Guidelines: Please turn in neat and clean exam solutions that give all the formulae that you have used as well as details that are required for the grader to understand your solution. In the calculations, assume $Pr = 0.7$ and a calorically perfect gas with $\gamma = 1.4$, $R_g = 286$ J/kgK, and $c_p = 1$ kJ/kgK unless stated otherwise. Attach these sheets to your solutions.

Student’s Name:.......................................................... Student’s ID:..........................

PART I: Closed notes, calculators allowed, compressible-flow tables allowed

Time: 60 mins

Questions (50 pts)

1. (20 pts) Describe under what conditions may a flow be regarded as hypersonic. This is an open-ended question, and therefore both creativity and rigor will be graded positively.

2. (10 pts) Describe the general non-equilibrium structure of a hypersonic normal shock wave in air, including the characteristic zones where each degree of freedom of molecular motion, or chemical process, may be in equilibrium or out of equilibrium. In your response, include sketches of typical temperature and density distributions across the shock in non-equilibrium conditions.

3. (10 pts) Provide a definition for the adiabatic wall temperature $T_{a,w}$ and describe the shapes of characteristic static temperature profiles that can be encountered in a hypersonic laminar boundary layer depending on whether the wall temperature $T_w$ is higher, lower or equal to $T_{a,w}$.

4. (10 pts) The temperature of a flat plate is $T_w = 4T_e$, where $T_e$ is the static temperature of a high Reynolds-number free stream of gas flowing parallel to the plate. At very small Mach numbers, $Ma_e \ll 1$, is the plate cooled or heated by the gaseous free stream? What characteristic minimum value of the free-stream Mach number $Ma_e$ needs to be attained for the plate to be heated by the gas, or equivalently, for high-speed aerodynamic heating to become more important than low-speed convective cooling?
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PART II: Open notes, calculators allowed, compressible-flow tables allowed

Time: 120 mins

Problem 1 (50 pts)

A long time before the theory of hypersonics, and during many centuries of history of civilization, the problem of aerodynamic heating, as we call it today, occupied the attention of great physicists, mathematicians, and philosophers such as Aristotle, Seneca, Saint Thomas Aquinas, Averroes, and Galileo. For instance, in 350 BC, Aristotle attempts to explain how the heavenly bodies generate heat and light and transmit them to the terrestrial world by linking heat with the mechanics of motion:

“The warmth and light which proceed from them are caused by the friction set up in the air by their motion. Movement tends to create fire in wood, stone, and iron; and with even more reason should it have that effect on air; a substance which is closer to fire than these. An example is that of missiles \( \text{[arrows]} \), which as they move are themselves fired so strongly that leaden balls are melted; and if they are fired the surrounding air must be similarly affected. Now while the missiles are heated by reason of their motion in air, which is turned into fire by the agitation produced by their movement, The Upper Bodies are carried on a moving sphere, so that, though they are not themselves fired, yet the air underneath the sphere of the revolving body is necessarily heated by its motion, and particularly in that part where the Sun is attached to it.”

Aristotle, in De Caelo (On The Heavens), Book II, Part 7 (350 BC).

A similar topic also caused a heated exchange between Galileo Galilei and Orazio Grassi, a priest best noted as a mathematician, astronomer and architect. In order to support his own explanation of the nature of comets and their bright tails with Aristotle’s theory that motion creates heat, Grassi authoritatively cites ancient wisdom collected in the Suda, a 10th-century Byzantine encyclopedia, to indicate that the Babylonians cooked eggs by whirling them very fast through the air in slings (Libra Astronomica Ac Philosophica, p. 57, 1619). Galileo did not particularly like the work of Grassi and his theory of heat created by motion, and in 1623 he published an extensive treatise in response to Grassi called Il Saggiatore (i.e., The Assayer). There, in a literary masterpiece, Galileo tore Grassi’s analysis to pieces for being groundless and solely based on the wisdom of earlier generations\(^1\):

