ME 356: Hypersonic Aerothermodynamics, Spring 2018
Stanford University
Homework 2: Hypersonic Inviscid Flows (II)
Due Thursday, May 3, in class.

Guidelines: Please turn in a neat and clean homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. In the calculations, assume a calorically perfect gas with $\gamma = 1.4$ unless stated otherwise.

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Questions (100 pts)

1. A supersonic air stream at $Ma_\infty = 3.0, T_\infty = 300$ K and $P = 1$ atm flows over a two-dimensional wedge of semi-angle $\delta = 20^\circ$. Calculate the angle of incidence $\beta$ of the resulting oblique shock and compare it against the one obtained if the wedge is replaced by a cone of semi-angle $\delta = 20^\circ$ (to answer this, you could compute the numerical solution of the conical flow or you could also research the class notes). Explain whether the flow is hypersonic and the reasons that justify your conclusion.

2. In the figure below, the surface pressure coefficient at point Q in the fuselage of projectile #1 is $C_p = 0.3$. Using van Dyke’s hypersonic similarity rule studied in class, compute the surface pressure coefficient at point Q in the fuselage of projectile #2. In the calculations, use the aspect ratio $c_1/\lambda = 0.2$ for projectile #1, with $c_2 = c_1/2$ for projectile #2.

3. Describe the character of the Mach-8 flow around a two-dimensional slender wedge of semi-angle $\delta$ as $\delta$ is increased from $2^\circ$ to $50^\circ$, indicating whether the flow is subsonic, supersonic, hypersonic, and whether it can be described using small-disturbance theories for the velocity perturbations.

4. Compare the hypersonic wave drag coefficients on a circular cylinder of radius $R$ at Mach $Ma_\infty = 7.0$ at 35 km altitude obtained using the straight Newtonian theory and the modified Newtonian theory, and provide an estimate of the pressure in the fore stagnation point in each case.

5. Explain the reasons why the straight Newtonian theory is more accurate in three-dimensional flows (i.e. conical flows) than in two-dimensional flows (i.e. wedge flows).
1. From the $\beta - 8$ chart $\Rightarrow \beta = 30^\circ$ for an oblique shock.

2. For a cone, from Fig. 24 in the class notes $\Rightarrow \beta = 30^\circ$.

In both cases, the flow is not hypersonic since $Ma_1 \sin^2 \beta$ is not a number much larger than unity.

3. PROJECTILE #1: $\beta_1 = 0.2$, $\c_1 = 0.2$, $Ma_1 = 6$, $\gamma = 1.4$

PROJECTILE #2: $\beta_2 = 2^\circ$, $\c_2 = 0.1$, $Ma_1 = 12$, $\gamma = 1.4$

$\Rightarrow \alpha = \c_1 Ma_1 = \c_2 Ma_2 = 1.2$, the similarity rule can be applied.

4. $\mu_m = \cos \sin (\gamma/\c) = 7.10$, $\beta = 2^\circ - 5^\circ$

Weak shocks (Mach waves)

$Ma_1 \sin \beta < (\sqrt[\gamma]{Ma_1})$

Linear supersonic flow

5. Straight Newtonian theory:

\[ c_4 = 2 \sin^2 \beta = 2 \cos^2 \theta \]

\[ c_4 = \frac{2}{\gamma - 1} \int_0^{\pi/2} c_1 \cos \theta \sin \theta \frac{R \theta}{M_\infty} \frac{d\theta}{R \sin \theta} \]

\[ = \frac{R}{M_\infty} \left( \frac{\gamma \frac{1}{2} \frac{1}{2} \frac{1}{2}}{3} \right) = \frac{1}{3} \int_0^{\pi/2} \frac{2 \cos^2 \theta}{2 \cos^2 \theta} \frac{R \theta}{M_\infty} \frac{d\theta}{R \sin \theta} \]

\[ = Z - \left( \frac{1+ tan^2 \theta}{3} \right) = 4/3 \cdot 1.33 \]

\[ P_2 = P_0 + \frac{\rho_0 U_0^2}{2} = 0.39 \text{ bar} \]

Modified Newtonian:

\[ c_4 = \frac{c_{4\text{max}} \sin^2 \beta}{2} \]

\[ = \frac{c_{4\text{max}} \sin^2 \beta}{2} \]

\[ P_0 = P_0 + \frac{3}{2} \frac{\rho_0 U_0^2}{2} \frac{c_{4\text{max}}}{2} = 0.36 \text{ bar} \]

Because the cone flow leads to a weaker shock that envelops more closely the body surface due to a 3D relieving effect.