The Optimal Tax Rate for Capital Income is Negative

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Abstract. Optimal income tax systems subsidize the purchase of capital goods to offset markups of price over marginal cost arising from imperfect competition. Optimal subsidies are economically significant even if pure profits are not taxed away and the alternative distortionary taxes have large efficiency costs. If markups are greater for equipment than for construction, as indicated by some evidence, this analysis justifies the Investment Tax Credit’s discrimination in favor of equipment over structures.
1. Introduction

The problem of optimal taxation of income in dynamic economies has been analyzed extensively, but primarily under the assumption of perfect competition. In this paper, we examine the nature of optimal taxation in a dynamic model with preexisting distortions due to imperfect competition. Our main finding is that while labor income, pure profits, and consumption should be taxed, capital income and firms’ purchases of capital goods should, in the long run, be subsidized at the margin to overcome distortions due to imperfect competition which cause prices of capital goods to exceed marginal cost. Since many sources of capital income are taxed, the gains from capital tax reductions are even larger than the usual estimates.

The basic idea is intuitively a combination of three well-known ideas. First, in markets where price exceeds marginal cost because of market power, subsidies can be used to offset the distortions if a lump-sum tax is available. This is an old argument, going back to Robinson (1934). Second, Diamond and Mirrlees (1971) tell us that only final goods should be taxed, not intermediate goods. More precisely, they show that with a flexible set of tax instruments any intermediate good distortion can be replaced with less damaging final good distortions. Third, efficient taxation implies that distortions between marginal rates of substitution and marginal rates of substitution should be bounded; Judd (1999) shows that this implies a long-run average rate of zero on capital income taxation. In combination, these principles indicate that final goods should be taxed to finance corrective subsidies of any intermediate good, particularly any capital good, which is sold at a price above marginal cost.

These results are obvious if there is a lump-sum tax to finance the subsidies\footnote{Barro and Sala-i-Martin (1992) also assumed an imperfectly competitive capital goods sector and argued for a negative tax on capital income. However, they assumed the presence of a lump-sum tax, an inelastic labor supply, and a perfectly competitive final goods market. Therefore, their analysis was only an instance of the general Robinson result where all goods with a markup should be subsidized.}; however, the assumption of lump-sum taxation is unrealistic, and leads to misleading conclusions. For example, if there were a lump-sum tax available then there is no need to distinguish between final and intermediate goods since the optimal tax policy would use revenues to eliminate all distortions in markets for both intermediate and final goods. The real-world question is how should tax policy manage the interactions between distortionary taxes and the distortions of imperfect competition.

In this paper we formally examine these arguments in dynamic models of economic growth, and discuss their implications for tax policy in models with distortionary taxation. We compute their quantitative importance using estimates of the distortionary cost of taxation. The special considerations for intermediate goods are important for income taxation theory once we make a key observation: taxation of capital income is the same as a tax on the purchase of capital goods. Specifically, a tax rate of $\tau$ on the net cash flow
generated by a unit of capital is equivalent to a sales tax of \(1/(1 - \tau)\) on the purchase the capital. This paper combines this view of capital income taxation with the presence of market power to arrive at a striking result: the optimal long-run tax on capital income is negative, even if the distortionary cost of raising revenue is infinite!

Many authors have investigated optimal factor income taxation in perfectly competitive economies, generally finding that capital income should not be taxed in the long run if social and private rates of time preference are equal and the government has a flexible set of tax instruments. This has been established in a wide variety of models; Atkinson and Sandmo (1980) discuss the conditions for the zero tax result in two-period overlapping generations models, and Judd (1985) examines models with infinitely-lived agents. The OLG and representative agent analyses are very different. The OLG results revolve around the separability of the utility function and the Slutsky matrix for the goods over the two periods, and assumes that all agents are identical except for time of birth. In contrast Judd (1985) shows that long-run zero tax result holds for a heterogeneous population of infinitely-lived agents with intertemporally nonseparable Uzawa utility functions. Judd (1999) shows that the key intuition is that a positive tax on asset income generates exponentially growing MRS/MRT distortions among goods over time, a feature which is inconsistent with Ramsey style rules for optimal taxation. Since such explosive distortions are inconsistent with commodity taxation, as shown in Atkinson and Stiglitz (1972), the long-run tax on capital income must be zero.

The observation that capital income taxation is an intermediate good tax has not previously been a focus of income tax analysis but it helps to put a number of results in perspective.

There are important qualifiers to both the zero long-run tax result and to the Diamond-Mirrlees production efficiency result. The Diamond–Mirrlees rule against intermediate good taxation disappears if the set of permissible final goods is restricted. Similarly, Jones et al. (1995) show that the optimal long-run rate may be nonzero if wealth is in the utility function, in which case wealth is a final good as well as an intermediate good. The other qualifier is that the Diamond-Mirrlees production efficiency result is modified when there are rents; however, production efficiency still obtains when there is 100% taxation of pure profits. Similarly, the optimal tax on capital income is positive in the short-run when capital income is mostly a quasi-rent earned by the initial capital stock.

There has also been work on optimal taxation with imperfect competition; however, the focus has been on incidence in static models with only final goods. For example, Myles (1989) examined optimal taxation with imperfect competition, but did not examine general equilibrium with both intermediate and final goods. We focus on the role of intermediate goods and dynamic entry in the determination of optimal tax policy in dynamic general equilibrium.

We add imperfect competition to a conventional infinitely-lived representative agent
The optimal tax rate for capital income is negative model. The representative agent framework will imply that social and private discount rates are equal, but the price distortions from imperfect competition will generate rents which may alter the production efficiency arguments. We examine these issues in two dynamic general equilibrium models. First, we show exactly how monopolistic distortions affect capital allocation in a completely specified general equilibrium model of monopolistic competition. The basic structure is taken from the model of monopolistic competition of Dixit and Stiglitz (1977) but generalized to allow both differentiated consumption goods and differentiated intermediate goods. The second model we examine is a more general one with multiple capital goods and preexisting distortions in their allocation. While the second model assumes a reduced, but rather general, form for the distortions, it allows us to examine issues related to the tax treatment of various kinds of capital and the role of entry assumptions.

The basic intuition of this paper is that market distortions act like a privately imposed tax on purchasers of intermediate goods, and that optimal tax policy will offset the privately created distortions if the set of tax instruments is sufficiently flexible. We show that this is exactly true when all pure profits are taxed away, or when there is free entry, or when the marginal efficiency cost of tax revenues is zero, or when the intermediate good sectors satisfy certain symmetry conditions. The intuition is clear: if profits are consumed by fixed costs, then the zero profit, free-entry oligopoly entry equilibrium is equivalent to a competitive market with tax revenues financing fixed costs. Such a policy would, according to Diamond–Mirrlees, be suboptimal for intermediate goods since their price should be equal to marginal cost. Since no extreme case is realistic, we show that in general the tax system should move a substantial direction in this direction even when we make conservative assumptions concerning profits taxation, entry and the cost of funds.

These results are intuitive but they differ substantially from the conventional wisdom on taxation and imperfect competition. For example, Alan J. Auerbach and James R. Hines, Jr. (2001, p. 59) in their survey of optimal taxation present the Robinson argument, but assert “other policy instruments (such as antitrust enforcement) are also typically available and may be more cost-effective at correcting the problem.” This view of imperfect competition has no support in the industrial organization literature since there is no evidence that most, or even a substantial part, of pricing above marginal cost is related to violations of antitrust law. For example, the purpose of patent and copyright law is to grant an innovator the ability to charge prices in excess of production costs. If a firm is using intellectual property rights in a legal fashion, it is hard to think of any policy instrument other than taxation which could alleviate the distortions due to imperfect competition. The same is true for markups over marginal cost arising from product differentiation, increasing returns to scale, and trade secrecy.

This model has several implications for various tax policy issues. If all intermediate
goods were affected symmetrically by market power, a uniform subsidy of capital goods purchases would be an appropriate corrective policy. This subsidy could take many forms; appropriate instruments include an investment tax credit, accelerated depreciation, and expensing of investment expenditures. We will make no essential distinction between tax credits and alternative policies; hence, when we speak of tax credits we include equivalent alternatives. The subsidy could also be a subsidy of capital income if we distinguish between income to capital goods and pure profits. For example, if there were a tax on business income, businesses interest were deductible, all capital were debt-financed, and there was a subsidy on interest and dividend income at the personal level, then the net result would be subsidy on capital formation but a tax on pure profits. The main result is that capital formation should be subsidized but not pure profits; we do not focus on which method is used.

It is unlikely that all capital goods’ prices display the same price-cost margin, implying that a nonuniform subsidy be implemented. The conventional argument is that there should be essentially uniform taxation of capital goods; see Auerbach (1989) and Goulder and Thalmann (1993) for recent statements of this principle. These arguments typically assume perfect competition in all markets, an assumption violating much empirical evidence. Our analysis argues against the conventional “level playing field” approach to tax design as conventionally pursued, deriving instead the true “level playing field” criterion. Furthermore, there is some (albeit mixed) empirical work arguing that equipment industries have substantial price-cost margins, whereas the construction industry is close to being competitive. This view is also supported by the observation that many equipment firms make substantial R&D expenditures and hold patents whereas construction firms do not. This view implies that the investment tax credit as implemented since its introduction in 1962 correctly discriminated in favor of equipment. We argue that this approach constitutes a more robust foundation for such equipment-focussed pro-growth policies than Keynesian stimulus arguments.

These results also have many implications for optimal policy in growing economies. While the growth models we examine converge to a steady-state output level, it is clear that the results have nothing to do with the absence of steady-state growth, and, like every other result on tax and fiscal policy, apply equally to models with and without steady-state growth. In particular, some results concerning productive efficiency hold along the path as well as at the steady state. Therefore, the arguments have several implications for R&D and growth policies. We argue that subsidizing the purchase of new capital goods subsidies is a more appropriate tool for encouraging R&D than an R&D tax credit.

While the analysis below takes a simple approach to the problem of market power and optimal taxation, it demonstrates that tax policy discussions should be reoriented in their focus. Income tax analyses usually proceed along the lines of the Haig-Simons approach: define economic income correctly and tax it. Consumption tax advocates generally appeal
to the homily that one should be taxed according to what one takes from the economy, not by what one produces. Both approaches generally assume competitive markets. The optimal tax analysis of this paper indicates that we should think of capital income taxation as a tax on intermediate goods, remember that such goods are often sold in imperfectly competitive markets, and take heed of the Diamond-Mirrlees principle against price-cost distortions in markets for intermediate goods.

