CHAPTER 12

Statistical Parsing

The characters in Damon Runyon’s short stories are willing to bet “on any proposition whatever”, as Runyon says about Sky Masterson in *The Idyll of Miss Sarah Brown*, from the probability of getting aces back-to-back to the odds against a man being able to throw a peanut from second base to home plate. There is a moral here for language processing: with enough knowledge we can figure the probability of just about anything. The last two chapters have introduced sophisticated models of syntactic structure and its parsing. Here, we show that it is possible to build probabilistic models of syntactic knowledge and use some of this probabilistic knowledge to build efficient probabilistic parsers.

One crucial use of probabilistic parsing is to solve the problem of disambiguation. Recall from Chapter 11 that sentences on average tend to be syntactically ambiguous because of phenomena like coordination ambiguity and attachment ambiguity. The CKY parsing algorithm can represent these ambiguities in an efficient way but is not equipped to resolve them. A probabilistic parser offers a solution to the problem: compute the probability of each interpretation and choose the most probable interpretation. Thus, due to the prevalence of ambiguity, most modern parsers used for natural language understanding tasks (semantic analysis, summarization, question-answering, machine translation) are of necessity probabilistic.

The most commonly used probabilistic grammar formalism is the probabilistic context-free grammar (PCFG), a probabilistic augmentation of context-free grammars in which each rule is associated with a probability. We introduce PCFGs in the next section, showing how they can be trained on Treebank grammars and how they can be parsed. We present the most basic parsing algorithm for PCFGs, which is the probabilistic version of the CKY algorithm that we saw in Chapter 11.

We then show a number of ways that we can improve on this basic probability model (PCFGs trained on Treebank grammars). One method of improving a trained Treebank grammar is to change the names of the non-terminals. By making the non-terminals sometimes more specific and sometimes more general, we can come up with a grammar with a better probability model that leads to improved parsing scores. Another augmentation of the PCFG works by adding more sophisticated conditioning factors, extending PCFGs to handle probabilistic subcategorization information and probabilistic lexical dependencies.

Heavily lexicalized grammar formalisms such as Lexical-Functional Grammar (LFG) (Bresnan, 1982), Head-Driven Phrase Structure Grammar (HPSG) (Pollard and Sag, 1994), Tree-Adjoining Grammar (TAG) (Joshi, 1985), and Combinatory Categorial Grammar (CCG) pose additional problems for probabilistic parsers. Section 12.7 introduces the task of supertagging and the use of heuristic search methods based on the A* algorithm in the context of CCG parsing.

Finally, we describe the standard techniques and metrics for evaluating parsers and discuss some relevant psychological results on human parsing.
12.1 Probabilistic Context-Free Grammars

The simplest augmentation of the context-free grammar is the Probabilistic Context-Free Grammar (PCFG), also known as the Stochastic Context-Free Grammar (SCFG), first proposed by Booth (1969). Recall that a context-free grammar \( G \) is defined by four parameters \( (N, \Sigma, R, S) \); a probabilistic context-free grammar is also defined by four parameters, with a slight augmentation to each of the rules in \( R \):

- \( N \) a set of non-terminal symbols (or variables)
- \( \Sigma \) a set of terminal symbols (disjoint from \( N \))
- \( R \) a set of rules or productions, each of the form \( A \rightarrow \beta \ [p] \), where \( A \) is a non-terminal, \( \beta \) is a string of symbols from the infinite set of strings \( (\Sigma \cup N)^* \), and \( p \) is a number between 0 and 1 expressing \( P(\beta|A) \)
- \( S \) a designated start symbol

That is, a PCFG differs from a standard CFG by augmenting each rule in \( R \) with a conditional probability:

\[
A \rightarrow \beta \ [p]
\]

(12.1)

Here \( p \) expresses the probability that the given non-terminal \( A \) will be expanded to the sequence \( \beta \). That is, \( p \) is the conditional probability of a given expansion \( \beta \) given the left-hand-side (LHS) non-terminal \( A \). We can represent this probability as

\[
P(A \rightarrow \beta)
\]

or as

\[
P(A \rightarrow \beta | A)
\]

or as

\[
P(RHS | LHS)
\]

Thus, if we consider all the possible expansions of a non-terminal, the sum of their probabilities must be 1:

\[
\sum_{\beta} P(A \rightarrow \beta) = 1
\]

Figure 12.1 shows a PCFG: a probabilistic augmentation of the \( L_1 \) miniature English CFG grammar and lexicon. Note that the probabilities of all of the expansions of each non-terminal sum to 1. Also note that these probabilities were made up for pedagogical purposes. A real grammar has a great many more rules for each non-terminal; hence, the probabilities of any particular rule would tend to be much smaller.

A PCFG is said to be **consistent** if the sum of the probabilities of all sentences in the language equals 1. Certain kinds of recursive rules cause a grammar to be inconsistent by causing infinitely looping derivations for some sentences. For example, a rule \( S \rightarrow S \) with probability 1 would lead to lost probability mass due to derivations that never terminate. See Booth and Thompson (1973) for more details on consistent and inconsistent grammars.
Probabilistic Context-Free Grammars

12.1 PCFGs for Disambiguation

A PCFG assigns a probability to each parse tree $T$ (i.e., each derivation) of a sentence $S$. This attribute is useful in disambiguation. For example, consider the two parses of the sentence “Book the dinner flight” shown in Fig. 12.2. The sensible parse on the left means “Book a flight that serves dinner”. The nonsensical parse on the right, however, would have to mean something like “Book a flight on behalf of ‘the dinner’” just as a structurally similar sentence like “Can you book John a flight?” means something like “Can you book a flight on behalf of John?”

The resulting probability $P(T, S)$ is both the joint probability of the parse and the sentence and also the probability of the parse $P(T)$. How can this be true? First, by the definition of joint probability:

$$P(T, S) = P(T)P(S|T)$$

How are PCFGs used? A PCFG can be used to estimate a number of useful probabilities concerning a sentence and its parse tree(s), including the probability of a particular parse tree (useful in disambiguation) and the probability of a sentence or a piece of a sentence (useful in language modeling). Let’s see how this works.

Figure 12.1 A PCFG that is a probabilistic augmentation of the $L_1$ miniature English CFG grammar and lexicon of Fig. 12.1. These probabilities were made up for pedagogical purposes and are not based on a corpus (since any real corpus would have many more rules, so the true probabilities of each rule would be much smaller).

<table>
<thead>
<tr>
<th>Grammar</th>
<th>Lexicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S \rightarrow NP \ VP$</td>
<td>[ .80 ] $Det \rightarrow that \ [.10] \</td>
</tr>
<tr>
<td>$S \rightarrow Aux \ NP \ VP$</td>
<td>[ .15 ] $Noun \rightarrow book \ [.10] \</td>
</tr>
<tr>
<td>$S \rightarrow VP$</td>
<td>[ .05 ] meal \ [.015] \</td>
</tr>
<tr>
<td>$NP \rightarrow Pronoun$</td>
<td>[ .35 ] flight \ [.40] \</td>
</tr>
<tr>
<td>$NP \rightarrow Proper-Noun$</td>
<td>[ .30 ] $Verb \rightarrow book \ [.30] \</td>
</tr>
<tr>
<td>$NP \rightarrow Det \ Nominal$</td>
<td>[ .20 ] $Verb \rightarrow prefer \ [.40]$</td>
</tr>
<tr>
<td>$NP \rightarrow Nominal$</td>
<td>[ .15 ] $Pronoun \rightarrow I \ [.40] \</td>
</tr>
<tr>
<td>Nominal $\rightarrow$ Nominal Noun</td>
<td>[ .75 ] $me \ [.15] \</td>
</tr>
<tr>
<td>Nominal $\rightarrow$ Nominal PP</td>
<td>[ .20 ] Proper-Noun $\rightarrow$ Houston \ [.60]$</td>
</tr>
<tr>
<td>$VP \rightarrow Verb$</td>
<td>[ .35 ] Aux $\rightarrow$ does \ [.60] \</td>
</tr>
<tr>
<td>$VP \rightarrow Verb \ NP$</td>
<td>[ .20 ] Preposition $\rightarrow$ from \ [.30] \</td>
</tr>
<tr>
<td>$VP \rightarrow Verb \ NP \ PP$</td>
<td>[ .10 ] on \ [.20] \</td>
</tr>
<tr>
<td>$VP \rightarrow Verb \ PP$</td>
<td>[ .15 ] through \ [.05]$</td>
</tr>
<tr>
<td>$VP \rightarrow Verb \ NP$</td>
<td>[ .05 ]</td>
</tr>
<tr>
<td>$VP \rightarrow VP \ PP$</td>
<td>[ .15 ]</td>
</tr>
<tr>
<td>PP $\rightarrow$ Preposition PP</td>
<td>[ .10 ]</td>
</tr>
</tbody>
</table>

12.1.1 PCFGs for Disambiguation

How are PCFGs used? A PCFG can be used to estimate a number of useful probabilities concerning a sentence and its parse tree(s), including the probability of a particular parse tree (useful in disambiguation) and the probability of a sentence or a piece of a sentence (useful in language modeling). Let’s see how this works.
But since a parse tree includes all the words of the sentence, \( P(S|T) \) is 1. Thus,

\[
P(T,S) = P(T)P(S|T) = P(T)
\]

