ME 355: Compressible Flows, Spring 2016

Stanford University

Midterm Exam

Tuesday, May 10

Guidelines: Please turn in *neat* and *clean* exam solutions that give all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. Assume $\gamma = 1.4$ and $c_p = 1.0$ KJ/KgK for all problems.

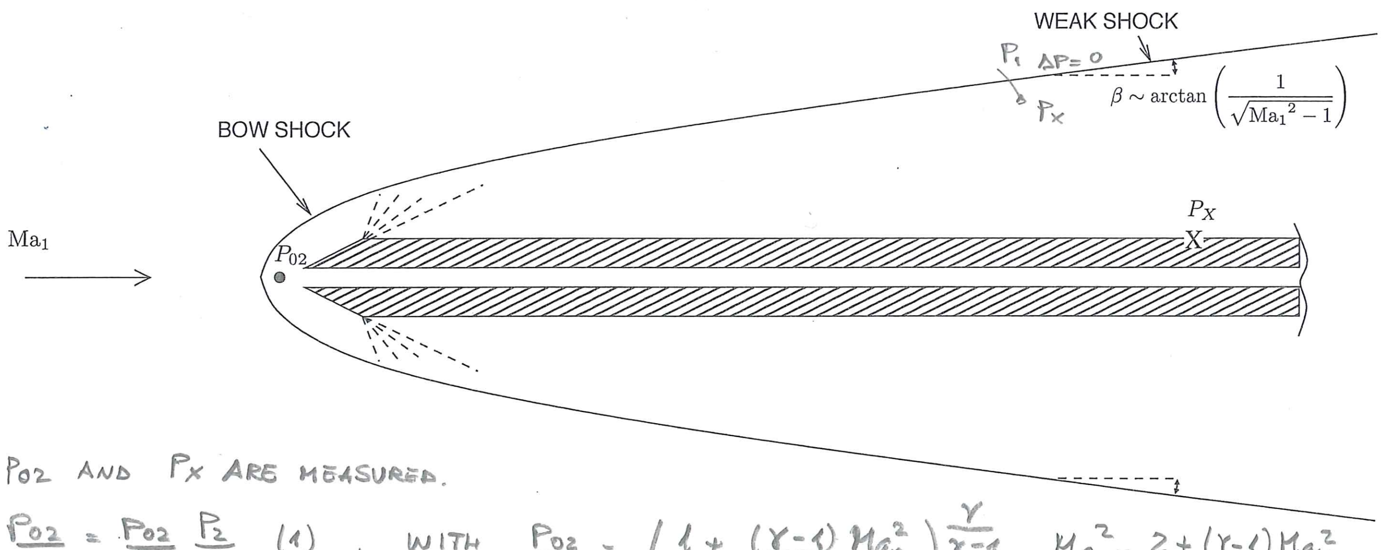
Student's Name:.......JAVIER URZAY.................. **Student's ID:**...........................

PART I: Closed books, closed notes, calculators allowed

Time: 20 mins

Questions (30 pts)

1. Explain what is supersonic wave drag and how it differs from viscous drag. ⇒ SEE PAGE 28 OF MY NOTES
2. The stagnation temperature upstream from a normal shock wave at $\mathrm{Ma}_1 = 1.5$ is $T_{01} = 335$ K. The static pressure downstream is $P_2 = 3$ bar. What is the stagnation enthalpy downstream h_{02}? ⇒ $h_0 = \text{CONST.}$ $h_{02} = c_p T_{02} = c_p T_{01} = 335$ KJ/Kg
3. A Machmeter for a supersonic aircraft consists of a very long central duct connected to a manometer that measures the post-shock stagnation pressure P_{02}, along with a lateral probe far away from the tube nose that measures the static pressure P_X at that position X (see fig. below). Based on those two quantities, how would you compute the Mach number of the free stream Ma_1?



