURE 10.12 Illustrating the Median Voter Theorem—A Centrist Electorate



h. Suppose now that some polarizing event occurs that causes several voters to adopt more extreme positions on the left-right issue dimension. The new distribution of voters is shown in Figure 10.13. Where will parties P_1 and P_2 locate in the left-right space, given the polarized nature of the electorate? Why?

FIGURE 10.13 **Illustrating the Median Voter Theorem—A Polarized Electorate**



i. Based on your answers to the two previous questions, does the result from the Median Voter Theorem stating that parties will converge to the position of the median voter depend on the distribution of voter ideal points? Explain.

j. Suppose now that three parties instead of just two are competing in the election. Imagine that all three of the parties locate at the position of the median voter. Would any of the parties want to change their position? If so, why? If not, why not? If it helps, you can think of all three parties locating at the position of the median voter in any of the three figures (10.11, 10.12, or 10.13).

k. Based on your answer to the previous question, does the result from the Median Voter Theorem stating that parties will converge to the position of the median voter depend on there being only two parties? Explain.

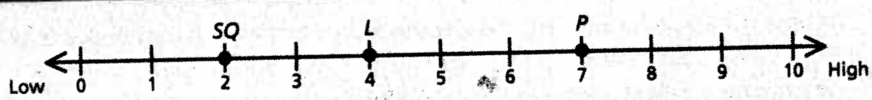
Spatial Models

7. The Median Voter Theorem is an example of a larger class of models known as spatial models. The primary characteristic of spatial models is that the preferences of actors can usefully be conceived as points in some kind of policy "space" (Hinich and Munger 1997, 5). Political scientists have employed spatial models to examine a diverse array of political situations, from leaders of countries negotiating territorial conflicts, to the relations between Congress, the president, and the Supreme Court in the United States, to party factions choosing a policy platform, to a policy adviser making recommendations to an elected official, and so on.

We now employ a simple spatial model to examine a situation in which a president (*P*) and a legislature (*L*) are considering whether to change the current level of public goods provision in a country. We can think of the level of public goods provision in a country as a single-issue dimension ranging from low public goods provision (0) to high public goods provision (10). The current level of public goods provision is referred to as the status quo (*SQ*). Both the president and the legislature have preferences over what the level of public goods provision should be. We will assume that the legislature gets to make proposals on what the level of public goods provision should be and that the president can either accept or veto these proposals. If the president vetoes the legislature's proposal, then the status quo policy is maintained. If the president approves the legislature's proposal, then the proposal is implemented and becomes the new status quo.

In Figure 10.14, we illustrate one possible scenario in which a president and a legislature could find themselves. The status quo level of public goods provision is 2. The president's most preferred level of public goods provision is 7, and the legislature's most preferred level is 4. Recall that in spatial models, actors are assumed to prefer policy outcomes that are closer to their ideal points than ones that are farther away. In other words, if an actor has to vote over two alternatives, then she will choose the one that is closer to her ideal point. If an actor has to choose between two policy outcomes that are equidistant from her ideal point, then she is indifferent between them and could choose either as a best response. For example, the president would be indifferent between having public goods provision at 6 or 8 because both of these outcomes are one unit away from her ideal point, 7.

FIGURE 10.14 Choosing a Level of Public Goods Provision: Scenario 1



Note: *SQ* = the current level of public goods provision; *L* = the ideal point of the legislature; *P* = the ideal point of the president.

a. Given that the status quo is at 2, what is the range of policy outcomes that the president prefers to the status quo? Recall that the president prefers any policy that is closer to her ideal point than the status quo.

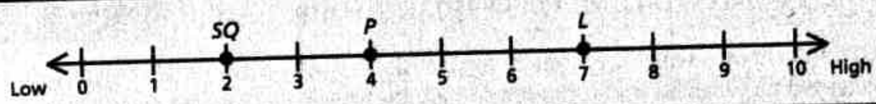
b. Given that the status quo is at 2, what is the range of policy outcomes that the legislature prefers to the status quo? Recall that the legislature prefers any policy that is closer to its ideal point than the status quo.

c. Do the ranges of policy outcomes that the president and legislature prefer to the status quo overlap? If they do overlap, how would you interpret the set of points where they overlap?

d. If the legislature proposes a new policy, and if it needs the president's approval for the new policy to be implemented, what level of public goods provision do you think the legislature will propose? Why? *Hint:* If the legislature proposes a new policy, it will want to choose the level of public goods provision that is closest to its ideal point and that is acceptable to the president.

Now imagine that the ideal points of the president and the legislature are reversed. In other words, the president's ideal point is now 4 and the legislature's ideal point is now 7. This scenario is illustrated in Figure 10.15.