(12.4)

<table>
<thead>
<tr>
<th>Rules</th>
<th>( P )</th>
<th>Rules</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S \rightarrow VP )</td>
<td>.05</td>
<td>( S \rightarrow VP )</td>
<td>.05</td>
</tr>
<tr>
<td>( VP \rightarrow ) Verb NP</td>
<td>.20</td>
<td>( VP \rightarrow ) Verb NP NP</td>
<td>.10</td>
</tr>
<tr>
<td>( NP \rightarrow ) Det Nominal</td>
<td>.20</td>
<td>( NP \rightarrow ) Det Nominal</td>
<td>.20</td>
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<tr>
<td>( Nominal \rightarrow Nominal )</td>
<td>.20</td>
<td>( Nominal \rightarrow Nominal )</td>
<td>.20</td>
</tr>
<tr>
<td>( Nominal \rightarrow Noun )</td>
<td>.75</td>
<td>( Nominal \rightarrow Noun )</td>
<td>.75</td>
</tr>
<tr>
<td>( Verb \rightarrow book )</td>
<td>.30</td>
<td>( Verb \rightarrow book )</td>
<td>.30</td>
</tr>
<tr>
<td>( Det \rightarrow the )</td>
<td>.60</td>
<td>( Det \rightarrow the )</td>
<td>.60</td>
</tr>
<tr>
<td>( Noun \rightarrow dinner )</td>
<td>.10</td>
<td>( Noun \rightarrow dinner )</td>
<td>.10</td>
</tr>
<tr>
<td>( Noun \rightarrow flight )</td>
<td>.40</td>
<td>( Noun \rightarrow flight )</td>
<td>.40</td>
</tr>
</tbody>
</table>

**Figure 12.2** Two parse trees for an ambiguous sentence. The parse on the left corresponds to the sensible meaning “Book a flight that serves dinner”, while the parse on the right corresponds to the nonsensical meaning “Book a flight on behalf of ‘the dinner’”.

We can compute the probability of each of the trees in Fig. 12.2 by multiplying the probabilities of each of the rules used in the derivation. For example, the probability of the left tree in Fig. 12.2a (call it \( T_{left} \)) and the right tree (Fig. 12.2b or \( T_{right} \)) can be computed as follows:

\[
P(T_{left}) = .05 \times .20 \times .20 \times .20 \times .75 \times .30 \times .60 \times .40 = 2.2 \times 10^{-6}
\]

\[
P(T_{right}) = .05 \times .10 \times .20 \times .15 \times .75 \times .75 \times .30 \times .60 \times .40 = 6.1 \times 10^{-7}
\]

We can see that the left tree in Fig. 12.2 has a much higher probability than the tree on the right. Thus, this parse would correctly be chosen by a disambiguation algorithm that selects the parse with the highest PCFG probability.

Let’s formalize this intuition that picking the parse with the highest probability is the correct way to do disambiguation. Consider all the possible parse trees for a given sentence \( S \). The string of words \( S \) is called the **yield** of any parse tree over \( S \).
Thus, out of all parse trees with a yield of $S$, the disambiguation algorithm picks the parse tree that is most probable given $S$:

$$\hat{T}(S) = \arg\max_{T:s.t.T=\text{yield}(T)} P(T|S)$$

By definition, the probability $P(T|S)$ can be rewritten as $P(T,S)/P(S)$, thus leading to

$$\hat{T}(S) = \arg\max_{T:s.t.T=\text{yield}(T)} \frac{P(T,S)}{P(S)}$$

Since we are maximizing over all parse trees for the same sentence, $P(S)$ will be a constant for each tree, so we can eliminate it:

$$\hat{T}(S) = \arg\max_{T:s.t.T=\text{yield}(T)} P(T,S)$$

Furthermore, since we showed above that $P(T,S) = P(T)$, the final equation for choosing the most likely parse neatly simplifies to choosing the parse with the highest probability:

$$\hat{T}(S) = \arg\max_{T:s.t.T=\text{yield}(T)} P(T)$$

### 12.1.2 PCFGs for Language Modeling

A second attribute of a PCFG is that it assigns a probability to the string of words constituting a sentence. This is important in language modeling, whether for use in speech recognition, machine translation, spelling correction, augmentative communication, or other applications. The probability of an unambiguous sentence is $P(T,S) = P(T)$ or just the probability of the single parse tree for that sentence. The probability of an ambiguous sentence is the sum of the probabilities of all the parse trees for the sentence:

$$P(S) = \sum_{T:s.t.T=\text{yield}(T)} P(T,S)$$

$$= \sum_{T:s.t.T=\text{yield}(T)} P(T)$$

An additional feature of PCFGs that is useful for language modeling is their ability to assign a probability to substrings of a sentence. For example, suppose we want to know the probability of the next word $w_i$ in a sentence given all the words we’ve seen so far $w_1,\ldots,w_{i-1}$. The general formula for this is

$$P(w_i|w_1,w_2,\ldots,w_{i-1}) = \frac{P(w_1,w_2,\ldots,w_{i-1},w_i)}{P(w_1,w_2,\ldots,w_{i-1})}$$

We saw in Chapter 3 a simple approximation of this probability using $N$-grams, conditioning on only the last word or two instead of the entire context; thus, the bigram approximation would give us

$$P(w_i|w_1, w_2, \ldots, w_{i-1}) \approx \frac{P(w_{i-1},w_i)}{P(w_{i-1})}$$
But the fact that the N-gram model can only make use of a couple words of context means it is ignoring potentially useful prediction cues. Consider predicting the word \textit{after} in the following sentence from Chelba and Jelinek (2000):

\begin{equation}
\text{(12.13) the contract ended with a loss of 7 cents after trading as low as 9 cents}
\end{equation}

A trigram grammar must predict \textit{after} from the words \textit{7 cents}, while it seems clear that the verb \textit{ended} and the subject \textit{contract} would be useful predictors that a PCFG-based parser could help us make use of. Indeed, it turns out that PCFGs allow us to condition on the entire previous context \(w_1, w_2, \ldots, w_{i-1}\) shown in Eq. 12.11.

In summary, this section and the previous one have shown that PCFGs can be applied both to disambiguation in syntactic parsing and to word prediction in language modeling. Both of these applications require that we be able to compute the probability of parse tree \(T\) for a given sentence \(S\). The next few sections introduce some algorithms for computing this probability.

### 12.2 Probabilistic CKY Parsing of PCFGs

The parsing problem for PCFGs is to produce the most-likely parse \(\hat{T}\) for a given sentence \(S\), that is,

\begin{equation}
\hat{T}(S) = \arg\max_{T \vdash S = \text{yield}(T)} P(T)
\end{equation}

The algorithms for computing the most likely parse are simple extensions of the standard algorithms for parsing; most modern probabilistic parsers are based on the probabilistic CKY algorithm, first described by Ney (1991).

As with the CKY algorithm, we assume for the probabilistic CKY algorithm that the PCFG is in Chomsky normal form. Recall from page ?? that grammars in CNF are restricted to rules of the form \(A \rightarrow BC\), or \(A \rightarrow w\). That is, the right-hand side of each rule must expand to either two non-terminals or to a single terminal.

For the CKY algorithm, we represented each sentence as having indices between the words. Thus, an example sentence like

\begin{equation}
\text{(12.15) Book the flight through Houston.}
\end{equation}

would assume the following indices between each word:

\begin{equation}
\text{(12.16) 0 Book 1 the 2 flight 3 through 4 Houston 5}
\end{equation}

Using these indices, each constituent in the CKY parse tree is encoded in a two-dimensional matrix. Specifically, for a sentence of length \(n\) and a grammar that contains \(V\) non-terminals, we use the upper-triangular portion of an \((n+1) \times (n+1)\) matrix. For CKY, each cell \(\text{table}[i,j]\) contained a list of constituents that could span the sequence of words from \(i\) to \(j\). For probabilistic CKY, it’s slightly simpler to think of the constituents in each cell as constituting a third dimension of maximum length \(V\). This third dimension corresponds to each non-terminal that can be placed in this cell, and the value of the cell is then a probability for that non-terminal/constituent rather than a list of constituents. In summary, each cell \([i,j,A]\) in this \((n+1) \times (n+1) \times V\) matrix is the probability of a constituent of type \(A\) that spans positions \(i\) through \(j\) of the input.