P_{02} AND P_X ARE MEASURED.

$$\frac{P_{02}}{P_X} = \frac{P_{02}}{P_2}\frac{P_2}{P_X} \quad (1), \quad \text{WITH} \quad \frac{P_{02}}{P_2} = \left(1 + \left(\frac{\gamma-1}{2}\right) Ma_2^2\right)^{\frac{\gamma}{\gamma-1}} \quad (2), \quad Ma_2^2 = \frac{2+(\gamma-1)Ma_1^2}{2+\gamma Ma_1^2+1-\gamma} \quad (3)$$

SINCE THE SHOCK VERY FAR FROM THE TUBE NOSE IS WEAK: ΔP IS VERY SMALL ⇒ $P_X \sim P_1$

$$\Rightarrow \frac{P_2}{P_X} \sim \frac{P_2}{P_1} = 1 + \frac{2}{\gamma+1}\left(Ma_1^2 - 1\right) \quad (4)$$

WITH (1), (2), (3) AND (4) ⇒ $Ma_1 = f\left(\frac{P_{02}}{P_X}\right)$

$$\left\{ \frac{P_{02}}{P_X} = \frac{\left(\frac{\gamma+1}{2} Ma_1^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{2\gamma}{\gamma+1} Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}} \right\}$$

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PART II: Open books, open notes, calculators allowed
Time: 60 mins

Problem 1 (60 pts)

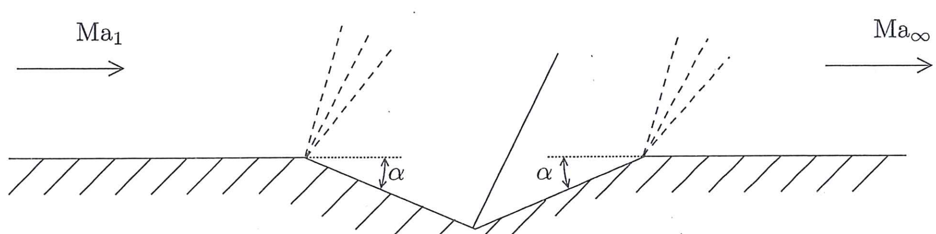
A supersonic stream at $\mathrm{Ma}_1 = 4.0$ flows parallel to a wall and encounters a geometrical disturbance of type A or B, as depicted below. Type A is an indentation and type B is a protrusion, both having the same deflection angle $\alpha = 20°$. Determine which configuration (type A or type B) yields

a) the maximum decrease in static pressure, $(P_1 - P_\infty)/P_1$

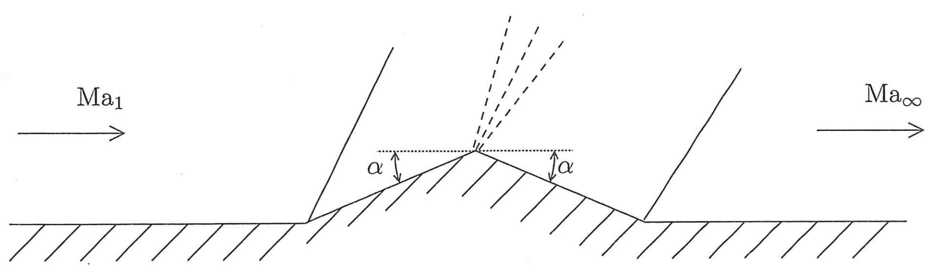
b) the maximum decrease in stagnation pressure, $(P_{01} - P_{0\infty})/P_{01}$

c) the maximum exit Mach number, Ma_∞

d) the minimum entropy increase, $(s_\infty - s_1)/c_v$

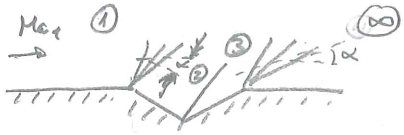


TYPE A



TYPE B

TYPE A :



• 1 → 2 : PRANDTL – MEYER EXPANSION. (TABLE III)

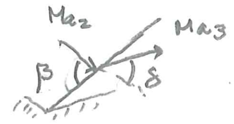
$$\Theta = \nu(Ma_2) - \underbrace{\nu(Ma_1)}_{65.78} = \alpha = 20^\circ$$

$$\Rightarrow \nu(Ma_2) = 85.78^\circ \Rightarrow \boxed{Ma_2 = 6.1}$$

ISENTROPIC FLOW: ($S_2 = S_1$) ($P_{02} = P_{01}$)

$$\frac{P_2}{P_1} = \left(\frac{1+\frac{(\gamma-1)}{2}Ma_1^2}{1+\frac{(\gamma-1)}{2}Ma_2^2}\right)^{\frac{\gamma}{\gamma-1}} = \boxed{0.087}$$

