Guidelines: Please turn in neat and clean exam solutions that give all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions.

Student’s Name: .......................................................... Student’s ID: ........................................

PART I: Closed notes, calculators allowed, compressible-flow tables allowed
Time: 20 mins

Question (40 pts)
1. The velocity/altitude diagram below shows three different re-entry trajectories across the terrestrial atmosphere.

a) (20 pts) Determine what type of aerospace vehicle (intercontinental ballistic missile, manned re-entry winged vehicle, or experimental hypersonic cruise aircraft) is most likely associated with each trajectory and justify your response.

b) (20 pts) Sketch on the diagram the approximate boundaries for the onset of the characteristic thermo-chemical effects arising at hypersonic flight speeds.

Notes: 1 kft = 304.8 m; 1kft/s = 304.8 m/s = 681.8 mph = 1097.3 km/h; speed of sound at 35 km altitude ~ 1 kft/s; temperature at 35 km altitude ~ 236 K.
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PART II: Open notes, calculators allowed, compressible-flow tables allowed
Time: 60 mins

Problem 1 (40 pts)

a) (30 pts) Using the straight Newtonian theory of hypersonic flows, in which the shock layer is infinitesimally thin, obtain expressions for the drag and lift coefficients on the sphere of radius $R_p$ sketched in Fig. 1 (use the spherical coordinate system shown in the figure to perform the corresponding integrations of the surface pressure coefficient $C_p$ along the spherical surface).

b) (10 pts) The experimental drag coefficient measured by Hodges (1957) corresponds to $C_D = 0.91$. Compare this value to the one obtained in part (a) and identify possible sources of discrepancy between both results.

Figure 1: Hypersonic flow over a sphere in the Newtonian theory.
Problem 2 (20 pts)

Consider the flow schematically illustrated in Fig. 2, where a high-speed gaseous stream at velocity $U_1$, pressure $P_1$ and density $\rho_1$ crosses a two-dimensional stationary curved shock wave. The velocity and thermodynamic variables of the pre-shock gases, including the entropy $s$, are uniform. As a result, in the pre-shock gases, Crocco’s equation

$$ T \nabla s = \omega \times u, $$

requires that the vorticity $\omega$ be zero there. In Eq. (1), $u$ is the velocity vector.

Since the shock wave is curved, different streamlines experience different jumps of entropy across the shock. As a result, the post-shock gas has constant entropy along the streamlines but the entropy varies from streamline to streamline, thereby rendering an isentropic flow. Because of Crocco’s equation (1), however, the post-shock gas is *rotational*.

The goal of this problem is to obtain an expression for the vorticity $\omega_2$ in the post-shock gas immediately behind the shock in the hypersonic limit. To this end, follow the steps below.

![Figure 2: Hypersonic flow across a two-dimensional curved shock wave.](image)

a) (4 pts) Obtain an expression for the derivative of the post-shock pressure $P_2$ with respect to the local incidence angle $\beta$ using the Rayleigh line

$$ \frac{P_2 - P_1}{\frac{1}{\rho_2} - \frac{1}{\rho_1}} = -\dot{m}''^2, $$

where $\dot{m}'' = \rho_1 U_1 \sin \beta$. 

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b) (4 pts) Obtain an expression for the derivative of the post-shock static enthalpy $h_2$ with respect to the local incidence angle $\beta$ using the Hugoniot equation

$$h_2 - h_1 = \frac{P_2 - P_1}{2\rho_1} \left(1 + \frac{\rho_1}{\rho_2}\right).$$

(3)

c) (4 pts) Utilize the expressions for $dP_2/d\beta$ and $dh_2/d\beta$ derived above, along with first principle of thermodynamics $T_2 ds_2 = dh_2 - dP_2/\rho_2$ in the post-shock gas, and show that the derivative of the post-shock entropy $s_2$ with respect to the local incidence angle $\beta$ of the shock is given by

$$T_2 \frac{ds_2}{d\beta} = U_\infty^2 (1 - \epsilon)^2 \sin \beta \cos \beta,$$

(4)

where $\epsilon = \rho_1/\rho_2$ is the density ratio across the shock.

d) (4 pts) Crocco’s equation (1) states that the gradient of the post-shock entropy across the streamlines is related to the cross product of the post-shock velocity ($U_2$) and vorticity ($\omega_2$), namely

$$T_2 |\nabla s_2| = \omega_2 U_2.$$  

(5)

Equivalently, since the post-shock flow is isentropic, $\nabla s_2$ is perpendicular to the streamlines, and correspondingly the derivative of the post-shock entropy along the direction $\hat{s}$ locally parallel to the shock is

$$T_2 \frac{ds_2}{d\hat{s}} = T_2 |\nabla s_2| \sin(\beta - \delta),$$

(6)

where $\delta$ is the local deflection angle of the streamlines, as shown in Fig. 2.

Using Eqs. (4)-(6), show that the vorticity immediately behind the shock is given by the expression

$$\omega_2 = U_\infty \frac{(1 - \epsilon)^2}{\epsilon} \cos \beta \frac{d\beta}{d\hat{s}},$$

(7)

where the direction of the vorticity points into the plane in Fig. 2 when $d\beta/d\hat{s} < 0$ (as in the upper portion of a bow shock around a blunt body). Note that 1) the derivation of Eq. (7) does not require the gas to be calorically perfect, and 2) the factor $d\beta/d\hat{s}$ represents the inverse of the radius of curvature of the shock front. In this way, the post-shock vorticity becomes zero either for $\beta = \pi/2$ (i.e., normal shock) or for $d\beta/d\hat{s} = 0$ (i.e., straight shock), as anticipated in class.

e) (4 pts) Simplify expression (7) for a calorically-perfect gas at hypersonic speeds $Ma_1^2 \sin^2 \beta \gg 1$, for which the post-shock density $\rho_2$ is uniform and the density ratio $\epsilon$ becomes a constant that is a sole function of the adiabatic coefficient $\gamma$. 

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