Figure 12.3 gives pseudocode for this probabilistic CKY algorithm, extending the basic CKY algorithm from Fig. ??.
12.3 Ways to Learn PCFG Rule Probabilities

Where do PCFG rule probabilities come from? There are two ways to learn probabilities for the rules of a grammar. The simplest way is to use a treebank, a corpus of already parsed sentences. Recall that we introduced in Chapter 10 the idea of treebanks and the commonly used Penn Treebank (Marcus et al., 1993), a collection of parse trees in English, Chinese, and other languages that is distributed by the Linguistic Data Consortium. Given a treebank, we can compute the probability of each expansion of a non-terminal by counting the number of times that expansion
The flight includes a meal

\[ \text{Det: .40} \]

\[ \text{NP: .30 \times .40 \times .02} = .0024 \]

\[ \text{N: .02} \]

\[ \text{V: .05} \]

\[ \text{NP: .30 \times .40 \times .02} = .0024 \]

\[ \text{Det: .40} \]

\[ \text{N: .01} \]

---

**Figure 12.4** The beginning of the probabilistic CKY matrix. Filling out the rest of the chart is left as Exercise 12.4 for the reader.

occurs and then normalizing.

\[
P(\alpha \rightarrow \beta | \alpha) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\sum_{\gamma} \text{Count}(\alpha \rightarrow \gamma)} = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}
\] (12.18)

If we don’t have a treebank but we do have a (non-probabilistic) parser, we can generate the counts we need for computing PCFG rule probabilities by first parsing a corpus of sentences with the parser. If sentences were unambiguous, it would be as simple as this: parse the corpus, increment a counter for every rule in the parse, and then normalize to get probabilities.

But wait! Since most sentences are ambiguous, that is, have multiple parses, we don’t know which parse to count the rules in. Instead, we need to keep a separate count for each parse of a sentence and weight each of these partial counts by the probability of the parse it appears in. But to get these parse probabilities to weight the rules, we need to already have a probabilistic parser.

The intuition for solving this chicken-and-egg problem is to incrementally improve our estimates by beginning with a parser with equal rule probabilities, then parse the sentence, compute a probability for each parse, use these probabilities to
weight the counts, re-estimate the rule probabilities, and so on, until our probabilities converge. The standard algorithm for computing this solution is called the inside-outside algorithm; it was proposed by Baker (1979) as a generalization of the forward-backward algorithm for HMMs. Like forward-backward, inside-outside is a special case of the Expectation Maximization (EM) algorithm, and hence has two steps: the expectation step, and the maximization step. See Lari and Young (1990) or Manning and Schütze (1999) for a complete description of the algorithm.

This use of the inside-outside algorithm to estimate the rule probabilities for a grammar is actually a kind of limited use of inside-outside. The inside-outside algorithm can actually be used not only to set the rule probabilities but even to induce the grammar rules themselves. It turns out, however, that grammar induction is so difficult that inside-outside by itself is not a very successful grammar inducer; see the Historical Notes at the end of the chapter for pointers to other grammar induction algorithms.

12.4 Problems with PCFGs

While probabilistic context-free grammars are a natural extension to context-free grammars, they have two main problems as probability estimators:

Poor independence assumptions: CFG rules impose an independence assumption on probabilities, resulting in poor modeling of structural dependencies across the parse tree.

Lack of lexical conditioning: CFG rules don’t model syntactic facts about specific words, leading to problems with subcategorization ambiguities, preposition attachment, and coordinate structure ambiguities.

Because of these problems, most current probabilistic parsing models use some augmented version of PCFGs, or modify the Treebank-based grammar in some way. In the next few sections after discussing the problems in more detail we introduce some of these augmentations.

12.4.1 Independence Assumptions Miss Structural Dependencies Between Rules

Let’s look at these problems in more detail. Recall that in a CFG the expansion of a non-terminal is independent of the context, that is, of the other nearby non-terminals in the parse tree. Similarly, in a PCFG, the probability of a particular rule like $NP \rightarrow Det N$ is also independent of the rest of the tree. By definition, the probability of a group of independent events is the product of their probabilities. These two facts explain why in a PCFG we compute the probability of a tree by just multiplying the probabilities of each non-terminal expansion.

Unfortunately, this CFG independence assumption results in poor probability estimates. This is because in English the choice of how a node expands can after all depend on the location of the node in the parse tree. For example, in English it turns out that $NPs$ that are syntactic subjects are far more likely to be pronouns, and $NPs$ that are syntactic objects are far more likely to be non-pronominial (e.g., a proper noun or a determiner noun sequence), as shown by these statistics for $NPs$ in the
Switchboard corpus (Francis et al., 1999).\(^1\)

<table>
<thead>
<tr>
<th></th>
<th>Pronoun</th>
<th>Non-Pronoun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>91%</td>
<td>9%</td>
</tr>
<tr>
<td>Object</td>
<td>34%</td>
<td>66%</td>
</tr>
</tbody>
</table>

Unfortunately, there is no way to represent this contextual difference in the probabilities in a PCFG. Consider two expansions of the non-terminal \(NP\) as a pronoun or as a determiner+noun. How shall we set the probabilities of these two rules? If we set their probabilities to their overall probability in the Switchboard corpus, the two rules have about equal probability.

\[
NP \rightarrow DT\ NN \quad .28 \\
NP \rightarrow PRP \quad .25
\]

Because PCFGs don’t allow a rule probability to be conditioned on surrounding context, this equal probability is all we get; there is no way to capture the fact that in subject position, the probability for \(NP \rightarrow PRP\) should go up to .91, while in object position, the probability for \(NP \rightarrow DT\ NN\) should go up to .66.

These dependencies could be captured if the probability of expanding an \(NP\) as a pronoun (e.g., \(NP \rightarrow PRP\)) versus a lexical \(NP\) (e.g., \(NP \rightarrow DT\ NN\)) were conditioned on whether the \(NP\) was a subject or an object. Section 12.5 introduces the technique of parent annotation for adding this kind of conditioning.

### 12.4.2 Lack of Sensitivity to Lexical Dependencies

A second class of problems with PCFGs is their lack of sensitivity to the words in the parse tree. Words do play a role in PCFGs since the parse probability includes the probability of a word given a part-of-speech (i.e., from rules like \(V \rightarrow sleep\), \(NN \rightarrow book\), etc.).

But it turns out that lexical information is useful in other places in the grammar, such as in resolving prepositional phrase (\(PP\)) attachment ambiguities. Since prepositional phrases in English can modify a noun phrase or a verb phrase, when a parser finds a prepositional phrase, it must decide where to attach it into the tree. Consider the following example:

(12.19) Workers dumped sacks into a bin.

Figure 12.5 shows two possible parse trees for this sentence; the one on the left is the correct parse; Fig. 12.6 shows another perspective on the preposition attachment problem, demonstrating that resolving the ambiguity in Fig. 12.5 is equivalent to deciding whether to attach the prepositional phrase into the rest of the tree at the \(NP\) or \(VP\) nodes; we say that the correct parse requires **VP attachment**, and the incorrect parse implies **NP attachment**.

Why doesn’t a PCFG already deal with \(PP\) attachment ambiguities? Note that the two parse trees in Fig. 12.5 have almost exactly the same rules; they differ only in that the left-hand parse has this rule:

\[
VP \rightarrow VBD\ NP\ PP
\]

---

\(^1\) Distribution of subjects from 31,021 declarative sentences; distribution of objects from 7,489 sentences. This tendency is caused by the use of subject position to realize the topic or old information in a sentence (Givón, 1990). Pronouns are a way to talk about old information, while non-pronominal (“lexical”) noun-phrases are often used to introduce new referents. We talk more about new and old information in Chapter 22.
12.4  •  PROBLEMS WITH PCFGS

Figure 12.5 Two possible parse trees for a prepositional phrase attachment ambiguity. The left parse is the sensible one, in which “into a bin” describes the resulting location of the sacks. In the right incorrect parse, the sacks to be dumped are the ones which are already “into a bin”, whatever that might mean.

Figure 12.6 Another view of the preposition attachment problem. Should the PP on the right attach to the VP or NP nodes of the partial parse tree on the left?

while the right-hand parse has these:

\[
\begin{align*}
VP & \rightarrow VBD NP \\
NP & \rightarrow NP PP
\end{align*}
\]

Depending on how these probabilities are set, a PCFG will always either prefer NP attachment or VP attachment. As it happens, NP attachment is slightly more common in English, so if we trained these rule probabilities on a corpus, we might always prefer NP attachment, causing us to misparse this sentence.

But suppose we set the probabilities to prefer the VP attachment for this sentence. Now we would misparse the following sentence, which requires NP attachment:

(12.20) fishermen caught tons of herring
What information in the input sentence lets us know that (12.20) requires NP attachment while (12.19) requires VP attachment?

It should be clear that these preferences come from the identities of the verbs, nouns, and prepositions. It seems that the affinity between the verb dumped and the preposition into is greater than the affinity between the noun sacks and the preposition into, thus leading to VP attachment. On the other hand, in (12.20) the affinity between tons and of is greater than that between caught and of, leading to NP attachment.

Thus, to get the correct parse for these kinds of examples, we need a model that somehow augments the PCFG probabilities to deal with these lexical dependency statistics for different verbs and prepositions.

Coordination ambiguities are another case in which lexical dependencies are the key to choosing the proper parse. Figure 12.7 shows an example from Collins (1999) with two parses for the phrase dogs in houses and cats. Because dogs is semantically a better conjunct for cats than houses (and because most dogs can’t fit inside cats), the parse [dogs in [NP houses and cats]] is intuitively unnatural and should be dispreferred. The two parses in Fig. 12.7, however, have exactly the same PCFG rules, and thus a PCFG will assign them the same probability.