2. A Dynamic Model of Monopolistic Competition

We first examine a model where differentiated goods are used in consumption and investment. We assume a continuum of goods, indexed by \( i \in [0, 1] \). Each of these goods can be used for both consumption and investment. Output of good \( i \) is produced by labor input, \( l_i \), and a continuum of capital goods inputs. Let \( k^j_i \), \( j \in [0, 1] \), denote the amount of capital stock of good \( j \) used to produce good \( i \), and define \( X_i \equiv \left( \int_0^1 (k^j_i)^{1-\eta} \, dj \right)^{1-\eta} \) to be an aggregate of capital used to produce good \( i \). The rate of output of good \( i \) is \( y_i = f (X_i, l_i) \). The production function \( f \) does not depend on \( i \), a convenient symmetry which we will exploit heavily in this first model. The CES specification for each \( X_i \) implies that the elasticity of demand for each capital good is \( \eta - 1 \). The total stock of capital good \( i \) is \( K_i = \int_0^1 k^j_i \, dj \).

Each good \( i \in [0, 1] \) is also consumed. The representative agent has utility \( \int_0^\infty e^{-\rho t} u(C(t), G(t)) \, dt \) where \( l \) is labor supply, \( C \) is a consumption aggregate defined by \( C(t) = \left( \int_0^1 c_i(t)^{1-\eta} \, dt \right)^{1-\eta} \) where \( c_i(t) \) is the consumption of good \( i \) at time \( t \), and \( G \) is a public goods aggregate defined by \( G(t) = \left( \int_0^1 g_i(t)^{1-\eta} \, dt \right)^{1-\eta} \) where \( g_i(t) \) is the public good expenditure on good \( i \) at time \( t \). The elasticity of consumption demand for each good is \( \eta - 1 \), independent of price and the level and allocation of consumption. The equality and constancy of the elasticity of substitution is assumed for simplicity and not crucial for any of the results.

2.1. The Firm’s Problem. Each firm is a monopoly producer of a single good. Firm \( i \) produces net output of good \( i \) at the rate of \( y_i \), using labor at the rate \( l_i \) and renting \( k^j_i \) units of good \( j \) from firm \( j \). Firm \( i \) puts output \( y_i \) to two uses. First, part of \( y_i \) is used to adjust the capital stock of its good, \( K_i \), and rents this stock\(^2\) to other firms, where it is used in production. Second, it sells the remainder of \( y_i \) to consumers who consume it immediately. In summary, \( y_i = K_i + c_i \) where we permit negative \( K_i \). We allow physical depreciation of capital, implying that \( y_i = f (X_i, l_i) \) is net output.

\(^2\)By assuming rental of the stock to other firms, we avoid the durable goods monopoly problem (see Stokey, 1981, and Bulow, 1982). We want to assume a market structure which implies that the price of the output is the monopolistically competitive price, and the rental assumption makes the analysis below a bit simpler. We also want to avoid tax issues regarding leasing versus ownership. As long as all firms face the same tax environment and rates, the actual ownership has no effect on anything.
Suppose that at time $t$, $R_i(K_i, t)$ is the rental rate for good $i$ when a firm rents $K_i$ units of the stock, and $P_i(c_i, t)$ is the price of consumption sales when firm $i$ sells $c$ units of output to consumers. Firm $i$’s objective at time $t = 0$ is to choose time paths for $c_i$, $k_j$, and $l_i$ to maximize the market value of the firm’s net cash flow:

$$
\max_{c_i, L_i, k_j} \int_0^\infty e^{-\int_0^t r(s) \, ds} \left( R_i(K_i, t)K_i + P_i(c_i, t)c_i - wL_i - I_i \right) dt
$$

where $I_i$ is the total rentals paid by firm $i$ to other firms for equipment rental, $r(s)$ is the market interest rate at $t = s$. We assume that the firm holds a patent (or some other equivalent exclusive right) for good $i$, issues debt paying interest $r$ to finance equipment rental and wage expenditures, and pays out all net earnings to its equity holders. The Hamiltonian for firm $i$ is

$$
H(K_i, \phi_i, l_i, c_i, k_j) = R_i(K_i, t)k_i + P_i(c_i, t)c_i - wL_i - I_i + \phi_i \left( f(X_i, l_i) - c_i \right)
$$

where $\phi_i$ is the marginal value of the stock of good $i$ to firm $i$. The first-order conditions for firm $i$’s choice of each $k_j$ is

$$
0 = -R_j(K_j, t) + \phi_j f_X(X_i, l_i) \left( k_j \right)^{-\eta} X_j^\eta
$$

which shows that the elasticity of rental for capital good $j$ by firm $i$ is $\eta$ for all capital goods by all firms. The first-order condition for firm $i$’s choice of $c_i$ is

$$
0 = (P_i(c_i, t) + c_i P'_i(c_i, t)) - \phi_i
$$

where $P'(c, t) \equiv \frac{\partial P}{\partial c}$. The costate equation is

$$
\dot{\phi}_i = r\phi_i - (R_i(K_i, t) + k_i R'_i(K_i, t))
$$

where $R'(k, t) \equiv \frac{\partial R}{\partial c}$.

### 2.2. The Consumer-Investor.

The representative individual faces the problem

$$
\max_{c_t} \int_0^\infty e^{-\rho t} u(C, l, G)
$$

$$
\dot{A} = \bar{r}A - \int p_i c_i dt + \bar{w}l + (1 - \tau_H)\Pi + S
$$

---

3 We allow net earnings to be negative, but this plays no role in any results.
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where \( A \) is his interest-paying assets, \( \Pi \) is the rate of dividends on his ownership of the monopolistic firms, \( \tau_\Pi \) is the rate of taxation on dividends (or, equivalently, on pretax corporate profits), \( \bar{\tau} \) is the after-tax rate of return on interest-paying assets, \( \bar{w} \) is the after-tax wage rate, \( c_i \) is consumption of the good \( i \) priced at \( p_i \), \( l \) is his labor supply, \( g \) is public good expenditure by the government, and \( S \) equals government transfers to the consumer. We are assuming that all agents hold the per capita amount of equity and debt in each firm, and that there is no trade in equity\(^4\).

If we let \( \lambda \) denote the shadow price of assets \( A \) for the representative individual, his solution satisfies

\[
\hat{\lambda} = \lambda (\rho - \bar{\tau}).
\] (3)

We assume that the individual pays tax on both labor and investment income, the latter being comprised of both corporate debt and corporate equity. Consumption demand and labor supply must satisfy the conditions

\[
\begin{align*}
0 &= \ u_C (C, l, G) C^n c_i^{-\eta} - p_i \lambda \\
0 &= \ u_l (C, l, g) + \lambda \bar{w}
\end{align*}
\] (4)

From these first-order conditions we can solve for \( l \) and \( c \) in terms of \( \lambda, \bar{w}, \) and the vector of prices, \( p \), implying that

\[
\begin{align*}
c_i &= \Psi (\lambda, \bar{w}, p, G) \\
l &= L (\lambda, \bar{w}, p, G)
\end{align*}
\] (5)

The consumers’ inverse demand function, \( P_i (c_i, t) \), is the same for each good and becomes

\[
P_i (c_i, t) = u_C (C (t), l (t), G (t)) C (t)^n c_i^{-\eta} \lambda (t)^{-1}
\]

which is a constant-elasticity demand function.

2.3. Equilibrium. We have assumed symmetry among all firms, all inputs, and all consumption goods. Therefore, in equilibrium all firms will charge the same price to all other firms for equipment rental, and the same price to all consumers. Furthermore, the demand conditions above showed that the elasticity of demand is \( \eta^{-1} \) in both markets for all firms\(^5\). Therefore, the common price equals marginal cost times \((1 - \eta)^{-1}\). All this

\(^4\)The no-trade assumption is largely a matter of convenience. The only trading we have to worry about is firms repurchasing their equity or other firm’s equity to convert dividends into capital gains for individuals. We rule out these transactions (or, more precisely, we assume that the prohibitions on these tax-avoidance strategies are enforced) to avoid complications not central to this paper’s concerns.

\(^5\)We have assumed that the elasticity of substitution across goods is the same for both consumption and production uses; this is done only to keep the analysis simple. In the general case with different elasticities for consumption and production use, the foregoing would be essentially unchanged by making the (reasonable) assumption that each firm could price discriminate between final good consumption and intermediate good use.
implies that in equilibrium, all firms use the same amount of inputs, produce the same level of output, accumulate the same level of capital for rental, and that the capital stock equals the value of debt-paying assets. Hence, $k_i^j = X_i = k = A$ where $k$ is both the aggregate average capital stock per type and, since the measure of goods is one, the aggregate capital stock. In equilibrium, the consumption aggregate equals the average consumption across goods, $C(t) = c_i(t)$. Without loss of generality, we can choose consumption good 1 to be numeraire; since all goods sell at the same price, we conclude $p_i = 1$ for each good $i$. Therefore, labor supply is $L(\lambda, \bar{w}, 1, G)$ and consumption is $\Psi(\lambda, \bar{w}, G)$. To economize on notation, we will drop the price vector $1$, and use $L(\lambda, \bar{w}, G)$ and $\Psi(\lambda, \bar{w}, G)$ to denote labor supply and consumption demand.

