MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems — Fall 2008

Quiz 1 Review Problems

Problem #1 *The purpose of this problem is to test your ability to interpret expressions involving impulses.*
Evaluate the expression

$$\int_0^\infty \delta(t-2)t^2 dt,$$

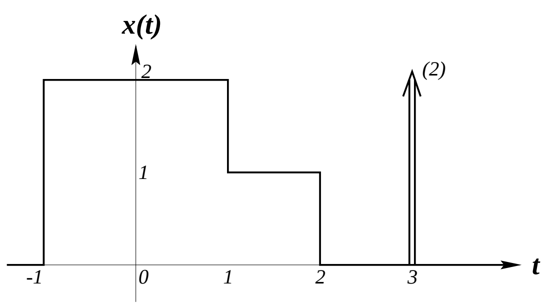
where $\delta(t)$ is the unit impulse function.

Problem #2 *The purpose of this problem is to test your understanding of continuous-time convolution, filtering, and impulses.*
The output $y(t)$ of a particular LTI system is the moving average of the last 1 second of the input $x(t)$. That is,

$$y(t) = \int_{t-1}^{t} x(\tau)d\tau \qquad (1)$$

(a) What is the impulse response $h(t)$ of this system?

(b) Provide a sketch of the output of this system when the input is as given in the figure below. Make sure to carefully label your sketch.



(c) Determine the output $y(t)$ if the input for all time is

$$x(t) = \cos(\pi t) + \sin(2\pi t + \pi/4).$$

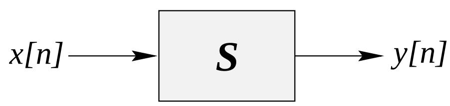
Problem #3 *The purpose of this problem is to test your understanding of discrete-time convolution.*
Compute $y[n] = x[n] * h[n]$ for all values of n, where

$$\begin{aligned} x[n] &= \left(\frac{1}{2}\right)^n u[n] \\ h[n] &= u[-n] \end{aligned}$$

Problem #4 *The purpose of this problem is to test your understanding of system properties.*
The discrete-time system S shown in the figure below has the input-output relation

$$y[n] = \text{Odd}\{x[n+1]\},$$

where $\text{Odd}\{f[n]\}$ takes the odd part of $f[n]$.



Is this system

(a) linear?

(b) time-invariant?

(c) causal?

In each part (a) through (c), show that the system has that property or give a counter-example.

Problem #5 *The purpose of this problem is to test your understanding of linear, time-invariant systems.*
Consider a discrete-time LTI system with impulse response

$$h[n] = 2^n u[4-n].$$

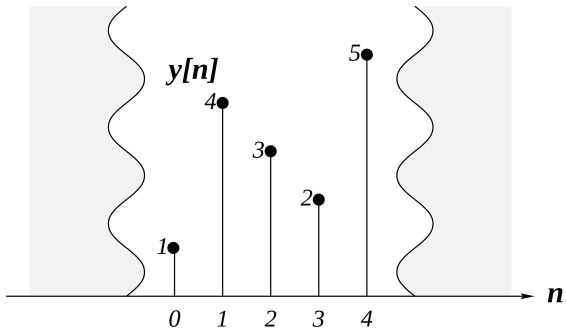
(a) Is this system causal? Justify your answer.

(b) Is this system stable? Justify your answer.

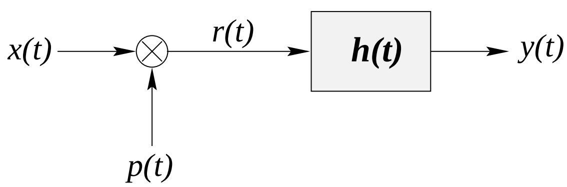
Problem #6 *The purpose of this problem is to test your understanding of the analysis of an LTI system.*
A **causal**, discrete-time LTI system has an input

$$x[n] = \left(\frac{1}{2}\right)^n u[n],$$

and an output $y[n]$ that is known only for the interval $0 \le n \le 4$ as shown in the figure below (*i.e.* the output is not known for other values of n). Determine $h[1]$.



Problem #7 *The purpose of this problem is to test your understanding of the continuous-time Fourier transform.*
Consider the system depicted below,



where

$$x(t) = \frac{\sin(4\pi t)}{\pi t}$$
$$p(t) = 2\cos(2\pi t),$$

and the impulse response $h(t)$ is given by

$$h(t) = 1 + 3\sin(4\pi t) + 2\cos(8\pi t).$$

(a) Provide a labeled sketch of $R(j\omega)$, the Fourier transform of $r(t)$.

(b) Determine $y(t)$.