In summary, we have shown in this section and the previous one that probabilistic context-free grammars are incapable of modeling important structural and lexical dependencies. In the next two sections we sketch current methods for augmenting PCFGs to deal with both these issues.

12.5 Improving PCFGs by Splitting Non-Terminals

Let’s start with the first of the two problems with PCFGs mentioned above: their inability to model structural dependencies, like the fact that NPs in subject position tend to be pronouns, whereas NPs in object position tend to have full lexical (non-pronominal) form. How could we augment a PCFG to correctly model this fact?

One idea would be to split the NP non-terminal into two versions: one for sub-
I 12.5 Improving PCFGs by Splitting Non-Terminals

jects, one for objects. Having two nodes (e.g., $NP_{\text{subject}}$ and $NP_{\text{object}}$) would allow us to correctly model their different distributional properties, since we would have different probabilities for the rule $NP_{\text{subject}} \rightarrow \text{PRP}$ and the rule $NP_{\text{object}} \rightarrow \text{PRP}$.

One way to implement this intuition of splits is to do parent annotation (Johnson, 1998), in which we annotate each node with its parent in the parse tree. Thus, an $NP$ node that is the subject of the sentence and hence has parent $S$ would be annotated $NP^S$, while a direct object $NP$ whose parent is $VP$ would be annotated $NP^VP$. Figure 12.8 shows an example of a tree produced by a grammar that parent-annotates the phrasal non-terminals (like $NP$ and $VP$).

Figure 12.8 A standard PCFG parse tree (a) and one which has parent annotation on the nodes which aren’t pre-terminal (b). All the non-terminal nodes (except the pre-terminal part-of-speech nodes) in parse (b) have been annotated with the identity of their parent.

In addition to splitting these phrasal nodes, we can also improve a PCFG by splitting the pre-terminal part-of-speech nodes (Klein and Manning, 2003b). For example, different kinds of adverbs (RB) tend to occur in different syntactic positions: the most common adverbs with ADVP parents are also and now, with VP parents n’t and not, and with NP parents only and just. Thus, adding tags like RB$^$ADVP, RB$^$VP, and RB$^$NP can be useful in improving PCFG modeling.

Similarly, the Penn Treebank tag IN can mark a wide variety of parts-of-speech, including subordinating conjunctions (while, as, if), complementizers (that, for), and prepositions (of, in, from). Some of these differences can be captured by parent annotation (subordinating conjunctions occur under S, prepositions under PP), while others require specifically splitting the pre-terminal nodes. Figure 12.9 shows an example from Klein and Manning (2003b) in which even a parent-annotated grammar incorrectly parses works as a noun in to see if advertising works. Splitting pre-terminals to allow if to prefer a sentential complement results in the correct verbal parse.

To deal with cases in which parent annotation is insufficient, we can also handwrite rules that specify a particular node split based on other features of the tree. For example, to distinguish between complementizer IN and subordinating conjunction IN, both of which can have the same parent, we could write rules conditioned on other aspects of the tree such as the lexical identity (the lexeme that is likely to be a complementizer, as a subordinating conjunction).

Node-splitting is not without problems; it increases the size of the grammar and hence reduces the amount of training data available for each grammar rule, leading to overfitting. Thus, it is important to split to just the correct level of granularity for a particular training set. While early models employed hand-written rules to try to find an optimal number of non-terminals (Klein and Manning, 2003b), modern models
automatically search for the optimal splits. The split and merge algorithm of Petrov et al. (2006), for example, starts with a simple X-bar grammar, alternately splits the non-terminals, and merges non-terminals, finding the set of annotated nodes that maximizes the likelihood of the training set treebank. As of the time of this writing, the performance of the Petrov et al. (2006) algorithm was the best of any known parsing algorithm on the Penn Treebank.

### 12.6 Probabilistic Lexicalized CFGs

The previous section showed that a simple probabilistic CKY algorithm for parsing raw PCFGs can achieve extremely high parsing accuracy if the grammar rule symbols are redesigned by automatic splits and merges.

In this section, we discuss an alternative family of models in which instead of modifying the grammar rules, we modify the probabilistic model of the parser to allow for lexicalized rules. The resulting family of lexicalized parsers includes the well-known Collins parser (Collins, 1999) and Charniak parser (Charniak, 1997), both of which are publicly available and widely used throughout natural language processing.

We saw in Section ?? that syntactic constituents could be associated with a lexical \textbf{head}, and we defined a \textbf{lexicalized grammar} in which each non-terminal in the tree is annotated with its lexical head, where a rule like \( VP \rightarrow VBD NP PP \) would be extended as

\[
VP(dumped) \rightarrow VBD(dumped) NP(sacks) PP(into) \tag{12.21}
\]

In the standard type of lexicalized grammar, we actually make a further extension, which is to associate the \textbf{head tag}, the part-of-speech tags of the headwords, with the non-terminal symbols as well. Each rule is thus lexicalized by both the

![Figure 12.9](image-url) An incorrect parse even with a parent-annotated parse (left). The correct parse (right), was produced by a grammar in which the pre-terminal nodes have been split, allowing the probabilistic grammar to capture the fact that \textit{if} prefers sentential complements. Adapted from Klein and Manning (2003b).
headword and the head tag of each constituent resulting in a format for lexicalized rules like

\[ VP(\text{dumped}, \text{VBD}) \rightarrow \text{VBD}(\text{dumped}, \text{VBD}) \text{ NP}(\text{sacks}, \text{NNS}) \text{ PP}(\text{into}, \text{P}) \]  \hspace{1cm} (12.22)

We show a lexicalized parse tree with head tags in Fig. 12.10, extended from Fig. ??.

![Lexicalized Parse Tree](image)

**Figure 12.10** A lexicalized tree, including head tags, for a WSJ sentence, adapted from Collins (1999). Below we show the PCFG rules that would be needed for this parse tree, internal rules on the left, and lexical rules on the right.

To generate such a lexicalized tree, each PCFG rule must be augmented to identify one right-hand constituent to be the head daughter. The headword for a node is then set to the headword of its head daughter, and the head tag to the part-of-speech tag of the headword. Recall that we gave in Fig. ?? a set of hand-written rules for identifying the heads of particular constituents.

A natural way to think of a lexicalized grammar is as a parent annotation, that is, a simple context-free grammar with many copies of each rule, one copy for each possible headword/head tag for each constituent. Thinking of a probabilistic lexicalized CFG in this way would lead to the set of simple PCFG rules shown below the tree in Fig. 12.10.

Note that Fig. 12.10 shows two kinds of rules: **lexical rules**, which express the expansion of a pre-terminal to a word, and **internal rules**, which express the other rule expansions. We need to distinguish these kinds of rules in a lexicalized grammar because they are associated with very different kinds of probabilities. The lexical rules are deterministic, that is, they have probability 1.0 since a lexicalized pre-terminal like \( \text{NN}(\text{bin}, \text{NN}) \) can only expand to the word \( \text{bin} \). But for the internal rules, we need to estimate probabilities.
Suppose we were to treat a probabilistic lexicalized CFG like a really big CFG that just happened to have lots of very complex non-terminals and estimate the probabilities for each rule from maximum likelihood estimates. Thus, according to Eq. 12.18, the MLE estimate for the probability for the rule \( P(VP(dumped, VBD) \rightarrow VBD(dumped, VBD) NP(sacks, NNS) PP(into, P)) \) would be

\[
\frac{\text{Count}(VP(dumped, VBD) \rightarrow VBD(dumped, VBD) NP(sacks, NNS) PP(into, P))}{\text{Count}(VP(dumped, VBD))} \tag{12.23}
\]

But there’s no way we can get good estimates of counts like those in (12.23) because they are so specific: we’re unlikely to see many (or even any) instances of a sentence with a verb phrase headed by \( dumped \) that has one \( NP \) argument headed by \( sacks \) and a \( PP \) argument headed by \( into \). In other words, counts of fully lexicalized PCFG rules like this will be far too sparse, and most rule probabilities will come out 0.

The idea of lexicalized parsing is to make some further independence assumptions to break down each rule so that we would estimate the probability

\[ P(VP(dumped, VBD) \rightarrow VBD(dumped, VBD) NP(sacks, NNS) PP(into, P)) \]

as the product of smaller independent probability estimates for which we could acquire reasonable counts. The next section summarizes one such method, the Collins parsing method.

### 12.6.1 The Collins Parser

Modern statistical parsers differ in exactly which independence assumptions they make. In this section we describe a simplified version of Collins’s worth knowing about; see the summary at the end of the chapter.

The first intuition of the Collins parser is to think of the right-hand side of every (internal) CFG rule as consisting of a head non-terminal, together with the non-terminals to the left of the head and the non-terminals to the right of the head. In the abstract, we think about these rules as follows:

\[
LHS \rightarrow L_n L_{n-1} \ldots L_1 H R_1 \ldots R_{n-1} R_n \tag{12.24}
\]

Since this is a lexicalized grammar, each of the symbols like \( L_1 \) or \( R_3 \) or \( H \) or \( LHS \) is actually a complex symbol representing the category and its head and head tag, like \( VP(dumped, VBD) \) or \( NP(sacks, NNS) \).