These facts further imply that the private shadow value of wealth equals the marginal utility of consumption, $0 = u_C - \lambda$ which in turn implies that the inverse demand function faced by each producer reduces to $P_i(c_i, t) = C(t)^\eta c_i^{-\eta}$. For $\Gamma$ rms, the symmetry implies a simple rental demand equation for $\Gamma_{rm}^j$ equal to $R_i(K_j, t) = (1 + \eta) f_X(k(t), l(t)) k(t)^\eta K_j^{-\eta}$ where $l$ is the average labor use across firms. From the firm’s consumer sales decision, we find that the firms’ common shadow price on capital, $\phi$, equals

$$ \phi = cP(c, t) + P(c, t) = 1 - \eta $$

which, since $\eta$ is constant, implies $\phi = 0$. These expressions, combined with the firm’s costate equation, (2), and demand equation, (1), implies

$$ r(1 - \eta) = R_i(K_i, t) + k_i R_i^c(K_i, t) = (1 - \eta)^2 f_X(k(t), l(t)) K_j^{-\eta} k(t)^\eta $$

After imposing the equilibrium condition $K_i = k$, this implies $r = (1 - \eta) f_X(k, l)$ in equilibrium, showing that the marginal product of capital equals the interest rate grossed up by the markup factor $(1 - \eta)^{-1}$. Efficiency would set $r = f_X$; hence, the monopolistic competition generates a distortion in the demand for capital goods. If we further define $\tau_D$ to be the personal tax on interest income, then $\tau = r(1 - \tau_D)$ and we find that

$$ (1 - \eta)(1 - \tau_D) f_X(k, l) = \pi $$

expresses the equilibrium cost of capital. This expression shows that monopolistic competition is equivalent to a second tax rate of $\eta$, justifying our analogy between taxation and monopolistic competition.

The equilibrium is summarized by a pair of equations. First, net investment is

$$ \dot{k} = f(k, L(\lambda, \bar{w}, G)) - \Psi(\lambda, \bar{w}, G) - G $$

(7)
The second equilibrium equation states that the individual rates of return on assets must equal the cost of capital to firms:

$$\dot{\lambda} = \lambda(\rho - (1 - \tau_D)(1 - \eta)f_X(k, L(\lambda, \bar{w}, g)))$$  

(8)

The two equations, (7) and (8), together with the definitions of $$\Psi$$ and $$L$$ in (5), and boundedness of $$k$$ and $$\lambda$$, define the dynamic equilibrium path for consumption, labor supply, and output for any dynamic path of $$\tau_D$$ and $$\bar{w}$$.

The other salient feature of equilibrium is the presence of pure rents, specifically the monopolistic rents collected in the form of dividends by equity owners, $$\Pi$$. The presence of rents modifies the Diamond-Mirrlees prescription against intermediate good taxation. In fact, Stiglitz and Dasgupta (1971) demonstrated the more general proposition that if pure profits are taxed away then there should be no taxation of intermediate goods even if there were decreasing returns in production. Strictly speaking, the Stiglitz-Dasgupta result does not apply here since the pure profits here are due to imperfect competition, not decreasing returns. However, we will see that the same result holds.

Since the canonical consumption good is the numeraire, the price of $$c$$ is 1. Since price equals $$(1 - \eta)^{-1}$$ times marginal cost, marginal cost is $$1 - \eta$$ in equilibrium. We need to compute equilibrium profits. The profits on consumption sales is $$\eta C(\lambda, \bar{w}, G)$$ and pre-tax profits on capital rentals is $$\eta kf(k, L(\lambda, \bar{w}, G))$$; therefore, total pre-tax profits is $$\Pi(k, \lambda, \bar{w}, G) = \eta C(\lambda, \bar{w}, G) + \eta kf(k, L(\lambda, \bar{w}, G))$$. This implies that if $$B$$ is the stock of government debt, then $$B$$ evolves according to

$$\dot{B} = \bar{r}B - f(k, L(\lambda, \bar{w}, G)) + \bar{r}k + \bar{w}L(\lambda, \bar{w}, G) + (1 - \tau_\Pi)\Pi(k, \lambda, \bar{w}, G) + G$$  

(9)

Equation (9) states that the deficit must equal interest payments minus total output plus net-of-tax factor payments and net-of-tax dividend distributions. This expression is useful since it distinguishes between the competitive return on debt-financed capital investment, $$k$$, and the income stream which is associated with the monopoly rents, $$\eta f(k, L)$$.

2.4. The Cost of Capital with Imperfect Competition. The equilibrium cost of capital equation (6) expressed the total distortion created by the interaction of taxation and imperfect competition. We next illustrate important implications of this distortion for intertemporal allocations and find the basic intuition for the theoretical results below.

We shall follow the logic in Judd (1999), which extends insights in Atkinson and Stiglitz (1972) to dynamic models. Judd (1999) argues that the best way to view income taxation is as a time-dependent commodity tax. Figure 1 represents the demand for consumption at time $$t > 0$$ where time $$t = 0$$ consumption is the numeraire. Income taxation implies a pattern of distortions across consumption and leisure at various dates. For example, if we save some money at time 0 for consumption at time $$t$$, then a tax on investment income essentially taxes consumption at time $$t$$. Suppose $$r$$ is the before-tax interest rate, and $$\tau$$
is the interest tax rate. The social cost of one unit of consumption at time $t$ in units of the time 0 good is $(1+r)^{-t}$ and the after-tax price is $(1+(1-\tau)r)^{-t}$. This implies a tax distortion between $MRS$, the marginal rate of substitution between time $t$ consumption and time 0 consumption, and $MRT$, the corresponding marginal rate of transformation, equal to

$$\left(\frac{MRS}{MRT}\right)_{c_0,c_t} = \left(\frac{1 + r}{1 + (1 - \tau)r}\right)^t$$

(10)

This distortion is the same as if we taxed consumption at time $t$ at the rate

$$\tau^*_{c} = \left(\frac{1 + r}{1 + (1 - \tau)r}\right)^t - 1$$

(11)

The key fact illustrated by this formula is that the commodity tax equivalent is exploding exponentially in time!

The situation is displayed in Figure 2. Figure 2 shows the demand for the time $t$ consumption good relative to some untaxed good $c_0$ (such as time 0 leisure). This income tax is equivalent to a commodity tax on time $t$ consumption equal to $\tau^*_{c}$ per unit of the time $t$ good. We make the common assumption that the consumption demand curves are identical and independent across time and not affected by leisure. The optimal tax system would impose the same tax on consumption at each different time. Instead, a constant positive interest tax is equivalent to an exponentially growing tax on time $t$ consumption, strongly violating the inverse elasticity rule. Notice in Figure 2 that as $t$ increases, the deadweight loss triangle, $H$, grows and squeezes the revenue box, $R$.

The exponential explosion in (11) appears dramatic, but we need to check that it is quantitatively important over reasonable horizon. Table 1 displays the consumption tax equivalents, $\tau^*_{c}$, for various combinations of $r$ and $\tau$. We see that the results depend substantially on the magnitude of $r$. For $r = .01$, the mean real return on safe assets, the effects are small. For example, even a 50% tax on interest income implies only a 22% tax on consumption tax 40 years hence compared to a 0.1% tax on consumption a year away. However, the situation is much different when $r = .10$, which is the average marginal product of capital. When $\tau = .3$ (which is less than the tax rate on equity-financed capital), the effective consumption tax over a one-year horizon is 3%, but it is 59% over a ten-year horizon, and a whopping 543% over a 40-year horizon! It is hard to imagine any government passing a 59% sales tax in 2008, but that is effectively what we do to many investors if we continue with an income tax system between 1998 and 2008.
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Figure 1: Intertemporal tax distortion with imperfect competition

Table 1: Consumption Tax Equivalents, $\tau^*_c$

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<td>543</td>
<td></td>
</tr>
</tbody>
</table>

The implications of this analysis are clear. If utility is separable across time and between consumption and leisure, and the elasticity of demand for consumption does not change over time, the best tax system would have a constant commodity tax equivalent. This can be accomplished by a constant consumption tax. However, any nonzero asset income tax produces substantial violations.

While the exposition above focuses on special cases, the result is robust. The results in Judd (1985, 1999) show that the optimal tax on asset income is zero in the long-run, even when preferences are far more general than those used in dynamic tax analyses or when the economy does not converge to a steady state. The key idea is that exploding consumption tax rates are not efficient and that the explosion is quantitatively important.

This result is not just an aggregate result assuming everyone is the same. It is true for each individual if his tastes do not change significantly over time. Therefore, even if
tastes vary across individuals, each individual will prefer a constant consumption tax over an income tax which extracts the same revenue from him.

The inverse elasticity rule argues for a different tax on all goods, whereas consumption tax proposals actually propose a single tax rate. While this may appear to be a serious difficulty, we will ignore it. This approach is supported by the arguments in Balcer et al. (1983). They show that while an optimal commodity tax system would have very different rates across goods, there is little welfare difference between that tax system and a revenue-equivalent flat tax. Given the extra complexity and administrative cost of a tax system which charges different tax rates on different goods, it seems sensible to stay with a uniform consumption tax.

This analysis does not necessarily imply that there be no taxation of asset income. Suppose that tastes depend on age. If we assume that the elasticity of demand for consumption fell with age in just the right way, then a constant interest rate tax would be optimal; this would require the demand curve in Figure 2 for the time t good to become less elastic as t increases. Such an age-dependence could result if one had just the right interaction between consumption demand and leisure. However, I have not seen advocates of asset income taxation use this approach. My suspicion is that these arguments would be very fragile since our knowledge of the critical elasticities is too imprecise for such a purpose. In any case, it is hard to imagine demand elasticities changing enough to justify substantial asset income taxation. In particular, Table 1 tells us that to justify a 30% income tax if \( r = 0.1 \) over a twenty year horizon, we would need consumption elasticity to fall by a factor of 25 over those 20 years, a rather implausible situation. Therefore, the constant elasticity case is a reasonable one to use.

The distinction between factor income taxation and commodity taxation is misleading since none of the problems in Figure 2 applies to wage income taxation. If \( \tau_L \) is a constant wage tax and \( \tau_K \) a constant interest rate tax, the MRS/MRT distortion between time 0 consumption and time t leisure is

\[
\left( \frac{\text{MRS}}{\text{MRT}} \right)_{c_0,t} = \left( \frac{1}{1 - \tau_L} \right) \left( \frac{1 + r}{1 + (1 - \tau_K)r} \right)^t \tag{12}
\]

The distortion in (12) represents the distortion of decisions to sacrifice consumption at time 0 to gain extra leisure at time t. This distortion also grows over time but only because of the interest rate tax. Wage taxation does not aggravate the distortions in savings but asset income taxation does aggravate distortions between consumption and leisure at different dates.

The commodity tax interpretation of taxation and the inverse elasticity rule reveal many features of factor taxation. This view shows us how distortionary asset income taxation...

\[\text{Simulations of tax policy analysis may stumble on this if they assume tastes which lead to time-varying consumption demand elasticities.}\]
taxation is, and hints at the value of removing it from tax systems.

