Numerical Methods in Economics

MIT Press, 1998

Notes for Chapter 1
Introduction

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The Role of Computation in Economic Analysis

- Traditional roles
  - Empirical analysis
  - Applied general equilibrium
- Nontraditional roles
  - Substitute for theory
  - Complement for theory
- Questions:
  - What can computational methods do?
  - Where does computation fit into economic methodology?

Computation and Science

- The Scientific Method
  - Experimentation – detect patterns
  - Theories, models, and deductive methods – produce theorems, closed-form solutions
  - Computations
- Computational Successes in Science
  - The Red Spot of Jupiter
  - Origin of the moon
  - Shape of Galaxies
- Computation in Science and Economics
  - Astronomy and economics – observational sciences
  - Red Spot $\sim$ Kydland-Prescott RBC success
  - Common challenge: "Visualization" problems
  - Differences: Precise theories of science versus qualitative theories in economics

What can we compute now?

- Optimization
  - Dynamic programming
  - Mechanism design
- General equilibrium
  - Arrow-Debreu general equilibrium
  - General equilibrium with incomplete markets
  - General equilibrium with imperfect competition
  - Dynamic, perfect foresight models
  - Dynamic, stochastic recursive models
- Asset markets
  - Asymmetric information - Grossman-Stiglitz, Radner
  - Imperfectly competition - Kyle model
- Games
  - Finite games- Lemke-Howson, Wilson, McKelvey
  - Supergames - Cronshaw-Luenberger, Judd-Yeltekin-Conklin
- Dynamic games
  - Closed-loop (a.k.a., Markov perfect) - Kotlikoff-Shoven, Wright-Williams, Mirand Rui, Vedenov-Miranda, Sibert
  - Supergames with states - Judd-Yeltekin

Progress in Numerical Analysis

- Linear programming - Interior point methods
- Nonlinear equations, complementarity problems
- Only small amount of numerical analysis is used in economics

Hardware Progress

- Moore's law for semiconductors
- Optical computing
- DNA computing
- Quantum computing

Software Progress

- Parallelism: Combine many cheap processors
- Program development tools

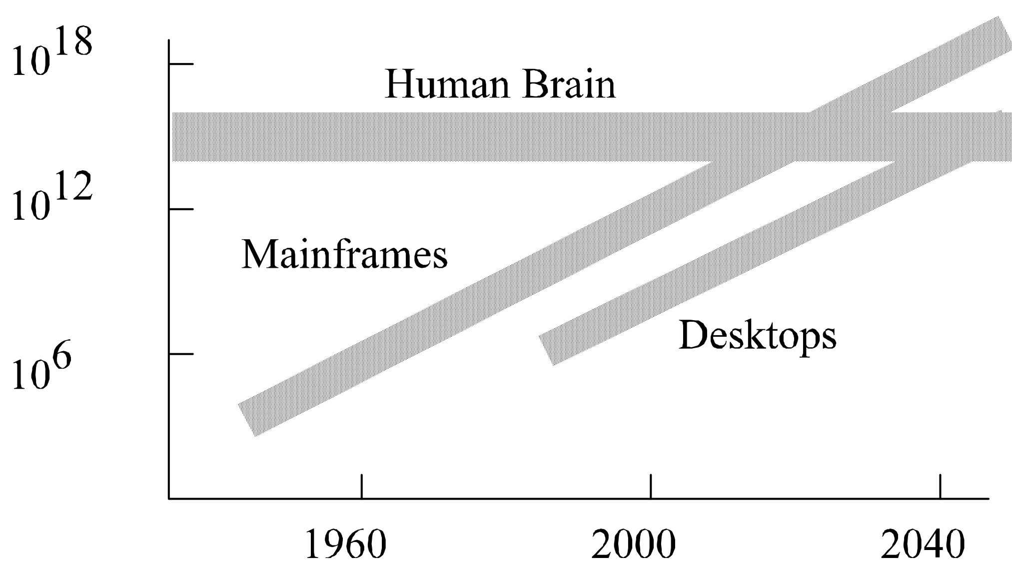

Figure 1: Trends in computation speed: flops vs. year

Modes of Theoretical Analysis

- Theory: A Definition
  - Define terms, concepts
  - State assumptions
  - Determine the implications of the theory
- Two ways to ascertain implications of a theory
  - Deductive Theory
    - Prove general theorems about general case
    - Add auxiliary assumptions to make tractable
    - Prove more precise theorems about tractable cases
  - Computational Theory
    - Specify parameterized versions of the theory
    - Compute specific models, i.e., fix parameter values
    - Summarize results of computations
- Examples
  - I.O. – oligopoly, theory of firm, info.
  - Labor – principal-agent, compensation
  - Finance – market microstructure, info.
  - Poli Sci – legislative, election models

