Economics 288

Fall 2002

Problem Set 5

Due November 26, 2002

1. Consider the optimal growth problem with adjustment costs

$$
\begin{aligned}
& \max_{c} \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \\
s.t.\ k_{t+1} &= k_t + I_t \\
0 &= f\left(k_t\right) - c_t - I_t - \phi I^2/k
\end{aligned}
$$

for

$$
\begin{aligned}
\beta &= 0.95, .99 \\
u(c) &= \log c \\
f(k) &= 0.2k^{.25} \\
\phi &= 0, 10, 100.
\end{aligned}
$$

over the range $k \in [.6, 1.4]$

(a) Discretize the state space and apply the value function iteration, policy function iteration, and upwind Gauss-Seidel method to solve the dynamic programming problem. Use at least 81 capital stocks. Which method is fastest? (By the way, if a method takes forever to converge, stop your computer and report the slow convergence; don't waste time waiting for slow methods to converge.) Can any of these methods solve the problem with 8001 capital stocks? 800,001 capital stocks?

(b) Use value function iteration with polynomial approximation of $V(k)$. Use at least a degree 5 polynomial. How high can you go? What is the implied policy function $C(k)$

(c) Derive the Euler equation and use projection methods (least squares, Galerkin, and collocation) to solve for the policy function $C(k)$. Use at least a fifth-order polynomial. Can you compute a degree 10 polynomial approximation? Which method worked best?

(d) Use the perturbation method to compute third-order Taylor series approximation for $V(k)$ around the steady state. (I suggest that you derive formulas - by hand unless you have Mathematica or something similar - for general ϕ and β first and then plug in the values.)

(e) Compare the policy functions computed in (a-d). What are the L_2 and L_∞ norms of the differences?

(f) Which policy functions appear to do best? How can you judge?