2.5. The Optimal Taxation Problem. The optimal taxation problem is to maximize the dynamic utility of the representative agent given the limited tax instruments and the competitive equilibrium they produce. Let

\[ v(\lambda, \bar{w}, g) \equiv u(\Psi(\lambda, \bar{w}), L(\lambda, \bar{w}), G) \]

be the utility function expressed as a function of the current marginal utility of consumption and the after-tax wage rate. The government chooses income tax rates so as to raise revenue sufficient to finance the initial stock of debt, \( B_0 \) and government expenditures, \( G \). The optimal tax problem becomes

\[
\max_{g, \bar{r}, \bar{w} \geq 0} \int_0^\infty e^{-\rho t} v(\lambda, \bar{w}, g) \quad (13)
\]

\[
k = f(k, L(\lambda, \bar{w}, G)) - \Psi(\lambda, \bar{w}, G) - G
\]

\[
\dot{\lambda} = \lambda(\rho - \bar{r})
\]

\[
\dot{B} = \bar{r}B - f(k, L(\lambda, \bar{w}, G)) + \bar{r}k + \bar{w}L(\lambda, \bar{w}, G)
\]

\[
+ (1 - \tau_\Pi)\Pi(k, \lambda, \bar{w}, G) + G
\]

\[
\lim_{t \to \infty} |B| < \infty
\]

The restriction on the growth of \( B \) prevents Ponzi schemes.

The current-value Hamiltonian\(^7\) of the optimal tax problem is

\[
H^{\text{tax}} = v(\lambda, \bar{w}, G) + \theta(f(k, L(\lambda, \bar{w}, G)) - \Psi(\lambda, \bar{w}, G) - G)
\]

\[
+ \psi \lambda(\rho - \bar{r})
\]

\[
+ \mu(\bar{r}B - f(k, L(\lambda, \bar{w}, G)) + \bar{r}k + \bar{w}L(\lambda, \bar{w}, G)
\]

\[
+ (1 - \tau_\Pi)\Pi(k, \lambda, \bar{w}, G)
\]

\[
+ \nu \bar{r}
\]

where \( \theta, \psi, \) and \( \mu \) are the planner’s shadow prices of \( k, \lambda, \) and \( B \). The costate equation for \( \theta \) is

\[
\dot{\theta} = \rho \theta - \theta f_X - \mu(\bar{r} - f_X + (1 - \tau_\Pi)\Pi_k(k, \lambda, \bar{w}, G)) \quad (14)
\]

We assume that the government can give away bonds; therefore, \( \mu \leq 0 \). We make the standard assumption that \( \theta \), the public shadow price of the capital stock, is positive. We let \( m = -\mu/\theta > 0 \) denote the marginal efficiency cost of funds. The term \( m \) is also known as the marginal excess burden of taxation; the social cost of funds would equal \( 1 + m \), that is, the private cost of a dollar of taxation equals the dollar of revenue plus \( m \).

\(^7\)The more proper way to proceed is to rewrite the bond equation as an integral equation imposing the natural present value condition for the government’s budget constraint, and analyzing the resulting isoperimetric problem. This procedure yields the same answer.
The costate equation for $\psi$ is

$$
\dot{\psi} = \rho \psi - v \lambda - \theta (f L \lambda - \Psi \lambda) - \psi (\rho - \bar{r}) - \mu ((-f + \bar{w}) L \lambda + (1 - \tau \Pi) \eta f L \lambda)
$$

The first-order condition for $\bar{r}$ is

$$
0 = -\psi \lambda + \mu k + \nu \bar{r}
$$

As is typical in these problems, there will be an initial period of time where the $\bar{r} \geq 0$ constraint will bind but after some finite amount of time, the optimal choice of $\bar{r}$ will be positive. See Judd (1999) for a presentation of this which can be adapted straightforwardly to yield the same result for this model; since it is not crucial to the points in this paper, we leave out the details. In all of the analysis below, we will assume that we have left that initial period and that the $\bar{r} \geq 0$ constraint does not bind.

We have not yet explained how $\Pi$ (which fixes $\tau_D$ since $(1 - \tau_D)(1 - \eta) f X = \Pi$) and $\tau \Pi$ are related. We will examine two cases regarding the determination of $\tau \Pi$. First, assume that $\tau \Pi$ can be chosen independently of any other tax rate. In particular, we assume that $\tau_D$ and $\tau \Pi$ can be set independently. For example, $\tau \Pi$ can represent a business profits tax such as that in the Hall-Rabushka Flat Tax proposal. Specifically, if business income is defined as income minus debt and dividend payments as well as wage and material costs, then the tax base of $\tau \Pi$ is only pure profit and does not affect the return to investors. In contrast, we can view $\tau_D$ as the individual level tax on dividends and interest income.

Since $\mu \Pi (k, \lambda, \bar{w}, G) < 0$, the optimal choice for $\tau \Pi$ in this model, as in other similar models, is 100%. Since 100% taxation of pure profits is an unrealistic assumption, we assume that there is a fixed profits tax rate. We examine the steady state of the costate equation (14) to find the optimal choice for $\tau_D$. The key fact is that

$$
\Pi_k (k, \lambda, \bar{w}, G) = \eta (f_k + k f_{kk}) = \eta f_k \left(1 + \frac{k f_{kk}}{f_k}\right)
$$

which depends only on the monopolistic competition in the capital goods sector. When we combine (14) with the equilibrium expressions for $\bar{r}$ and $f_k$, we arrive at the following theorem for the case of Cobb-Douglas production.

**Theorem 1.** The optimal tax on profits is 100%. If the profits tax rate is $\tau \Pi$, and the solution to (13) converges to a steady state, then the optimal tax rate on interest in the steady state equals

$$
\tau_D^{opt} = -\frac{\eta}{1 - \eta} \left(1 - \frac{m}{1 + m} (1 - \tau \Pi) \left(1 - \frac{\theta L}{\sigma}\right)\right)
$$

(15)

In particular, if $\sigma \equiv 1$ (the Cobb-Douglas production function), then
1. $\tau_{opt}^D < 0$ if $\eta > 0$ and $m < \infty$;

2. If $m = \infty$, then

$$\tau_{opt}^D = -\frac{\eta}{1 - \eta} (1 - (1 - \tau_{\Pi}) \theta_K)$$
$$= -\frac{\eta}{1 - \eta} (\theta_L + \tau_{\Pi} \theta_K) < 0$$

3. In general,

$$-\frac{\eta}{1 - \eta} < \tau_{opt}^D < -\frac{\eta}{1 - \eta} (\theta_L + \tau_{\Pi} \theta_K) < 0$$

The optimal tax rate in Theorem 1 is simple. If the efficiency cost of taxation is zero then the optimal tax completely neutralizes the monopolistic price distortion. If one can implement the optimal tax on pure profits then the optimal policy is to eliminate the monopolistic price distortion.

In any case, the optimal policy is almost always a subsidy. Even when $m = \infty$, the point where revenue is being maximized, the optimal subsidy equals the product of $\tau_{\Pi}$ and the subsidy rate which would eliminate the price-cost distortion. This is quite remarkable since this says that one finances a sizable subsidy even if the cost of funds to finance the subsidy is infinite! This nicely illustrates the strength of the Diamond–Mirrlees prohibition against distortions in intermediate goods. Only when $m = \infty$ and $\tau_{\Pi} = 0$ does the subsidy disappear.

2.6. Quantitative Importance. While our optimal tax formula (15) is clean, it is not clear that the subsidy is economically significant when we use reasonable values for $\eta$, the markup, and the marginal excess burden $m$. Barro and Sala-i-Martin assume that $m = 0$ and that labor supply is perfectly inelastic; both assumptions are implausible and lead to nonrobust results. If we assume that $m = 0$ in our model with elastic labor supply we would not only get (15) but conclude that the labor tax rate should be negative to counter the monopolistic distortions in the consumption goods market. In our model with equal markups in the intermediate and final goods markets we would come to the conclusion that every marketed good should be subsidized at the rate $-\eta/(1 - \eta)$; if we had assumed that consumption goods were less elastically demanded and marked up more, than they would receive the higher subsidy. Therefore, under the $m = 0$ case there is no favoritism shown for capital goods. However, the $m = 0$ case is an unrealistic one to look at since it would imply everything is being subsidized, a situation inconsistent with dynamic budget balance and the usual condition of a positive stock of public debt.

To determine whether the optimal subsidy is significant, we compute it for reasonable values of $\eta$ and $m$. Since the key parameters are only poorly estimated by the empirical literature, it would be irresponsible to examine just one set of parameter values. Instead, we examine the implications of Theorem 1 for a broad range of cases. We assume $\eta \in$
The optimal tax rate for capital income is negative; this range is suggested by the empirical literature on price-cost margins (we will discuss this literature in more detail below). The range for $m$ is taken from Judd (1987). The dynamic equations above are essentially the same as for the competitive model in Judd (1987) where we add the price-cost margins to the explicit tax rates to get the total effective tax rate. Therefore, the results in Judd (1987) for a competitive model are roughly appropriate. Since there is often only a small capital goods distortion in the long-run of the optimal tax policy, $m$ is nearly the same as the marginal excess burden in a competitive model with only labor taxation, and is therefore confined to the interval $[0, 1]$ for almost all estimates of labor supply elasticities, price-cost margins, and tax rates in U.S. experience.

Table 2 shows that even if the shadow price of funds is nontrivial, the optimal tax substantially reduces the monopolistic distortion. In Table 2 we assume that the profits tax is zero or 0.5.