Now, instead of computing a single MLE probability for this rule, we are going to break down this rule via a neat generative story, a slight simplification of what is called Collins Model 1. This new generative story is that given the left-hand side, we first generate the head of the rule and then generate the dependents of the head, one by one, from the inside out. Each of these generation steps will have its own probability.

We also add a special STOP non-terminal at the left and right edges of the rule; this non-terminal allows the model to know when to stop generating dependents on a given side. We generate dependents on the left side of the head until we’ve generated STOP on the left side of the head, at which point we move to the right side of the head and start generating dependents there until we generate STOP. So it’s as if we
are generating a rule augmented as follows:

\[ P(\text{VP}(\text{dumped, VBD}) \rightarrow \text{STOP VBD(dumped, VBD)} \text{ NP(sacks,NNS)} \text{ PP(into,P)} \text{ STOP}) \]  

Let’s see the generative story for this augmented rule. We make use of three kinds of probabilities: \(P_H\) for generating heads, \(P_L\) for generating dependents on the left, and \(P_R\) for generating dependents on the right.

1. Generate the head VBD(dumped,VBD) with probability
   \[ P(H|\text{LHS}) = P(\text{VBD(dumped,VBD)} \mid \text{VP(dumped,VBD)}) \]

2. Generate the left dependent (which is STOP, since there isn’t one) with probability
   \[ P(\text{STOP} \mid \text{VP(dumped,VBD)}, \text{VBD(dumped,VBD)}) \]

3. Generate right dependent NP(sacks,NNS) with probability
   \[ P_R(\text{NP(sacks,NNS)} \mid \text{VP(dumped,VBD)}, \text{VBD(dumped,VBD)}) \]

4. Generate the right dependent PP(into,P) with probability
   \[ P_R(\text{PP(into,P)} \mid \text{VP(dumped,VBD)}, \text{VBD(dumped,VBD)}) \]

5) Generate the right dependent STOP with probability
   \[ P_R(\text{STOP} \mid \text{VP(dumped,VBD)}, \text{VBD(dumped,VBD)}) \]

In summary, the probability of this rule

\[ P(\text{VP(dumped,VBD}) \rightarrow \text{VBD(dumped, VBD)} \text{ NP(sacks,NNS)} \text{ PP(into,P)}) \]

is estimated as

\[ P_H(\text{VBD} \mid \text{VP, dumped}) \times P_L(\text{STOP} \mid \text{VP}, \text{VBD}, \text{dumped}) \times P_R(\text{NP(sacks,NNS)} \mid \text{VP}, \text{VBD}, \text{dumped}) \times P_R(\text{PP(into,P)} \mid \text{VP}, \text{VBD}, \text{dumped}) \times P_R(\text{STOP} \mid \text{VP}, \text{VBD}, \text{dumped}) \]  

Each of these probabilities can be estimated from much smaller amounts of data than the full probability in (12.26). For example, the maximum likelihood estimate
for the component probability $P_R(NP(sacks, NNS)|VP, VBD, dumped)$ is

$$\text{Count}(\ VP(dumped, VBD) \ with \ NNS(sacks)\ as \ a \ daughter \ somewhere \ on \ the \ right ) \over \text{Count}(\ VP(dumped, VBD) )$$

(12.28)

These counts are much less subject to sparsity problems than are complex counts like those in (12.26).

More generally, if $H$ is a head with head word $hw$ and head tag $ht$, $lw/lt$ and $rw/rt$ are the word/tag on the left and right respectively, and $P$ is the parent, then the probability of an entire rule can be expressed as follows:

1. Generate the head of the phrase $H(hw, ht)$ with probability:

$$P_H(H(hw, ht)|P, hw, ht)$$

2. Generate modifiers to the left of the head with total probability

$$\prod_{i=1}^{n+1} P_L(L_i(lw_i, lt_i)|P, H, hw, ht)$$

such that $L_{n+1}(lw_{n+1}, lt_{n+1}) = \text{STOP}$, and we stop generating once we’ve generated a STOP token.

3. Generate modifiers to the right of the head with total probability:

$$\prod_{i=1}^{n+1} P_R(R_i(rw_i, rt_i)|P, H, hw, ht)$$

such that $R_{n+1}(rw_{n+1}, rt_{n+1}) = \text{STOP}$, and we stop generating once we’ve generated a STOP token.

### 12.6.2 Advanced: Further Details of the Collins Parser

The actual Collins parser models are more complex (in a couple of ways) than the simple model presented in the previous section. Collins Model 1 includes a distance feature. Thus, instead of computing $P_L$ and $P_R$ as follows,

$$P_L(L_i(lw_i, lt_i)|P, H, hw, ht)$$

(12.29)

$$P_R(R_i(rw_i, rt_i)|P, H, hw, ht)$$

(12.30)

Collins Model 1 conditions also on a distance feature:

$$P_L(L_i(lw_i, lt_i)|P, H, hw, ht, dist_L(i-1))$$

(12.31)

$$P_R(R_i(rw_i, rt_i)|P, H, hw, ht, dist_R(i-1))$$

(12.32)

The distance measure is a function of the sequence of words below the previous modifiers (i.e., the words that are the yield of each modifier non-terminal we have already generated on the left).

The simplest version of this distance measure is just a tuple of two binary features based on the surface string below these previous dependencies: (1) Is the string of length zero? (i.e., were no previous words generated?) (2) Does the string contain a verb?
Collins Model 2 adds more sophisticated features, conditioning on subcategorization frames for each verb and distinguishing arguments from adjuncts.

Finally, smoothing is as important for statistical parsers as it was for N-gram models. This is particularly true for lexicalized parsers, since the lexicalized rules will otherwise condition on many lexical items that may never occur in training (even using the Collins or other methods of independence assumptions).

Consider the probability \( P_R(R_i(rw_i, rt_i)|P, hw, ht) \). What do we do if a particular right-hand constituent never occurs with this head? The Collins model addresses this problem by interpolating three backed-off models: fully lexicalized (conditioning on the headword), backing off to just the head tag, and altogether unlexicalized.

\[
P_R(\ldots) = \lambda_1 e_1 + (1 - \lambda_1)(\lambda_2 e_2 + (1 - \lambda_2)e_3)
\]

The values of \( \lambda_1 \) and \( \lambda_2 \) are set to implement Witten-Bell discounting (Witten and Bell, 1991) following Bikel et al. (1997).

The Collins model deals with unknown words by replacing any unknown word in the test set, and any word occurring less than six times in the training set, with a special \texttt{UNKNOWN} word token. Unknown words in the test set are assigned a part-of-speech tag in a preprocessing step by the Ratnaparkhi (1996) tagger; all other words are tagged as part of the parsing process.

The parsing algorithm for the Collins model is an extension of probabilistic CKY; see Collins (2003). Extending the CKY algorithm to handle basic lexicalized probabilities is left as Exercises 14.5 and 14.6 for the reader.

### 12.7 Probabilistic CCG Parsing

Lexicalized grammar frameworks such as CCG pose problems for which the phrase-based methods we’ve been discussing are not particularly well-suited. To quickly review, CCG consists of three major parts: a set of categories, a lexicon that associates words with categories, and a set of rules that govern how categories combine in context. Categories can be either atomic elements, such as \( S \) and \( NP \), or functions such as \( (S\backslash NP)/NP \) which specifies the transitive verb category. Rules specify how functions, their arguments, and other functions combine. For example, the following rule templates, **forward** and **backward function application**, specify the way that functions apply to their arguments.

\[
X/Y \ Y \Rightarrow X
\]

\[
Y \ X\backslash Y \Rightarrow X
\]

The first rule applies a function to its argument on the right, while the second looks to the left for its argument. The result of applying either of these rules is the
category specified as the value of the function being applied. For the purposes of
this discussion, we’ll rely on these two rules along with the **forward and backward**
**composition rules** and **type-raising**, as described in Chapter 10.

### 12.7.1 Ambiguity in CCG

As is always the case in parsing, managing ambiguity is the key to successful CCG
parsing. The difficulties with CCG parsing arise from the ambiguity caused by the
large number of complex lexical categories combined with the very general nature of
the grammatical rules. To see some of the ways that ambiguity arises in a categorial
framework, consider the following example.

(12.34) United diverted the flight to Reno.

Our grasp of the role of the *flight* in this example depends on whether the prepo-
sitional phrase *to Reno* is taken as a modifier of the *flight*, as a modifier of the entire
verb phrase, or as a potential second argument to the verb *divert*. In a context-free
grammar approach, this ambiguity would manifest itself as a choice among the fol-
lowing rules in the grammar.

\[
\begin{align*}
\text{Nominal} & \rightarrow \text{Nominal PP} \\
\text{VP} & \rightarrow \text{VP PP} \\
\text{VP} & \rightarrow \text{Verb NP PP}
\end{align*}
\]

In a phrase-structure approach we would simply assign the word *to* to the cate-
gory *P* allowing it to combine with *Reno* to form a prepositional phrase. The sub-
sequent choice of grammar rules would then dictate the ultimate derivation. In the
categorial approach, we can associate *to* with distinct categories to reflect the ways
in which it might interact with other elements in a sentence. The fairly abstract
combinatoric rules would then sort out which derivations are possible. Therefore,
the source of ambiguity arises not from the grammar but rather from the lexicon.

