MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems — Spring 2008

Problem Set 1

Issued: February 7, 2008 Due: February 13, 2008

Exercise for home study (not to be turned in, although we will provide solutions):

O&W 1.54

Problems to be turned in:

Problem 1 Answer the questions asked in Problem 1.49 of O&W 1.49 for each of the following complex numbers:

(a) $(1 - j\sqrt{3})^7 e^{j2\pi/3}$

(b) $\frac{e^{-j\pi/3}-1}{1+j\sqrt{3}}$

Problem 2 Consider the signal $x(t)$ in Figure P1.21 on p. 60 of O&W. Sketch and label carefully each of the following signals:

(a) $2x(3 - \frac{t}{2}) + 1$

(b) $x(t-1)[\delta(t - \frac{4}{3}) - 2\delta(t + \frac{1}{2}) - u(1 - t)]$

Problem 3 Consider the signal $x[n]$ in Figure P1.22 on p. 60 of O&W. Sketch and label carefully each of the following signals:

(a) $x[6 - 2n]$

(b) $(1 - e^{j\pi n})x[n - 1] - x[2n + 1]$

Problem 4 Refer to the signal below:

$$x[n] = 2^n u[1 - n] \tag{1}$$

(a) Sketch the signal, with labeling and scaling for both axes

(b) Find the total energy in the signal

Problem 5 Determine whether each of the following signals is periodic and, if it is, determine its fundamental period:

(a) $x(t) = \mathrm{e}^{j\pi/3} + \sin(20t)$

(b) $x(t) = 5.2 + 3\sin(4t + 3) - 6.1\cos(10t - 2)$

(c) $x[n] = \sin(\pi - 2n)$

(d) $x[n] = 3\cos(\pi n/8) - \sin(\pi n/16 - 3) + 2\sin(2\pi n - \pi/4)$

Problem 6 For each system, determine which of the following four properties hold: time-invariance, linearity, causality and stability. Justify your answers with a proof or counterexample.

(a) $y(t) = x(\sin t)$

(b) $y(t) = \sin(x(t))$

(c) $y(t) = \int_t^{t^2} x(\tau - 1)\,\mathrm{d}\tau$

(d) $y[n] = x[n]\mathrm{e}^{j\pi n/3}$

(e) $y[n] = 2^{x[n]}$

Problem 7 O&W 1.42

Problem 8 The DT system A maps the following two inputs to the corresponding outputs:

$$x_1[n] = (-1)^n \quad \longrightarrow \quad y_1[n] = 1, \quad \text{for all } n \tag{2}$$
$$x_2[n] = (-1)^{n+1} \quad \longrightarrow \quad y_2[n] = 1, \quad \text{for all } n \tag{3}$$

The DT system B maps the following two inputs to the corresponding outputs:

$$x_3[n] = (-1)^n \quad \longrightarrow \quad y_3[n] = 1, \quad \text{for all } n \tag{4}$$
$$x_4[n] = (-1)^{n+1} \quad \longrightarrow \quad y_4[n] = -1, \quad \text{for all } n \tag{5}$$

(a) Could the system A be linear? Justify your answer.

(b) Could the system A be time-invariant? Justify your answer.

(c) Could the system B be linear? Justify your answer.

(d) Could the system B be time-invariant? Justify your answer.

Laboratory Assignment BDS 1.3; 1.6 (a), (b), (c)

To receive full credit, all plots must be fully labeled. As an example, try running the following script:

```
% Example plots

% DT signal
n = [1:5];
x1 = [6, 2, -1, 3, 5];
figure;     % create new figure
stem(n, x1);
grid on;
axis([-2, 8, -2, 8]);     % readjust axes
title('Example Plot 1');
xlabel('time n');
ylabel('signal x_{1}[n]');

% CT signal
t = linspace(0, 10, 101);
x2 = t.*exp(j*2*pi*t);
figure;
plot(t, real(x2), 'b-');     % solid blue line
hold on;     % do not erase previous plot
plot(t, imag(x2), 'r-.');     % dash-dotted red line
grid on;
title('Example Plot 2: Real and Imaginary Parts Separate');
xlabel('time t');
ylabel('signal x_{2}(t)');
legend('Real Part of e^{j2\pi t}', 'Imag Part of e^{j2\pi t}');

% Greek and mathematical symbols in labels and title
% MATLAB understands many LaTeX commands
figure;
title({'\it{Italicized Title} \rightarrow {\bf Boldfaced Title}',
 '2^{nd} Line'});
xlabel('Upper case: \Omega ; Lower case: \omega')
ylabel('X_{subscript}^{superscript}, i.e. x_{0}^{2}[n]');
```

Use the command `help FUNCTIONNAME` to get information about using `FUNCTIONNAME`.

The problems are from *Computer Explorations in Signals and Systems Using MATLAB, Second Edition* by Buck, Daniel and Singer (BDS). It is recommended that you read and experiment with BDS Section 1.1 and the MATLAB tutorials available on the 6.003 website.

Keep in mind that laboratory assignments are graded separately from the problem sets and successful completion is **mandatory** to pass 6.003 (review the grading criteria as explained in the

course information packet). It is **strongly recommended** that you begin these assignments early, especially if you are unfamiliar with MATLAB. Please feel free to contact your TA if you have questions about using MATLAB or the grading policy.

Reminder: The first 20 problems in each chapter of O&W have answers included at the end of the text. Consider using these for additional practice, either now or as you study for tests.