### Table 2: Optimal Tax Rates

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\tau_{II}$: 0</th>
<th>.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m$: .2 .4 .7 1.0</td>
<td>.2 .4 .7 1.0</td>
<td>$[0, \infty)$</td>
</tr>
<tr>
<td>-.1</td>
<td>-.09 -.08 -.07 -.06</td>
<td>-.10 -.10 -.09 -.08</td>
<td>-.11</td>
</tr>
<tr>
<td>-.2</td>
<td>-.21 -.18 -.15 -.12</td>
<td>-.23 -.21 -.20 -.19</td>
<td>-.25</td>
</tr>
<tr>
<td>-.3</td>
<td>-.36 -.31 -.25 -.21</td>
<td>-.39 -.37 -.34 -.32</td>
<td>-.47</td>
</tr>
<tr>
<td>-.4</td>
<td>-.56 -.48 -.39 -.33</td>
<td>-.61 -.57 -.53 -.50</td>
<td>-.67</td>
</tr>
</tbody>
</table>

Table 2 illustrates a number of points. First, the zero profits tax case is the extreme case where the subsidy is lowest. When the profits tax is 100% the subsidy is highest and brings price down to social cost. Second, the optimal subsidy is nontrivial in most cases. Even in the most pessimistic case where there is a zero profits tax and $m$ is 1.00 along the optimal tax policy, the optimal subsidy still eliminates half of the monopolistic price-cost margin and is large relative to conventional tax experience.

### 2.7 Interpretations

While our model made some simplifying assumptions, a number of equivalent results are also clear. The subsidies we found above are paid directly to the investors. That is clearly not necessary. These subsidies could also be paid to the firms in the form of investment tax credits or accelerated depreciation schedules. In this simple model, we did not make these distinctions; the more general model in the next section will allow us to make these distinctions.

Our simple, symmetric monopolistically competitive model clearly demonstrates a number of important points. First, optimal tax policy should put a priority on countering
the monopolistic price distortions in capital good markets, even at the expense of aggravating the distortions in final goods markets. Our model makes this particularly clear since each good has a final good use and intermediate good use, and only the intermediate good use is generally subsidized.

Second, the desired subsidies are quantitatively important even when pure profits are lightly taxed and the social cost of funds is large. We next examine a generalization of these points to somewhat more general situations, which will allow us to examine more detailed issues of optimal tax design.

3. General Distortions

The previous section examined a specific model with particular pattern of product differentiation and oligopoly behavior, and was strongly symmetric. While it made the main points in a simple fashion, it ignores many factors which we would like to bring in to the discussion. In particular, it is difficult to distinguish between the tax treatment of capital as an intermediate good and the tax treatment of capital as a form of saving. We now examine a more general model with multiple capital stocks and potentially more complex distortions. This model allows us to focus on heterogeneities across capital goods and distinguish between subsidies to saving in general and good-specific subsidies. We also examine the non-steady state behavior of tax policy as well as the long-run policy. To avoid game-theoretic complexities, we do not solve any specific model of imperfect competition; instead we specify market distortions in a simple reduced form, but general, fashion. While the reduced form specification for the pre-existing market distortions is not based on a completely specified model of market structure and conduct, the reduced form is general, presumably representing several alternative specifications of imperfect competition, and allows us to examine a much wider range of tax policy issues. In fact, this approach determines the features of optimal policy which are robust across alternative specifications of market structure and conduct.

3.1. Model. We assume that there is one good used for both consumption and investment, one capital stock with several capital uses, and an elastic labor supply. Net output is $y = f(K, l)$ where $K \in \mathbb{R}^n$ is the vector of capital inputs and $l$ is the labor input. We assume that capital is putty-putty; that is, each period begins with an aggregate stock of capital, $k$, which is allocated over the $n$ types of capital; hence, $k = \sum_i K_i$ at all times. Since our focus is on the long-run tax policy, this helpful simplification should not affect our results relative to more realistic specifications incorporating adjustment costs. Net output $y$ is divided between consumption, $c$, and net investment, $\dot{k}$.

We assume that there are several tax instruments. First, labor income is taxed at the rate $\tau_L$. Second, individuals pay taxes on income from owning physical capital at the rate $\tau_K$. Third, there is a tax, $\tau_i$, on the income of type $i$ capital. We also assume that there is a
distortion in the allocation of each type of capital; specifically, the imperfectly competitive market structure for the capital goods causes an equilibrium distortion for capital of type $i$ to reduce the private return of using factor $i$ by the factor $\pi_i(k, \tau, l, c)$ if the aggregate capital stock is $k$, the aggregate labor supply is $l$, and aggregate consumption is $c$. If $\bar{r}$ is the after-tax return to capital for an individual, then capital is allocated so as to satisfy the equations

$$\bar{r} = (1 - \tau_K)(1 - \pi_i(k, \tau, l, c))f_{K_i}(K, l)$$

$$k = \sum_i K_i$$

These equations fix both $K$, the allocation of capital, and $\tau_K$, the general capital income tax rate given the capital-specific taxes, $\pi$, the pattern of distortions, and the total amount of capital and labor. The model behind the optimal tax problem in (13) was an example of a constant distortion; this formulation allows various forms which may arise with different modes of imperfect competition, different tastes and technologies, and different sources of distortions.

We make no assumptions about $\pi$, except those implicitly necessary for the technical details of the arguments below. This is very important to note. We have made no assumptions concerning Cournot or Bertrand or other modes of oligopolistic competition. We have not assumed a fixed number of firms; in fact, $\pi$ includes the possibility that there is entry of firms if there are pure profits to be made. Such entry can be differentiated or undifferentiated. The only assumption we are making is that the imperfect competition distortion depends solely on the current factor supplies and the current prices and taxes. This is not without loss of generality; in particular, it rules out dynamic fixed costs of the oligopolistic firms and related phenomenon such as learning curves.

Let $\bar{w}$ be the after-tax wage rate. As above, the representative individual faces the problem

$$\max_{c_i} \int_0^\infty e^{-\rho t} u(c, l) dt$$

$$\bar{A} = \bar{r} A - c + \bar{w} l + (1 - \tau_\Pi) \Pi$$

The individual’s Hamiltonian is

$$u(c, l) + \lambda(\bar{r} A - c + \bar{w} l + (1 - \tau_\Pi) \Pi)$$

where we let $\lambda$ denote the shadow price of assets for the representative individual. The solution satisfies the intertemporal Euler equation $\dot{\lambda} = \lambda(\rho - \bar{r})$. Labor supply and consumption demand in each period is determined by the first-order conditions

$$-\Pi u_c(c, l) = u_l(c, l)$$

$$u_c(c, l) = \lambda$$

(17)
The net result of the first-order conditions for consumption, labor supply, (17), and capital allocation, (16) is that the momentary equilibrium values for consumption, labor supply, capital allocation, and the capital income tax are functions of $\tau, \bar{w}, \lambda, k$:

$$ c = \Psi(\bar{w}, \lambda) \quad (18) $$
$$ l = L(\bar{w}, \lambda) $$
$$ K = K(\tau, \bar{w}, \lambda, k) $$
$$ \tau_K = T_K(\tau, \bar{w}, \lambda, k, r) $$

We are implicitly assuming that these momentary equilibrium functions exist in the distorted economy. We also assume that the momentary equilibria are locally determinate and smooth. Then the dynamic evolution of the economy can be expressed by two equations. First, net investment equals output minus consumption,

$$ \dot{k} = f(K(\tau, \bar{w}, \lambda, k), L(\lambda, \bar{w})) - \Psi(\lambda, \bar{w}) $$

Second, the personal shadow price of capital obeys

$$ \dot{\lambda} = \lambda(\rho - \bar{r}) $$

These two equations plus boundedness on $k$ and $\lambda$ determine the dynamic evolution of the economy once the dynamic pattern of tax rates and net-of-tax factor returns are fixed.

### 3.2. Optimal Taxation.

Define $v(\lambda, \bar{w}) \equiv u(C(\lambda, \bar{w}), L(\lambda, \bar{w}))$. The optimal tax problem chooses the after-tax factor returns and the type-specific capital taxes, summarized by the vector $\tau = (\tau_\Pi, \tau_D, \tau_i)$, to maximize the representative agent’s utility subject to the economy following an equilibrium dynamic path and long-run budget balance. This is expressed as the optimal control problem

$$ \max_{\bar{r}, \bar{w}, \tau_i} \int_0^\infty e^{-\rho t} v(\lambda, \bar{w}) $$
$$ \dot{k} = f(K(\tau, \bar{w}, \lambda, k), L(\lambda, \bar{w})) - \Psi(\lambda, \bar{w}) $$
$$ \dot{\lambda} = \lambda(\rho - \bar{r}) $$
$$ \dot{B} = \bar{r}B - f(K(\tau, \bar{w}, \bar{r}, \lambda, k), L(\lambda, \bar{w})) + \bar{r}k + \bar{w}L(\lambda, \bar{w}) + \Pi(\tau, \bar{w}, \lambda, k) $$

$$ \lim_{t \to \infty} |B| < \infty $$

where $\Pi(\tau, \bar{w}, \lambda, k)$ is the after-tax dividends (pure profits) received by households. If $\tau_\Pi = 1$ then $\Pi = 0$.

As before, we form the Hamiltonian of the problem ignoring the boundedness condition.
on bonds\textsuperscript{9},

\[
H = v(\lambda, \bar{w}) + \theta(f(K(\tau, \bar{w}, \bar{r}, \lambda, k), L(\lambda, \bar{w})) - \Psi(\lambda, \bar{w})) + \psi \lambda (\rho - \bar{r}) + \mu (\bar{r}B - f(K(\tau, \bar{w}, \bar{r}, \lambda, k), L(\lambda, \bar{w})) + \bar{r}k + \bar{w}L(\lambda, \bar{w}) + \Pi(\tau, \bar{w}, \lambda, k))
\]

The costate equation for $\theta$ is\textsuperscript{10}

\[
\dot{\theta} = \rho \theta - \theta f_j K^j_k + \mu \left( f_j K^j_k - \Pi_k \right)
\]

(19)

and the costate equation for $\mu$ is

\[
\dot{\mu} = \mu (\rho - \bar{r})
\]

(20)

The first-order condition with respect to $\tau_i$ is

\[
f_j K^j_i = \frac{\mu}{\mu - \theta} \Pi_{r_i}
\]

(21)

which states that the marginal productive inefficiency as tax rate $\tau_i$ is changed, $f_j K^j_i$, is to be proportional to the marginal pure profit, with more productive inefficiency when the marginal cost of funds, $\mu$, is larger relative to the social value of output, $\mu - \theta$. Note also that the sign of the productive inefficiency, $f_j K^j_i$, is the same as the sign of the marginal profit effect; that is, if an increase in a tax increases pure profits, then an increase in that tax will, at the optimum, increase output.

