ME 356: Hypersonic Aerothermodynamics, Spring 2020

Stanford University

Homework 1: Inviscid Hypersonic Flows (I)

Due Friday, April 24.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. In the calculations, assume a calorically perfect gas unless stated otherwise.

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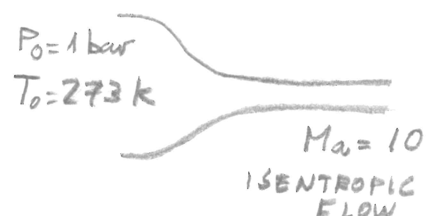
Questions (40 pts)

1. Sketch the inviscid hypersonic flow over a slender two-dimensional wedge at a free-stream Mach number $Ma_\infty = 15$ in the stratosphere. Outline possible thermochemical effects that might develop in this case as a result of the high temperatures prevailing in the shock layer.

   SEE FIGS. 5 AND 9. IN THE CLASS NOTES.

2. Consider a Mach-10 blowdown wind tunnel ingesting atmospheric air at standard conditions. Calculate the velocity, static temperature, and static pressure in the test section, along with the flux of stagnation enthalpy there. Compare the value of the stagnation enthalpy in this wind tunnel with the stagnation enthalpy around the X-43A at maximum flight speed of Mach 10 at 40 km of altitude, and provide an explanation of the discrepancy.



$C_p = 1$ kJ/kg K

$R_g = 286$ J/kg K

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right) Ma^2$$

$T \approx 13$ K (UNREALISTIC) — A HEATER WOULD BE REQUIRED AT THE ENTRANCE TO PREVENT LIQUEFACTION.

$$\frac{P_0}{P} = \left(1 + \left(\frac{\gamma-1}{2}\right) Ma^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P = 2.3 \cdot 10^{-5} \text{ bar}$$

(UNREALISTIC)

$$U = Ma\sqrt{\gamma R_g T} = 722 \text{ m/s}$$

→ RELATIVELY LOW SPEED BUT HIGH MACH BECAUSE OF LOW T

FLUX OF STAGNATION ENTHALPY: $\rho U h_0 = \frac{P}{R_g T} U C_p T_0 = 122$ kW/m^2

FOR THE X-43A AT 40 km ALTITUDE:

$\rho_\infty \simeq 4.0 \cdot 10^{-3}$ kg/m^3

$T_\infty \simeq 250$ K

} FROM US STANDARD ATMOSPHERE:

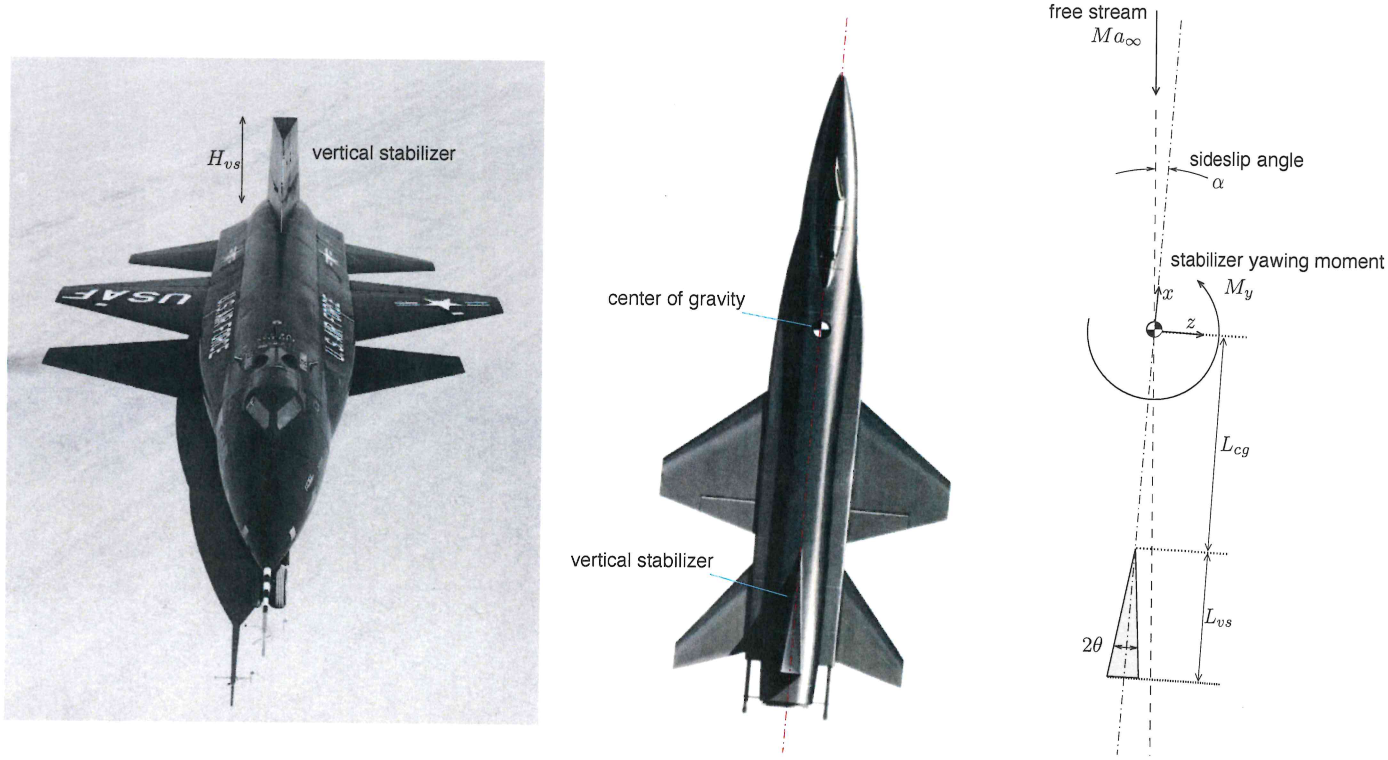
VELOCITY: $U_\infty = \sqrt{\gamma R_g T_\infty}\, Ma_\infty = 3164$ m/s