• 2 → 3



$\delta = 2\alpha = 40^\circ$, $Ma_2 = 6.1$ } OBLIQUE – SHOCK CHART $\beta \sim 57^\circ$

$$\Rightarrow Ma_{2n} = Ma_2 \sin\beta = 5.11,$$

TABLE II : $Ma_{3n} = 0.41 = Ma_3 \sin(\beta - \delta)$

$$\Rightarrow \boxed{Ma_3 = 1.4}$$

TABLE II : $\boxed{P_3/P_2 = 30.1}$, $\boxed{\rho_3/\rho_2 = 5.08}$, $\boxed{P_{03}/P_{02} = 0.057}$

ENTROPY VARIATION: $\frac{S_3 - S_2}{C_v} = \ln\left(\frac{P_3}{P_2}\left(\frac{\rho_2}{\rho_3}\right)^\gamma\right) = \boxed{1.15}$

• 3 → ∞, PRANDTL – MEYER EXPANSION WAVE (TABLE III)

$$\Theta = \nu(Ma_\infty) - \underbrace{\nu(Ma_3)}_{8.9^\circ} = 20^\circ \Rightarrow \nu(Ma_\infty) = 28.9^\circ$$

$$\Rightarrow \boxed{Ma_\infty = 2.1}$$

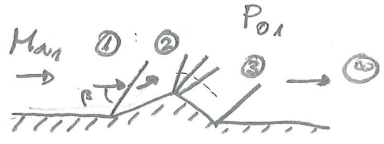
ISENTROPIC FLOW ($S_3 = S_\infty$) ($P_{03} = P_{0\infty}$)

$$\frac{P_\infty}{P_3} = \left(\frac{1+\frac{(\gamma-1)}{2}Ma_3^2}{1+\frac{(\gamma-1)}{2}Ma_\infty^2}\right)^{\frac{\gamma}{\gamma-1}} = \boxed{0.35}$$

$$\Rightarrow \frac{P_1 - P_\infty}{P_1} = 1 - \frac{P_\infty}{P_1} = 1 - \frac{P_\infty}{P_3}\frac{P_3}{P_2}\frac{P_2}{P_1} = \boxed{0.083}$$

$$\frac{P_{01} - P_{0\infty}}{P_{01}} = 1 - \frac{P_{0\infty}}{P_{01}} = 1 - \cancelto{1}{\frac{P_{0\infty}}{P_{03}}}\frac{P_{03}}{P_{02}}\cancelto{1}{\frac{P_{02}}{P_{01}}} = \boxed{0.94}$$

TYPE B :



• 1 → 2 OBLIQUE SHOCK CHART

$\delta = \alpha = 20^\circ$, $Ma_1 = 4.0$ } $\beta = 32^\circ$

$$Ma_{1n} = Ma_1 \sin\beta = 2.11$$

TABLE II : $Ma_{2n} = 0.56$, $\boxed{\frac{P_2}{P_1} = 4.97}$, $\boxed{\frac{\rho_2}{\rho_1} = 2.81}$

$$\Rightarrow \frac{S_2 - S_1}{C_v} = \ln\left(\frac{P_2}{P_1}\left(\frac{\rho_1}{\rho_2}\right)^\gamma\right) = \boxed{0.15}$$

$$Ma_2 = \frac{Ma_{2n}}{\sin(\beta - \delta)} = \boxed{2.69} \qquad \boxed{\frac{P_{02}}{P_{01}} = 0.67}$$

• 2 → 3 : PRANDTL – MEYER EXPANSION WAVE (TABLE III)

$$\Theta = \nu(Ma_3) - \underbrace{\nu(Ma_2)}_{43.6^\circ} = 2\alpha = 40^\circ \Rightarrow \nu(Ma_3) = 83.6^\circ \Rightarrow \boxed{Ma_3 = 5.8}$$

ISENTROPIC FLOW $\Rightarrow$ $\dfrac{P_3}{P_2} = \left(\dfrac{1 + \frac{(\gamma-1)}{2} Ma_2^2}{1 + \frac{(\gamma-1)}{2} Ma_3^2} \right)^{\frac{\gamma}{\gamma-1}} = \boxed{0.017}$

$\begin{pmatrix} P_{O2} = P_{O3} \\ S_2 = S_3 \end{pmatrix}$

- $3 \rightarrow \infty$ OBLIQUE SHOCK

Ma_3

$\left.\begin{matrix} Ma_3 = 5.8 \\ \delta = \alpha = 20^\circ \end{matrix}\right\}$ CHART: $\beta \sim 28^\circ$.