The Hamiltonian reveals an important detail. Since $\mu \leq 0$, the Hamiltonian reveals that anything which increases the pure profits term, holding fixed the pure profits tax, $\tau$, will reduce the value of the Hamiltonian. Intuitively, this occurs because an increase in profits, holding fixed the state variables, takes income away from investment income and wage income, which form the other components of the tax base. Another way of viewing it is that pure profits are, in this formulation, similar to lump-sum transfers from the government to agents, a transfer which increases the necessary tax burden. In any case, the results below are easier to understand when we remember that an increase in pure profits is bad.

**Short-Run Optimal Policy.** The first-order conditions in (21) are the short-run optimality conditions and must hold at all times when. We first consider its implications. Equation (21) is clear when we remember that pure profits are bad. The first term is the product of $\theta - \mu \geq 0$, the marginal social value of an increase in output\textsuperscript{11}, and the change in output due to a momentary change in $\tau_i$. The second term is the social cost of the

\textsuperscript{9}Again, the more proper way to proceed is to rewrite the bond equation as an integral equation imposing the natural present value condition, and analyzing the resulting isoperimetric problem. This procedure yields the same answer.

\textsuperscript{10}We use the Einstein summation notation; that is $a^i b_i \equiv \sum_i a^i b_i$.

\textsuperscript{11}Formally, we can prove $\theta - \mu \geq 0$ by assuming that the government has ways to throw away output without buying it, such as through wasteful regulation.
change in pure profits due to a change in $\tau_i$. The condition (21) says that the value of extra output due to a tax change must be balanced by the value of the impact on pure profits. For example, if a tax reduction increased pure profits, the optimal policy would choose a tax rate where further reduction would increase output.

With (21), we can analyze the importance for the optimal tax rates of entry into oligopolistic markets, even though we do not assume any particular oligopoly model. In the previous model, we assumed a fixed collection of firms; this is clearly unrealistic. Pure profits do lead to entry, which will substantially reduce the flow of pure profits, $\Pi(\tau, \bar{w}, \lambda, k)$, to households. We will first consider the case of free instantaneous entry, where $\Pi(\tau, \bar{w}, \lambda, k) \equiv 0$. This implies $\Pi_k = \Pi_{\tau_i} = 0$, which reduces the first-order condition, (21), to $0 = (\theta - \mu) f_j K^i_{\tau_i}$ for each $\tau_i$. However, at the optimal choice for $\tau_i$ the Hamiltonian is concave in $\tau_i$, implying that $\theta - \mu \neq 0$. Therefore,

$$f_j K^i_{\tau_i} = 0$$

for each $\tau_i$. This condition can be interpreted as saying that the reallocation of capital which results from a marginal tax on capital of type $i$, $K^i_{\tau_i}$, should lead to no change in total output, where that change equals $f_j K^i_{\tau_i}$.

We can say much more under the assumption of local determinacy of equilibrium. Since total capital is fixed at any moment, the vector of all 1’s is a solution for $x_i$ to $x_j K^i_{\tau_i} = 0$, as is any multiple; therefore, the matrix $K^i_{\tau_i}$ is singular. If momentary equilibria are locally determinate, the matrix $K^i_{\tau_i}$ is of rank $n - 1$. Since $K^i_{\tau_i}$ is of rank $n - 1$, the vector of all 1’s and its multiples must exhaust the solutions to $x_j K^i_{\tau_i} = 0$. But we saw above that optimality implied (22). Therefore, the $f_j$ vector must be proportional to a vector of ones, which is the same as saying that the $f_j$’s are equated. Therefore, if there is free entry, the optimal tax policy permits no productive distortion in the allocation of capital at any point in time, with taxes and subsidies continuously neutralizing preexisting distortions.

Without free entry, the implications are muddier, but the optimal tax condition for $\tau_i$ still has some interesting implications. If $K^i_{\tau_i}$ is a symmetric matrix and $\Pi_{\tau_i}$ is proportional to a vector of 1’s, a situation which would arise in symmetric specifications of the several sectors, then the $f_j$ are again equaled. This is implied by (22) and the $n - 1$ rank of $K^i_{\tau_i}$. This condition is particularly surprising since we get the efficiency result without free entry and without making any particular game theory assumptions about imperfect competition. While this kind of symmetry is not a compelling description of reality, it does argue that the important factor is not the presence of pure profits but the deviation from symmetry.

More generally, we get less precise results without zero-profit free entry. If the marginal rent terms, $\Pi_{\tau_i}$, are small, then the average marginal productivity effect of a tax change, $f_j K^i_{\tau_i}$, must also be small, implying that we get close to productive efficiency. Third, if the shadow price of funds is small, we still find that $f_j K^i_{\tau_i}$ must be small for each $\tau_i$, again
implying near productive efficiency at all times. The final special case is taxing away all pure profits, implying $\Pi(\tau, \overline{w}, \lambda, k) = 0$ as in the free entry case.

**Theorem 2.** Assume that the equilibrium under the optimal tax policy is locally determinate at all times. If either there is free entry into the markets for capital goods ($\Pi = 0$), or the cost of funds, $\mu$, is zero or if the technology and pre-existing distortions are symmetric ($K^i_j$ is a symmetric matrix, and $\Pi^i_j = \Pi^i_j$ for all $i, j$), then the optimal tax policy equates the marginal product of capital across all uses at all times.

Therefore, there will be substantial deviation from productive efficiency only if the shadow price of funds, entry barriers, light taxation of profits, and asymmetries are all significant. The fact that we need all of these factors for substantial productive inefficiency strengthens the case for productive efficiency. Of course, the quantitative importance of this proviso remains to be investigated, requiring the solution of more completely specified imperfectly competitive markets.

**Long-Run Optimal Policy.** We next turn to the long-run properties of the optimal tax policy. The steady state equation for $\theta$ implied by (19)

$$0 = \theta \left( \rho - f_i K^i_k \right) + \mu \left( f_i K^i_k - \bar{r} - \Pi_k \right)$$

This condition says that the social value of the extra output from extra capital, $\theta f_i K^i_k$, and the social value of its net contribution to revenue, $\mu \left( f_i K^i_k - \bar{r} - \Pi_k \right)$, must be balanced against the time cost of investment, $\theta \rho$. In the steady state, $\rho = \bar{r}$. Under (instantaneous) free entry, $\Pi_k = 0$ and the $f_i$ are equalized. The steady-state equation reduces to $0 = (\theta - \mu) \left( \rho - f_i K^i_k \right)$. Since $\theta > 0 > \mu$, we conclude that

$$\rho = \bar{r} = f_i$$

for each capital stock type $i$. Therefore, all type-specific distortions in the capital allocation are neutralized by the type-specific capital taxes and there is no net taxation of capital investment in the long run. If we don’t have free entry, we still have the conditions

$$\rho - f_i K^i_k = \frac{\mu}{\theta - \mu} \Pi_k$$

which states that the gap between $\rho$ and the social product of an extra unit of capital, $(\rho - f_i K^i_k)$, is to be proportional to $\Pi_k$, the incremental effect of capital on pure profits, with a common proportionality constant across types of capital.

The following theorem summarizes our arguments.

**Theorem 3.** Assume that the equilibrium under the optimal tax policy is locally determinate at all times. If either there is free entry into the markets for capital goods ($\Pi = 0$), or the cost of funds, $\mu$, is zero or if the technology and pre-existing distortions
are symmetric ($K_j^\tau_i$ is a symmetric matrix, and $\Pi_j^\tau_i = \Pi_j^\tau_j$ for all $i, j$), then the optimal tax policy neutralizes all preexisting distortions of capital allocation in the steady state, producing intertemporal productive efficiency with $\rho - f_i = 0$ for all capital uses $i$.

Theorem 3 find that taxes should neutralize both allocative and intertemporal distortions in the capital goods market under free entry or symmetry or zero cost of funds. While none of these conditions are likely to hold in reality, deviations from $\rho - f_i = 0$ will can occur only when the cost of funds is large, there is substantially restricted entry, substantial light taxation of pure profits, and substantial asymmetry, a set of conditions which appear to be strong.

However, it must be acknowledged at this point that a more quantitative discussion of these points similar to that above for the simpler model would be a more complicated exercise for this model and is beyond the scope of this paper. While we cannot argue that all models with distortions will reduce to dynamic general equilibrium models of this form, it is a reasonable conjecture to believe that we have covered an important class of such models. In the sections below we will turn to interpreting the general principle, address comparisons with the static Diamond-Mirrlees principle, discuss the likely magnitude and structure of the optimal subsidies, and the implications for tax policy debate.

4. Interpretations and Generalizations

The general principal of these results is that intermediate goods and capital income should not be taxed in the long-run, and if there are distortions arising from imperfect competition, subsidies should be used to subsidize each intermediate good so that the post-subsidy price is equal to the social cost of the good. In this section, we shall discuss how this compares with previous results, and various interpretations of the results.

4.1. Implications for Research and Development Policy. When we consider the possible sources of market power, the rationale for subsidizing certain kinds of capital accumulation becomes clearer. Suppose that the markups are just manifestations of patent-holding innovators reaping the profits necessary to offset their R&D costs. Remember what a patent is. Whether a patent holder actually produces a good or licenses it, the essential fact is that a patent grants the holder the right to assess a tax on the purchasers of the patented good. A patent is essentially a modern form of tax farming.

Therefore, a patent taxes a good’s users to finance the fixed costs of innovation. This, however, contradicts the spirit of the Diamond-Mirrlees principle. The Diamond–Mirrlees principle says that any pure public good, such as the fixed cost of inventing an intermediate good, should be financed by taxation of final goods. It is not important that, under a patent system, the tax revenues from taxing a good goes to cover its fixed costs, and that the taxing is done by private agents. The result of a patent system as it applies to intermediate goods is inappropriate taxation of intermediate goods. Therefore, the real
result here is that there should be no net taxation of intermediate goods in the long run, and that the explicit government subsidies should neutralize the taxes implicit in a patent.

These arguments show that subsidizing the purchase of new capital goods subsidies is a more appropriate tool for encouraging R&D than an R&D tax credit. It is the purchase subsidy, not the development subsidy, which corrects the price-cost gap in the market for technologically advanced equipment. The R&D credit may then be used to create the correct share of resources allocated to R&D, but since the rate of R&D expenditure may be less than, equal to, or greater than the optimal expenditure level (see Judd, 1985) it is unclear if R&D should be subsidized or taxed. Also note that the subsidy we derive here was optimal even though it does not encourage innovation. If we added innovation to our model, such as in Judd (1985), then the subsidy would have even greater social value, likely strengthening the case for the subsidy.

