Economics 288

Fall 2002

Problem Set 4

Due November 12, 2002

1. Compute the degree n, m-point Chebyshev regression approximation for x^{α} on $[a,b]$ for $n = 3, 6, 15$, $m = n+1, 2n$, $\alpha \in \{0.5, .9\}$, $a \in \{0.2, 1\}$, and $b \in \{2, 5\}$. Compute the L^{∞} and L^2 errors for f and the L^{∞} error for f' over $[a,b]$. Report the results in a table like

a	b	α	n	m	L^{∞} error for f	L^2 error for f	L^{∞} error for f'
.2	2	.5	3	4	?	?	?
			3	6	?	?	?
			6	7	?	?	?
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Compute the L^{∞} and L^2 errors by using 1001 uniformly distributed points on $[a,b]$

2. Let

$$V(k_1, k_2, k_3) = (k_1 + k_2 + k_3)^{1/2} - (k_1 - k_2)^2 - (k_1 - 3k_3)^2 - (k_2 - 2k_3)^2.$$

Compute the three-dimensional Chebyshev polynomial tensor product approximation on $[1,3]^3$ using 11 points in each dimension. Use 41 uniformly distributed points in each dimension to compute the L^2 and L^{∞} norms of the error. Then compute the complete polynomial approximation. How much accuracy do you lose? What is the relative computational cost of the two approximations?

3. Write a program to implement Schumaker's quadratic shape-preserving approximation described in the book. Apply it to the function $V(k) = k^{\alpha}$ on $[0, 10]$ for $\alpha = .01, .25, .80$. Compare the results when you use slope information to the results when you just approximate the slopes. Compare the results with Chebyshev interpolation.

4. Compute $\int_0^{\infty} e^{-\rho t} u(1 - e^{-\lambda t}) dt$ for $\rho \in \{0.04, 0.25\}$, $u(c) = -e^{-ac}$, $a \in \{0.25, 1.0, 5.0\}$, $\lambda \in \{0.01, 0.05, 0.20\}$, using Gauss-Laguerre quadrature with 3, 7, and 10 nodes. Use the change of variables $t(z) = \frac{z}{1-z}$ to find an equivalent integral over $[0, 1)$, and apply Simpson's rule using 3, 7, and 11 points to the transformed problem. Compare the Gauss-Laguerre and Newton-Cotes approaches.

5. Suppose that there are three assets, Z_i, $i = 1, 2, 3$, each with log Normal distribution; specifically, assume $\log Z_i \sim N(\mu_i, \sigma_i^2)$ with $\mu = (0.03, 0.04, 0.05)$ and $\sigma = (.1, .2, .3)$. Suppose that an individual owns $1/i$ units of asset i and has von Neumann-Morgenstern utility function $u(c) = \log c$. Compute expected utility, $E\{u(1 + Z_1 + Z_2 + Z_3)\}$, using (a) product Gauss-Hermite quadrature using 5, 7, and 11 points in each direction, (b) an appropriately modified version of (7.5.11), (c) Monte Carlo (use any random number generator your software provides) using 100, 10^3, and 10^5 points, and (d) three-dimensional Weyl sequence using 100, 10^3, and 10^5 points. Some of these will require an appropriate change of variables. Assume that the 11-point Gauss-Hermite result is the truth. Report the error of each of the other methods.

6. Suppose X and Y are independent and both $U[0, 1]$, and $Z = (X^{.2} + Y^{.5})^{.3}$. Compute polynomial approximations of the conditional expectation $E\{Y|Z\}$ using Monte Carlo sampling and regression. What size sample do you need to get the average error down to .01? Now repeat this with a Weyl sequence. What size sample do you need to get the average error down to .01?

7. Consider the Solow growth model $\dot{k} = sf(k) - \delta k$ with $s = 0.2$, $\delta = 0.1$, $f(k) = k^{\alpha}$, $\alpha = 0.25$, and $k(0) = 0.5$. Compute the numerical solution for $t \in [0, 100]$ using the Euler method, Runge-Kutta, and RK4 with $h = 1, .1, .01$. Compare these solutions with the exact solution.