Economics 288

Fall 2002

Problem Set 3

Due Oct. 29, 2002

1. Solve the dynamic life-cycle consumption problem (14.1) where $w_t = 1 + .2t - 0.003t^2$ in year t, $T = 50$, $\beta = .96$, $r = .06$, and t is understood to be the year. Next assume utility is $u(c) + v(\ell)$ where c is consumption and ℓ is labor supply. The consumer's problem is then to choose c_t and ℓ_t so as to maximize

$$\sum_{t=1}^{t=T} \beta^t \left(u(c_t) + v(\ell_t)\right)$$

with the constraint that final savings $S_T = 0$ where

$$\begin{aligned} S_{t+1} &= (1+r)S_t + w_{t+1}\ell_{t+1} - c_{t+1} \\ S_0 &= 0 \end{aligned}$$

Let $u(c) = c^{1-\gamma}/(1-\gamma)$, $\gamma = .5, 2, 3$, and let $v(\ell) = -\ell^{1+\eta}/(1+\eta)$ where $\eta = 2, 10$.

a. Compute the solutions of these life-cycle problems. Write your own code using any optimization method we discussed.

b. Next incorporate flat tax rates on wage and interest income, that is, there is a tax rate τ_K on interest income and a tax rate τ_L on labor income. Suppose the government needs to raise revenue with a present value of 3; that is, the tax policy must satisfy the constraint

$$\sum_{t=1}^{t=T} (1+r)^{-t} \left(\tau_L w_t \ell_t + \tau_K r S_{t-1})\right) \geq 3$$

What choice of tax rates is best for the life-cycle consumer? How does the answer depend on the choices of γ and η? For this part, use any optimization software you have.

If $T = 50$ is too large a problem for you (this may be the case when you do the optimal tax exercise) then try the following assumptions: $T = 5$, $w_t = -0.75 + 23t - 3t^2$ in period t, $\beta = .67$, and $r = .79$.

2. Write a program to solve the principal-agent problem with the following assumptions: Output will be either 0 or 5: $y_1 = 0,\ y_2 = 5$. The agent chooses one of two effort levels: $L \in \{0,\ 1\}$. High output occurs with probability 0.8 if the effort level is high; that is $\text{Prob}\{y = 5 \mid L = 1\} = 0.8$. High output occurs with probability 0.4 if effort level is low; that is, $\text{Prob}\{y = 5 \mid L = 0\} = 0.4$. We assume that the agent's utility is $U^A(w, L) = u(w) - d(L) = -e^{-2w} + 1 - d(L)$ and that the disutility of effort is $d(0) = 0,\ d(1) = 0.01$. We assume that the agent's best alternative job is equivalent to $w = 2.5$ and $L = 1$ surely, implying that the reservation utility level, R, is $u(2.5) - d(1)$.

This problem is slightly different and is better behaved than the one in the book. You may use any software you have.

3. (Withdrawn)

4. Consider the CGE problem in section 5.10 with

$$u^i(x) = \sum_{j=1}^{m} a^i_j \, x_j^{\nu^i_j+1} \, (1+\nu^i_j)^{-1}.$$

For $\nu^i_j = -1$, we replace $x_j^{\nu^i_j+1} \, (1+\nu^i_j)^{-1}$ with $\ln x_j$; if one does not want to program this special case, one can approximate $\nu^i_j = -1$ with, for example, $\nu^i_j = -1.01$. Assume that $a^i_j, \, e^i_j > 0 > \nu^i_j$, $i = 1, \cdots, n$, $j = 1, \cdots, m$. Write programs that take the a^i_j, ν^i_j, and e^i_j coefficients as inputs and compute the pure exchange equilibrium price vector p_j, $j = 1, \cdots, m$, and consumption of good j for agent i, c^i_j. Let $m = 2$ and $n = 3$, and

$$\begin{aligned} a^1 &= (1,1), \; \nu^1 = (-1.1, -1.1), \; e^1 = (1,1) \\ a^2 &= (1,2), \; \nu^2 = (-2, -3), \; e^2 = (2,3) \\ a^3 &= (2,1), \; \nu^3 = (-.5, -.8), \; e^3 = (1,5) \end{aligned}$$

Compute equilibrium using your own coding of any of the methods described in the lecture.

5. Compute the degree 3 and 4 Taylor series expansions for $f(x) = (x^{1/2} + 1)^{2/3}$ near $x_0 = 1$. Compute the (2,2) Padé approximation based at $x_0 = 1$. Compare the quality of these approximations for $x \in [0.2, 3]$. Do not use finite differences to compute derivatives. By the way, you are not allowed to use Taylor series or Padé programs; you write the programs.

6. Compute the natural cubic spline approximation to $x^{1/4}$ on $[1, 11]$ using 6 and 11 nodes. Plot the errors and compute the L^2 and L^∞ norms of the errors for each case. Where do the spline approximations do poorly? Write your own code, but you may use other code for linear algebra.

7. Write a program that takes an increasing function $f(x)$ and computes the inverse function. Use any approximation scheme you deem reasonable. Test the program on the functions x^3 and e^x on $[0, 2]$ using the points $\{0, 0.5, 1.0, 1.5, 2.0\}$. Write your own code, but you may use other code for linear algebra.

8. Let $f(k, \ell) = (k^{1/2} + \ell^{1/2})^2$. Compute its linear, quadratic, and cubic Taylor series expansions around $(1, 1)$. Compute the L^2 and L^∞ errors over all points in $[0, 2.5] \times [0, 2.5]$ of the form $(.025\, i, .025\, j)$, i, j integers.