Guidelines: Please turn in a neat and clean homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. In the calculations, assume a calorically perfect gas unless stated otherwise.

Questions (40 pts)

1. Consider a normal shock in a Mach-10 wind tunnel operating with air at temperature $T_1 = 80$ K and pressure $P_1 = 1$ bar in the test section. Calculate the stagnation enthalpy $h_0$, the ratio of the post-shock temperature $T_2$ to the stagnation temperature $T_0$, and the ratio of the post-shock pressure $P_2$ to twice the upstream dynamic pressure $\rho_1 U_1^2$. Compare the value of $h_0$ to the value of typical stagnation enthalpies involved in the re-entry of the Shuttle Orbiter from LEO.

2. Sketch the hypersonic flow over a slender two-dimensional wedge of semi-angle $\delta = 35^\circ$ at a free-stream Mach number $Ma_1 = 8$ in the stratosphere. Briefly describe the main characteristics of the flow and outline possible thermochemical effects that might develop in this case as a result of the high temperatures prevailing in the shock layer.
Problem 1 (60 pts)

Reusable first-stage boosters have recently become relevant assets for decreasing the cost and increasing the operational lifetime of space launch systems. Panel (a) in the figure below provides an approximate velocity-altitude trajectory of the reusable first stage of the Falcon-9 rocket that can be described as follows. The trajectory begins with an ascent of the first and second stages up to the separation point at an approximate altitude of 90 km, where the first stage is jettisoned and the second stage continues in an ascending trajectory to orbit. The first stage is then reoriented for re-entry using cold-gas thrusters, and its main rocket engines perform a boostback burn to decrease the speed. The boostback burn is the first of the three retrofirings performed during the landing sequence. A supplementary reorientation maneuver is then commanded to the cold-gas thrusters while grid fins are deployed for aerodynamic guidance [see panel (b)]. Grid fins are control surfaces used for steering and stabilizing the first stage, and they have been traditionally used in high-speed aerodynamics as stabilizers for control of thermobaric weapons such as the GBU-43/B [see panel (c)]. At 4,500 km/h and 55 km of altitude, an entry burn decreases the speed by half during 20 km, whereas the landing burn at approximately 10 km altitude brings down the first stage safely to the ground.
The remainder of this exercise will be focused on point “N” of the descent trajectory in panel 1(a), which corresponds approximately to an altitude of $z = 60$ km above sea level while descending at a velocity $U_\infty = 4,400$ km/h with a negligible angle of attack. Assume that the length and width of the first stage are $L = 45$ m and $D = 3.7$ m, respectively, and that the grid fins are located at $L_f = 3$ m from the top of the first stage.

a) Calculate the Mach number $Ma_\infty$ corresponding to the relative motion of the air around the first stage at the aforementioned conditions of velocity and altitude.

b) For simplicity, assume that the bottom side of the first stage resembles a planar wedge of semiangle $\alpha = 30^\circ$ and depth $D$, as shown in panel (d). Calculate the shock-incidence angle $\beta$, along with the Mach number $Ma_2$ and the pressure $P_2$ downstream of the oblique shock emanating from the geometry.

c) Calculate the turning angle through the expansion fans emanating from the junction between the wedge and the fairing, along with the Mach number $Ma_3$ and pressure $P_3$ downstream.

d) The high-speed flow through the grid fins is complex and beyond the scope of this exercise. For further details, the reader is referred to the NATO report RTO-MP-AVT-135 (2006) available for download at this repository. For simplicity, assume that the first stage deploys two grid fins of side length $L_g = 1.5$ m normal to the flow, and their leading edges resemble planar wedges of width $w = 5$ cm and semiangle $\theta/2 = 20^\circ$, which create oblique shocks as sketched in panel (e). Calculate the post-shock pressure $P_4$ acting on the leading edges of the grid fins.

e) Using the results obtained from parts (a), (b), (c), and (d), determine the maximum height $L_{cg}$ of the center of gravity such that the first stage remains aerodynamically stable to yaw rotation during flight at point “N” of the velocity-altitude trajectory.

f) Redo part (e) using the standard Newtonian theory and compare the results. In doing so, assume that the incident Mach number upstream of the grid fins corresponds to the value of $Ma_3$ calculated in part (c).