- Theory as Exploration: The Typical Scenario
  - A theory defines a general class of models
  - Deductive theory can examine, without error, a thread containing a continuum of instances of the theory
  - Computation can examine, with error, a finite collection of general instances
  - The properties of the model are piecewise continuous in the parameters

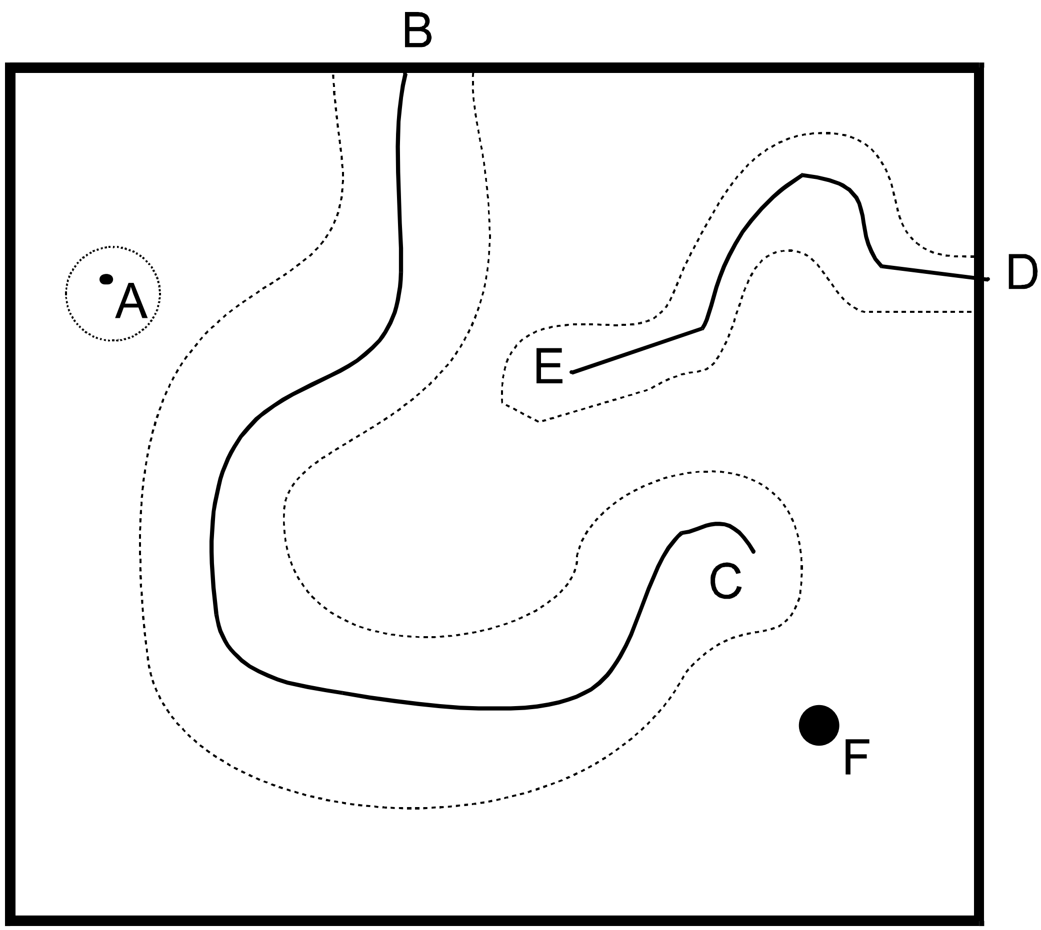

Figure 2: Typical graph of tractable cases

- Theoretical Practice: Science vs. Economics
  - Professional rewards: E.g., Einstein credited with general relativity, but he never proved a general existence theorem, produced no nontrivial solution.
  - Approximate "computational" methods pervade physics
  - Ad hoc mathematics often used in theoretical physics
  - Economic theorists follow "Bourbaki", pure math
- Computational versus Deductive Theory
  - Deductive theory produces absolute truths
  - Deductive methods can build a "deep" theory, as in mathematics
  - Deductive theory focusses on very broad qualititative issues or a narrow collection of excessively simple models
  - Is mathematics, Bourbaki-style, appropriate for economics?

