Computational Economics

Stanford University Department of Economics, Econ 288 Autumn Quarter 2002-2003 11:00 a.m. - 12:50, Tuesday and Thursday Economics 218

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Course Description

Objective

This course introduces computational approaches for solving economic models. It focuses on a broad range of numerical methods and then applies them to economic problems. We formulate economic problems in computationally tractable forms, and use numerical analysis techniques to solve them. We will study examples of computational techniques in the current economic literature as well as discuss areas of economic analysis where numerical analysis may be useful in future research of dynamic economic problems. The substantive applications will cover a wide range of problems including public finance, macroeconomics, game theory, mechanism design, finance, industrial organization, agricultural economics, and econometrics.

Overview

We begin with an overview of the necessary numerical analysis and approximation theory. First, we review standard numerical analysis - interpolation and approximation techniques, numerical optimization, numerical solutions to systems of nonlinear equations, quadrature formulas for numerical integration, and basic solution techniques for ordinary and partial differential equations. Second, we examine some less standard material from numerical analysis which is very useful for economic problems. That includes projection approaches to ordinary and partial differential equations as well as topics in high-dimensional integration. Third, we cover perturbation and bifurcation techniques which are useful in economics problems.

We give economic examples including Nash equilibrium and general equilibrium calculations using nonlinear equation methods and optimal incentive mechanisms using numerical optimization. We use numerical methods to solve dynamic optimization problems and applications to economic problems, particularly optimal growth, life-cycle models of consumption and savings, portfolio management, and equilibrium with taxation. We will develop techniques for numerical solutions to a broad range of recursive equilibrium problems.

Prerequisites

It is best if students are familiar with basic dynamic optimization theory, and have a good undergraduate background in mathematics. In particular, students should be familiar with linear algebra, finite- dimensional optimization theory, and linear differential equations. However, any student who is interested in the course and has passed the first year core econometrics, microeconomics, and macroeconomics courses will be able to follow the course.

Course Grade

Doing computation is the only way to learn computation. The course grade will be based mostly on computational exercises assigned about every two weeks. You will be encouraged to work in groups of two or three.

Computing Languages

Students will need to know some computational language. FORTRAN is still used frequently in scientific computations since it offers the greatest range of packaged routines. Students are welcomed to use other languages, such as C, C++, Matlab, and Gauss. Some people use statistical software such as TSP, but it is better to learn a more flexible language. (No, Excel will *not* be adequate.) In fact, if you do not know any suitable language, I recommend that you use Matlab since it is relatively easy to learn, adequate for this course, often useful in research, and available on Stanford computer systems.

While the range of topics is broad, the course aims to acquaint students with the range of techniques that have been useful in economic analysis as well as expose students to other techniques that have potential use in economic applications. I want to develop a basic understanding of numerical techniques, demonstrating their use in economic examples, show how existing techniques fit into the broader numerical literature, and point the students to potentially useful numerical techniques.

Scheduling Comment

In the past, Economics 288 has been taught every other year. It is unlikely that I will be teaching it in academic year 2003-2004.

Textbook

The text is *Numerical Methods in Economics* by Judd, published by MIT Press. I will be particularly appreciative of any comments you may have on the text (typos, lack of clarity, etc.) since I am beginning to prepare the second edition.

Winter quarter extension

This course can only begin to examine the literature on numerical methods in economics. I plan on offering extra lectures on advanced topics (computing Nash equilibrium of supergames, models with heterogeneous agents, advanced methods for dynamic programming, advanced perturbation methods, and asset market methods) in the Winter or Spring quarter if there is sufficient interest.

Course Outline and Schedule

I. Numerical Analysis: General Considerations - September 26

Chapters 1 and 2. General ideas of convergence rates, computational errors, error analysis.

II. Solution of Linear Equations - October 1, 3

Chapter 3. LU, QR, and Cholesky decomposition, condition numbers, Gauss-Jacobi and Gauss-Seidel methods.

III. Optimization Methods - October 3, 8

Chapter 4. Search methods, bisection, Newton's method, BFGS and DFP updates. Applications to consumer demand and incentive problems.

IV. Solving Nonlinear Equations - October 10, 15

Chapter 5. Bisection methods, Newton's method, Broyden updates, Powell hybrid method, continuation and homotopy methods. Applications to general equilibrium and Nash equilibrium.

V. Approximation of Functions - October 17, 22

Chapter 6. Orthogonal polynomials, splines, interpolation.

VI. Numerical Quadrature and Monte Carlo simulation - October 24, 29

Chapter 7, 8, and 9. Integration methods for single- and multiple-dimensional integrals. Monte Carlo simulation methods. Applications to portfolio choice and dynamic problems.

VII. Differential, Integral, and Functional Equations - October 31, November 5

Chapter 10 and 11. Methods for solving ordinary differential equations as well as the more complex equations arising in dynamic economic models. Applications to dynamic optimization.

VIII. Numerical Dynamic Programming - November 7, 12

Chapter 12. Solutions to deterministic and stochastic dynamic programming problems using approximation, integration, and optimization methods. Applications to savings-consumption problems.

IX. Perturbation Methods - November 14, 19

Chapter 13, 14, and 15. Taylor series approximations to find numerical solutions, linearizing around a steady state. Simple bifurcation methods. Applications to dynamic equilibrium models.

X. Solutions of Deterministic Dynamic Models - November 21

Chapter 16. Solution of perfect foresight models. Fair-Taylor method, parametric path method.

XI. General Methods for Solving Rational Expectations Models - Nov. 26, Dec. 3, 5

Chapter 17. Rational expectations models, Lucas asset pricing model, Brock-Mirman growth models.