\(^1\)It is thought that this and other clashes alienated many of the Jesuits who had previously been sympathetic to Galileo’s ideas, and that later these might have played a role when he was brought to trial by The Inquisition [e.g., see Koestler A., “The Sleepwalkers: A History of Man’s Changing Vision of the Universe”, Ch. 2 (1959)].
“But it is wrong to say, as Sarsi [Grassi’s secret pseudonym] does, that Guiducci [Galileo’s friend] and I would laugh and joke at the experiences adduced by Aristotle. We merely do not believe that a cold arrow shot from a bow can take fire in the air; rather, we think that if an arrow were shot when afire, it would cool down more quickly than it would if it were held still. This is not derision; it is simply the statement of our opinion. Sarsi goes on to say that since this experience of Aristotle’s has failed to convince us, many other great men also have written things of the same sort. To this I reply that if in order to refute Aristotle’s statement we are obliged to represent that no other men have believed it, then nobody on earth can ever refute it, since nothing can make those who have believed it not believe it. But it is news to me that any man would actually put the testimony of writers ahead of what experience shows him. To adduce more witnesses serves no purpose, Sarsi, for we have never denied that such things have been written and believed. We did say they are false, but so far as authority is concerned yours alone is as effective as an army’s in rendering the events true or false. You take your stand on the authority of many poets against our experiments. I reply that if those poets could be present at our experiments they would change their views, and without disgrace they could say they had been writing hyperbolically—or even admit they had been wrong.”

Galileo, in Il Saggiatore (1623).

Coincidentally, in Il Saggiatore Galileo also established the modern scientific method by arguing that mathematics and experimentation, rather than testimonials of famous poets and philosophers, should be the basis of science and nothing else. Galileo was particularly skeptical of the Babylonians way of cooking eggs by whirling them fast in circular orbits in slings through the air, and in this way he sarcastically wrote:

“If Grassi wants me to believe with Suidas that the Babylonians cooked their eggs by whirling them in slings, I shall do so; but I must say that the cause of this effect was very different from what he suggests. To discover the true cause I reason as follows: “if we do not achieve an effect which others formerly achieved, then it must be that in our operations we lack something that produced their success. And if there is just one single thing we lack, then that alone can be the true cause. Now we do not lack eggs, nor slings, nor sturdy fellows to whirl them; yet our eggs do not cook, but merely cool down faster if they happen to be hot. And since nothing is lacking to us except being Babylonians, then being Babylonians is the cause of the hardening of eggs, and not friction of the air.” And this is what I wished to discover. Is it possible that Sarsi has never observed the coolness produced on his face by the continual change of air when he is riding post? If he has, then how can he prefer to believe things related by other men as having happened two thousand years ago in Babylon rather than present events which he himself experiences?”

Galileo, in Il Saggiatore (1623).

Galileo did not believe in ancient authorities and performed the experiment himself. He took an egg and whirled it in a sling until his arm was tired, and then he asked his graduate students at the University of Padova to whirl the egg for a bit longer. However, despite their efforts, the egg remained raw, but they all ended up with sore arms. Lastly, Galileo boiled an egg and whirled it through the air, and found that the egg was cooler after whirling due to what we call today convective cooling. Galileo therefore concluded that an egg would actually cool down by moving it fast in air.

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3This is a mechanism by which a hot plate can be rapidly cooled by an overriding cold fluid at high Reynolds numbers and low Mach numbers, whereby large negative temperature gradients develop across the resulting boundary layer.
In this problem, we return to the question addressed by Galileo and attempt to calculate whether it would have been possible for the Babylonians to cook eggs by moving them fast in air. However, here we hypothesize that the Babylonians had some sort of advanced technology available to accelerate eggs to hypersonic speeds, and we investigate whether an egg could be cooked by aerodynamic heating.

To this end, follow the steps below.

a) (10 pts) Let us begin by assuming that an egg is an axisymmetric body of revolution as depicted in Fig. 1(a). In particular, the egg consists of two conics, namely, a paraboloid \( x = Br^2 \) with \( r \) in cm and \( B = 1 \text{ cm}^{-1} \), and a sphere with radius \( R_s \), which are tangent at a point 4 cm downstream from the leading edge of the sharpest side of the egg. The density of the egg is \( \rho_b = 1000 \text{ kg/m}^3 \) and its specific heat is \( c_b = 4 \text{ kJ/kgK} \).