Three Examples of Computational Theory

- Haubrich
  - Question: Executive rewards
  - Model: principal-agent model
  - Observations: executives get only about $3 per thousand
  - Conventional wisdom: executives should get more for incentives - supported by risk-neutral agent specification
  - Computations: compute optimal contract for reasonable tastes and technology
  - Results: optimal share is often about $3 per thousand
- Spear-Srivastava and Phelan-Townsend
  - Computation and theory as "Tag Team" partners
  - A theory begins with an enormous collection of possible results: payments in repeated moral hazard problems can depend on entire history
  - Theory can narrow range of possibilities: S-S reduced problem to 1-D dynamic programming problem
  - Computation uses theoretical analysis to construct efficient computational methods: P-T papers

- Quirmbach
  - Question: What ex post market structure best encourages ex ante innovation among competitors?
  - Model: A two-period model of innovation then production
  - Computations: Search across demand functions, costs, R&D success rates, game forms
  - Results: Bertrand and Cournot were roughly the same, better than ex post collusion.
  - There are no reasonable theorems.

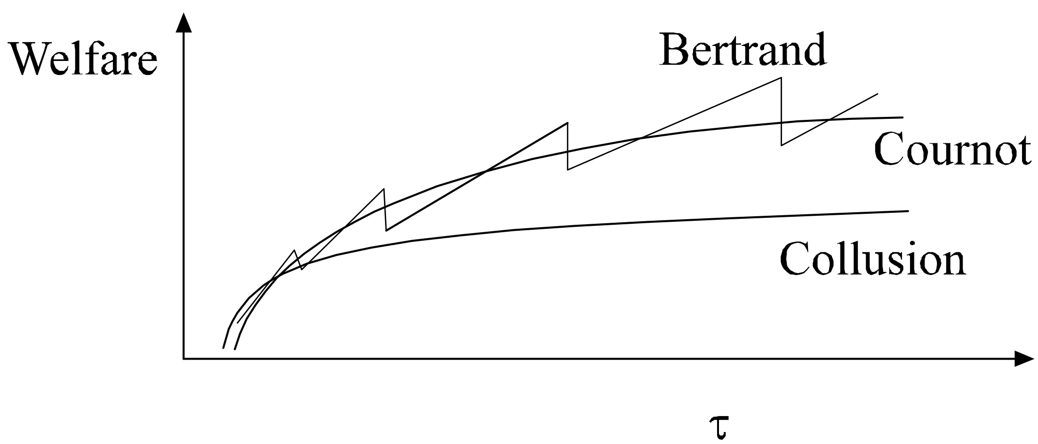

Figure 3: Welfare vs. probability of innovation success

Systematic Approaches to Computational Theory

- Perturbation Methods: Fattening the Thread
  - $F(x, \delta, \epsilon) = 0$ expresses a theory, with general solution $x(\delta, \epsilon)$
  - Deductive theory solves $F(x, \delta, 0) = 0$ to get $x(\delta, 0)$ – the thread of special cases.
  - Perturbation methods compute $x(\delta, \epsilon)$ for small ϵ – fattens the thread.
  - Example: consumption function in growth models

$$C(k, \sigma^2) \doteq C(k^*, 0) + C_k(k^*, 0) + C_{\sigma^2}(k^*, 0)\sigma^2 + \cdots$$