Let’s see how this works by considering several possible derivations for this
example. To capture the case where the prepositional phrase *to Reno* modifies the *flight*,
we assign the preposition *to* the category \((NP\setminus NP)/NP\), which gives rise to
the following derivation.

<table>
<thead>
<tr>
<th>United</th>
<th>diverted</th>
<th>the</th>
<th>flight</th>
<th>to</th>
<th>Reno</th>
</tr>
</thead>
<tbody>
<tr>
<td>NP</td>
<td>(S\setminus NP)/NP</td>
<td>NP/N</td>
<td>N</td>
<td>(NP\setminus NP)/NP</td>
<td>NP</td>
</tr>
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</tbody>
</table>

Here, the category assigned to *to* expects to find two arguments: one to the right as
with a traditional preposition, and one to the left that corresponds to the *NP* to be
modified.

Alternatively, we could assign *to* to the category \((S\setminus S)/NP\), which permits the
following derivation where to Reno modifies the preceding verb phrase.
A third possibility is to view *divert* as a ditransitive verb by assigning it to the category \((S\backslash NP)/PP)/NP\), while treating *to Reno* as a simple prepositional phrase.

While CCG parsers are still subject to ambiguity arising from the choice of grammar rules, including the kind of spurious ambiguity discussed in Chapter 10, it should be clear that the choice of lexical categories is the primary problem to be addressed in CCG parsing.

### 12.7.2 CCG Parsing Frameworks

Since the rules in combinatory grammars are either binary or unary, a bottom-up, tabular approach based on the CKY algorithm should be directly applicable to CCG parsing. Recall from Fig. 12.3 that PCKY employs a table that records the location, category and probability of all valid constituents discovered in the input. Given an appropriate probability model for CCG derivations, the same kind of approach can work for CCG parsing.

Unfortunately, the large number of lexical categories available for each word, combined with the promiscuity of CCG’s combinatoric rules, leads to an explosion in the number of (mostly useless) constituents added to the parsing table. The key to managing this explosion of zombie constituents is to accurately assess and exploit the most likely lexical categories possible for each word — a process called supertagging.

The following sections describe two approaches to CCG parsing that make use of supertags. Section 12.7.4, presents an approach that structures the parsing process as a heuristic search through the use of the A* algorithm. The following section then briefly describes a more traditional maximum entropy approach that manages the search space complexity through the use of *adaptive supertagging* — a process that iteratively considers more and more tags until a parse is found.

### 12.7.3 Supertagging

Chapter 8 introduced the task of part-of-speech tagging, the process of assigning the correct lexical category to each word in a sentence. Supertagging is the corresponding task for highly lexicalized grammar frameworks, where the assigned tags often dictate much of the derivation for a sentence.
CCG supertaggers rely on treebanks such as CCGbank to provide both the overall set of lexical categories as well as the allowable category assignments for each word in the lexicon. CCGbank includes over 1000 lexical categories, however, in practice, most supertaggers limit their tagsets to those tags that occur at least 10 times in the training corpus. This results in an overall total of around 425 lexical categories available for use in the lexicon. Note that even this smaller number is large in contrast to the 45 POS types used by the Penn Treebank tagset.

As with traditional part-of-speech tagging, the standard approach to building a CCG supertagger is to use supervised machine learning to build a sequence classifier using labeled training data. A common approach is to use the maximum entropy Markov model (MEMM), as described in Chapter 8, to find the most likely sequence of tags given a sentence. The features in such a model consist of the current word \( w_i \), its surrounding words within \( l \) words \( w_{i-l}^{i+l} \), as well as the \( k \) previously assigned supertags \( t_{i-k}^{i-1} \). This type of model is summarized in the following equation from Chapter 8.

\[
\hat{T} = \arg\max_T P(T|W) = \arg\max_T \prod_i P(t_i|w_{i-l}^{i+l}, t_{i-k}^{i-1}) = \arg\max_T \prod_i \frac{\exp \left( \sum_i w_if_i(t_i, w_{i-l}^{i+l}, t_{i-k}^{i-1}) \right)}{\sum_{t' \in \text{tagset}} \exp \left( \sum_i w_if_i(t', w_{i-l}^{i+l}, t_{i-k}^{i-1}) \right)}
\]

Word and tag-based features with \( k \) and \( l \) both set to 2 provides reasonable results given sufficient training data. Additional features such as POS tags and short character suffixes are also commonly used to improve performance.

Unfortunately, even with additional features the large number of possible supertags combined with high per-word ambiguity leads to error rates that are too high for practical use in a parser. More specifically, the single best tag sequence \( \hat{T} \) will typically contain too many incorrect tags for effective parsing to take place.

To overcome this, we can instead return a probability distribution over the possible supertags for each word in the input. The following table illustrates an example distribution for a simple example sentence. In this table, each column represents the probability of each supertag for a given word in the context of the input sentence. The “...” represent all the remaining supertags possible for each word.

<table>
<thead>
<tr>
<th>United</th>
<th>serves</th>
<th>Denver</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/N: 0.4</td>
<td>(S\NP)/NP: 0.8</td>
<td>NP: 0.9</td>
</tr>
<tr>
<td>NP: 0.3</td>
<td>N: 0.1</td>
<td>N/N: 0.05</td>
</tr>
<tr>
<td>S/S: 0.1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>S\S: 0.05</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

In a MEMM framework, the probability of the optimal tag sequence defined in Eq. 12.35 is efficiently computed with a suitably modified version of the Viterbi algorithm. However, since Viterbi only finds the single best tag sequence it doesn’t
provide exactly what we need here; we need to know the probability of each possible word/tag pair. The probability of any given tag for a word is the sum of the probabilities of all the supertag sequences that contain that tag at that location. A table representing these values can be computed efficiently by using a version of the forward-backward algorithm used for HMMs.

The same result can also be achieved through the use of deep learning approaches based on recurrent neural networks (RNNs). Recent efforts have demonstrated considerable success with RNNs as alternatives to HMM-based methods. These approaches differ from traditional classifier-based methods in the following ways:

- The use of vector-based word representations (embeddings) rather than word-based feature functions.
- Input representations that span the entire sentence, as opposed to size-limited sliding windows.
- Avoiding the use of high-level features, such as part of speech tags, since errors in tag assignment can propagate to errors in supertags.

As with the forward-backward algorithm, RNN-based methods can provide a probability distribution over the lexical categories for each word in the input.

### 12.7.4 CCG Parsing using the A* Algorithm

The A* algorithm is a heuristic search method that employs an agenda to find an optimal solution. Search states representing partial solutions are added to an agenda based on a cost function, with the least-cost option being selected for further exploration at each iteration. When a state representing a complete solution is first selected from the agenda, it is guaranteed to be optimal and the search terminates.

The A* cost function, \( f(n) \), is used to efficiently guide the search to a solution. The \( f \)-cost has two components: \( g(n) \), the exact cost of the partial solution represented by the state \( n \), and \( h(n) \) a heuristic approximation of the cost of a solution that makes use of \( n \). When \( h(n) \) satisfies the criteria of not overestimating the actual cost, A* will find an optimal solution. Not surprisingly, the closer the heuristic can get to the actual cost, the more effective A* is at finding a solution without having to explore a significant portion of the solution space.

When applied to parsing, search states correspond to edges representing completed constituents. As with the PCKY algorithm, edges specify a constituent’s start and end positions, its grammatical category, and its \( f \)-cost. Here, the \( g \) component represents the current cost of an edge and the \( h \) component represents an estimate of the cost to complete a derivation that makes use of that edge. The use of A* for phrase structure parsing originated with (Klein and Manning, 2003a), while the CCG approach presented here is based on (Lewis and Steedman, 2014).

Using information from a supertagger, an agenda and a parse table are initialized with states representing all the possible lexical categories for each word in the input, along with their \( f \)-costs. The main loop removes the lowest cost edge from the agenda and tests to see if it is a complete derivation. If it reflects a complete derivation it is selected as the best solution and the loop terminates. Otherwise, new states based on the applicable CCG rules are generated, assigned costs, and entered into the agenda to await further processing. The loop continues until a complete derivation is discovered, or the agenda is exhausted, indicating a failed parse. The algorithm is given in Fig. 12.11.
Heuristic Functions

Before we can define a heuristic function for our A* search, we need to decide how to assess the quality of CCG derivations. For the generic PCFG model, we defined the probability of a tree as the product of the probability of the rules that made up the tree. Given CCG’s lexical nature, we’ll make the simplifying assumption that the probability of a CCG derivation is just the product of the probability of the supertags assigned to the words in the derivation, ignoring the rules used in the derivation. More formally, given a sentence $S$ and derivation $D$ that contains supertag sequence $T$, we have:

$$P(D, S) = P(T, S)$$

$$= P(T, S)$$

$$= \prod_{i=1}^{n} P(t_i|s_i)$$

To better fit with the traditional A* approach, we’d prefer to have states scored by a cost function where lower is better (i.e., we’re trying to minimize the cost of a derivation). To achieve this, we’ll use negative log probabilities to score derivations; this results in the following equation, which we’ll use to score completed CCG derivations.

$$P(D, S) = P(T, S)$$

$$= \sum_{i=1}^{n} - \log P(t_i|s_i)$$

Given this model, we can define our $f$-cost as follows. The $f$-cost of an edge is the sum of two components: $g(n)$, the cost of the span represented by the edge, and
12.7 Probabilistic CCG Parsing

$h(n)$, the estimate of the cost to complete a derivation containing that edge (these are often referred to as the **inside** and **outside costs**). We’ll define $g(n)$ for an edge using Equation 12.39. That is, it is just the sum of the costs of the supertags that comprise the span.

