James L. Sweeney Winter, 1995/96
EES 212B February 14, 1996

MIDTERM EXAM

Open book, open notes. Your answers should be concise and to the point. In all cases where numerical answers are requested, you should demonstrate how you reached your results. The total possible points for each section are given to the left of the section number.

Question 1 (30 Points) Assume there are two people and two goods in a pure exchange economy. They have different initial endowments of the two goods and different preferences.

Person A has an initial endowment of 1000 units of good 1 and 2000 units of good 2.
Person B has an initial endowment of 4000 units of good 1 and 3000 units of good 2.

Person A has preferences that can be represented as a Cobb-Douglas utility function as follows:

$$U_A = 10\, X_{A1}\, X_{A2}$$

Person B has preferences that can be represented as a Cobb-Douglas utility function as follows:

$$U_B = 50\, (X_{B1})^2\, X_{B2}$$

Here X_{A1} and X_{A2} represent the quantities of goods 1 and 2, respectively, consumed by person A and X_{B1} and X_{B2} represent the quantities of goods 1 and 2, respectively, consumed by person B.

a) (10 points) Find a mathematical expression determining the combinations of X_{B1} and X_{B2} which are part of Pareto Optimal allocations of these goods. Be sure to show the minimum and maximum values for X_{B1} and X_{B2} that are Pareto Optimal.

b) (10 points) Find a mathematical expression determining the combinations of X_{B1} and X_{B2} which are part of the core of this exchange economy. Be sure to show the minimum and maximum values for X_{B1} and X_{B2} that are in the core.

c) (10 points) Find a mathematical expression determining the combinations of X_{B1} and X_{B2} which are part of Walrasian equilibrium allocations of these goods. Be sure to show the minimum and maximum values for X_{B1} and X_{B2} that are Walrasian equilibrium allocations.

In your answers to parts a), b), and c), diagram and label your answers. Use a single diagram for the three parts.

Question 2 (30 Points) Let $Q = F(A) * (B-P)$ be the demand function of a homogeneous product, where A is total advertising expenditure, P is the price, and F(.) is a concave, increasing, and positive function of A.

(a) (10 points) Consider a monopolist facing a non-decreasing marginal cost (as a function of quantity) and a linear cost for advertising. State the first and second order necessary conditions for the optimal level of advertising and the optimal price and quantity in this case.

(b) (10 points) Assume the monopolist is broken up and organized into three different firms. Each firm faces the same cost structure as that of the monopolist. The industry faces the same demand function, now with A equal to the sum $A_1 + A_2 + A_3$, the advertising expenditures of the three firms. Under the assumption that the firms have cournot conjectures on quantity and advertising, state the first and second order necessary conditions which would determine the optimal quantities, price, and total advertising.

(c) (10 points) Show that the total level of advertising in part b) is lower than that computed in part a).

Question 3 (40 Points). Several firms are planning to develop a new product that has inherent physical risks. The risk is described by the probability Pr that any particular unit of the product will fail and will significantly damage other equipment owned by the customer. Each firm has the choice of making the product less risky, reducing the probability from Pr to Ps (where Ps < Pr). However, making the product less risky would increases the total development cost. In particular, if C is the total cost of developing the risky product, then C + D is the cost of developing the safe product, where D > 0. In case of an accident, the loss incurred L is the same for both types of products.

The manufacturer will sell N units of the product, so that the expected value of losses is Pr N L or Ps N L. Assume that total expected costs would be smaller if a firm developed the safer product. That is, assume that D < (Pr - Ps) N L.

If an accident occured, the manufacturer of the new product would be liable for any damages. But these firms could purchase insurance which would pay the cost of the damages. The insurance industry cannot tell which firms develop the riskier product and therefore all firms buying insurance pay the same insurance premium. The insurance industry is perfectly competitive.

Each insurance company is risk neutral. There is no administrative cost for the insurance policies. The manufacturer is risk averse, so that the certain equivalent of any uncertain loss exceeds the expected value of the loss. For this problem, make the simplifying assumption that the certain equivalent of an uncertain loss is 25% larger than the expected value of that loss.

Insurance companies could offer full liability insurance coverage, which would pay all costs of accidents, or partial coverage, which would pay only a fraction of the costs of accidents.

(a) (10 points) Assume that there are many firms manufacturing the product, and that all such firms purchase full liability insurance coverage. Show that all firms will choose to develop the risky product if they develop any product at all.

(b) (10 points) Assume now that there are few such manufacturing firms. Develop a condition to show which firms in Nash equilibrium would develop the safer product and which would develop the risky product. In particular, your condition should derive a particular expected market share, such that firms with a larger expected market share than the particular share will develop the safer product and those with a smaller expected market share will develop the riskier product. Express the particular market share that divides the two groups as a function of the parameters of the problem.

(c) (10 points) Assume again that there is a large number of small firms. Assume that firms have different development costs such that D varies uniformly from 0 to L N. Suppose the insurance companies change their policies, and now offer to pay only 60% of the losses, should an accident occur. Show that, in equilibium, some, but not all, of the firms will choose to develop the safe product, and the rest will develop the risky one. Write down the criterion that defines which firms will develop the safe product.

d) (10 points) Assume again that there is a large number of small firms. Assume now that the insurance companies offered both types of coverage, full liability or 60% coverage. Show that the only market equilibria that are possible will be of Type a) or Type b), as follows. In your demonstration, you must eliminate all other types of equilibria.

Type a) All firms with low enough D will develop the safer product and will purchase the 60% insurance coverage. All firms with higher D will develop the riskier product and will purchase full liability coverage.

Type b) All firms will develop the riskier product and will purchase full liability coverage.