$Ma_{3n} = Ma_3 \sin\beta = 2.72$

FROM TABLE II:

$\dfrac{P_\infty}{P_3} = 8.33$, $\dfrac{\rho_\infty}{\rho_3} = 3.60$,

$Ma_{\infty n} = 0.49$, $\dfrac{P_{O\infty}}{P_O} = 0.42$

$$\Rightarrow Ma_\infty = \frac{Ma_{\infty n}}{\sin(\beta - \delta)} = \boxed{3.52}$$

$$\frac{S_\infty - S_3}{c_v} = \ln\left(\frac{P_\infty}{P_3} \frac{\rho_3^\gamma}{\rho_\infty^\gamma} \right) = \boxed{0.32}$$

THEN:

$$\frac{P_1 - P_\infty}{P_1} = 1 - \frac{P_\infty}{P_3}\frac{P_3}{P_2}\frac{P_2}{P_1} = \boxed{0.29}$$

$$\frac{P_{O1} - P_{O\infty}}{P_{O1}} = 1 - \frac{P_{O\infty}}{P_{O3}} \overset{1}{\cancel{\frac{P_{O3}}{P_{O2}}}} \frac{P_{O2}}{P_{O1}} = \boxed{0.71}$$

$$\frac{S_\infty - S_1}{c_v} = \frac{S_\infty - S_3}{c_v} + \frac{S_2 - S_1}{c_v} = \boxed{0.47}$$

ANSWERS:

a) MAX. DECREASE IN STATIC PRESSURE: TYPE B

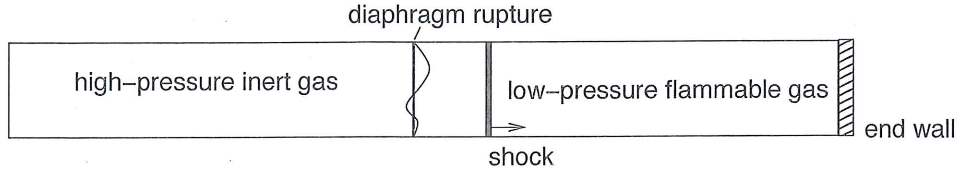
b) MAX. DECREASE IN STAGNATION PRESSURE: TYPE A

c) MAX. MACH NUMBER: TYPE B

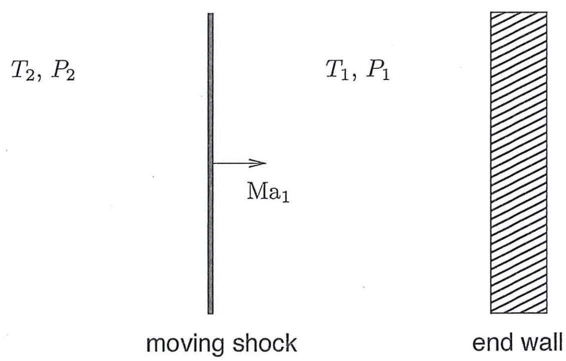
d) MIN ENTROPY INCREASE: TYPE B.

Problem 2 (10 pts)

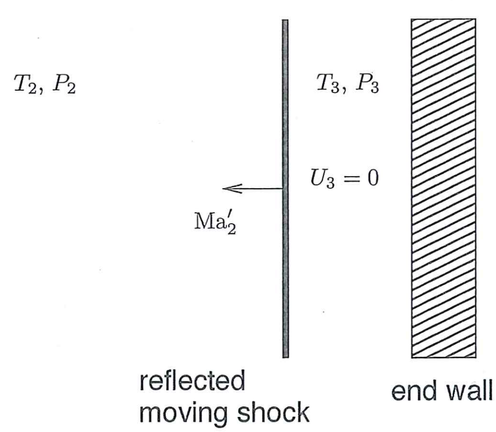
Shock tubes are typically used for studies of combustion chemical kinetics. In a shock tube, a diaphragm separates high pressure inert gas from a lower-pressure, flammable gas mixture at temperature $T_1 = 300\text{K}$ and pressure $P_1 = 0.1$ bar. At $t = 0$, the diaphragm is ruptured and the over-pressure created by the inert gas produces a shock wave that propagates at $\text{Ma}_1 = 2$ into the reacting gaseous mixture at rest. The reflection of the shock wave plays an important role in the ignition of the reactants. In the first approximation, however, neglect the combustion chemical processes in the shock tube.