Using the straight Newtonian theory of hypersonic flows, in which the shock layer is infinitesimally thin, obtain expressions for the hypersonic drag coefficients \( C_D \) on the egg at zero angle of attack depending on whether the free stream approaches from the left (case A) or from the right (case B). In case A, ignore the small portion of the spherical cap that is exposed to the incident hypersonic stream.

b) (10 pts) Which configuration (case A or B) would lead to more intense heating? Justify your response based on elementary heat transfer theory for hypersonics, and disregard any concerns related to longitudinal static stability.

c) (10 pts) The egg is placed at the entrance of a long tunnel filled with still air at pressure \( P_\infty = 1 \)
bar and temperature $T_\infty = 300$ K. The initial temperature of the egg is the same as that of the surrounding air $T_\infty$. As depicted in Fig. 1(b), the egg is then shot horizontally along the tunnel at an initial velocity $U_0$ that is 10 times larger than the speed of sound and is attained by using an advanced railgun\(^4\). Obtain an expression for the velocity of the egg $U$ as a function of the downrange $s$ (the horizontal distance traveled by the egg) by integrating the equation of motion as it moves through the tunnel. In the calculations, assume that the distortion of the trajectory due to gravity is negligible, or equivalently, that $U_0/\sqrt{gL} \gg 1$, where $g$ is the acceleration of gravity and $L$ is the length of the tunnel.

d) (10 pts) Consider the expression of the heat flux $[\text{W/m}^2]$ on a blunt body

$$q_w = 0.763 \Pr^{-0.6} (\rho e \mu e A)^{1/2} c_p (T_{0,\infty} - T_w), \quad (1)$$

where $A = (dU_e/d\ell)|_{\ell=0}$ is the local strain rate, with $\ell$ a curvilinear coordinate emanating from the stagnation point (see Fig. 1a). Additionally, the subindex $e$ represents conditions at the edge of the boundary layer (i.e., post-shock conditions behind a normal shock at pre-shock velocity $U$), $T_{0,\infty}$ is the stagnation temperature of the free stream in the egg reference frame, and $T_w$ is the wall temperature of the egg. Using the Newtonian theory in the vicinity of the egg’s blunt nose (whichever one you have chosen in part b), estimate the strain rate $A$ and derive from (1) the equivalent form

$$q_w = 0.381 \varepsilon^{1/4} \Pr^{-0.6} \left[ \epsilon \mu_{\infty} \mu_{\infty} \left( \frac{c_p T_{\infty}}{2} R_0 \right)^{1/2} U_0^3 \right]$$

by assuming that $T_{0,\infty}/T_w \gg 1$ because of the high Mach numbers, and also that $\mu_e/\mu_{\infty} = (T_e/T_\infty)^{1/2}$, with $T_e \simeq T_{0,\infty}$ due to the low velocities in the post-shock region. In this formulation, $\epsilon = \rho_e/\rho_{\infty}$ is a density ratio that can be obtained from the hypersonic normal-shock jump conditions, $\mu_{\infty} = 1.8 \cdot 10^{-5}$ Ns/m\(^2\) is the dynamic viscosity of the air in the tunnel, and $R_0$ is the radius of curvature in the vicinity of the axis of symmetry of the blunt side of the egg that you have previously chosen in part b).

e) (10 pts) Equation (2), along with the relation $U(s)$ obtained in part c), provide the dominant aerodynamic heat flux into the egg as a function of time. Let us assume that the heat flux enters the egg uniformly through an effective characteristic area of order $\pi R_0^2$. Assuming also that the temperature $T_b$ inside a volume $V_0 = 4\pi R_0^3/3$ of the egg next to the shell is instantaneously uniform, derive an expression for $T_b$ as a function of the downrange $s$ integrating the conservation of energy, and state whether $T_b$ will ever reach the critical value 338 K for the egg to get cooked in the tunnel\(^5\).

\(^4\)It is assumed that the egg is shielded from any possible aerodynamic pressure loads that could cause it to crack by an advanced resistant armor that has negligible thermal inertia and fits perfectly to the shape of the egg so as to not modify the drag coefficient. Similarly to a railgun, the armor together with the egg slide through an incident electromagnetic field that enables the high accelerations.

\(^5\)Assume $C_D = 1$ if you got stuck in part a).