For $h(n)$, we need a score that approximates but **never overestimates** the actual cost of the final derivation. A simple heuristic that meets this requirement assumes that each of the words in the outside span will be assigned its *most probable supertag*. If these are the tags used in the final derivation, then its score will equal the heuristic. If any other tags are used in the final derivation the $f$-cost will be higher since the new tags must have higher costs, thus guaranteeing that we will not overestimate.

Putting this all together, we arrive at the following definition of a suitable $f$-cost for an edge.

\[
f(w_{i,j}, t_{i,j}) = g(w_{i,j}) + h(w_{i,j})
\]

\[
= \sum_{k=i}^{j} -\log P(t_k|w_k) + \sum_{k=1}^{i-1} \max_{t \in \text{tags}} (-\log P(t|w_k)) + \sum_{k=j+1}^{N} \max_{t \in \text{tags}} (-\log P(t|w_k))
\]

As an example, consider an edge representing the word *serves* with the supertag $N$ in the following example.

\[ \text{(12.41) United serves Denver.} \]

The $g$-cost for this edge is just the negative log probability of the tag, or $X$. The outside $h$-cost consists of the most optimistic supertag assignments for *United* and *Denver*. The resulting $f$-cost for this edge is therefore $x+y+z = 1.494$.

**An Example**

Fig. 12.12 shows the initial agenda and the progress of a complete parse for this example. After initializing the agenda and the parse table with information from the supertagger, it selects the best edge from the agenda — the entry for *United* with the tag $N/N$ and $f$-cost 0.591. This edge does not constitute a complete parse and is therefore used to generate new states by applying all the relevant grammar rules. In this case, applying forward application to *United: N/N* and *serves: N* results in the creation of the edge *United serves: N[0,2], 1.795* to the agenda.

Skipping ahead, at the third iteration an edge representing the complete derivation *United serves Denver, S[0,3], .716* is added to the agenda. However, the algorithm does not terminate at this point since the cost of this edge (.716) does not place it at the top of the agenda. Instead, the edge representing *Denver* with the category $NP$ is popped. This leads to the addition of another edge to the agenda (type-raising *Denver*). Only after this edge is popped and dealt with does the earlier state representing a complete derivation rise to the top of the agenda where it is popped, goal tested, and returned as a solution.

The effectiveness of the $A^*$ approach is reflected in the coloring of the states in Fig. 12.12 as well as the final parsing table. The edges shown in blue (including all the initial lexical category assignments not explicitly shown) reflect states in the search space that never made it to the top of the agenda and, therefore, never
contributed any edges to the final table. This is in contrast to the PCKY approach where the parser systematically fills the parse table with all possible constituents for all possible spans in the input, filling the table with myriad constituents that do not contribute to the final analysis.
12.8 Evaluating Parsers

The standard techniques for evaluating parsers and grammars are called the PARSEVAL measures; they were proposed by Black et al. (1991) and were based on the same ideas from signal-detection theory that we saw in earlier chapters. The intuition of the PARSEVAL metric is to measure how much the constituents in the hypothesis parse tree look like the constituents in a hand-labeled, gold-reference parse. PARSEVAL thus assumes we have a human-labeled “gold standard” parse tree for each sentence in the test set; we generally draw these gold-standard parses from a treebank like the Penn Treebank.

Given these gold-standard reference parses for a test set, a given constituent in a hypothesis parse $C_h$ of a sentence $s$ is labeled “correct” if there is a constituent in the reference parse $C_r$ with the same starting point, ending point, and non-terminal symbol.

We can then measure the precision and recall just as we did for chunking in the previous chapter.

\[
labeled\text{precision: } \frac{\# \text{ of correct constituents in hypothesis parse of } s}{\# \text{ of total constituents in hypothesis parse of } s}
\]

\[
labeled\text{recall: } \frac{\# \text{ of correct constituents in hypothesis parse of } s}{\# \text{ of correct constituents in reference parse of } s}
\]

As with other uses of precision and recall, instead of reporting them separately, we often report a single number, the $F$-measure (van Rijsbergen, 1975): The $F$-measure is defined as

\[
F_\beta = \frac{(\beta^2 + 1)PR}{\beta^2P + R}
\]

The $\beta$ parameter differentially weights the importance of recall and precision, based perhaps on the needs of an application. Values of $\beta > 1$ favor recall and values of $\beta < 1$ favor precision. When $\beta = 1$, precision and recall are equally balanced; this is sometimes called $F_{\beta=1}$ or just $F_1$:

\[
F_1 = \frac{2PR}{P + R}
\]

The $F$-measure derives from a weighted harmonic mean of precision and recall. Remember that the harmonic mean of a set of numbers is the reciprocal of the arithmetic mean of the reciprocals:

\[
\text{HarmonicMean}(a_1, a_2, a_3, a_4, \ldots, a_n) = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n}}
\]

and hence the $F$-measure is

\[
F = \frac{1}{\alpha \frac{1}{P} + (1 - \alpha) \frac{1}{R}} \quad \text{(or with } \beta^2 = \frac{1 - \alpha}{\alpha}) \quad F = \frac{(\beta^2 + 1)PR}{\beta^2P + R}
\]

We additionally use a new metric, crossing brackets, for each sentence $s$:

**cross-brackets:** the number of constituents for which the reference parse has a bracketing such as ((A B) C) but the hypothesis parse has a bracketing such as (A (B C)).
As of the time of this writing, the performance of modern parsers that are trained and tested on the *Wall Street Journal* treebank was somewhat higher than 90% recall, 90% precision, and about 1% cross-bracketed constituents per sentence.

For comparing parsers that use different grammars, the PARSEVAL metric includes a canonicalization algorithm for removing information likely to be grammar-specific (auxiliaries, pre-infinitival “to”, etc.) and for computing a simplified score (Black et al., 1991). The canonical implementation of the PARSEVAL metrics is called evalb (Sekine and Collins, 1997).

Nonetheless, phrasal constituents are not always an appropriate unit for parser evaluation. In lexically-oriented grammars, such as CCG and LFG, the ultimate goal is to extract the appropriate predicate-argument relations or grammatical dependencies, rather than a specific derivation. Such relations are also more directly relevant to further semantic processing. For these purposes, we can use alternative evaluation metrics based on measuring the precision and recall of labeled dependencies, where the labels indicate the grammatical relations (Lin 1995, Carroll et al. 1998, Collins et al. 1999).

Finally, you might wonder why we don’t evaluate parsers by measuring how many sentences are parsed correctly instead of measuring component accuracy in the form of constituents or dependencies. The reason we use components is that it gives us a more fine-grained metric. This is especially true for long sentences, where most parsers don’t get a perfect parse. If we just measured sentence accuracy, we wouldn’t be able to distinguish between a parse that got most of the parts wrong and one that just got one part wrong.

### 12.9 Human Parsing

Are the kinds of probabilistic parsing models we have been discussing also used by humans when they are parsing? The answer to this question lies in a field called human sentence processing. Recent studies suggest that there are at least two ways in which humans apply probabilistic parsing algorithms, although there is still disagreement on the details.

One family of studies has shown that when humans read, the predictability of a word seems to influence the reading time: more predictable words are read more quickly. One way of defining predictability is from simple bigram measures. For example, Scott and Shillcock (2003) used an eye-tracker to monitor the gaze of participants reading sentences. They constructed the sentences so that some would have a verb-noun pair with a high bigram probability (such as (12.45a)) and others a verb-noun pair with a low bigram probability (such as (12.45b)).

\[(12.45)\]  
\[a) \text{HIGH PROB: One way to avoid confusion is to make the changes during vacation}\]  
\[b) \text{LOW PROB: One way to avoid discovery is to make the changes during vacation}\]

They found that the higher the bigram predictability of a word, the shorter the time that participants looked at the word (the initial-fixation duration).

While this result provides evidence only for N-gram probabilities, more recent experiments have suggested that the probability of an upcoming word given the syntactic parse of the preceding sentence prefix also predicts word reading time (Hale 2001, Levy 2008).
The second family of studies has examined how humans disambiguate sentences that have multiple possible parses, suggesting that humans prefer whichever parse is more probable. These studies often rely on a specific class of temporarily ambiguous sentences called *garden-path* sentences. These sentences, first described by Bever (1970), are sentences that are cleverly constructed to have three properties that combine to make them very difficult for people to parse:

1. They are **temporarily ambiguous**: The sentence is unambiguous, but its initial portion is ambiguous.
2. One of the two or more parses in the initial portion is somehow preferable to the human parsing mechanism.
3. But the dispreferred parse is the correct one for the sentence.