STAGNATION ENTHALPY: $h_{0\infty} = C_p T_\infty \left(1 + \left(\frac{\gamma-1}{2}\right) Ma_\infty^2\right) = 5.2$ MJ/kg

$\Rightarrow$ FLUX: $\rho_\infty U h_{0\infty} \approx 66$ MW/m^2 MUCH LARGER THAN THE COLD TUNNEL! → ALWAYS A PROBLEM....

Problem 1 (60 pts)

A vertical stabilizer is a rudder mounted on the aft end of the fuselage that provides directional stability to the aircraft when it sideslips with respect to the free stream. For instance, the X-15 hypersonic rocket plane used a vertical stabilizer that had the shape of a wedge of semi-angle θ. When the rocket plane sideslipped by an angle α, the aerodynamic forces on the vertical stabilizer induced a yawing moment M_y around the center of gravity that restored the lateral attitude. In the notation below, L_{cg} is the distance from the vertical stabilizer's edge to the center of gravity, whereas L_{vs} and H_{vs} are, respectively, the length and height of the vertical stabilizer.



Assuming that the free-stream Mach number is $Ma_\infty = 8$, and the sideslip angle is $\alpha = 12°$, and $L_{vs}/L_{cg} \ll 1$, calculate the yawing moment coefficient

$$C_y = \frac{2M_y}{\rho_\infty U_\infty^2 H_{vs} L_{vs} L_{cg}}$$

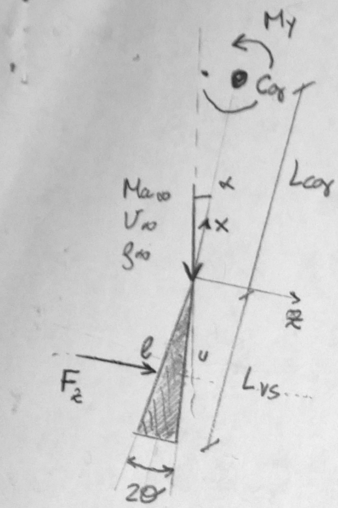
in the following cases:

a) $\theta = 20°$.

b) $\theta = 8°$.

c) Redo parts (a) and (b) using the Newtonian theory of hypersonics and compare the solutions.

SOLUTION



* SIDEWAYS (LIFT) FORCE ON THE VS:

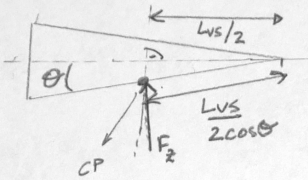
$$F_z = \frac{1}{2}\rho_\infty U_\infty^2 \left(C_{p_l} - C_{p_u}\right) \frac{L_{VS}}{\cancel{\cos\theta}} H_{VS} \cancel{\cos\theta}$$

WITH $C_{p_l} = \dfrac{p_l - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$ AND $C_{p_u} = \dfrac{p_u - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$

THE PRESSURE COEFFICIENTS ON THE LOWER AND UPPER SURFACES.