4.2. Antitrust Policy. The presence of monopolistic competition is the key source of inefficiency in our model, and the subsidy to investment is a way of bringing price down to marginal cost. Another way would be to eliminate the market power through antitrust policy and let the competitive market force price down to marginal cost. However, if there were fixed costs of production, then competition cannot push price down to marginal cost, and having firms specialize in differentiated goods is desirable. Therefore, a conventional antitrust policy would not be appropriate. While we did not explicitly model fixed costs, it is obvious that if we added fixed costs the optimal tax results would be unchanged since no marginal condition is affected by fixed costs. As noted above, extending the model to include innovation would allow for a richer analysis; antitrust policy would also be of questionable value since the point of a patent is to give incentives for innovation. Therefore, it is often not appropriate to attack the price-cost distortions through antitrust policy.

Robinson disparaged the kind of subsidy policy supported by this analysis. Her criticism focussed on the income distribution consequences of subsidizing monopolists’ income. However, if we assume that fixed costs eat up most of the gross profits, then these concerns are of far less importance.

On the other hand, if the market power had nothing to do with fixed costs or innovation, then the model does seem to say something about antitrust policy. Our analysis appears to indicate that distortions in capital goods markets are more damaging than distortions in consumer goods markets, implying that antitrust policy should give priority to intermediate goods markets. While this observation is interesting, it is beyond the scope of this paper to investigate further.

4.3. The Transition to the Long Run. Our main results concern only the long run of the optimal tax policy. A legitimate criticism of this kind of focus is that it ignores the
transition process. Transition analysis is generally ignored because it is difficult to treat precisely and because we expect that the results depend on special assumptions. While we do not endeavor here to give a complete analysis of the transition path, and future work will be necessary to determine quantitative aspects, we do know the qualitative features of the transition, and have some evidence of its importance.

In our model, the optimal short-run tax rate on capital will surely be high because it is largely a tax on the inelastically supplied capital stock in place at the initial time. The only limit is that a tax on income in excess of 100% will cause capitalists to withdraw their capital from the market. The optimal tax rate will initially be 100%, but then will fall to zero. There are good reasons to believe that the transition to the long-run will be relatively fast. First, Judd(1987) showed that when we use conventional estimates for the critical parameters, the welfare cost of capital taxation is high even over small horizons of a few years. Jones et al.(1993) demonstrate rapid convergence to the long-run tax policy for reasonably calibrated examples. While these studies assumed competitive models, our analysis shows that the dynamics of the distorted economies are similar to competitive economies, indicating that the period during which optimal policy differs significantly from the long-run policy is also short.

The transition will affect the long run outcome. If there is an initial period of high tax rates on capital income, there will be an initial surplus which essentially endows part of the long-run subsidy on capital income. In the extreme case, this initial surplus may finance all of the long-run subsidy. However, this is unlikely because of the problems with a high-tax-rate initial phase. More typically there will be continuing taxation of consumption and/or labor income which will finance most of the long-run subsidy.

4.4. Long-Run Growth. The models above display no steady state growth. However, that is of little importance since, by proper choices of production functions and other structural elements, the differences between our model and a model with steady state growth can be made arbitrarily small over any finite horizon. As long as the social and private objective includes discounting, the policy differences between our model and similar ones with steady state growth will also be arbitrarily small.

For some issues the transition phase looks like the steady state. This was seen in the general model in Section 3, where many of the policy results were true at all times, not just in the steady state. This was particularly true of the desirability to use tax policy to offset differential price-cost margins across various capital goods. Since few of our results actually depended on steady-state conditions, it is there is no reason to think that extensions which have long-run growth would produce different results. Only if our instruments are restricted in relevant and economically reasonable ways will the Diamond-Mirrlees result be substantially altered when we move to more complex growth models.
4.5. Current vs. Optimal Tax Policy. Even if the optimal policy is infeasible, these results do indicate the costs of alternative feasible policies and the correct direction for tax policy reform. The literature on tax reform has nearly exclusively focused on the costs of capital taxation in a competitive economy and argued that the optimal tax rate on capital income is zero, whereas current U.S. tax policy imposes a substantial tax on many forms of capital income. The results in this paper indicate that current policy is even farther from optimal policy than indicated by the competitive model. Since the efficiency costs of a tax are roughly equal to the square of the tax, our results also indicate that the efficiency costs of current tax policy are greater than those implied by the competitive model. For example, if the current effective tax rate is 30%, but the optimal subsidy is 30%, then (using our quadratic rule-of-thumb) the gain from an optimal policy is four times the gain from eliminating the 30% tax in a competitive model. Furthermore, even if the only feasible policy reform is reducing the capital income tax rate to zero, this analysis increases the estimated benefits of that reform. In our example, the gain from moving the 30% tax down to zero in our monopolistically competitive model is three times the gain from such a change in the competitive model. This quadratic approximation is of course rough, but it points out the substantial change in our evaluation of even conventional tax policy changes when we move from a competitive model to a noncompetitive one.

4.6. Labor Union Markups. We have assumed market power in the capital equipment market. Labor markets also have elements of market power, particularly in unionized sectors. This need not affect any of our results. Suppose, for example, that there is a uniform markup in labor markets equal to \( m_u \) (as would be implied if each individual faced a constant elasticity of labor supply and had a constant “cost”). Then labor supply would be \( L(\lambda, \bar{w} / (1 + m_u)) \); that is, workers receive a wage of \( \bar{w} \) but supply only \( L(\lambda, \bar{w} / (1 + m_u)) \) units. When we substitute that into the optimal tax problem, the new Hamiltonian is

\[
H_{tax} = v(\lambda, \bar{w}) + \theta \left( f(k, L(\lambda, \bar{w} / (1 + m_u))) - C(\lambda, \bar{w} / (1 + m_u)) \right) \\
+ \psi \lambda (\rho - \bar{r}) \\
+ \mu (\bar{r}B - f(k, L(\lambda, \bar{w} / (1 + m_u)))) + \bar{k} + \bar{w}L(\lambda, \bar{w} / (1 + m_u)) \\
+ (1 - \tau_H) \eta f(k, L(\lambda, \bar{w} / (1 + m_u))) \\
+ \nu \bar{r}
\]

The costate equation for \( \theta \) in this new Hamiltonian is still (14), the costate equation when \( m_u = 0 \). Since this version of a union premium has no effect on (14), our results for the optimal tax are qualitatively unchanged.

4.7. International Trade Implications. While the models above ignored international trade explicitly, some simple aspects of trade in capital goods can be analyzed.
In the general model, we could imagine that some of the domestic capital stock, $k$, and domestic labor are used to produce an exported good which is sold to finance the importation of the services of type $j_M$ capital. The distortion in the sale of type $j_M$ capital, $\pi_{j_M}$, would be nonzero only if the importers had market power over the rental of that capital in the importing country. The price-cost gap of the foreign producer is not relevant since the import price is the true cost of the machine to the importing country.

If a country is a small country then the world price of an imported capital good is the social cost for the country. In this case, the small country should not subsidize the imported capital goods as long as the internal price equals the world price. Even if it produced capital goods which were perfect substitutes for the imported goods, standard trade theory still applies and there should still be no subsidy. Therefore, the analysis above gives little support for investment tax credits and the like in small or developing countries.

However, if the small country merged with a (presumably large) country which produces intermediate goods, then our analysis indicates that the optimal policy for the merged country would subsidize the capital used in the former independent small country. This indicates that there would be some incentive for the two countries to coordinate investment policies. Furthermore, this coordination proposition obtains also for coalitions of large countries. Therefore, there should be an international agreement to subsidize investment affected by imperfect competition.

Such coordination is not always possible. The final interesting question is the optimal policy for a capital-producing and -exporting country which also imports some capital goods. If it had no effect on terms of trade, then the answer is again clear: the capital subsidies should be limited to domestically produced capital goods which are sold at prices above their marginal cost. However, if it has market power in the export markets for capital goods, as would be natural to assume here, then it does have an effect on the terms of trade.

All of these arguments are complicated by multinational firms. If foreigners own a domestic firm which produces a monopolized capital service, the rent goes to the foreigners and the true social cost to the domestic government is the monopoly price; hence, no subsidy is justified. Symmetrically, if domestic citizens own a foreign factory which then imports the capital good back to the owners’ country, then such imports should be subsidized because the true marginal cost to the country is the production cost at the foreign factory, not the monopolized price. A complete analysis of optimal policy must take into account international structure of ownership, and is beyond the scope of this paper.

These arguments indicate that there is much to be explored regarding trade policy. International trade economists have investigated both the positive and normative aspects of trade policy in the face of oligopolistic international markets, but they generally work in
partial equilibrium frameworks which ignore distinctions between intermediate and final goods. Further work is needed to answer the important related trade policy questions.

4.8. Materials. Our analysis has not explicitly treated materials. This is a limitation since materials constitute a substantial portion of any firm’s costs (roughly half for manufacturing), are also sold in imperfectly competitive markets, but would not qualify for capital subsidies. We have, however, treated them implicitly. The aggregate production function \( f(K, \ell) \) could be thought of as a reduced form expressing output as a function of inputs, treating materials as intermediate goods which have been produced but then consumed in the production of the final goods. In that case, \( f(k, \ell) \) assumes efficiency in the materials markets and the distortion terms in our general model would include any distortions in private returns to investment which are due to imperfections in the materials goods markets. Our formulas for efficiency remain unchanged but must be reinterpreted. While a complete analysis remains to be done, some conjectures seem to be safe.

The primary concern is that a subsidy for capital but no parallel subsidy for materials would twist inputs inefficiently towards capital. While this would be a concern, it ultimately depends on the elasticity of substitution between materials and capital. If they were perfect complements, then subsidies would produce no inefficient change in the material/capital mix. As long as materials and capital are complementary inputs, one would expect the net effect of any capital subsidy to be an increase in materials demand, which would improve efficiency whenever those materials are priced above marginal social cost. In this case, the general case for a capital subsidy would not be altered substantially even if a parallel subsidy for materials is not possible. In fact, the case is strengthened; if the materials markets are distorted but the capital subsidy increases materials use, then the materials goods distortions are also partially relieved and efficiency improves. Therefore, the optimal capital subsidy is arguably greater if capital and materials are complements (as is indicated by most empirical studies) and subsidies for materials are not offered.