a) Compute the pressure P_2 and temperature T_2 of the flammable gas after the shock has passed (see fig. below).



b) When the moving shock reaches the wall, it is reflected back towards the left leaving the gas at rest behind, $U_3 = 0$, in accordance with the non-penetration boundary condition at the wall. Calculate the Mach number of the reflected shock Ma_2', along with the pressure P_3 and temperature T_3 of the gas between the reflected shock and the end wall.



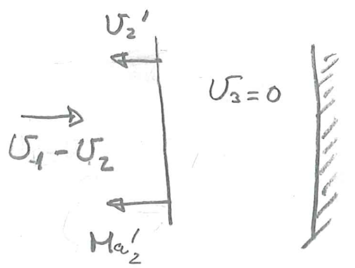
a) LAB FRAME WAVE FRAME

SINCE $Ma_1 = 2$, FROM TABLE II NORMAL SHOCK RELATIONS:

$$\frac{P_2}{P_1} = 4.5 \,, \quad \frac{T_2}{T_1} = 1.7 \,, \quad \underline{\underline{Ma_2 = 0.57}}.$$

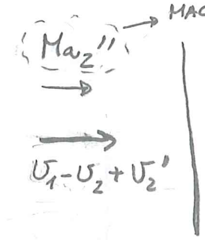
$$\Rightarrow \boxed{P_2 = 0.45 \text{ bar}} \quad \boxed{T_2 = 510\text{ K}}$$

b) LAB FRAME



WAVE FRAME

MACH NORMAL SHOCK FOR JUMP RELATIONS



WITH : $Ma_2'' = \dfrac{U_1 - U_2 + U_2'}{a_2} =$

$$= Ma_1 \left(\frac{T_1}{T_2}\right)^{1/2} - Ma_2 + Ma_2'$$

THEREFORE :

$$Ma_2'' = Ma_1 \left(\frac{T_1}{T_2}\right)^{1/2} - Ma_2 + Ma_2' \quad (1) \qquad \text{(REFLECTION + } U_3 = 0\text{)}$$

AND FROM THE NORMAL-SHOCK RELATIONS:

$$Ma_3^2 = \frac{2 + (\gamma - 1) Ma_2''^2}{2\gamma Ma_2''^2 + 1 - \gamma} \quad (2)$$

(POST-SHOCK MACH) $Ma_3 = U_2'/a_3$

WITH $Ma_3 = \dfrac{U_2'}{a_3} = \left(\dfrac{T_2}{T_3}\right)^{1/2} \cdot Ma_2' \quad (3)$

AND
$$\frac{T_3}{T_2} = \frac{\left[2\gamma Ma_2''^2 - (\gamma - 1)\right]\left[2 + (\gamma - 1) Ma_2''^2\right]}{(\gamma + 1)^2 Ma_2''^2} \quad (4)$$

THE PROBLEM IS CLOSED BY SOLVING (1) - (4) WITH $Ma_1 = 2$, $\frac{T_2}{T_1} = 1.7$, AND $Ma_2 = 0.57$ FROM PART a)

TABLE II OR EQS (2) AND (4) EQ (3)

ITERATION: $Ma_2' = 1.8$, $Ma_2'' = 2.76$, $Ma_3 = 0.49$, $\frac{T_3}{T_2} = 2.4 \Rightarrow Ma_2' = 0.76$ ⊗

$Ma_2' = 0.8$ $Ma_2'' = 1.80$ $Ma_3 = 0.61$, $\frac{T_3}{T_2} = 1.23 \Rightarrow Ma_2' = 0.67$ ⊗

$Ma_2' = 0.74$ $Ma_2'' = \underline{\underline{1.70}}$ $Ma_3 = \underline{\underline{0.64}}$ $\frac{T_3}{T_2} = \underline{\underline{1.46}} \Rightarrow Ma_2' = 0.77$ ✓

$\Rightarrow T_3 = 1.46 \cdot 510 = \underline{\underline{744.6\text{ K}}}$, $\quad P_3 = 3.2 \cdot 0.45 = \underline{\underline{1.44\text{ bar}}}$