* YAWING MOMENT: $M_y = F_z \left(L_{cg} + \dfrac{L_{VS}}{2}\right) \approx F_z L_{cg}$ $L_{VS}/L_{cg} \ll 1$

LINE PASSING THROUGH THE CENTER OF PRESSURE

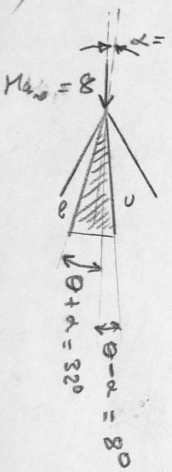


* YAWING MOMENT COEFFICIENT

$$C_y = \frac{M_y}{\frac{\rho_\infty U_\infty^2}{2} L_{VS} L_{cg} H_{VS}} = \frac{\cancel{1/2\,\rho_\infty U_\infty^2}\left(C_{p_l} - C_{p_u}\right)\cancel{L_{VS} H_{VS} L_{cg}}}{\cancel{1/2\,\rho_\infty U_\infty^2}\,\cancel{L_{VS} L_{cg} H_{VS}}}$$

$$= \left(C_{p_l} - C_{p_u}\right) \qquad (1)$$

a) $\theta = 20^\circ$



$Ma_\infty = 8$, $\delta_l = \theta + \alpha = 32^\circ$ } OBLIQUE SHOCK CHART: $\beta_l = 42^\circ$

$$\Rightarrow \frac{p_l}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}\left(Ma_\infty^2 \sin^2\beta_l - 1\right) = 33.2$$

$$C_{p_l} = \frac{2}{\gamma Ma_\infty^2}\left(\frac{p_l}{p_\infty} - 1\right) = \underline{\underline{0.72}}$$

$Ma_\infty = 8$, $\delta_u = \theta - \alpha = 8^\circ$ } OBLIQUE SHOCK CHART $\beta_u = 12^\circ$

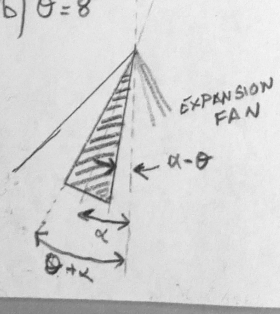
$$\Rightarrow \frac{p_u}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}\left(Ma_\infty^2 \sin^2\beta_u - 1\right) = 3.1$$

$$C_{p_u} = \frac{2}{\gamma Ma_\infty^2}\left(\frac{p_u}{p_\infty} - 1\right) = \underline{\underline{0.05}}$$

$\Rightarrow$ SUBSTITUTING C_{p_l} AND C_{p_u} INTO (1): $C_y = \underline{\underline{0.67}}$

FROM NEWTONIAN THEORY: $C_{p_l} = 2\sin^2\delta_l = 0.56$, $C_{p_u} = 2\sin^2\delta_u = 0.04$ } $C_y = \underline{\underline{0.52}}$

b) $\theta = 8^\circ$



$Ma_\infty = 8$, $\delta_l = \theta + \alpha = 20^\circ$ } OBLIQUE CHART: $\beta_l = 26.5^\circ$

$$\frac{p_l}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}\left(Ma_\infty^2 \sin^2\beta_l - 1\right) = 14.8$$

$$C_{p_l} = \frac{2}{\gamma Ma_\infty^2}\left(\frac{p_l}{p_\infty} - 1\right) = \underline{\underline{0.31}}$$

$\delta_U = \alpha - \theta = 4^\circ$
$Ma_\infty = 8$

FROM TABLES OF PRANDTL-MEYER EXPANSION FAN

$\nu(Ma_\infty) = 95.6^\circ$ FROM TABLE

$\Rightarrow \nu(Ma_U) = \nu(Ma_\infty) + \delta_U = 99.6^\circ \Rightarrow Ma_U \simeq 9.1$

THEREFORE $$\frac{P_U}{P_\infty} = \left(\frac{1+\left(\frac{\gamma-1}{2}\right)Ma_\infty^2}{1+\left(\frac{\gamma-1}{2}\right)Ma_U^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.43$$

$$\Rightarrow C_{P_U} = \frac{2}{\gamma Ma_\infty^2}\left(\frac{P_U}{P_\infty} - 1\right) = -4.8 \cdot 10^{-3}$$

SUBSTITUTING C_{PU} AND C_{PL} INTO (1): $C_Y = 0.31$

FROM THE NEWTONIAN THEORY:

$$\left.\begin{aligned} C_{P_L} &= 2\sin^2\delta_L = 0.23 \\ C_{PU} &= 0 \end{aligned}\right\} \quad C_Y \simeq 0.23$$