4.9. Intermediate versus Final good innovation. The differences between our results here and those in Judd (1985) also indicate the importance of distinguishing between intermediate and final goods when discussing R&D policy. In the model of Judd (1985), infinite-life patents resulted in implementing the first-best allocation between invention and consumption; however, it assumed only final good innovation. Here, we find inefficiently low levels of intermediate good output. The natural conjecture is that R&D is biased towards final goods with relatively too little incentive for intermediate good innovation, further strengthening the case of intermediate good subsidies. Further analysis is needed to establish that conclusion.
5. The Investment Tax Credit

We next apply our results to evaluate the Investment Tax Credit (ITC). If all capital goods entered symmetrically into production and the distortions were uniform across the various types of capital, then a savings subsidy would be an adequate policy in a closed economy\textsuperscript{12}. However, this is probably not the case. In this section we will examine the ITC as an instrument which partially implements the corrective tax policy.

The investment tax credit (ITC) was first introduced in 1962. The ITC gives firms a tax credit proportional to their purchase of equipment, but not structures. The ITC fluctuated between zero and ten percent, and was eliminated in 1986. Initially, its justification was to improve productivity by replacing old equipment with new, and to stimulate ‘autonomous demand’. Because of the implementation lags, few would still take the Keynesian business cycle fine-tuning arguments seriously. In contrast, we provide a purely microeconomic foundation for the ITC, unrelated to business cycle considerations\textsuperscript{13}.

5.1. Structures versus Equipment. Critics of the ITC have argued that the ITC biases investment inefficiently against structures. Our analysis suggests otherwise. The analysis above shows that the subsidy should be higher for intermediate goods which have higher margins. Both the consideration of the structure and conduct of equipment and construction industries and empirical estimates of price-cost margins suggest that the margins in equipment exceed those in construction.

First, equipment makers engage in substantial R&D effort, and equipment often embodies new technology protected by patents and/or trade secrecy. The ratio of private R&D expenditures to sales is highest, being about 4\%, in the SIC categories of machinery, electrical equipment, and instruments. Equipment can also be substantially differentiated, enhancing market power. The production of new equipment is also likely to exhibit learning-by-doing.

Construction firms, however, engage in very little R&D. While buildings are differentiated, the construction firms do not specialize to the extent equipment manufacturers do. Typically, several firms will bid on each construction job. The structure and conduct of the construction industry therefore indicates a competitive outcome. The ITC does enhance productivity in construction because construction firms may increase their productivity by using new equipment for which they receive a tax credit, a benefit which is passed onto the buyer of a structure in a competitive market. However, if the construction

\textsuperscript{12}Since the U. S. is not a closed economy, savings incentives would be an inadequate policy in any case. 
\textsuperscript{13}The ITC has often been thought of as a countercyclical tool. While the analysis above focused on the long run, the generalization to a stochastic model is clear. The basic point is that tax policy should neutralize price-cost margins. Therefore, if margins are procyclical, as conventional wisdom argues (see Scherer), then the neutralizing subsidies should also be procyclical. However, the evidence on this is weak (see Domowitz, et al.). Furthermore, the ITC is a weak countercyclical tool because of implementation lags in policy and in investment.
services are themselves supplied competitively, then they should receive no subsidy.

This view of the equipment and construction industries is consistent with the empirical estimates of Hall (1986). In fact, the competitive hypothesis cannot be rejected for construction whereas it is for the machinery categories and instruments. Both the Hall study and that of Domowitz et al. (1988) indicate that the margins in the equipment sectors are substantial in size, lying generally between 15% and 40% of the price. These margins may seem large and other work in the macroeconomic literature has argued for smaller margins. However, the perfectly competitive assumption is not generally supported. Furthermore, the empirical Industrial Organization literature contains some industry specific studies on price-cost margins. These studies also produce high estimates for price-cost margins (see, e.g., Appelbaum, 1982).

Fortunately, our discussion here does not rely critically on these estimates of price–cost margins. R&D expenditures are 3–6% of sales for many types of equipment (see Scherer, 1980), implying (under the assumption of nonnegative profits and nondecreasing long–run returns to scale) an equivalent lower bound for the gap between long–run marginal cost and price. This lower bound plus a conservative estimate for learning curve and economies of scale effects, and other long–run fixed costs puts us in a range relevant for our policy discussions. Therefore, even under conservative readings of the empirical evidence, the gap between industries with small R&D expenditures, such as construction, and industries with high R&D costs, such as most equipment industries, appears substantial.

Critics of the ITC have relied on the competitive model for their analysis. The competitive assumption is surely incorrect for many manufactured goods, particularly many of the types of equipment associated with technological advancement. The basic argument here is that if the construction industry is competitive but equipment industries are imperfectly competitive, then the market allocation is inefficient and the ITC for equipment which has been often implemented partially corrected the problem, albeit unintentionally.

### 5.2. Income versus Consumption Taxation.

Many authors of tax policy tracts have argued for consumption taxation over income taxation; however, their arguments often ignore the optimal tax literature. Instead they emphasize the simplicity of a uniform consumption tax, and preach that it is morally superior to tax people for what they take from society instead of taxing them for what they produce. The Diamond–Mirrlees rule against intermediate good taxation provides us with a strong theoretical economic principle supporting the consumption taxation argument. However, this paper shows

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14 Two prominent examples of such advocacy are Bradford, and Hall and Rabushka. While I am sure that most of these authors were aware of the Diamond–Mirrlees results, neither book includes that paper, nor any paper on optimal taxation, in their citations of the academic literature.

15 Strictly speaking, we have been implicitly advocating a uniform distortion between consumption and leisure whereas the Diamond–Mirrlees result may need commodity–specific consumption taxation. Since all consumption goods in our models entered into utility symmetrically, we ignored the possibility
that we need to go further than eliminate intermediate goods taxation. Since we instead want to implement intermediate goods subsidies, it would appear that we want to keep an income tax structure to facilitate such subsidies.

Consumption taxation alone cannot practically achieve the optimal tax policy when there are pre-existing distortions in the intermediate goods markets. First, it would be more difficult to ascertain the cumulative gap between price and social marginal cost for any good since that gap depends on the total collection of intermediate goods used. The policy we outlined above just needs to know the difference between the price and the producer’s marginal cost for an intermediate good, not the entire input-output structure of the economy. Also, if the margins for intermediate goods are unchanged but the mix of intermediate goods used to produce a final good changes, then the commodity tax would need to be adjusted whereas the ITC would not need to be changed.

While our discussion has focused on the ITC, accelerated depreciation could also be used to accomplish the same effects. Our focus on the ITC is driven only by the ways in which the ITC and depreciation rules have been conventionally used. The key observation is that tax relief should be related to the price-cost margins.

5.3. The ITC, New Equipment, and the Equipment Replacement Cycle. The original ITC proposal was only for the purchase of new equipment, not used equipment. A straightforward extension of our analysis provides an economic rationale for the restriction to new equipment. A new type of equipment is likely to incorporate the newest technology and to be differentiated with respect to used equipment, to new production of old models, and to other new equipment. Since imitation is less likely, the producer of a new model is better able to charge above marginal cost. The used equipment market is more likely to be competitive. Therefore, a tax preference for newly produced equipment is partially a preference for new types of equipment, which are more likely to be priced above marginal cost.

Even if old varieties are no longer available, the ITC will still help. In deciding when to replace an old piece of equipment no longer being produced with a new variety, a firm will compare the present value of the extra output with the price of the new machine. Typically, the new machine will be priced above marginal cost, and that margin will decline over time. The result will be a replacement time later than is socially optimal, implying that the capital stock will be older and less productive than is socially optimal, as was argued by the initial proponents of the ITC.

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of heterogeneous tax rates. Our focus on such simple systems is empirically reasonable. In particular, Balcer and Sadka (2007) show that the utility gain from a fully optimal commodity tax system (where tax rates roughly obey the inverse-elasticity rule) relative to a uniform commodity tax (or wage tax) is small for empirically reasonable demand systems despite the dramatically different tax rates.
5.4. The ITC as Countercyclical Policy. The ITC has often been thought of as a countercyclical tool. However, critics have pointed out that the ITC cannot be a useful countercyclical tool because of implementation lags in policy and in investment. In fact, the net effect may be that it has often been procyclical. None of what we have said disputes this.

While the analysis above focused on the long run, the generalization to a stochastic model is clear. The basic point is that tax policy should neutralize price-cost margins. Therefore, if margins are procyclical, as conventional wisdom argues (see Scherer), then the neutralizing subsidies should also be procyclical. This point shows how our approach is substantively very different from the Keynesian style of argument which has been used to support the ITC.

When we combine the basic intuition with empirical evidence, it is unclear what the proper cyclicity is. Domowitz et al. argue that margins are generally procyclical, but that durable goods’ margins tend to be countercyclical. However, the margin to which our argument applies is to the gap between the marginal cost and price of the rental service. Since the price of a durable good is the present value of its rental services, countercyclical margins on durable goods’ prices is consistent with procyclical margins on the rental services with appropriate interest rate movements; hence, the empirical work does not address the correct issue for our purposes. If both rental service margins and nondurable goods’ margins are procyclical then the ITC should be procyclical; otherwise, the nature of the proper ITC is unclear. In any case, the weak empirical evidence does not currently support a countercyclical ITC under the theory developed above.

6. Conclusions

We have examined dynamic models with distortions. The robust finding is that distortions of intermediate goods markets, both tax induced as well as market structure induced, are to be ameliorated in the long run. In the face of permanent market power, tax policy should subsidize capital formation in proportion to the distortion. Since equipment markets are more distorted by market power, partly because of R&D expenditures, the tax subsidy should look similar to the investment tax credit, which has been occasionally part of the U.S. tax code, and should be implemented on a permanent basis. We also show that entry into oligopolistic markets strengthens these conclusions. While the models we examined were simple and further analysis is needed, it is clear that these considerations will have a significant impact on the optimal tax policy.
References


