For a function of period $2T$,

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\frac{\pi}{T}x}$$

$$\text{where } c_n = \frac{1}{2T}\int_{-T}^{T} e^{-in\frac{\pi}{T}t} f(t)\,dt$$

Let $\omega_n = \frac{n\pi}{T}$ & $\Delta\omega = \omega_n - \omega_{n-1} = \frac{\pi}{T}$

$$\Rightarrow f(x) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty}\left(e^{i\omega_n x}\int_{-T}^{T} e^{-i\omega_n t} f(t)\,dt\right)(\Delta\omega)$$

Now, let $T \to \infty$ (and hence $\Delta\omega \to 0$), we have the above expression resemble a Riemann Sum for a definite integral.

$$\boxed{\therefore f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(e^{i\omega x}\int_{-\infty}^{\infty} e^{-i\omega t} f(t)\,dt\right)d\omega}$$

The boxed equation motivates the definition of the Fourier Transform!