Problem #8 *The purpose of this problem is to test your understanding of the continuous-time Fourier transform.*
Consider the signal

$$x(t) \quad = \quad \cos(2\pi t) + \sin(6\pi t).$$

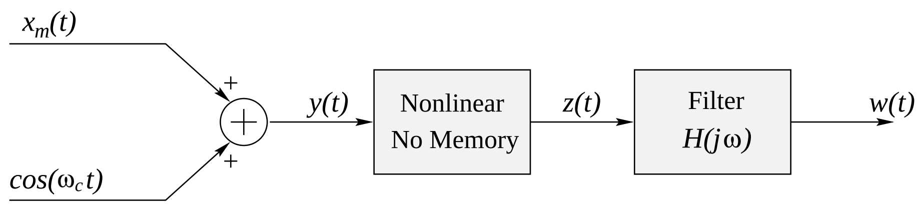
Suppose that this signal is the input to each of the LTI systems with impulse responses given below. Determine the output in each case.

(a) $h(t) = \dfrac{\sin(4\pi t)}{\pi t}$

(b) $h(t) = \dfrac{[\sin(4\pi t)]\,[\sin(8\pi t)]}{\pi^2 t^2}$

(c) $h(t) = \dfrac{[\sin(4\pi t)]\,[\cos(8\pi t)]}{\pi t}$

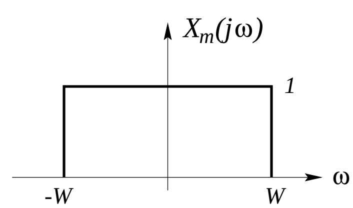
Problem #9 *The purpose of this problem is to test your understanding of continuous-time modulation.*



In the modulator shown above, the modulating signal $x_m(t)$ and a sinusoid at the intended carrier frequency are <u>added</u> to produce $y(t) = x_m(t) + \cos(\omega_c t)$, which is then passed through a non-linear device to yield

$$z(t) \quad = \quad 5y(t) + y^2(t).$$

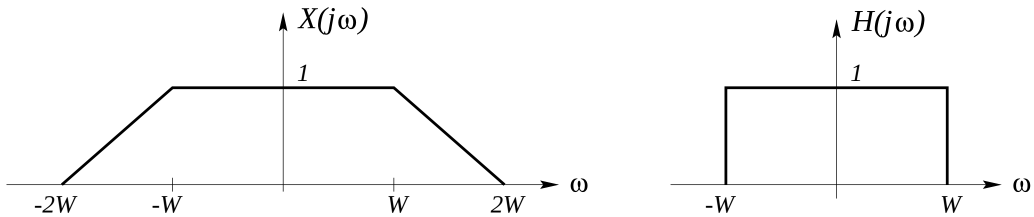
(a) Assume $x_m(t)$ is a real, even function having the spectrum shown below, where $W \ll \omega_c$.



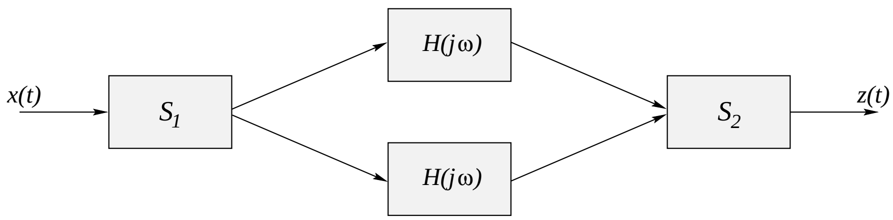
Make a carefully labeled sketch of $Z(j\omega)$ over the range $-3\omega_c < \omega < 3\omega_c$.

(b) Describe the frequency response $H(j\omega)$ of the filter such that $w(t)$ has the form of $x_m(t)$ double-sideband amplitude-modulated (with carrier) on a carrier at ω_c.

Problem #10 *The purpose of this problem is to test your understanding of continuous-time modulation.*
We would like to transmit the signal $x(t)$ with the Fourier transform depicted on the left side of the figure below. Unfortunately, the only available communications channels have limited bandwidth. Specifically, each such channel can be viewed as an LTI system with frequency response $H(j\omega)$ depicted on the right side of the figure below.



Fortunately, we have two such channels at our disposal, and thus, it is possible to design systems S_1 and S_2, depicted below, so that $z(t) = x(t)$.



Both S_1 and S_2 can be constructed using:

(1) signal generators that can produce signals of the form $\cos(\omega_0 t)$ at any fixed frequency ω_0;

(2) multipliers and adders;

(3) ideal filters.

Specify the designs of S_1 and S_2.
