Guidelines: Please turn in neat and clean exam solutions that give all the formulae that you have used as well as details that are required for the grader to understand your solution. In the calculations, assume Pr = 0.7 and a calorically perfect gas with $\gamma = 1.4$, $R_g = 286$ J/kgK, and $c_p = 1$ kJ/kgK unless stated otherwise. Attach these sheets to your solutions.

Student’s Name: ...........................................  Student’s ID: .........................

PART I: Closed notes, calculators allowed, compressible-flow tables allowed
Time: 80 mins

Questions (50 pts)

1. (10 pts) Describe under what conditions may a flow be regarded as hypersonic. This is an open-ended question, and therefore both creativity and rigor will be graded positively.

2. (20 pts) Describe the structure of a normal shock wave in thermochemical equilibrium air in the stratosphere, including characteristic internal-energy modes and typical chemical species found in the post-shock gas depending on the Mach number.

3. (20 pts) The temperature of a water-cooled flat plate is uniform and equal to $T_w$. A high-Reynolds-number hypersonic air stream at Mach number $Ma_e = 10$ and static temperature $T_e = T_w/4$ flows over the plate creating a laminar boundary layer with negligible pressure gradient, as sketched in Fig. 1. Assuming that the Prandtl number Pr is close to unity, $Pr \simeq 1$, answer the following questions:

   (a) Show that the plate temperature $T_w$ is lower than the adiabatic wall temperature $T_{aw}$.

   (b) Estimate the local dimensionless streamwise velocity $u^*/U_e$, the local temperature $T^*/T_e$, and the local Mach number $Ma^*$ where the temperature within the boundary layer attains its maximum value, as indicated in Fig. 1.

   (c) What is the asymptotic value of the local Mach number $Ma^*$ where the temperature becomes maximum in the limit $Ma_e \to \infty$?

Figure 1: Hypersonic boundary layer over an isothermal flat plate.
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Student’s Name:.......................................................... Student’s ID:............................

PART II: Open notes, calculators allowed, compressible-flow tables allowed
Time: 100 mins

Problem (50 pts)

The Titan-II was a multi-stage liquid-fueled intercontinental ballistic missile (ICBM) in service in the United States from 1963 to 1987 (Fig 1a). Besides flying several Gemini missions for NASA during the 60’s, the Titan-II was primarily designed to carry a 4,000 kg nuclear warhead known as W-53 with 9 megatons of explosive yield (approx. 38,000 TJ) and with a range of 16,000 km over the North Pole. The nuclear warhead was contained within a Mark-VI reentry vehicle or nose cone (Fig. 1b,c).

Figure 2: Titan-II ICBM.

A characteristic intercepting elliptic orbit followed by the Titan-II ICBM is shown in Fig. 3. The trajectory consists of the following phases:

i) A boost phase where the ICBM gains altitude while powered by rocket engines until a burnout altitude $z_{bo} = 150$ km, where the velocity is $U_{bo} = 7$ km/s.

ii) A midcourse unpowered phase where the nose cone separates from the booster and coasts in space along the intercepting elliptic orbit.

iii) A terminal re-entry phase where the nose cone enters ballistically the atmosphere at an approximate velocity $U_E \simeq U_{bo}$ and entry angle $\gamma_E$ equal in magnitude to the burnout flight-path angle $\gamma_{bo}$.
a) (10 pts) Consider the launch (A) and target (B) points over the surface of the Earth, as sketched in Fig. 3. If the geodesic distance $\ell_{AB}$ between A and B is $\ell_{AB} = 8,000$ km, estimate the free-flight range angle $\Psi$ (in doing so, neglect the horizontal distance traveled by the ICBM during the boost and re-entry phases). Calculate the burnout flight-path angle $\gamma_{bo}$ by using the following expression (see Bate et al., Fundamentals of Astrodynamics, 1971):

$$\sin(\gamma_{bo} + \Psi/2) = \frac{2 - Q_{bo}}{Q_{bo}} \sin(\Psi/2),$$  \hspace{1cm} (1)

where $Q_{bo} = U_{bo}^2 r_{bo}/\mu_\oplus$ is the ratio of the burnout velocity to the circular orbit velocity at the burnout altitude. Pick the solution that yields the largest $\gamma_{bo}$, or equivalently, the highest apogee.

b) (10 pts) The Mark-VI re-entry vehicle can be described by the sphere-cone geometry provided in Fig. 4. Calculate the drag coefficient $C_D$ on the vehicle during the re-entry phase using the straight Newtonian theory of hypersonics.

c) (10 pts) Calculate the ratio of the kinetic energy at the impact point B to the explosive yield.

d) (10 pts) Calculate the maximum dynamic pressure $q_d = \rho_\infty U_\infty^2/2$ acting on the Mark-VI vehicle during the re-entry phase along with the corresponding altitude where that maximum is attained. In this formulation, $\rho_\infty$ is the local air density and $U_\infty$ the nose-cone velocity (Fig. 4).

e) (10 pts) As shown in class, the heat flux $q''_{nose}$ entering the nose region is a function of the nose curvature radius $R_o$. After some algebra, $q''_{nose}$ can be written as

$$q''_{nose} = \frac{0.381}{\epsilon^{1/4}} Pr^{-0.6} \left[ \frac{\rho_\infty \mu_\infty}{(c_p T_\infty)^{1/2} R_0} \right]^{1/2} U_\infty^3 \text{ in } \left[ \frac{W}{m^2} \right].$$  \hspace{1cm} (2)

In this formulation, $\epsilon = (\gamma - 1)/(\gamma + 1)$ is the density ratio in the hypersonic limit, $\mu_\infty = 1.8 \cdot 10^{-5}$ Ns/m$^2$ is the free-stream dynamic viscosity, and $T_\infty = 273$ K is the free-stream temperature, with $\mu_\infty$ and $T_\infty$ being assumed to be constant throughout the re-entry phase. Estimate the maximum wall temperature $T_w$ by balancing the maximum value of $q''_{nose}$ attained during the re-entry phase with the heat flux radiated from the nose surface $q''_{rad} = \sigma \varepsilon T_w^4$ knowing that the ablative surface material of the Mark-VI was phenolic nylon (emissivity $\varepsilon = 0.87$) and neglecting gasification of the material.

Figure 3: Titan-II ICBM trajectory over the North Pole.

Figure 4: Mark-VI re-entry nose-cone model.