FIGURE 10.15 **Choosing a Level of Public Goods Provision: Scenario 2**

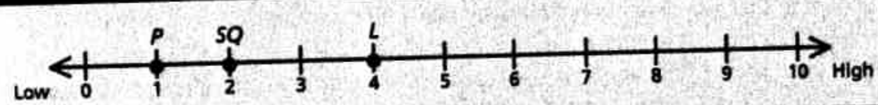


Note: SQ = the current level of public goods provision; *L* = the ideal point of the legislature; *P* = the ideal point of the president.

e. Given that the status quo is at 2, what is the range of policy outcomes that the president prefers to the status quo?
f. Given that the status quo is at 2, what is the range of policy outcomes that the legislature prefers to the status quo?
g. Do the ranges of policy outcomes that the president and legislature prefer to the status quo overlap? If they do overlap, how would you interpret the set of points where they overlap?
h. If the legislature proposes a new policy, and its implementation needs the president's approval, what level of public goods provision do you think the legislature will propose? Why?

Now imagine that the ideal points of the president and the legislature are on opposite sides of the status quo. Let's assume that the president's ideal point is now 1 and the legislature's ideal point is now 4. This scenario is illustrated in Figure 10.16.

FIGURE 10.16 **Choosing a Level of Public Goods Provision: Scenario 3**



Note: SQ = the current level of public goods provision; *L* = the ideal point of the legislature; *P* = the ideal point of the president.

i. Given that the status quo is at 2, what is the range of policy outcomes that the president prefers to the status quo?
j. Given that the status quo is at 2, what is the range of policy outcomes that the legislature prefers to the status quo?
k. Do the ranges of policy outcomes that the president and legislature prefer to the status quo overlap? If they do overlap, how would you interpret the set of points where they overlap? If they do not overlap, what does this mean?
l. Can the legislature make a successful proposal to change the status quo? If so, why? If not, why not?
m. Given your analysis of the three scenarios above, what can you say about the conditions under which policy will change versus when it will be stable?

Box 10.2

STABILITY IN TWO-DIMENSIONAL MAJORITY-RULE VOTING

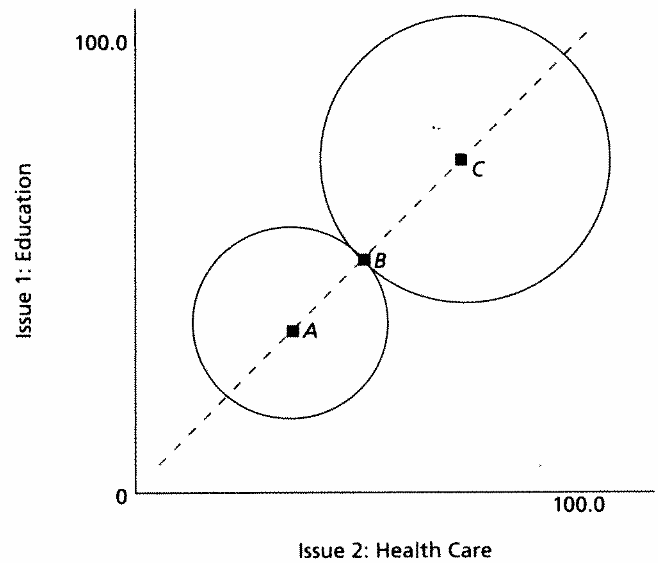
Earlier in this chapter, we argued that multidimensional voting was almost always going to be characterized by instability and cyclical majorities. Recall that this was the central insight gleaned from the Chaos Theorem. Only in extremely rare circumstances can stability be achieved. At the time, we did not present any evidence for why the conditions necessary to achieve stability in multidimensional voting were likely to be so rare. Although a full treatment of this subject exceeds the space and technical limits of this book, we now present some informal evidence to support this assertion. To keep things simple, we focus on two-dimensional voting. The logic of our argument, though, applies equally well to multidimensional scenarios in general.

One way instability is avoided in two-dimensional voting is if the individuals in a group have radially symmetric preferences (Plott 1967). This involves having a single individual be the median voter in both dimensions and all of the other voters be aligned symmetrically around this person. In Figure 10-17, we present a situation in which three individuals (voters *A*, *B*, and *C*) must make decisions on two issue dimensions simultaneously. For example, perhaps Issue 1 is the amount of money spent on education and Issue 2 is the amount of money spent on health care. As you can see, the voters in Figure 10-17 have radially symmetric preferences. Voter *B* is the median voter on both Issue 1 and Issue 2. For example, voter *B* has one voter to her left and one to her right on Issue 2, and one voter above her and one below her on Issue 1. As the dashed line indicates, the two other voters, *A* and *C*, are aligned symmetrically around voter *B*; that is, they are exactly opposite each other with voter *B* in between. Given this arrangement of voter preferences, if voter *B*'s ideal point were ever to become the status quo, there would be no majority that could displace it. In effect, voter *B*'s ideal point is an equilibrium in this two-dimensional majority-rule voting scenario.