The result of these three properties is that people are “led down the garden path” toward the incorrect parse and then are confused when they realize it’s the wrong one. Sometimes this confusion is quite conscious, as in Bever’s example (12.46); in fact, this sentence is so hard to parse that readers often need to be shown the correct structure. In the correct structure, *raced* is part of a reduced relative clause modifying *The horse*, and means “The horse [which was raced past the barn] fell”; this structure is also present in the sentence “Students taught by the Berlitz method do worse when they get to France”.

(12.46) The horse raced past the barn fell.

Other times, the confusion caused by a garden-path sentence is so subtle that it can only be measured by a slight increase in reading time. Thus, in (12.47) readers often misparse *the solution* as the direct object of *forgot* rather than as the subject of an embedded sentence. This misparse is subtle, and is only noticeable because experimental participants take longer to read the word *was* than in control sentences. This “mini garden path” effect at the word *was* suggests that subjects had chosen the direct object parse and had to reanalyze or rearrange their parse now that they realize they are in a sentential complement.

(12.47) The student forgot the solution was in the back of the book.
While many factors seem to play a role in these preferences for a particular (incorrect) parse, at least one factor seems to be syntactic probabilities, especially lexicalized (subcategory) probabilities. For example, the probability of the verb forgot taking a direct object ($VP \rightarrow V NP$) is higher than the probability of it taking a sentential complement ($VP \rightarrow V S$); this difference causes readers to expect a direct object after forgot and be surprised (longer reading times) when they encounter a sentential complement. By contrast, a verb which prefers a sentential complement (like hope) didn’t cause extra reading time at was. The garden path in (12.46) is at least partially caused by the low probability of the reduced relative clause construction.

12.10 Summary

This chapter has sketched the basics of probabilistic parsing, concentrating on probabilistic context-free grammars and probabilistic lexicalized context-free grammars.

- Probabilistic grammars assign a probability to a sentence or string of words while attempting to capture more sophisticated syntactic information than the $N$-gram grammars of Chapter 3.
- A probabilistic context-free grammar (PCFG) is a context-free grammar in which every rule is annotated with the probability of that rule being chosen. Each PCFG rule is treated as if it were conditionally independent; thus, the probability of a sentence is computed by multiplying the probabilities of each rule in the parse of the sentence.
- The probabilistic CKY (Cocke-Kasami-Younger) algorithm is a probabilistic version of the CKY parsing algorithm. There are also probabilistic versions of other parsers like the Earley algorithm.
- PCFG probabilities can be learned by counting in a parsed corpus or by parsing a corpus. The inside-outside algorithm is a way of dealing with the fact that the sentences being parsed are ambiguous.
- Raw PCFGs suffer from poor independence assumptions among rules and lack of sensitivity to lexical dependencies.
- One way to deal with this problem is to split and merge non-terminals (automatically or by hand).
• **Probabilistic lexicalized CFGs** are another solution to this problem in which the basic PCFG model is augmented with a **lexical head** for each rule. The probability of a rule can then be conditioned on the lexical head or nearby heads.

• Parsers for lexicalized PCFGs (like the Charniak and Collins parsers) are based on extensions to probabilistic CKY parsing.

• Parsers are evaluated with three metrics: **labeled recall**, **labeled precision**, and **cross-brackets**.

• Evidence from **garden-path sentences** and other on-line sentence-processing experiments suggest that the human parser uses some kinds of probabilistic information about grammar.

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**Bibliographical and Historical Notes**

Many of the formal properties of probabilistic context-free grammars were first worked out by Booth (1969) and Salomaa (1969). Baker (1979) proposed the inside-outside algorithm for unsupervised training of PCFG probabilities, and used a CKY-style parsing algorithm to compute inside probabilities. Jelinek and Lafferty (1991) extended the CKY algorithm to compute probabilities for prefixes. Stolcke (1995) drew on both of these algorithms in adapting the Earley algorithm to use with PCFGs.

A number of researchers starting in the early 1990s worked on adding lexical dependencies to PCFGs and on making PCFG rule probabilities more sensitive to surrounding syntactic structure. For example, Schabes et al. (1988) and Schabes (1990) presented early work on the use of heads. Many papers on the use of lexical dependencies were first presented at the DARPA Speech and Natural Language Workshop in June 1990. A paper by Hindle and Rooth (1990) applied lexical dependencies to the problem of attaching prepositional phrases; in the question session to a later paper, Ken Church suggested applying this method to full parsing (Marcus, 1990). Early work on such probabilistic CFG parsing augmented with probabilistic dependency information includes Magerman and Marcus (1991), Black et al. (1992), Bod (1993), and Jelinek et al. (1994), in addition to Collins (1996), Charniak (1997), and Collins (1999) discussed above. Other recent PCFG parsing models include Klein and Manning (2003a) and Petrov et al. (2006).

This early lexical probabilistic work led initially to work focused on solving specific parsing problems like preposition-phrase attachment by using methods including transformation-based learning (TBL) (Brill and Resnik, 1994), maximum entropy (Ratnaparkhi et al., 1994), memory-based Learning (Zavrel and Daelemans, 1997), log-linear models (Franz, 1997), decision trees that used semantic distance between heads (computed from WordNet) (Stetina and Nagao, 1997), and boosting (Abney et al., 1999).

Another direction extended the lexical probabilistic parsing work to build probabilistic formulations of grammars other than PCFGs, such as probabilistic TAG grammar (Resnik 1992, Schabes 1992), based on the TAG grammars discussed in Chapter 10, probabilistic LR parsing (Briscoe and Carroll, 1993), and probabilistic link grammar (Lafferty et al., 1992). An approach to probabilistic parsing called **supertagging** extends the part-of-speech tagging metaphor to parsing by using very complex tags that are, in fact, fragments of lexicalized parse trees (Bangalore and
Joshi 1999, Joshi and Srinivas 1994), based on the lexicalized TAG grammars of Schabes et al. (1988). For example, the noun purchase would have a different tag as the first noun in a noun compound (where it might be on the left of a small tree dominated by Nominal) than as the second noun (where it might be on the right).

Goodman (1997), Abney (1997), and Johnson et al. (1999) gave early discussions of probabilistic treatments of feature-based grammars. Other recent work on building statistical models of feature-based grammar formalisms like HPSG and LFG includes (Riezler et al. 2002, Kaplan et al. 2004), and Toutanova et al. (2005).

We mentioned earlier that discriminative approaches to parsing fall into the two broad categories of dynamic programming methods and discriminative reranking methods. Recall that discriminative reranking approaches require $N$-best parses. Parsers based on A* search can easily be modified to generate $N$-best lists just by continuing the search past the first-best parse (Roark, 2001). Dynamic programming algorithms like the ones described in this chapter can be modified by the elimination of the dynamic programming with heavy pruning (Collins 2000, Collins and Koo 2005, Bikel 2004), or through new algorithms (Jiménez and Marzal 2000, Charniak and Johnson 2005, Huang and Chiang 2005), some adapted from speech recognition algorithms such as those of Schwartz and Chow (1990) (see Section ??).

In dynamic programming methods, instead of outputting and then reranking an $N$-best list, the parses are represented compactly in a chart, and log-linear and other methods are applied for decoding directly from the chart. Such modern methods include (Johnson 2001, Clark and Curran 2004), and Taskar et al. (2004). Other reranking developments include changing the optimization criterion (Titov and Henderson, 2006).

Collins’ (1999) dissertation includes a very readable survey of the field and an introduction to his parser. Manning and Schütze (1999) extensively cover probabilistic parsing.

The field of grammar induction is closely related to statistical parsing, and a parser is often used as part of a grammar induction algorithm. One of the earliest statistical works in grammar induction was Horning (1969), who showed that PCFGs could be induced without negative evidence. Early modern probabilistic grammar work showed that simply using EM was insufficient (Lari and Young 1990, Carroll and Charniak 1992). Recent probabilistic work, such as Yuret (1998), Clark (2001), Klein and Manning (2002), and Klein and Manning (2004), are summarized in Klein (2005) and Adriaans and van Zaalen (2004). Work since that summary includes Smith and Eisner (2005), Haghighi and Klein (2006), and Smith and Eisner (2007).

**Exercises**

12.1 Implement the CKY algorithm.

12.2 Modify the algorithm for conversion to CNF from Chapter 11 to correctly handle rule probabilities. Make sure that the resulting CNF assigns the same total probability to each parse tree.

12.3 Recall that Exercise 13.3 asked you to update the CKY algorithm to handle unit productions directly rather than converting them to CNF. Extend this change to probabilistic CKY.

12.4 Fill out the rest of the probabilistic CKY chart in Fig. 12.4.
12.5 Sketch how the CKY algorithm would have to be augmented to handle lexicalized probabilities.

12.6 Implement your lexicalized extension of the CKY algorithm.

12.7 Implement the PARSEVAL metrics described in Section 12.8. Next, either use a treebank or create your own hand-checked parsed test set. Now use your CFG (or other) parser and grammar, parse the test set and compute labeled recall, labeled precision, and cross-brackets.


