5.60/BE.110: Thermodynamics and Kinetics (r08)
Recitation Handout for 10/17/2006

Quick summary of today's topics:

- Stirling's formula: $\ln(N!) \approx N\ln(N) - N$
- Boltzmann's constant: $k = \frac{R}{N_A} = 1.38 \times 10^{-23} \frac{J}{K}$
- The postulates of statistical mechanics:
  - "First law": $E = \sum_i E_i p_i$
  - "Second law": $S = k\ln(\Omega)$ (or, equivalently $S = -k\sum_i p_i \ln(p_i)$)
- **The Boltzmann Distribution:**
  - $p_j = \frac{e^{(\frac{-E_i}{kT})}}{\sum_i e^{(\frac{-E_i}{kT})}} = \frac{e^{(\frac{-E_i}{kT})}}{Q}$
  - The Boltzmann distribution gives us the probability distribution that maximizes the entropy within the energy constraint at a particular temperature, T.
- **The Partition Function, Q:**
  - $Q = \sum_i e^{(\frac{-E_i}{kT})}$
  - Interpretation: Q is a dimensionless number that is a function of T. It tells us the number of *"thermally accessible states"* at a given T.

Basic Counting Problems:

1. Suppose four (different sex) couples are lined up in a row:
   1. How many possible arrangements are there?
   2. How many arrangements are there if we require that all the women and all the men are grouped by themselves? (i.e. MMMMWWWW or WWWWMMMM)
   3. How many arrangements are possible if each couple must sit next to each other?
   4. Compute the "entropy" of the above cases by applying the Boltzmann formula for entropy.

2. Suppose that there are 10 policemen. If we need 5 to patrol, 3 to work traffic, and 2 to remain at the station (in case of emergencies!), how many ways are there to achieve these assignments?

3. Suppose there are 10 tennis players. If they are to be grouped in 2's for the first round of the tournament, how many ways are there to achieve these groupings?