To see why, we must examine the indifference curves of voters *A* and *C* with respect to voter *B*'s ideal point (the status quo)—these are the circles that surround the ideal points of voters *A* and *C*. These circles show all of the policy outcomes for which voters *A* and *C* would be indifferent between that policy and the status quo. Note that each indifference curve goes through *B*. Consequently, any point inside a circle is closer (and, therefore, preferred) to the relevant voter's ideal point than the status quo (represented by voter *B*'s ideal point. There is no circle with *B* as its center because there are no alternatives that voter *B* prefers to his own ideal point.[1] The two circles in Figure 10-17, therefore, represent the set of policy proposals that *A* and *C* prefer to the status quo. Majority rule means that two out of the three voters would have to prefer an alternative to *B*'s position for a policy proposal to dislodge *B*. The fact that the two circles never overlap, though, means that *A* and *C* cannot agree on an alternative to replace the status quo. If *C* proposed an alternative closer to his ideal point, *A* and *B* would vote against it. Similarly, if *A* proposed an alternative closer to his ideal point, *B* and *C* would vote against it. In other words, any alternative to *B*'s position

1. That is the definition of an ideal point, after all.

FIGURE 10.17 Stability in Two-Dimensional Voting

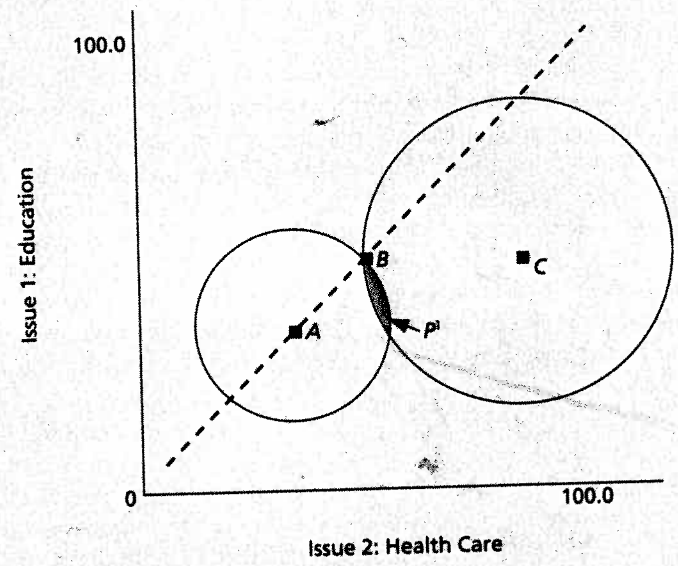


Note: Voter *B*'s position = the status quo policy; the two circles = the indifference curves for voters *A* and *C* with respect to the status quo policy *B*.

would never get more than one vote and would, therefore, lose. As this example illustrates, it is possible to obtain a stable outcome—in this case *B*'s position—in two-dimensional majority-rule voting.

Although stability is *possible* in two-dimensional majority-rule voting games, is it *likely*? Many scholars think not. To see why, imagine what happens if one of the voter's ideal points moves just a little. In Figure 10-18, we shift voter *C*'s ideal point by an arbitrary amount off of the dashed line to the southeast. You will immediately notice that there is now a lens-shaped area south of *B* in which the indifference curves of voters *A* and *C* overlap. This lens-shaped area is the winset of *B* and contains all of the alternative policy outcomes that both voters *A* and *C* prefer to *B*. This means that if either voter *A* or *C* has an opportunity to propose a change in policy to a point in the winset, say P_1, he will do so and the new policy proposal will win a majority vote. Once this happens, all the indifference curves will need to be redrawn so that they go through this new status quo point. If we were to do this, we would find another lens-shaped area, thereby demonstrating that at least two voters will prefer yet another policy combination to this new status quo. At this point we will be off to the races, and the instability of the Chaos Theorem will ensue because the condition that had previously made *B* a stable outcome no longer pertains.

FIGURE 10.18 Instability in Two-Dimensional Voting



Note: Voter *B*'s position = the status quo policy; the two circles = the indifference curves for voters *A* and *C* with respect to the status quo policy; the shaded oval area = the winset of *B* and represents the alternative policy outcomes that voters *A* and *C* prefer to voter *B*'s position. P^1 = a policy proposal that would defeat the status quo policy (*B*) in a majority rule vote.

Recall what made *B* stable in the first place. First, there was a voter whose ideal point made her a median voter in both issue dimensions. Second, all of the other voters' ideal points radiated from *B* in just the right way. Specifically, *A* and *C* were aligned on a straight line with *B* (the dashed line in Figure 10-17). Charles Plott (1967) proved that such "radial symmetry" is sufficient to guarantee the existence of a stable outcome in a two-dimensional majority-rule voting scenario. Radial symmetry, however, is not strictly necessary, and subsequent scholars have identified other conditions that are sufficient to produce stability.[2] These further conditions, like Plott's condition, though, are quite restrictive. As a result, it is reasonable to conclude that majority rule is unstable in multidimensional situations except under very unusual circumstances in which voter preferences line up in just the right way.

2. Hinich and Munger's *Analytical Politics* (1997) is a good resource for ambitious students who would like to pursue this topic further.