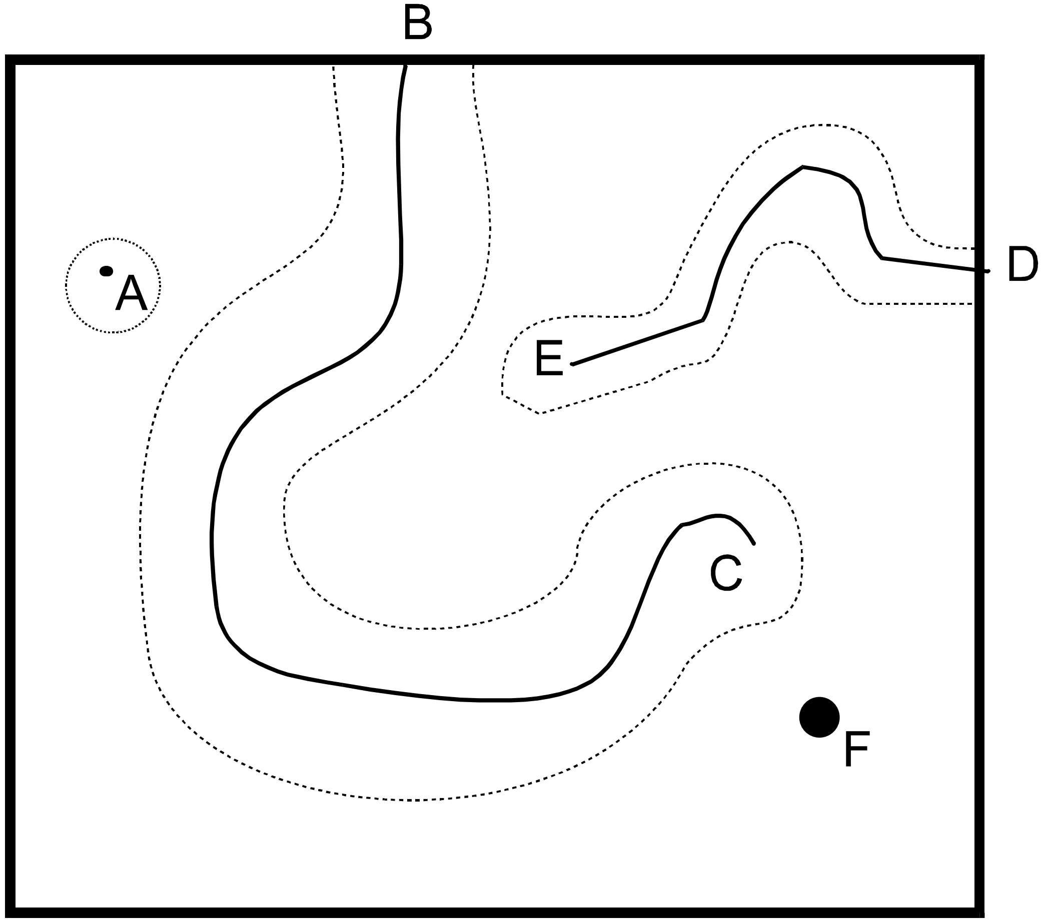

Figure 4: Typical graph of tractable cases

- Monte Carlo Sampling
  - Draw N points independently from the model space according to a probability measure μ.
  - Test proposition at each sampled instance
  - If proposition is true at each sampled instance, then state "We conclude with $1-(1-\epsilon)^N$ confidence that the set of counterexamples has μ-measure less than ϵ."

- Quasi-Monte Carlo Sampling
  - Construct a N-point set with low discrepancy (i.e., "uniformly spread") in a metric d. Let mesh be δ.
  - Test proposition at each sampled instance
  - If proposition is true at each sampled instance, then "No set of counterexamples contains a ball of diameter δ."
  - If proposition is true at each sampled instance, interval arithmetic is used, and Lipschitz bounds apply and can be computed a priori, then Proposition is *proven*!

- Regression Methods of Summarizing Results
  - Construct a N-point set of instances.
  - Compute quantities of interest (price, quantity, welfare, etd.) at each sampled instance
  - "Regress" the quantities of interest on the model parameters, compute "covariances" among model parameters and equilibrium outcomes.

- Presentation of Computational Results
  - Tables, Graphs – limited
  - "Confidence probabilities"
  - "Regression" results
  - Need ways to describe robust patterns

Computational Theory vs. Calibration

- Calibration
  - Specify a model
  - Consult empirical facts to choose a case
  - Analyze a single case
  - Inconsistent with Bayesian decision theory
- Computational theory
  - Only loosely constrained by data
  - Look for patterns - comparative statics

Deductive versus Computational Theory

	Deductive methods	Computational methods
Approach:	Prove theorems	compute examples
Validity:	absolute	limited by numerical error
Range:	continua	finite number of examples
Generality:	simplifications made for tractability	limited only by computational methods
Existence:	proven	present examples of ϵ-equilibrium
Efficiency:	(dis)proven	indicate quantitative importance
Comp. statics:	usually need special functional forms	impose empirically motivated restrictions
Errors:	specification errors	numerical errors
Inputs:	mathematical theory skills	computer time, computational skills

- Synergies
  - Numerical examples inform deductive analyses
  - Computational simplifications from deductive theory
- Tradeoffs and trends
  - Error type
    - Specification errors of deductive theory - trend?
    - Numerical errors of computational methods - falling rapidly
  - Input Costs
    - Human math skills and knowledge - some trend
    - Computation costs ($/Flop) - falling rapidly

The Future of Computational Economics

- Technology – Hardware and Software
  - Computing costs will continue to decrease
  - New computing environments and technologies can be exploited
  - Economists will catch up to numerical analysis frontier
  - Numerical analysis will develop better methods to exploit new technologies
  - Economists will develop of problem-specific methods (as in CGE)
- An Economic Theory of the Future
  - Inputs: Human time and computers
  - Outputs: Understanding of economic systems
  - Trend: Falling price of computation
  - Prediction: Comparative advantage principles imply
    - ∗ Substitution of computer power for human time and effort in analysis of specific models
    - ∗ Human activity will specialize on formulating theoretical concepts and models, and deciding which problems are most important.