Guidelines: Please turn in a neat and clean homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. In the calculations, assume a calorically perfect gas with $\gamma = 1.4$, $Pr = 0.72$, $R_g = 286$ J/kgK, and $c_p = 1$ kJ/kgK unless stated otherwise.

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Questions (100 pts)

1. (30pts) Explain under what conditions (a) the stagnation temperature is constant across a hypersonic laminar boundary layer, and (b) the Reynolds analogy factor is unity.

2. (30 pts) Calculate the critical free-stream Mach number at which a wedge of semi-angle $\delta = 10^\circ$, kept at 1400 K, and moving through air at 220 K, will start to be aerodynamically heated by viscous effects. Assume that the recovery factor is equal to the square root of the Prandtl number.

3. (40 pts) Download the Matlab code posted at

http://web.stanford.edu/~jurzay/ME356_files/HW3code.tar.gz

for integrating the problem of a compressible laminar boundary layer over a flat plate with zero streamwise gradients in the overriding free stream. In particular, the formulation numerically integrated by the code consists of Eqs. (225)-(226) in the class notes, with the viscosity following a temperature power law with exponent $\sigma = 0.7$. Assume that the edge conditions involve a Mach number $Ma_e = 6.0$. Use (and modify accordingly) this code to answer the following questions:

a) Determine the ratio of the adiabatic wall temperature $T_{aw}$ to the edge temperature $T_e$, along with the recovery factor $r$. State whether the approximation $r \approx \sqrt{Pr}$ is a good one for these conditions.

b) For an isothermal wall at $T_w/T_e = 3$, plot the dimensionless profiles of the static temperature $T/T_e$, stagnation temperature $T_0/T_e$, density $\rho/\rho_e$, and streamwise velocity $u/U_e$, all as a function of the wall-normal coordinate $y$ divided by the equivalent incompressible boundary-layer thickness $\delta_{B0} = x/\sqrt{Re_e,x}$, where $x$ is the streamwise coordinate and $Re_e,x$ is the edge Reynolds number based on $x$.

c) For the conditions described in b), calculate the products $C_f\sqrt{Re_e,x}$ and $St\sqrt{Re_e,x}$, along with the Reynolds analogy factor $2St/C_f$, where $C_f$ is the skin friction coefficient and $St$ is the Stanton number.

Attach a screenshot of your modified version of the main.m script with your solutions.
1) \( T_0 = \text{const when } Pr = 1, \left. \frac{dT}{dy} \right|_{y=3} = 0, \) and the gas is calorically perfect.

b) \( Ra = 1 \) when \( Pr = 1, T = T_w, \) \( dP/dx = 0 \) and

\[
\frac{T_w}{T_0} = 6.36
\]

2) The wedge will begin to be aerodynamically heated when \( T_w = T_{aw} = \) adiabatic wall temperature, from the definition of the recovery factor:

\[
Q = \frac{T_w - T_e}{T_0 - T_e} = \frac{T_w - T_0}{T_0 - T_e} = \sqrt{Pr}
\]

with
\[
\frac{T_{aw}}{T_e} = 1 + \left( \frac{y-1}{2} \right) M_{aw}^2
\]

and
\[
\frac{T_e}{T_0} = \left( \frac{2 + (y-1) M_{aw}^2}{(y+1)^2 M_{aw}^2 \sin^2 \beta} \right)^{\frac{1}{(y-1)}}
\]

where
\[
\tan \beta = \frac{2 \tan \alpha}{y \cos (2 \beta)}
\]

The system of equations (1)-(4) can be solved to obtain the values of \( \frac{T_{aw}}{T_e} \), \( T_{aw}/T_0 \), \( M_{aw} \) and \( \beta \): by iteration:

\[
M_{aw} \approx 5.6, \quad \beta \approx 18.2° \quad (M_{aw} = 1.75), \quad \frac{T_{aw}}{T_0} = 7.27, \quad \frac{T_e}{T_0} = 1.49
\]

3) a) \( \frac{T_{aw}}{T_0} = 7.04 \) and \( \beta = 0.84 \), with \( \sqrt{Pr} = 0.98 \) (good approximation)

b) see figures attached next page

c) \( \tilde{C_f} \sqrt{Re_{aw}} = 0.56 \)
\( Sl \sqrt{Re_{aw}} = 0.35 \)
\( Ra = 2St \sqrt{C_f} = 1.25 \)
% solve similarity equations for compressible flow
% Here Re is based on distance from cone vertex
clc;
clear
close all;
format long; close all; clc; clear all;

% code parameters
M = 6;  % Edge Mach number
flag_ad = 0;  % Flag for wall thermal boundary condition: 1=adiabatic wall; 0=isothermal wall;
Tw = 3;  % dimensionless wall temperature (not used in a
Pr = 0.72;  % Prandtl number
n = 0.7;  % power law coefficient for viscosity \mu/\mu_0=(T/T_0)^n
gamma = 1.4;  % adiabatic coefficient
etamax = 10;  % maximum \eta coordinate for integration

% solve boundary layer equations in similarity form:
[\eta, f]=solve_comp(M,flag_ad,Tw,Pr,gamma,n,etamax);

% get variables:
 u = f(:,2);  %dimensionless velocity
 T = f(:,4);  %dimensionless temperature
 rho = 1./T;  %dimensionless density

% plot the selfsimilar solution \f'(\eta), g(\eta)
figure(1);
plot(u, eta, 'r'); hold on;
plot(T, eta, 'b');
xlabel('\f (red) and g (blue)');
ylabel('\eta (self-similar variable)');

%dimensional wall normal distance
y = cumtrapz(eta, f(:,4))*sqrt(2);

%plot u,T and To
figure(2);
plot(u, y, 'r-'); hold on;
plot(1./T, y, 'k-'); hold on;
xlabel('u/U_e (red) and \rho/\rho_e (black)');
ylabel('y/\delta_{B0}');
set(gca, 'FontSize', 15);

figure(3);
plot(T, y, 'b-'); hold on;
plot(T+(gamma-1)*M^2/2*(u).^2, y, 'k-'); hold on;
xlabel('T/T_e (blue) and T_0/T_e (black)');
ylabel('y/\delta_{B0}');
set(gca, 'FontSize', 15);

%dimensional adiabatic temperature (if flag is 1)
if flag_ad==1;
    Taw=T(1);
end

%edge stagnation temperature
Toe=1+(gamma-1)*M^2/2;

%recovery factor
r=(Taw-1)/(Toe-1);

if flag_ad==0
    Taw=7.039349681637581;
end

%Skin friction
Cn=f(1,4)^(n-1);  %Chapman-Rubesin parameter at the wall (g(0)^n(n-1))
CfRe=sqrt(2)*Cn*f(1,3);
StRe=Cf/sqrt(2)*f(1,5)/(Taw-Tw)/Pr
RA2=2*StRe/CfRe

%Reynolds analogy factor
RA=f(1,5)/(Taw-Tw)/Pr/Pr